Curtin University – Department of Computing

Assignment Cover Sheet / Declaration of Originality

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- The above information is complete and accurate.
- The work I am submitting is *entirely my own*, except where clearly indicated otherwise and correctly referenced.
- I have taken (and will continue to take) all reasonable steps to ensure my work is *not accessible* to any other students who may gain unfair advantage from it.
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- It is my responsibility to ensure that my submission is complete, correct and not corrupted.

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Question 1:

Part A:

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

Therefore $p \oplus (p \vee q)$ is a contingency.

Part B:

p	q	$p \wedge q$	$p \rightarrow \neg q$	$(p \rightarrow \neg q) \wedge (p \wedge q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

Therefore $(p \rightarrow \neg q) \wedge (p \wedge q)$ is a contradiction.

Part C:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)]$	$(p \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	F	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	T	T	T

Therefore $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology

Question 2:

Part A:

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$ Logical equivalence for implications

$\equiv \neg p \vee (q \wedge r)$ Distributive

$\equiv p \rightarrow (q \wedge r)$ Equivalence for implication

Therefore, logically equivalent to RHS.

Part B:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ Logical equivalence for biconditional

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ Logical Equivalence for implications

$\equiv \neg p \wedge (\neg q \vee p) \vee q \wedge (\neg q \vee p)$ Distributive Law

$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p)$ Distributive Law

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$ Identity Law

Therefore, logically equivalent to RHS.

Question 3:

All the propositions can be made simultaneously true by the assignment of q = True, p = True and r = False.

This is because all but one of the propositions is a conjunction $(p \wedge q)$ while the rest of the proposition were disjunctions. Since the only conjunction involved only p and q, for the proposition to be true, p and q both had to be true. r had to be false because of the disjunction $\neg q \vee \neg r$, which meant that for it to be true r had to be false, and since r is only involved in all the disjunctions, this meant that it did not matter that r was false for the other propositions to be true.

Question 4:

If the program returns that a given proposition is unsatisfiable, this means that the negation of that proposition was a tautology. This program can be used to determine whether a given proposition is a tautology by first putting the negation of that proposition through the program. If the program returns that the negation of the proposition is unsatisfiable, this means that the original proposition is a tautology.

Question 5:

Part A)

Converse: If n^2 is greater than 9, then n is greater than 3.

Inverse: If n is not greater than 3, then n^2 is not greater than 9.

Contrapositive: If n^2 is not greater than 9, then n is not greater than 3.

Part B)

Converse: I will go fishing, only if it doesn't rain today.

Inverse: If it rains today, then I will not go fishing.

Contrapositive: I will not go fishing, only if it rains today.

Question 6:

a) There is a student at Curtin that likes Chinese cuisine and all students at Curtin likes Mexican cuisine.
b) There is a cuisine that either Monica or Jay likes.
c) Nobody except one student at Curtin likes a specific cuisine.
d) There is a student at Curtin that only likes a specific cuisine.
e) There are two students at Curtin that likes all the cuisine.
f) All students at Curtin like a cuisine.

Question 7:

a) $\neg$ F(Ross, Rachel)
b) $\forall x\ (\neg F(x, Joe))$
c) $\forall x\ (A(x) \rightarrow x \neq Monica)$
d) $\exists x\ \forall y\ (x \neq y \rightarrow F(x, y))$
e) $\forall x\ \exists y\ ((A(x) \land (x \neq y)) \rightarrow F(x, y))$
f) z domain is all people in computing
   $\exists x\ \exists y\ ((x \neq y) \land \forall z\ (F(x, z) \rightarrow \neg F(y, z)))$

Question 8:

a) $P(x)$ = x is a cat

$F(x)$ = Eric loves x

1) $\forall x, P(x) \rightarrow F(x)$
2) P(Sushi)

——————————

∴ F(Sushi)

From 1 and 2, Modus Ponens. Therefore, argument is valid.

b) $P(x)$ = x is an island
$F(x)$ = x is a man

1) $\forall x, P(x) \rightarrow \neg F(x)$
2) P(Manhattan)

——————————

∴¬F(Manhattan)

From 1 and 2, Modus Ponens. Therefore, argument is valid.

c) $P(x)$ = Chandler likes x
$F(x)$ = x is an Action Movie.

1) $\forall x, F(x) \rightarrow P(x)$
2) P(Twelve Angry Man)

——————————

∴ F(Twelve Angry Man)

Argument is invalid. The fallacy is affirming the conclusion.

d) $P(x)$ = x is a student in Computing.
$F(x)$ = x knows how to write programs in Python.
$T(x)$ = x can get a high-paying job

1) P(Phoebe) ∧ F(Phoebe)
2) $\forall x, F(x) \rightarrow T(x)$

——————————

∴ $\exists x, P(x) \wedge T(x)$

Argument is invalid. The fallacy is circular reasoning.

e) $P(x)$ = x is between 30 to 39
$F(x)$ = x can have Covid 19 Vaccination.

1) $\forall x, P(x) \rightarrow F(x)$
2) $\neg P(John)$

——————————

$\therefore \neg F(John)$

Argument is invalid. The fallacy is denying the hypothesis

f) A= Superman is able to prevent evil
W = Superman is willing to prevent evil
D = Superman prevents evil.
I = Superman is impotent
M = Superman is malevolent
E = Superman exists.

1) $(A \wedge W) \rightarrow D$
2) $\neg A \rightarrow I$
3) $\neg W \rightarrow M$
4) $\neg D$
5) $E \rightarrow \neg I \wedge \neg M$

——————————

$\therefore \neg E$

From 1 and 2, Modus Tollens

6) $\neg (A \wedge W)$ Apply De Morgan's
$\neg A \vee \neg W$

For 2 and 3, apply logical equivalence for implication

7) $\neg (\neg A) \vee I$ Apply Negation Rule
$A \vee I$
8) $\neg (\neg W) \vee M$ Apply Negation rule
$W \vee M$

From 6 and 7, Resolution Rule

9) $\neg W \vee I$

From 9 and 8, Resolution Rule

10) $M \vee I$

From 10 and 5, Modus Tollens

11) $\neg E$

Therefore, Argument is valid.

Question 9:

Part A)

An even number can be represented as 2k

An odd number can be represented as 2k + 1

*k is an integer

Odd Number 1: 2a + 1

Odd Number 2: 2b + 1

$$2a + 1 + 2b + 1 = 2a + 2b + 2$$

$$= 2(a + b + 1)$$

Therefore, sum of two odd numbers returns a number that is divisible by 2. Therefore, it is an even number.

Part B)

Contraposition: n is odd, then $n^3 + 5$ is even. | n is an integer.

n = 2k + 1

sub (2k + 1) into $n^3 + 5$

$= (2k + 1)^3 + 5$

$= 8k^3 + 12k^2 + 6k + 1 + 5$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2(4k^3 + 6k^2 + 3k + 3)$

Divisible by 2, therefore $n^3 + 5$ is even.

Contrapositive statement is true, so original statement must be true.

Part C)

Assume sum of an irrational number and rational number is rational.

Rational numbers can be shown as a ratio of two integers.

Let irrational number = x

Let rational number $= \frac{a}{b}$

let the sum of the two numbers $= \frac{m}{n}$ (Since we are assuming that the sum is a rational number)

$x + \frac{a}{b} = \frac{m}{n}$

$x = \frac{m}{n} - \frac{a}{b}$

$x = \frac{mb-an}{nb}$

Since all the values a,b,m,n are integers. That means that x can be put as ratio of two integers, making it a rational number.

Therefore, there is a contradiction as x is supposed to be an irrational number, but from above it is shown that its not.

Hence the sum of an irrational number and rational number must be irrational.

Part D)

Using Proof by Exhaustion.

- $n = 1$

  $(1)^2 + 1 \geq 2^1$

  $n = 1$ is true.

- $n = 2$

  $(2)^2 + 1 \geq 2^2$

  $n = 2$ is true.

- $n = 3$

  $(3)^2 + 1 \geq 2^3$

  $n = 3$ is true.

- $n = 4$

  $(4)^2 + 1 \geq 2^4$

  $n = 4$ is true.

All integers within the domain satisfies the statement, therefore the statement is true.

Part E)

Using method of generalization for universal statements.

Only need to test integers 0 - 3 for x and y. Because x and y are squared, so negative numbers will work the same as non-negative numbers and because any number above 3^2 will be greater than 14 already.

Therefore, the proof can be generalized for the domain 0 – 3.

$$2(0)^2 + 5(0)^2 = 0 \neq 14$$

$$2(0)^2 + 5(1)^2 = 5 \neq 14$$

$$2(0)^2 + 5(2)^2 = 20 \neq 14$$

$$2(0)^2 + 5(3)^2 = 45 \neq 14$$

$$2(1)^2 + 5(0)^2 = 2 \neq 14$$

$$2(1)^2 + 5(1)^2 = 7 \neq 14$$

$$2(1)^2 + 5(2)^2 = 22 \neq 14$$

$$2(1)^2 + 5(3)^2 = 47 \neq 14$$

$$2(2)^2 + 5(0)^2 = 16 \neq 14$$

$$2(2)^2 + 5(1)^2 = 13 \neq 14$$

$$2(2)^2 + 5(2)^2 = 28 \neq 14$$

$$2(2)^2 + 5(3)^2 = 53 \neq 14$$

$$2(3)^2 + 5(0)^2 = 18 \neq 14$$

$$2(3)^2 + 5(1)^2 = 23 \neq 14$$

$$2(3)^2 + 5(2)^2 = 38 \neq 14$$

$$2(3)^2 + 5(3)^2 = 63 \neq 14$$

No combinations satisfy the equation; therefore, statement is true.

Question 10)

a) $1^2 = \frac{(1)((1)+1)(2(1)+1)}{6}$

b) $1^2 = \frac{3*2}{6}$

$1*1 = \frac{6}{6}$

$1 = 1$

Therefore true for P(1).

c) Assume that P(k) is true.

d) That by assuming that P(k) is true, P(k+1) follows to be true too.

e)

$$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = (k+1)(k+1+1)(2(k+1)+1)/6$$

Sub LHS of P(k) into LHS of P(k+1)

$$\frac{k(k+1)(2n+1)}{6} + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{2k^3+3k^2+k}{6} + (k+1)^2 = \frac{2k^3+3k^2+4k^2+6k+2k^2+3k+4k+6}{6}$$

$$\frac{2k^3+3k^2+k}{6} + (k+1)^2 = \frac{2k^3+3k^2+k}{6} + \frac{4k^2+5k+2k^2+3k+4k+6}{6}$$

$$\frac{2k^3+3k^2+k}{6} + (k+1)^2 = \frac{2k^3+3k^2+k}{6} + \frac{6k^2+12k+6}{6}$$

$$\frac{2k^3+3k^2+k}{6} + (k+1)^2 = \frac{2k^3+3k^2+k}{6} + k^2+2k+1$$

$$\frac{2k^3+3k^2+k}{6} + (k+1)^2 = \frac{2k^3+3k^2+k}{6} + (k+1)^2$$

Therefore RHS = LHS.

f) By showing that it works for the smallest positive integer, 1 (Base Case) it provides a basis to the claim. And then the inductive step shows that by assuming it works for any integer above 1, if you add one to the integer it still works. That essentially covers any integer above 1. Therefore, proving that the formula is true whenever n is a positive integer.

Question 11)

Base Case: $1 + \frac{1}{(2)^2} < 2 - \frac{1}{2}$

$$1.25 < 1.5$$

True for base case.

Inductive Step:

Let n = k   k>1

Assume $1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} < 2 - \frac{1}{k}$ is true.

Prove $1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$

Add $\frac{1}{(k+1)^2}$ to both sides of inductive hypothesis.

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} + \frac{1}{(\mathrm{k}+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$$

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} + \frac{1}{(\mathrm{k}+1)^2} < 2 - \frac{1}{k+1}\left(\frac{k+1}{k} - \frac{1}{k+1}\right)$$

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} + \frac{1}{(\mathrm{k}+1)^2} < 2 - \frac{1}{k+1}\left(\frac{(k^2+k+1)}{k(k+1)}\right)$$

Since $\frac{(k^2+k+1)}{k(k+1)}$ is always greater than 1 for when k > 1.

$$1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{k^2}+\frac{1}{(\mathrm{k}+1)^2}<2-\frac{1}{k+1}\left(\frac{(k^2+k+1)}{k(k+1)}\right)< \ 2-\frac{1}{k+1}$$

This means that

$$1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{k^2}+\frac{1}{(\mathrm{k}+1)^2}<2-\frac{1}{k+1}$$

Therefore, inductive step is also true.

Hence statement is true.

Question 12)

Prove $P(x) = \ 3(x)+5(y)=n \ \mid n \geq 8$

Basis Step: $8 = 3(1) + 5(1)$

True for basis step.

Inductive Step:

Assume P(8) ∧ P(9) ∧ P(10) … ∧ P(k) is True.

Show P(k+1) follows to be true.

Since P(k-2) is assumed true under the inductive hypothesis. As $P(8) < P(k-2) < P(k)$

That means that P(k-2) can be formed with 3 and 5 cent stamps.

By adding a 3-cent stamp to P(k-2), it will give us P(k+1). Which means that P(k+1) can also be formed with just 3 and 5 cent stamps.

Therefore, Inductive step is also true.

Hence statement follows to be true too.