Unit #6 Exponential & Logarithmic Functions

(The Exponential function and its inverse)

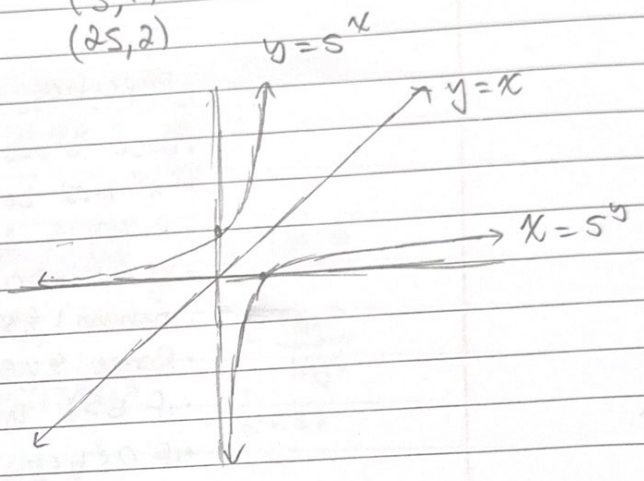
Lets compare $y = 5^x$ and it's inverse $x = 5^y$

x	$y = 5^x$	points:	for the inverse $x = 5^y$
-2	$5^{-2} = \frac{1}{25}$	$(-2, 1/25)$	$(1/25, -2)$
-1	$5^{-1} = 1/5$	$(-1, 1/5)$	$(1/5, -1)$
0	$5^0 = 1$	$(0, 1)$	$(1, 0)$
1	$5^1 = 5$	$(1, 5)$	$(5, 1)$
2	$5^2 = 25$	$(2, 25)$	$(25, 2)$

↑ the ratio of y-values is same as the base

$\frac{1}{5} = 5$

$\frac{1}{\frac{1}{5}} = 5$



Key Features

	$y = 5^x$	$x = 5^y$
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R} \mid x > 0\}$
range	$\{y \in \mathbb{R} \mid y > 0\}$	$\{y \in \mathbb{R}\}$
x-int	none	1
y-int	1	none
Asymptote	HA at $y = 0$	VA at $x = 0$
Interval where		
↳ positive/negative	Positive $(-\infty, \infty)$	positive $(1, \infty)$ negative $(0, 1)$
↳ increasing/decreasing	increasing $(-\infty, \infty)$	increasing $(0, \infty)$

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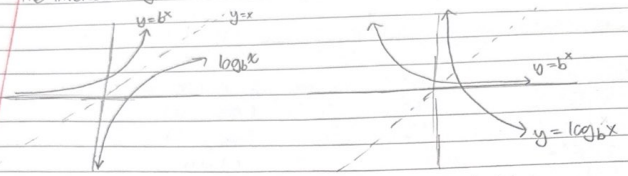
LOGARITHMS

this can be written as $y = \log_b x$

the inverse of $y = b^x$ is $x = b^y$ ✓



Properties of $y = \log_b x$

- Base "b" is positive
- "x" must be positive
- x-int is 1
- VA at x=0
- Domain: $\{x \in \mathbb{R} \mid x > 0\}$
- Range: $\{y \in \mathbb{R}\}$
- if $b > 1$ $y = \log_b x$ is increasing
- if $0 < b < 1$, $y = \log_b x$ is decreasing

Exponential Form	LOGARITHMIC Form
$x = b^y$	$y = \log_b x$

$4^3 = 64$ is the same as $\log_4 64 = 3$

☆ the common log base is 10, we need not write the base of 10

eg. $\log_{10} 5 = \log 5$

Examples:

① write $5^{-2} = \frac{1}{25}$ in

② write $\log 100 =$

③ Evaluate log

Power Law

$\log_b x^n =$

Examples:

$\log_b b = 1$

$\log_b b^k = k$

$\log_b 1 =$

② write

as

$=$

Examples:

① write $5^{-2} = \frac{1}{25}$ in logarithmic form $\log_5\left(\frac{1}{25}\right) = -2$

② write $\log 100 = -2$ in exponential form $10^2 = 100$

③ Evaluate $\log_3\left(\frac{1}{9}\right)$ let $x = \log_3\left(\frac{1}{9}\right)$

in exponential form: $3^x = \frac{1}{9}$

$3^x = 3^{-2}$

$\therefore x = -2$

Power Law

$\log_b x^n = n \log_b x$

change of base formula

$\log_b m = \frac{\log_n m}{\log_n b}$

Examples:

$\log_b b = 1$

$b^{\log_b k} = k$

$\log_b 1 = 0$

a) $\log_2 8$

$= \log_2 2^3$

$= 3 \log_2 2$

$= 3(1)$

$= 3$

b) $\log_7 \sqrt[3]{\frac{1}{343}}$

$= \log_7 (343^{-1})^{\frac{1}{3}}$

$= \log_7 (343)^{-\frac{1}{3}}$

$= -\frac{1}{3} \log_7 7^3$

$= 3\left(-\frac{1}{3}\right) \log_7 7$

$= -1(1)$

$= -1$

c) $\log_5 9$

$= \frac{\log 9}{\log 5}$

$= 1.37$

② write $\frac{\log 7}{\log 3}$

as a single logarithm

$= \log_3 7$

③ solve $4^n = 9$

$\log 4^n = \log 9$

$n \log 4 = \log 9$

$n = \frac{\log 9}{\log 4}$

$n = \log_4 9$ ← exact

$n \doteq 1.585$ ← approximate

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Product & Quotient Laws of Logarithms

Product Law
$\log_b [mn] = \log_b m + \log_b n$

Quotient Law
$\log_b \left[\frac{m}{n}\right] = \log_b m - \log_b n$

Examples:

① Evaluate a) $\log_5 80 - \log_5 16$

$= \log_5 \left(\frac{80}{16}\right)$

$= \log_5 5$

$= 1$

b) $3\log 6 + 2\log 5 - \log 54$

$= \log 6^3 + \log 5^2 - \log 54$

$= \log 216 + \log 25 - \log 54$

$= \log\left(\frac{216 \times 25}{54}\right)$

$= \log 100$

$= \log_{10} 10^2$

$= 2\log_{10} 10$

$= 2(1)$

$= 2$

② Simplify and state restrictions on the variables!

a) $2\log(3a) + 4\log y - 5\log(6w) \rightarrow = \log(3a)^2 + \log(y)^4 - \log(6w)^5$

$= \log\left(\frac{9a^2y^4}{7776w^5}\right)$ $a>0, y>0, w>0$

b) $\log(2x^2+7x+6) - \log(x^2-4)$

$= \log(x+2)(2x+3) - \log(x-2)(x+2)$

$= \log\left(\frac{(x+2)(2x+3)}{(x-2)(x+2)}\right)$

$= \log\left(\frac{2x+3}{x-2}\right)$

RESTRICTIONS

① $2x^2+7x+6 > 0$

$(2x+3)(x+2) > 0$

$-\frac{3}{2}$ -1.5

Intervals $(-\infty, -2)$ $(-2, -\frac{3}{2})$ $(-\frac{3}{2}, \infty)$

$(2x+3)$ $-$ $-$ $+$

$(x+2)$ $-$ $+$ $+$

$f(x)$ $+$ $-$ $+$

$\therefore (-\infty, -2)$

overlap

③ if $\log_a b = 3$

find a) $\log_a$

$= \log_a$

$= \log_a$

$= 1$

$= 6$

b) cont.

RESTRICTIONS

① $2x^2+7x+6>0$

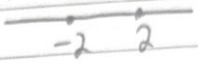
$(2x+3)(x+2)>0$

Interval	$(-\infty,-2)$	$(-2,-1.5)$	$(-1.5,\infty)$
$(2x+3)$	−	−	+
$(x+2)$	−	+	+
$f(x)$	+	−	+

$\therefore (-\infty,-2) \cup (-1.5,\infty)$

② $x^2-4>0$

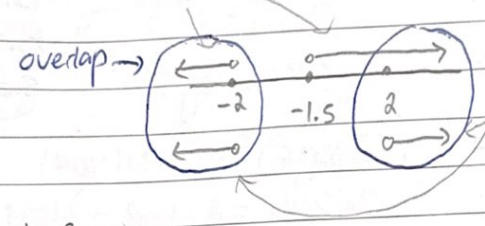
$(x-2)(x+2)>0$



Interval	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
$(x-2)$	−	−	+
$(x+2)$	−	+	+
$g(x)$	+	−	+

$\therefore (-\infty,-2) \cup (2,\infty)$

③ $x \neq -2$ pinhole



$\therefore x<-2, x>2$

③ if $\log_a b = 3$

find a) $\log_a ab^2$

$= \log_a a + \log_a b^2$

$= \log_a a + 2\log_a b$

$= 1 + 2(3)$

$= 6$

b) $\log_b a$ change of base formula

$= \frac{\log_a a}{\log_a b}$

$= \frac{1}{3}$

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Solving Exponential Equations

solve:

① $\left(\frac{1}{4}\right)^x = 8^{5/3}$

$(4^{-1})^x = 8^{5/3}$

$(2^2)^{-x} = (2^3)^{5/3}$ ... convert to same base

$2^{-2x} = 2^5$

$\therefore -2x = 5$

$x = -\frac{5}{2}$

② $2(4)^x - 17(2)^x + 8 = 0$

$2(2^2)^x - 17(2)^x + 8 = 0$

$2(2^{2x}) - 17(2)^x + 8 = 0$

let $m = 2^x$

$2m^2 - 17m + 8 = 0$

$(2m-1)(m-8) = 0$

$\therefore 2m - 1 = 0$ or $m - 8 = 0$

$m = \frac{1}{2}$ $m = 8$

$\therefore 2^x = 2^{-1}$ $\therefore 2^x = 8$

$x = -1$ $2^x = 2^3$

$x = 3$

⑤ $(5^x - 3)(5^x + 1) = 0$

$\therefore 5^x - 3 = 0$ or $\therefore 5^x + 1 = 0$

$5^x = 3$ $5^x = -1$

$\log 5^x = \log 3$ $\log 5^x = \log(-1)$

$x \log 5 = \log 3$ $x \log 5 = \log(-1)$

$x = \frac{\log 3}{\log 5}$ $x = \frac{\log(-1)}{\log 5}$

$x = \log_5 3$ not possible

extraneous root

$3^{x+1} - 3^x = 18$ two terms with power x

$(3^x)(3^1) - 3^x = 18$

$3^x[3^1 - 1] = 18$

$3^x(2) = 18$

$3^x = 9$

$3^x = 3^2$

$\therefore x = 2$

④ $6^{3x} = 4^{2x-3}$ cannot get same base so log both sides

$\log 6^{3x} = \log 4^{2x-3}$

$(3x)(\log 6) = (2x-3)(\log 4)$

$3x \log 6 = 2x \log 4 - 3 \log 4$

$3x \log 6 - 2x \log 4 = -3 \log 4$

$x(3 \log 6 - 2 \log 4) = -3 \log 4$

$x = \frac{-3 \log 4}{3 \log 6 - 2 \log 4}$

$x \doteq -1.6$

Solving Logarithmic

check for extrane

solve

① $\log(x+5) = 2\log(x-1)$

$\log(x+5) = \log(x-$

$\therefore (x+5) = (x-1)^2$

$x + 5 = x^2 - 2x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\therefore x - 4 = 0$ or

$x = 4$

$\therefore x = 4$

③ $\log(x-1) - 1 =$

$\log(x-1) + \log($

$\log[(x-1)(x+$

convert to

$10^1 = (x-$

$10 = x^2 -$

$0 = x^2$

$0 = (x$

$\therefore x$

extra

two terms with power x

not get same so log both

Solving Logarithmic Equations

* check for extraneous roots

log only on one side

solve

① $\log(x+5) = 2\log(x-1)$

$\log(x+5) = \log(x-1)^2$

log on both sides

$\therefore (x+5) = (x-1)^2$

$x+5 = x^2-2x+1$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$\therefore x-4=0$ or $x+1=0$

$x=4$ $x=-1$

Extraneous root cannot have log(-1)

$\therefore x=4$

② $\log(x-1)-1 = -\log(x+2)$

$\log(x-1)+\log(x+2)=1$ ← Product law

$\log[(x-1)(x+2)] = 1$

convert to exp. form

$10^1 = (x-1)(x+2)$

$10 = x^2+x-2$

$0 = x^2+x-12$

$0 = (x+4)(x-3)$

$\therefore x=-4$ or $x=3$

↑ extraneous root

$\therefore x=3$

③ $\log_5(6x+1) = 2$

write in exponential form

$5^2 = 6x+1$

$25 = 6x+1$

$24 = 6x$

$4 = x$

$\therefore x=4$

④ $\log\sqrt[3]{x^2+48x} = \frac{2}{3}$

$\log(x^2+48x)^{\frac{1}{3}} = \frac{2}{3}$

$\frac{1}{3}\log(x^2+48x) = \frac{2}{3}$

$\log(x^2+48x) = 2$

convert to exp form

$10^2 = x^2+48x$

$x^2+48x-100=0$

$(x+50)(x-2)=0$

$\therefore x=-50$ or $x=2$

Note: neither are extraneous roots

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⑤ $\log_3 x - \log_3(x-4) = 2$

$\log_3\left(\frac{x}{x-4}\right) = 2$

$3^2 = \frac{x}{x-4}$

$9(x-4) = x$

$9x - 36 = x$ $\quad \therefore x = \frac{9}{2}$

$8x = 36$

$x = \frac{9}{2}$

Applications

RICHTER SCALE
Magnitude of an

I: intensity of e
Io: Intensity o
M: Magnitude

pH scale
measures the c

H^+: concer

Decibel sca
Measures s

B: sound l
I: Intens

normal con

Applications

RICHTER SCALE

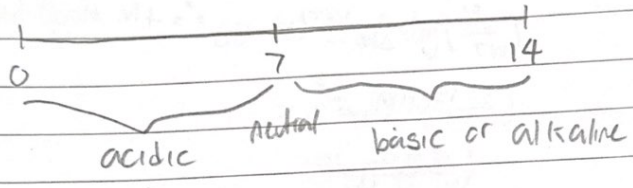
Magnitude of an earthquake $M = \log\left(\frac{I}{I_0}\right)$

I: intensity of earthquake measured
I_0: intensity of reference earthquake
M: Magnitude

pH scale

measures the acidity of a liquid · $pH = -\log[H^+]$

H^+: concentration of hydrogen ion in moles/litre



Decibel scale

Measures sound $B_2 - B_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

B: sound levels in decibels (dB)
I: intensities in watts per square metre

normal conversation: 60dB · jet engine: 140dB

Exam Prep

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3. Review the exam instruct

Instructions : You are allowe
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foolscap provided, write an ess
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Examples!

① How many times as intense as a standard earthquake measuring 2.4 on the richter scale?

Given: $M = 2.4$ Find: $\frac{I}{I_0}$

$M = \log\left(\frac{I}{I_0}\right)$

$2.4 = \log_{10}\left(\frac{I}{I_0}\right)$

$10^{2.4} = \left(\frac{I}{I_0}\right)$

ratio → $\frac{I}{I_0} \doteq 251.2$

∴ it is approximately 251 times more intense

② What is the magnitude of an earthquake 1000 times as intense as a standard earthquake?

Given: $\frac{I}{I_0} = 1000$ Find: M

$M = \log\left(\frac{I}{I_0}\right)$

$M = \log_{10} 1000$

$= \log_{10} 10^3$

$= 3\log_{10} 10$

$= 3$

∴ the magnitude is 3

③ Blood has a hydronium ion concentration of approximately 4×10^{-7} mol/L is blood acidic or alkaline

$PH = -\log[H^+]$

$= -\log[4 \times 10^{-7}]$

$= 6.4$

∵ $6.4 < 7$ ∴ blood is acidic

④ ~~A glass of juice has a pH of 3 what is the hydrogen ion concentration~~

$PH = -\log[H^+]$

$3 = -\log_{10}[H^+]$

$-3 = \log_{10}[H^+]$

$[H^+] = 10^{-3}$

$= 0.001$

∴ the hydrogen ion concentration is 0.001 mol/L

⑥ The sound level in normal city traffic is 85dB. the sound level while riding a snowmobile is about 32 times as intense. What is the sound level of riding a snow mobile

$B_2 - 85 = 10\log(32)$ ← $\frac{I_S}{I_N}$

$B_2 = 10\log(32) + 85$

$B_2 = 100$

∴ the sound level is approx 100 dB

⑤ How many times more intense as a whisper is the sound of a normal conversation, whisper is 30dB, normal conversation 60dB

B_2 = normal convo

B_1 = whisper

$B_2 - B_1 = 10\log\left(\frac{I_2}{I_1}\right)$

$60 - 30 = 10\log\left(\frac{I_C}{I_W}\right)$

$30 = 10\log\left(\frac{I_C}{I_W}\right)$

$3 = \log_{10}\left(\frac{I_C}{I_W}\right)$

$10^3 = \left(\frac{I_C}{I_W}\right)$

∴ A whisper is 1000 times more intense

⑦ A Bacteria culture doubles every 15 min. How long will it take 20 bacteria to grow to 163840

$A = A_0(r)^{\frac{t}{n}}$

$163840 = 20(2)^{\frac{t}{15}}$

$8192 = 2^{\frac{t}{15}}$

$2^{13} = 2^{\frac{t}{15}}$

$13 = \frac{t}{15}$

$t = 195$ minutes $\div 60 = 3.25$ hours

∴ It will take 3.25 hours

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⑧ $1000 is invested at 12% per annum, compounded monthly. How long will it take for the investment to grow to $5000?

$A = A_0(1+i)^n$

$5000 = 1000\left(1+\frac{0.12}{12}\right)^{12n}$ ∴ approx 13.5 years

$5 = (1.01)^{12n}$

$\log 5 = \log(1.01)^{12n}$

$\log 5 = (12n)\log(1.01)$

$n = \frac{\log 5}{12\log(1.01)}$

$n = 13.5$

Graphing Logar

Parent function

x	$y = \log_b x$
1	0
b	1
$x \neq 0$	

Graph $y = -3\log_2$

$-3\log_2 4(x+5)+$

x	$y = \log_2 x$
1	0
2	1
$x \neq 0$	

$y = -3\log_2 4(x+5)$

$y = -3\log_2 4(0+$

$= -12.9657\ldots$

$= -11.966$

Graphing Logarithms

Parent function to graph $y = a\log_b k(x-d)+c$

x	$y=\log_b x$
1	0
b	1
$x\neq 0$	

show mapping work!

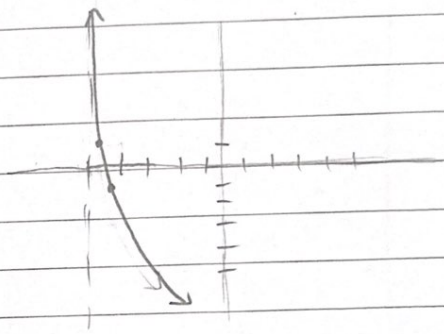
x	$y=\log_b x$	$\rightarrow$ $x\div k\mp d$	$ay\pm c$
1	0		
b	1		
$x\neq 0$			

* add 2 arrows + dashed line for asymptote.
remember equation of asmptote is $x=\#$

Graph $y=-3\log_2(4x+20)+1$ and state a) domain b) range c) equation of asymptote

$-3\log_2 4(x+5)+1$

x	$y=\log_2 x$	$\rightarrow$ $x\div 4-5$	$-3y+0$
1	0	-4.75	1
2	1	-4.5	-2
$x\neq 0$		$x\neq -5$	



~~$\{x\in\mathbb{R} \mid x>-5\}$~~

$\{y\in\mathbb{R}\}$

equation of asmptote: $x=-5$

$y=3\log_2 4(x+5)+1$

$y=-3\log_2 4(0+5)+1$

$=-12.9657\ldots+1$

$=-11.966$