Epitrochoid Mathematical Derivation

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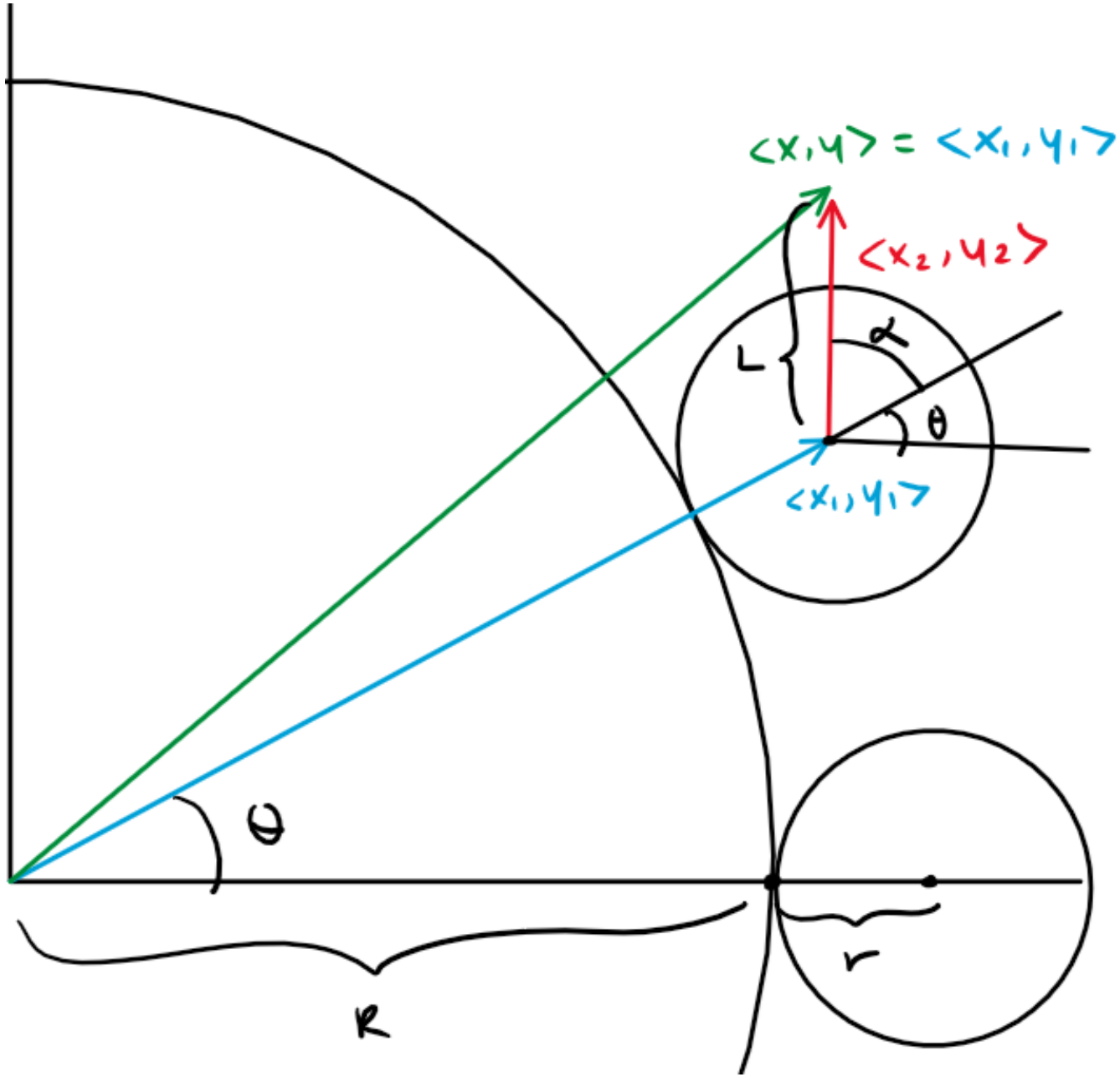
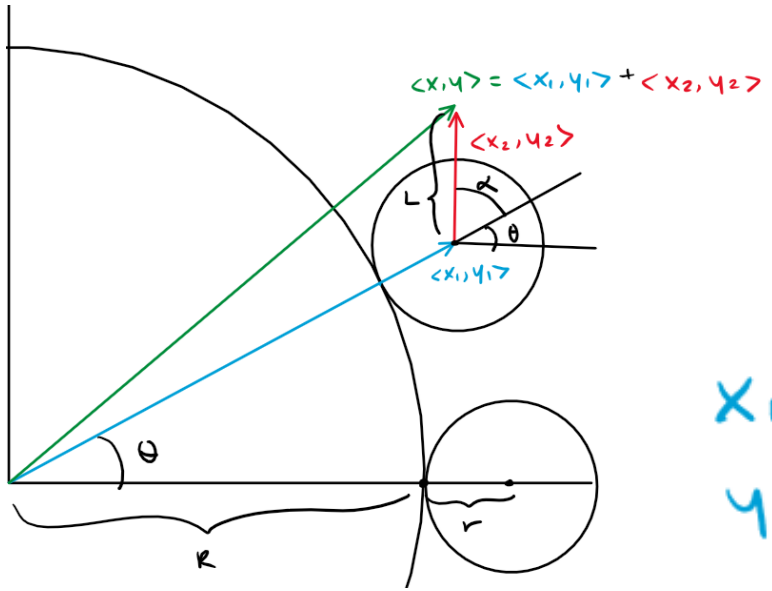
Multivariable Calculus

Grade 12



$$k\theta = r\alpha$$

$$\alpha = \frac{R}{r}\theta$$



$$x_1 = (R+r)\cos\theta$$
$$y_1 = (R+r)\sin\theta$$

$$x_2 = L\cos(\theta+\alpha)$$
$$y_2 = L\sin(\theta+\alpha)$$

$$x = x_1 + x_2 = (R+r)\cos\theta + L\cos(\theta+\alpha)$$
$$y = y_1 + y_2 = (R+r)\sin\theta + L\sin(\theta+\alpha)$$

$$x(\theta) = (R+r)\cos\theta + L\cos\left(\theta + \frac{R}{r}\theta\right)$$

$$y(\theta) = (R+r)\sin\theta + L\sin\left(\theta + \frac{R}{r}\theta\right)$$