STA8190 S2 2019: Nonparametric Statistics topic.

A very solid assignment and well presented. Remember that you need to always explain any result you obtained. And in particular your hypotheses need to be explained in context
89/100

Assignment 1 30% (100 marks) Due: 28/8/2019 11.55pm

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All analysis can be performed in either SPSS or R unless otherwise indicated. Alpha=0.05 should be used for all tests. **Clearly label each part of your work.**

Please remember that STA8190 is a postgraduate level course which requires that students demonstrate an advanced level of knowledge, skills, reasoning and problem-solving. Assessment items are your opportunity to demonstrate your full understanding of the concepts and methods covered in this topic. If you do not fully explain your answers you cannot receive full marks in this assessment.

Contents

Question 1 (10 marks):

Name the appropriate nonparametric and parametric tests in the table below.

	2 samples		
	related	unrelated	categorical
Parametric	t-Test for dependent samples	t-Test for independent samples	NONE
Nonparametric	Wilcoxon signed ranks test and sign test	Mann-Whitney U-test and Kolmogorov-Smirnov two-sample test	Chi Square (x2) tests and Fisher exact test
	>2 samples		
	related	unrelated	
Parametric	Repeated measures, analysis of variance (ANOVA)	One-way ANOVA	
Nonparametric	Friedman test	Kruskal-Wallis H-test	
	Correlation		
	2 rank-ordered variables	1 continuous dichotomous variable	1 discrete dichotomous variable
Parametric	Pearson Product-moment correlation	Pearson Product-moment correlation	Pearson Product-moment correlation
Nonparametric	Spearman Rank-Order Correlation	Biserial Correlation	Point-biserial Correlation

Well Done 10/10

Question 2 (10 marks):

a) Briefly define Type I and Type II errors associated with hypothesis testing. (2 marks)

Type I-Error (α): We reject the null hypothesis (H_0), but it is actually true.
Type II-Error (β): We do not reject the null Hypothesis (H_0), but it is actually false.

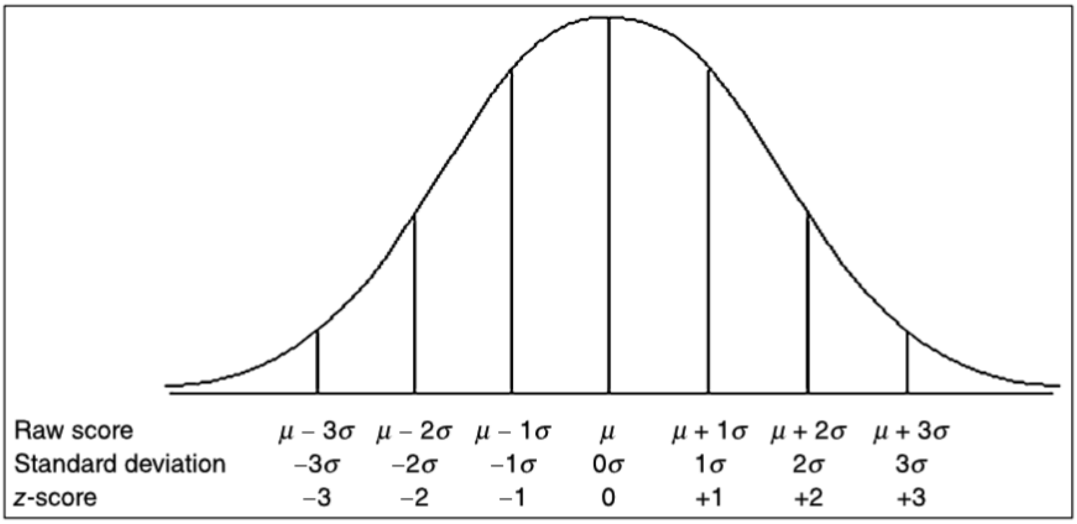
b) Describe the key properties of the normal curve/distribution and explain what z=2.5 means. (3 marks)

The three key properties of the normal distribution are[1]:

1. The mean, median and mode are equal.
2. The curve displays perfect symmetry about the mean.
3. The tails on the left and right side are asymptotic, which means they approach the horizontal (x) axis but never touch it.

These three properties allow for mathematical methods to be used to consistently describe the data set provided the mean (μ) and the standard deviation (σ) are known. The reason for this is that the normal distribution model, with its known mathematical properties, can be shaped using the mean and standard deviation of the data set to fit it.

A z-score of 2.5 means that a variable is 2.5 standard deviations above the mean. A simple way of understanding the standard deviation is that it is the average absolute difference from the mean of the data points in the data set. As such the z-score is a measure of the relative distance away from the mean along the x-axis, using the standard deviation as a measure[2]. The below diagram illustrates this point[3]:



[1] (Corder and Foreman, 2014, p.16)
[2] (Ibid, p.16)
[3] (Ibid, Fig. 2.3, p.16)

As the above diagram shows a z-score of 2.5 would be 2 standard deviations above the mean (μ). The implication of this relationship is that if z-score were used as a reference to height someone with a z-score of 2.5 would be 2.5 standard deviations taller than average, which translates to being taller than 98.7581% of the normally distributed population.

c) Which would have a greater standard deviation: data with a leptokurtic distribution or data with a platykurtic distribution? Explain your answer. (2 marks)

Platykurtic distributions will have a greater standard deviation as it is flat and less spikey than a leptokurtic distribution, which means less of the samples are near the mean (μ). The image below demonstrates this point[4]:

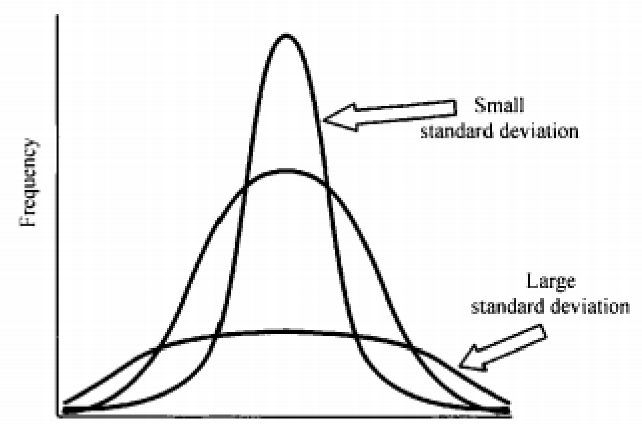


FIGURE 2.1

Lovely

One way to think of standard deviation is that it is the average of the absolute difference of each observation from the average. As such, standard deviation will decrease as kurtosis increases because more of the data is positioned near the mean, and standard deviation is based upon typical deviation from the mean. As kurtosis decreases the data is more spread out, so the standard deviation will increase because the 'typical' data point is further away.

[4] (Ibid, Fig. 2.1, p.14)

d) Discuss statistical power in the context of parametric and non-parametric methods (200 words maximum). (3 marks)

Parametric statistical methods are more powerful than non-parametric methods because the assumption of a normal distribution serves as a limiting factor. As non-parametric methods do not have this limiting factor there are not as many constraints on where the data points are expected to be, making the predictions offered more 'broad'[5]. Because of this they are less likely to reject the null hypothesis.

Another way of phrasing this is that when the alternative hypothesis is true a non-parametric test is less likely to reject the null hypothesis. In other words the likelihood of a Type II error is higher. This is what it means for a statistical test to be less powerful. Recall that the burden of proof is on the alternative hypothesis. The idea being that we are attempting to disprove the null hypothesis in favour of the alternative, but we want to be conservative and careful in our approach. If a test can disprove the null hypothesis while remaining conservative, it is a more powerful test.

In conclusion, parametric methods have more power than non-parametric methods as they are more likely to reject the null hypothesis when it is false.

But if the parametric test violates the assumptions then there is a greater risk of committing a type I error

7/10

[5] (Corder and Foreman, 2014, p. 60)

Question 3 (70 marks):

The data file ***twin.txt*** shows the height of 20 pairs of ewes. Pairs were formed at the time of weaning based on size. One ewe in each pair was fed a control diet and the other ewe in the pair was fed a treatment diet. The treatment diet was hoped to improve growth. At the end of the experiment height in inches was measured.

The data file ***goats.txt*** shows data for 30 goats. The goats were divided into two equal groups. One group was fed a control diet (treatment 1) and the other group was fed a treatment diet (treatment 2). The treatment diet was hoped to improve overall condition. At the end of the experiment the overall condition was judged (Judgement) on a scale that could range between 40 and 130 (units and exact measurement process are unknown).

All analyses can be performed in either SPSS or R. Alpha=0.05 should be used for all tests.

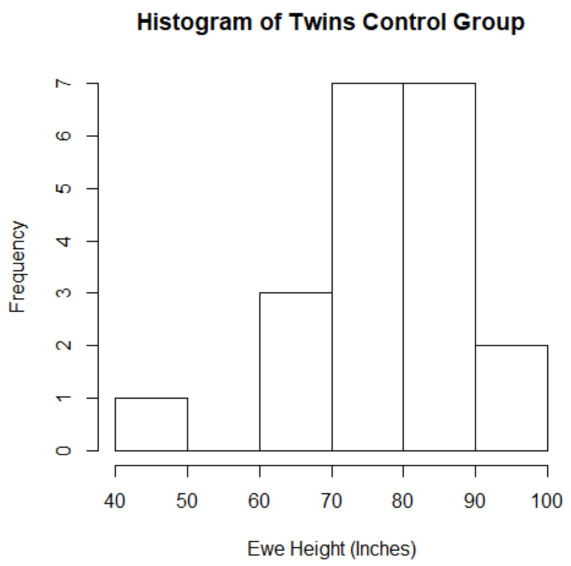
For each data set, answer each of the following sections. Present all parts (a – e) for the twin dataset first and then all parts (a - e) for the goats dataset in a separate section. **Clearly label each part of your work**:

Twin Dataset

Summary Statistics

a) Describe the distributions of the two groups (control and treatment) using histograms, summary statistics, and skewness and kurtosis statistics. (3 marks x 2 data sets)

Control Group



Summary Statistics	
Min	48.10
1^{st} Qu	70.67
Median	77.00
Mean	76.64
3^{rd} Qu	86.20
Max	92.80
Kurtosis	-0.4075253
Skewness	-0.6244909
Std Deviation	11.8158357167071

Make sure if you include any statistics that you round appropriately and include units (tell the reader exactly what you found) -1

Standard Error for Kurtosis:

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

Where n = 20

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

$$SE_K = \sqrt{\frac{480(19)^2}{(18)(17)(25)(23)}}$$

$$SE_K = \sqrt{\frac{173280}{175950}}$$

$$SE_K = 0.9923\ (rounded\ down)$$

Z-score for Kurtosis:

impressive you did via hand but you could also have used R to do this!

$$Z_K = \frac{K-0}{SE_k}$$

$$Z_K = \frac{-0.4075253-0}{SE_k}$$

$$Z_K = \frac{-0.4075253-0}{0.9923}$$

$$Z_K = \frac{-0.4075253-0}{0.9923}$$

$$Z_K = -0.4106\ (rounded\ down)$$

Conclusions concerning Kurtosis:

As this Z-score is between - 1.96 and + 1.96 it passes the test for normality Kurtosis at a critical value of 0.05. As such it is accepted that the distribution has a kurtosis sufficiently within the range of a normal distribution.

Standard Error for Skewness:

$$SEsk = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

Where n = 20

$$SEsk = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

$$SEsk = \sqrt{\frac{120(19)}{(18)(21)(23)}}$$

$$SEsk = \sqrt{\frac{2280}{8694}}$$

$$SEsk = \sqrt{\frac{2280}{8694}}$$

$$SEsk = \ 0.5121\ (rounded\ down)$$

Z-score for Skewness:

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$$

$$Z_{S_k} = \frac{-0.6244909 - 0}{0.5121}$$

$$Z_{S_k} = \ -1.2194\ (rounded\ down)$$

Conclusions concerning Skewness:

As this Z-score is between - 1.96 and + 1.96 it passes the assumption of normality test for Skewness.

Description Report:

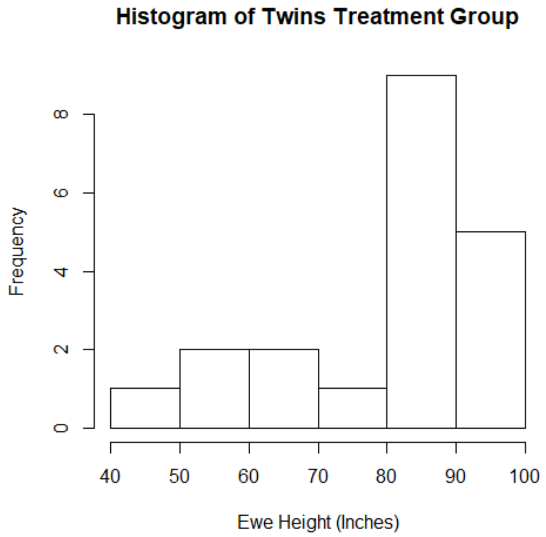
The control group of ewes passed the normality assumption for skewness and kurtosis, so overall the distribution is normal. The data is slightly left skewed with a skewness of -0.6244909 and slightly platykurtic with a kurtosis of -0.4075253.

Always include units

The mean of 76.64 and the median of 77.00 inform us that the data is quite symmetrical, given that the balancing value and the middle value are so close.

Overall the data is normally distributed, with slight deviancies but nothing that may fail a test.

Treatment Group



Summary Statistics	
Min	49.50
1st Qu	75.75
Median	81.50
Mean	79.66
3rd Qu	89.35
Max	97.30
Kurtosis	-0.576976928766213
Skewness	-0.727773388265493
Std Deviation	14.006031595446

As above - explain with units all summary stats

Standard Error for Kurtosis:

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

Where n = 20

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

$$SE_K = \sqrt{\frac{480(19)^2}{(18)(17)(25)(23)}}$$

$$SE_K = \sqrt{\frac{173280}{175950}}$$

$$SE_K = 0.9923\ (rounded\ down)$$

Z-score for Kurtosis:

$$Z_K = \frac{K - 0}{SE_k}$$

$$Z_K = \frac{-0.5769(\text{rounded down}) - 0}{0.9923}$$

$$Z_K = -0.5813\ (rounded\ down)$$

Conclusions concerning Kurtosis:

As the Z-score of -0.5813 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Kurtosis at a level of significance of 0.05. As such it is accepted that the distribution has a kurtosis sufficiently within the range of a normal distribution.

Standard Error for Skewness:

$$SEsk = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

Where n = 20

$$SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

$$SE_{S_k} = \sqrt{\frac{120(19)}{(18)(21)(23)}}$$

$$SE_{S_k} = \sqrt{\frac{2280}{8694}}$$

$$SE_{S_k} = \sqrt{\frac{2280}{8694}}$$

$$SE_{S_k} = 0.5121\ (rounded\ down)$$

Z-score for Skewness:

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$$

$$Z_{S_k} = \frac{-0.7277(\text{rounded down}) - 0}{0.5121}$$

$$Z_{S_k} = -1.4210\ (rounded\ down)$$

Conclusions concerning Skewness:

As the Z-score of -1.4210 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Skewness at a level of significance of 0.05. As such it is accepted that the distribution has a Skewness sufficiently within the range of a normal distribution.

Description Report:

The treatment group of ewes passed the normality assumption for skewness and kurtosis, so overall the distribution is normal. The data is slightly left skewed with a skewness of -0.7277 (rounded down) and slightly platykurtic with a kurtosis of -0.5769 (rounded down).

The mean of 79.66 and the median of 81.50 inform that the data is quite symmetrical, given that the balancing value and the middle value are so close.

Overall the data is normally distributed, with slight deviancies but nothing that may fail a test.

Kolmogorov-Smirnov One-Sample Test

b) Use the Kolmogorov-Smirnov one-sample test to test deviations from normality for each group. As part of your answer include the hypotheses, **relevant** test output from R or SPSS and report and interpret their meaning. What do you conclude about the normality of each data set? (7 marks x 2 data sets)

Control Group

Hypothesis:

H_0 = There is no difference in growth between the observed distribution of untreated ewes and a normally distributed empirical sample.

H_A = There is a difference in growth between the observed distribution of untreated ewes and a normally distributed empirical sample.

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
        One-sample Kolmogorov-Smirnov test

data:  controlVar

D = 0.11836, p-value = 0.9113

alternative hypothesis: two-sided
```

Interpretation:

Given a level of risk of α = 0.05 the p-value is larger at 0.9113. As such we cannot reject the null hypothesis. It can then be concluded that the data does not differ significantly from that which is normally distributed.

Report:

D= 0.11836

Degrees of Freedom = 20

p-value = 0.9113

Based on the analysis the control group of ewes is sufficiently normal where $D_{(20)} = 0.11836$, $p > 0.05$.

Conclusion:

It is concluded that the control group of ewes is approximately normally distributed.

Treatment Group

Hypothesis:

H_0 = There is no difference in growth between the observed distribution of treated ewes and a normally distributed empirical sample.

H_A = There is a difference in growth between the observed distribution of treated ewes and a normally distributed empirical sample.

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
One-sample Kolmogorov-Smirnov test

data:  treatmentVar

D = 0.21998, p-value = 0.2488

alternative hypothesis: two-sided
```

Interpretation:

The critical value for rejecting the null hypothesis is α = 0.05. As the obtained p-value is larger at 0.2488, we cannot reject the null hypothesis. It can then be concluded that the data does not differ significantly from that which is normally distributed.

Report:

D= 0.21998

Degrees of Freedom = 20

p-value = 0.2488

Based on the analysis the treatment group of goats is sufficiently normal where $D_{(20)} = 0.21998, p > 0.05$.

Conclusion:

It is concluded that the treatment group of ewes is approximately normally distributed.

Well done

Non-parametric Tests

c) Use the TWO appropriate non-parametric tests for the data set from the following list: Mann- Whitney U test, Wilcoxon signed rank test, the Kolmogorov-Smirnov two-sample test, Sign test. As part of your answer include the hypotheses (Ho and Ha) for each test, **relevant** test results from R or SPSS and report and interpret their meaning. (15 marks x 2 data sets)

Wilcoxon signed rank test

Hypothesis:

$H_0 = \mu_D = 0$

$H_a = \mu_D \neq 0$

If you write hypotheses in symbols make sure you let the reader know what mu_D is (the mean of what differences?) -1

Level of Risk:

α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
Wilcoxon signed rank test

data:  controlVar and treatmentVar

V = 87, p-value = 0.5217

alternative hypothesis: true location shift is not equal to 0

95 percent confidence interval:

 -9.5  3.6

sample estimates:

(pseudo)median

     -2.2

Sum of Positive Difference Ranks = 87

Sum of Negative Difference Ranks = 123
```

Interpretation:

A p-value of 0.5217 is above the critical value of 0.05, which means the null hypothesis cannot be rejected. This in turn suggests that the treatment did not result in a difference between the control and treatment groups. As such it is reasonable to conclude the treatment was ineffective.

Report:
T-statistic: 87
Sample Size (n): 20
P-value: 0.5217
$\Sigma R_{+} = 47.5$
$\Sigma R_{-} = 123$

The Wilcoxon signed rank test ($T = 87$, $n = 20$, $p > 0.05$) indicated that the heights of ewes was not significantly different. The sum of the positive difference ranks ($\Sigma R_{+} = 47.5$) was smaller than the sum of the negative difference ranks ($\Sigma R_{-} = 123$), showing a negative result in height growth from the treatment diet but not enough to reject the null hypothesis.

Conclusion:
The analysis concludes that there is no significant difference in height in the ewes fed the control diet and the ewes fed the treatment diet.

Sign test

Hypothesis:

H_0: p = 0.5

H_A: p ≠ 0.5

Again if in symbols explain what is meant by them -1

Level of Risk:

α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
Dependent-samples Sign-Test

data:  controlVar and treatmentVar

S = 8, p-value = 0.5034

alternative hypothesis: true median difference is
not equal to 0

95 percent confidence interval:

 -10.196353   4.176706

sample estimates:

median of x-y

      -1.65

Achieved and Interpolated Confidence Intervals:

                  Conf.Level   L.E.pt U.E.pt

Lower Achieved CI     0.8847  -4.1000 4.0000

Interpolated CI       0.9500 -10.1964 4.1767

Upper Achieved CI     0.9586 -11.0000 4.2000
```

Interpretation:

The p-value of 0.5034 is greater than the critical value of α = 0.05, therefore we cannot reject the null hypothesis. As such, there is no evidence that the treatment results in positive or negative differences.

Report:

Sample Size(n) = 20

Pluses = 8

Negatives = 12

Ties = 0

p-value = 0.5034

The obtained value of 0.5034 was greater than the critical value, α = 0.05. As a result it was not possible to reject he null hypothesis. This suggests that the treatment diet was not beneficial or harmful when compared to the control diet for increasing the ewe's heights. It is worth noting that though there is not a significant difference between the two groups, the treatment diet did result in a slight decrease with 8 positive growth examples and 12 negative growth examples. Not enough at the level of significance, but still present. ✓

Conclusion:
The analysis concludes that there is no significant difference in height in the ewes fed the control diet and the ewes fed the treatment diet. ✓

Well done

Parametric Test

d) Now reanalyse the data in the dataset using an appropriate parametric test, either the paired t-test or the independent samples t-test. As part of your answer include the hypotheses for the test, **relevant** test output from R or SPSS and report and interpret its meaning. (5 marks x 2 data sets)

Paired Samples T-test

Hypothesis: Explain in conext -1

H_0: $\mu_d = 0$ (the mean difference (μ_d) between paired observations is zero.)
H_A: $\mu_d \neq 0$ (the mean difference (μ_d) between paired observations is not equal to zero.)

Level of Risk:
$\alpha = 0.05$

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
        Paired t-test

data:  controlVar and treatmentVar
t = -0.96763, df = 19, p-value = 0.3454
alternative hypothesis: true difference in means is
not equal to 0
95 percent confidence interval:
 -9.552353  3.512353
sample estimates:
mean of the differences
              -3.02
```

Interpretation:

As the p-value of 0.3454 is greater than $\alpha = 0.05$ we cannot reject the null hypothesis. Thus, we can conclude that there is not a significant mean difference between the two groups.

Report:
T = -0.96763
Degrees of Freedom = 19
P-value = 0.3454

The Paired Samples T-test ($t(19) = -0.96763$, $p > 0.3454$) indicated that the heights of the ewes from the control group was not significantly difference than the heights of the treatment group.

Conclusion:

mean height -0.5

The analysis concludes that there is no significant difference in height in the ewes fed the control diet and the ewes fed the treatment diet.

Overall Conclusion

e) What is your overall conclusion about any differences between groups in each data set. (5 marks x 2 data sets).

Final Summary Statistics		
Single Tests		
	Control Group	Treatment Group
Skewness	Normal	Normal
Kurtosis	Normal	Normal
Kolmogorov-Smirnov One-Sample Test	p-value = 0.9113	p-value = 0.2488
Comparison Tests		
Wilcoxon signed rank test	V = 87, p-value = 0.5217	
Sign test	S = 8, p-value = 0.5034	
Paired Samples T-test	t = -0.96763, df = 19, p-value = 0.3454	

Well presented!

Overall, every single statistic suggests that there is no significant difference between the control group and the treatment group at α = 0. The parametric and non-parametric tests were in general agreement with quite high p-values, meaning that none of them were particularly close to rejecting the null hypothesis that there was no difference between the groups.

As such it is fair to conclude that there was no significant difference in height growth between the ewe's fed a control diet and a treatment diet. ✓

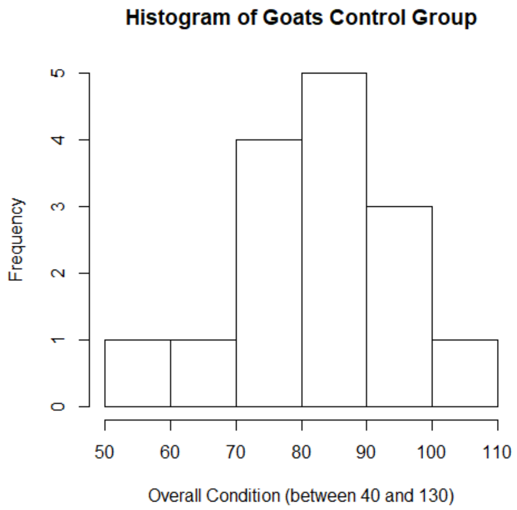
Twins
(a) 2/3
(b) 7/7
(c) 13/15
(d) 4.5/5
(e) 5/5

Goats Dataset

Summary Statistics

a) Describe the distributions of the two groups (control and treatment) using histograms, summary statistics, and skewness and kurtosis statistics. (3 marks x 2 data sets)

Control Group



Summary Statistics	
Min	52.30
1^{st} Qu	75.55
Median	83.30
Mean	81.82
3^{rd} Qu	89.35
Max	104.50
Kurtosis	-0.08308128
Skewness	-0.494891
Std Deviation	12.7396008011016

Same as above for summary statistics - round and interpret or refer to them -1

Standard Error for Kurtosis:

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

Where n = 15

$$SE_K = \sqrt{\frac{360(14)^2}{(13)(12)(20)(18)}}$$

$$SE_K = \sqrt{\frac{70560}{56160}}$$

$$SE_K = 1.1208\ (rounded\ down)$$

Z-score for Kurtosis:

$$Z_K = \frac{K - 0}{SE_k}$$

$$Z_K = \frac{-0.0830\ (\text{rounded down}) - 0}{1.1208}$$

$$Z_K = -0.0740\ (rounded\ down)$$

Conclusions concerning Kurtosis:

As the Z-score of -0.0740 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Kurtosis at a level of significance of 0.05. As such it is accepted that the distribution has a kurtosis sufficiently within the range of a normal distribution.

Standard Error for Skewness:

$$SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

Where n = 15

$$SE_{S_k} = \sqrt{\frac{1260}{(13)(16)(18)}}$$

$$SE_{S_k} = \sqrt{\frac{1260}{(13)(16)(18)}}$$

$$SE_{S_k} = \sqrt{\frac{1260}{3744}}$$

$$SE_{S_k} = 0.5801\ (rounded\ down)$$

Z-score for Skewness:

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$$

$$Z_{S_k} = \frac{-0.4948 - 0}{0.5801}$$

$$Z_{S_k} = -0.8529\ (rounded\ down)$$

Conclusions concerning Skewness:

As the Z-score of -0.8529 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Skewness at a level of significance of 0.05. As such it is accepted that the distribution has a Skewness sufficiently within the range of a normal distribution.

Description Report:

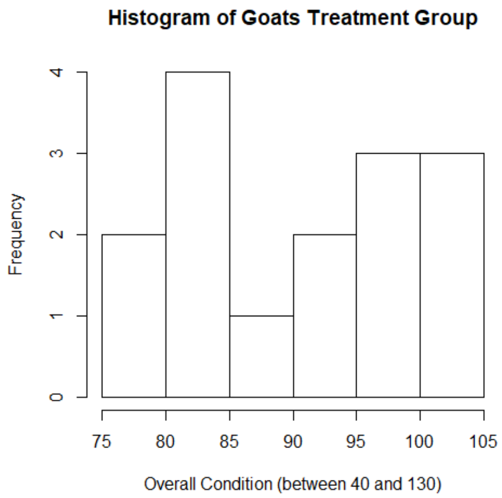
The control group of goats passed the normality assumption for skewness and kurtosis, so overall the distribution is normal. The data is slightly left skewed with a skewness of -0.4948 (rounded down) and slightly platykurtic with a kurtosis of -0.0830 (rounded down).

units

The mean of 81.82 and the median of 83.30 informs that the data is quite symmetrical, given that the balancing value and the middle value are so close.

Overall the data is normally distributed, with slight deviancies but nothing that may fail a test.

Treatment Group



Keep the x-axis the same (ie 10) ... -0.5

Summary Statistics	
Min	76.40
1st Qu	82.20
Median	92.40
Mean	90.74
3rd Qu	97.30
Max	104.60
Kurtosis	-1.5804105214569
Skewness	0.0530176229522323
Std Deviation	9.4379779916796

Standard Error for Kurtosis:

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

Where n = 15

$$SE_K = \sqrt{\frac{360(14)^2}{(13)(12)(20)(18)}}$$

$$SE_K = \sqrt{\frac{70560}{56160}}$$

$$SE_K = 1.1208\ (rounded\ down)$$

Z-score for Kurtosis:

$$Z_K = \frac{K - 0}{SE_k}$$

$$Z_K = \frac{-1.5804\ (\text{rounded down}) - 0}{1.1208}$$

$$Z_K = -1.4100\ (rounded\ down)$$

Conclusions concerning Kurtosis:

As the Z-score of -1.41 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Kurtosis at a level of significance of 0.05. As such it is accepted that the distribution has a kurtosis sufficiently within the range of a normal distribution.

Standard Error for Skewness:

$$SEsk = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$$

Where n = 15

$$SEsk = \sqrt{\frac{1260}{(13)(16)(18)}}$$

$$SEsk = \sqrt{\frac{1260}{(13)(16)(18)}}$$

$$SEsk = \sqrt{\frac{1260}{3744}}$$

$$SEsk = 0.5801\ (rounded\ down)$$

Z-score for Skewness:

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$$

$$Z_{S_k} = \frac{0.0530 - 0}{0.5801}$$

$$Z_{S_k} = 0.0913\ (rounded\ down)$$

Conclusions concerning Skewness:

As the Z-score of 0.0913 is between - 1.96 and + 1.96 it passes the test for the normality assumption for Skewness at a level of significance of 0.05. As such it is accepted that the distribution has a Skewness sufficiently within the range of a normal distribution.

Description Report:

The treatment group of goats passed the normality assumption for skewness and kurtosis, so overall the distribution is normal. The data is slightly right skewed with a skewness of 0.0530 (rounded down) and moderately platykurtic with a kurtosis of -1.5804 (rounded down).

units

The mean of 90.74 and the median of 92.40 inform that the data is quite symmetrical, given that the balancing value and the middle value are so close.

Overall the data is normally distributed, with slight deviancies but nothing that may fail a test.

Kolmogorov-Smirnov One Sample Test

b) Use the Kolmogorov-Smirnov one-sample test to test deviations from normality for each group. As part of your answer include the hypotheses, **relevant** test output from R or SPSS and report and interpret their meaning. What do you conclude about the normality of each data set? (7 marks x 2 data sets)

Control Group

Hypothesis:

H_0 = There is no difference in growth between the observed distribution of goats fed a control diet and a normally distributed empirical sample.

H_A = There is a difference in growth between the observed distribution of goats fed a control diet and a normally distributed empirical sample.

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
        One-sample Kolmogorov-Smirnov test

data:  goatControl

D = 0.13066, p-value = 0.9308

alternative hypothesis: two-sided
```

Given a level of risk of 0.05, the p-value is larger at 0.9308. This in turn means that the null hypothesis is correct and that the data is normally distributed.

It can then be concluded that the data does not differ significantly from that which is normally distributed.

Interpretation:

Given a level of risk of 0.05, the p-value is larger at 0.9308, we cannot reject the null hypothesis. It can then be concluded that the data does not differ significantly from that which is normally distributed.

Report:

D= 0.13066

Degrees of Freedom = 15

P = 0.9308

Based on the analysis the control group of goats is sufficiently normal where $D_{(20)}$ = 0.13066, p> 0.05.

Conclusion:

It is concluded that the control group of goats is normally distributed.

Conclusion that there is no evidence to suggest the control group isn't normally distributed (we didn't prove that it was normal, just that there wasn't enough evidence to suggest it wasn't) -0.5

Treatment Group

Hypothesis:

H_0 = There is no difference in growth between the observed distribution of goats fed a treatment diet and a normally distributed empirical sample.

H_a = There is a difference in growth between the observed distribution of goats fed a treatment diet and a normally distributed empirical sample.

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
One-sample Kolmogorov-Smirnov test

data:  goatTreat

D = 0.16569, p-value = 0.7458

alternative hypothesis: two-sided
```

Interpretation:

Given a level of risk of 0.05, the p-value is larger at 0.7458, we cannot reject the null hypothesis. It can then be concluded that the data does not differ significantly from that which is normally distributed.

Report:

D= 0.16569

Degrees of Freedom = 15

P = 0.7458

Based on the analysis the control group of goats is sufficiently normal where $D_{(20)} = 0.16569$, $p > 0.05$.

Conclusion:

It is concluded that the treatment group of goats is normally distributed. As above -0.5

Non-Parametric Tests

c) Use the TWO appropriate nonparametric tests for the data set from the following list: Mann- Whitney U test, Wilcoxon signed rank test, the Kolmogorov-Smirnov two-sample test, Sign test. As part of your answer include the hypotheses (Ho and Ha) for each test, **relevant** test results from R or SPSS and report and interpret their meaning. (15 marks x 2 data sets)

Mann-Whitney U-test

Hypothesis:

H_0 = There is no tendency for ranks of one method to be significantly higher (or lower) than those of the other.
H_A = The ranks of one method are systematically higher (or lower) than those of the other

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
        Wilcoxon rank sum test

data:  Judgement by Treatment
W = 66, p-value = 0.05553
alternative hypothesis: true location shift is not
equal to 0
```

Interpretation:

A p-value of 0.05553 is greater than the critical value of α = 0.05, therefore we cannot reject the null hypothesis. As such, there is no evidence that the treatment results in a positive or negative difference between the control and treatment groups.

Report:

Sample size (n): 15
U = 66
p-value = 0.1844
Sum of Ranks 1 = 186
Sum of Ranks 2 = 279

The obtained value of 0.1844 was greater than the critical value of α = 0.05. As a result the null hypothesis was unable to be rejected. The results from the Kolmogorov– Smirnov two-sample test (U = 66, n 1 = 15, n 2 = 15, p > 0.05) suggest that the treatment diet was not significantly different to the control diet when it came to improving the goats' condition. While the sum of ranks for the control diet ($\Sigma R_1 = 186$) was lesser than the sum of ranks for the treatment diet ($\Sigma R_2 = 279$), it was not a substantial enough difference to be considered significant.

Conclusion:

The analysis concludes that there is no significant difference in condition between the goats fed a control diet and a treatment diet.

Kolmogorov-Smirnov two-sample test

Hypothesis:

H_0 = [F (t) = G (t), for every t]
H_A = [F (t) ≠ G (t) for at least one value of t]

Explain in context -1

Level of Risk:
α = 0.05

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
        Two-sample Kolmogorov-Smirnov test

data:  goatControl and goatTreat
D = 0.4, p-value = 0.1844
alternative hypothesis: two-sided
```

Interpretation:

A p-value of 0.1844 is greater than the critical value of α = 0.05, therefore we cannot reject the null hypothesis. As such, there is no evidence that the treatment results in a positive or negative difference between the control and treatment groups.

Report:

Sample size (n): 15
D = 0.4
p-value = 0.1844

The obtained value of 0.1844 was greater than the critical value of α = 0.05. As a result the null hypothesis was unable to be rejected. The results from the Kolmogorov– Smirnov two-sample test (D = 0.4, $p < 0.05$) suggest that the treatment diet was not significantly different to the control diet when it came to improving the goats' condition.

Conclusion:

The analysis concludes that there is no significant difference in condition between the goats fed a control diet and a treatment diet.

Parametric Test

d) Now reanalyse the data in the dataset using an appropriate parametric test, either the paired t-test or the independent samples t-test. As part of your answer include the hypotheses for the test, **relevant** test output from R or SPSS and report and interpret its meaning. (5 marks x 2 data sets)

Independent samples t-test

Hypothesis:

H_0: $\mu_1 = \mu_2$ ("the two population means are equal")

H_A: $\mu_1 \neq \mu_2$ ("the two population means are not equal")

Explain in context -1

Level of Risk:

$\alpha = 0.05$

There is a 95% chance that any observed statistical difference will be real and not due to chance.

R Output:

```
          Two Sample t-test

data:  goatControl and goatTreat
t = -2.179, df = 28, p-value = 0.03791
alternative hypothesis: true difference in means is
not equal to 0
95 percent confidence interval:
 -17.305517  -0.534483
sample estimates:
mean of x mean of y
   81.82     90.74

Levene's Test for Homogeneity of Variance
(center = median)
      Df F value Pr(>F)
group  1  0.2452 0.6243
      28
```

Interpretation:

Given a level of risk of $\alpha = 0.05$ the p-value of 0.03791 is less, suggesting that the null hypothesis is likely to be false and thus the alternative hypothesis true. As such it is likely that there is significant difference in condition between the control group and the treatment group of goats as a result of diet.

Report:

t= -2.179

Degrees of Freedom = 28

p-value = 0.03791

Based on the analysis the control group of ewes is sufficiently normal where $D(20) = 0.11836$, $p > 0.05$.

Conclusion:

It is concluded that the there is significant difference in condition between the control group and the treatment group of goats as a result of the diet, with an overall decrease in the condition of the goats in the treatment group when compared to the control group.

Overall Conclusions

e) What is your overall conclusion about any differences between groups in each data set. (5 marks x 2 data sets).

Final Summary Statistics		
Single Tests		
	Control Group	Treatment Group
Skewness	Normal	Normal
Kurtosis	Normal	Normal
Kolmogorov-Smirnov One-Sample Test	p-value = 0.9308	p-value = 0.7458
Comparison Tests		
Wilcoxon rank sum test	W = 66, p-value = 0.05553	
Two-sample Kolmogorov-Smirnov test	D = 0.4, p-value = 0.1844	
Independent samples t-test	t = -2.179, df = 25.81, p-value = 0.03791	

Overall, most of the statistics supported the null hypothesis that there was not a difference between the control and treatment group at a significance level of α = 0.05. However, the parametric test applied in this particular case did not support the null hypothesis having a p-value of 0.03866. This means that the parametric test suggested that the alternative hypothesis that there is a difference between the control and treatment groups is true.

The non-parametric tests all suggest no difference between the groups, but the parametric suggests that there is. Given that both groups met all criteria for the assumption of normalcy, the parametric test ought to be used. As the samples have met the normality assumption it is reasonable to conclude that the result from the Independent Samples t-test is to be prioritized. Furthermore, the Levene test of Homogeneity of Variance generated a p-value of 0.6243 and so the null hypothesis that the two data sets had equal variances had to be accepted. As such the two data sets had similar enough variances to prevent ruling the results of the t-test out on those grounds. With all of the assumptions for a t-test thus met the result from it, being the more powerful test, has priority and must be used.

In conclusion, there was enough difference between the condition of the control group of goats and the treatment group to suggest the treatment had an effect. The Independent Samples t-test generated a t-score of -2.179, which means that there was a negative effect on the treatment group. As such it is suggested that the treatment diet resulted in a decrease in the condition of the goats.

Great summary well

Goats
(a) 1.5/3
(b) 6/7
(c) 14/15
(d) 4/5
(e) 5/5

Question 4 (10 marks):

a) For only the independent samples data set and associated analyses from Question 3, discuss the two *p*-values from Q3c) and additional *p*-value from Q3d) and why the three methods may lead to different conclusions for this data. (5 marks x 1 data set)

Interestingly, both of the non-parametric methods, the Mann-Whitney U-test and the Two-sample Kolmogorov-Smirnov test, supported the null hypothesis that there is no difference between the control and treatment groups. However, it is worth noting that the Mann-Whitney U-test was very close to rejecting the null hypothesis at $\alpha = 0.05$, given that its p-value was 0.05553. This is compared to the Two-sample Kolmogorov-Smirnov test with a p-value = 0.1844.

The reason for this discrepancy is the difference in the ways in which the test is calculated. The Two-sample Kolmogorov-Smirnov test searches for the point of maximum divergence in the cumulative frequency distribution functions of the two groups. The Mann-Whitney U-test is based on rank ordering the combined data sets and then determining if one group is clustered more toward one end than the other group.

As the Mann-Whitney U-test was already close to rejecting the null hypothesis it is not so surprising that its parametric equivalent, the independent samples t-test, rejected the null hypothesis at $\alpha = 0.05$ with a p-value of 0.03791. Parametric methods in general are more likely to reject the null hypothesis than non-parametric, and given that the p-value was already on the border of being low enough to reject the null hypothesis according to the Mann-Whitney U-test, its more powerful parametric equivalent was able to.

b) For the data set used for the sign test in Question 3, repeat the sign test analysis by hand. You can use R or Excel to follow the calculations as outlined on pages 51-53 of the textbook for small data samples. In your answer provide only the equation equivalent to the equation at the top of page 53 for this particular analysis. State whether your result agrees with your result from Q3c). (5 marks x 1 data set).

Hypothesis:

$H_0 : p = 0.5$

$H_A : p \neq 0.5$

In context

Level of Risk:

$\alpha = 0.05$

There is a 95% chance that any observed statistical difference will be real and not due to chance.

Pair	Control Group	Treatment Group	Control - Treatment	Sign of Difference
1	92.8	96.2	-3.4	-
2	86.9	89.1	-2.2	-
3	75.3	56.3	19	+
4	81.4	93.4	-12	-
5	62.5	49.5	13	+
6	48.1	84.1	-36	-
7	92.2	96.3	-4.1	-
8	74.3	97.3	-23	-
9	84.8	80.6	4.2	+
10	75.8	54.8	21	+
11	70.7	66.7	4	+
12	70.6	81.6	-11	-
13	86.1	81.4	4.7	+
14	86.5	86.3	0.2	+
15	78.2	81.2	-3	-
16	61.4	80.1	-18.7	-
17	89.1	90.1	-1	-
18	60.6	78.6	-18	-
19	74.2	67.2	7	+
20	81.2	82.3	-1.1	-
				8 positive, 12 negative, 0 neutral

Calculations:

$$P(X) = \frac{n!}{(n-X)!\,X!} \cdot p^{x} \cdot (1-p)^{n-X}$$

Where,
n = n_p + n_r = 8 + 12 = 20
p = 0.5

You had 8 and 12 yet calculated 6 and 14 .... hence why you got a discrepancy! -2

$$P(0) = \frac{2{,}432{,}902{,}008{,}176{,}640{,}000}{(2{,}432{,}902{,}008{,}176{,}640{,}000 - 20) * 0} \cdot 0.5^{0} \cdot (1-0.5)^{20-0}$$
$$= 0.0000009536743$$

$P(1) = 0.00001907349$
$P(2) = 0.0001811981$
$P(3) = 0.001087189$
$P(4) = 0.004620552$
$P(5) = 0.01478577$
$P(6) = 0.03696442$
$P(7) = 0.07392883$
$P(8) = 0.1201344$
$P(9) = 0.1601791$
$P(10) = 0.1761971$
$P(11) = 0.1601791$
$P(12) = 0.1201344$
$P(13) = 0.07392883$
$P(14) = 0.03696442$
$P(15) = 0.01478577$
$P(16) = 0.004620552$
$P(17) = 0.001087189$
$P(18) = 0.0001811981$
$P(19) = 0.00001907349$
$P(20) = 0.0000009536743$

P for pluses:

P(14,15,16,17,18,19,20) = 0.0000009536743 + 0.00001907349 + 0.0001811981 + 0.001087189 + 0.004620552 + 0.01478577 + 0.03696442 = 0.05765915626

P for minuses:

P(0,1,2,3,4,5,6) = 0.0000009536743 + 0.00001907349 + 0.0001811981 + 0.001087189 + 0.004620552 + 0.01478577 + 0.03696442 = 0.05765915626

P value overall:

P(14,15,16,17,18,19,20) + P(0,1,2,3,4,5,6) = 0.05765915626 + 0.05765915626

p = 0.11531831252

p = 0.1153 (rounded down)

Interpretation:

As the p-value of 0.1153 is above the critical value of 0.05, we cannot reject the null hypothesis. This in turn means that there is no significant difference between the two samples.

Report:

The obtained p-value = 0.1153 was greater than the critical value of 0.05, therefore the null hypothesis cannot be rejected. This in turn suggests that there is no difference between the control and treatment diet where improving the goats' condition was concerned.

Q3c Result Comparison:

There is broad agreement between the results as both p-values result in rejecting the null hypothesis. However, the hand calculated p-value of 0.1153 was different to the R-generated value of 0.5034 from question 3 c.

The reason for this discrepancy comes down to the way in which the programs must be calculating the tails, namely by rounding up from P(15,16,17,18,19,20) = 0.02069473626 to arrive at 0.025 and from P(0,1,2,3,4,5) = 0.02069473626 to arrive at 0.025. This would put them over the 0.05 threshold for the tails whereas my calculations did not round and so needed to include P(6) and P(20), which dramatically increased the overall calculation compared to the computed versions.

In conclusion, the results agreed with one another but the methods of dealing with rounding differed slightly, but enough to result in quite different p-values.

You shouldn't get a different answer so always check your results! 8/10

References

Corder, G. and Foreman, D. (2014). *Nonparametric statistics*. 2nd ed. Hoboken, New Jersey: John Wiley & Sons, Incorporated.

Appendices

R Analysis

Input

```
#Open Libraries

#library(e1071)

library(dplyr)
library(car)

library(lattice)
library(BSDA)
library(psych)
library(PerformanceAnalytics)


#skew(data$scores, na.rm = TRUE,type=2) #this is closes to SPSS
#kurtosi(data$scores, na.rm = TRUE,type=2)#this is closes to SPSS

#Declaring variables Section

twin<- read.table(file="C:/Users/Justin/Desktop/STA8190 Non-Parametrics/Assignments to Students 2019/Data Files/twin.txt", header=TRUE, sep="")
twinDf<-data.frame(twin)
controlVar=twinDf %>% pull(Control)
treatmentVar=twinDf %>% pull(Treatment)


goats<- read.table(file="C:/Users/Justin/Desktop/STA8190 Non-Parametrics/Assignments to Students 2019/Data Files/goats.txt", header=TRUE, sep="")
goatDf<-data.frame(goats)
goat_split <- split(goatDf, goatDf$Treatment)
goatControl <- goat_split$`1` %>% pull(Judgement)
goatTreat <-goat_split$`2` %>% pull(Judgement)


# Display output Section

print('#####################')
print('##### TWINS.txt #####')
print('#####################')

print('*** Control Group ***')
print(controlVar)
print(summary(controlVar))
print(paste0("Standard Deviation = ",sd(controlVar)))
hist(controlVar,main="Histogram of Twins Control Group",xlab="Ewe Height (Inches)")
print(kurtosi(controlVar,na.rm = TRUE,type=2))
print(skew(controlVar,na.rm = TRUE,type=2))
print(ks.test(controlVar,"pnorm",mean = mean(controlVar), sd= sd(controlVar)))


print('*** Treatment Group ***')
print(treatmentVar)
hist(treatmentVar,main="Histogram of Twins Treatment Group",xlab="Ewe Height (Inches)")
print(summary(treatmentVar))
```

```
print(paste0("Standard Deviation = ",sd(treatmentVar)))
print(paste0("Kurtosis = ", kurtosi(treatmentVar,na.rm = TRUE,type=2)))
print(paste0("Skewness = ", skew(treatmentVar,na.rm = TRUE,type=2)))
print(ks.test(treatmentVar,"pnorm", mean = mean(treatmentVar), sd= sd(treatmentVar)))

print('*** Treatment vs Control Group ***')
print(wilcox.test(controlVar,treatmentVar,paired=TRUE,conf.int = TRUE,conf.level = 0.95))
#paired=TRUE results in wilcox signed rank test for paired data, defaults to two tailed.


print(SIGN.test(x = controlVar, y = treatmentVar, alternative = "two.sided",conf.level = 0.95)) #
Two sided means two tailed

print(ks.test(controlVar,treatmentVar))
print(t.test(controlVar,treatmentVar,paired=TRUE,conf.level=0.95)) # Students t-test for
dependant samples

print('#####################')
print('##### GOATS.txt #####')
print('#####################')

#print(goatControl)
#print(goatTreat)

print('*** Control Group ***')
print(goatControl)
print(summary(goatControl))
print(paste0("Standard Deviation = ",sd(goatControl)))
hist(goatControl,main="Histogram of Goats Control Group",xlab="Overall Condition (between
40 and 130)")
print(kurtosi(goatControl,na.rm = TRUE,type=2))
print(skew(goatControl,na.rm = TRUE,type=2))
print(ks.test(goatControl,"pnorm", mean = mean(goatControl), sd= sd(goatControl)))


print('*** Treatment Group ***')
print(goatTreat)
hist(goatTreat,main="Histogram of Goats Treatment Group",xlab="Overall Condition (between
40 and 130)")
print(summary(goatTreat))
print(paste0("Standard Deviation = ",sd(goatTreat)))
print(paste0("Kurtosis = ", kurtosi(goatTreat,na.rm = TRUE,type=2)))
print(paste0("Skewness = ", skew(goatTreat,na.rm = TRUE,type=2)))
print(ks.test(goatTreat,"pnorm", mean = mean(goatTreat), sd= sd(goatTreat)))


print('***Treatment vs Control Group***')
#print(wilcox.test(goatControl,goatTreat,paired=FALSE))
print(wilcox.test(Judgement ~ Treatment,data=goatDf)) # Judgement by treatment. Mann
Whitney U-test, paired = false is default, defaults to two tailed.
print(ks.test(goatControl,goatTreat)) # Two sample Kolmogorov-Smirnoff Test
print(t.test(goatControl,goatTreat,paired=FALSE,conf.level=0.95, var.equal=TRUE)) #
Students t-test for independant samples

print(wilcox.test(Judgement ~ Treatment,data=goatDf))

#print(cat("Treatment Group Skewness = ", skewness(treatmentVar)))

print('***Levene Test of Homogeneity of Variance***')
```

```
y <- c(goatControl, goatTreat)

group <- as.factor(c(rep(1, length(goatControl)), rep(2, length(goatTreat))))

print(leveneTest(y, group))


#print(leveneTest(Judgement ~ Treatment, data=goatDf, center=mean))



print('########################################')
print('##### Hand Based Sign Calculation #####')
print('########################################')

print(goatControl)
print(goatTreat)

n = 20
x = 20
#factorial = 0

topHalf<-factorial(n)
bottomHalf<-(factorial(n-x)) * (factorial(x))
middle<-(0.5^x)
end<-((1-0.5)^(n-x))

print(topHalf)
print(bottomHalf)
print(middle)
print(end)

#print((factorial(n)/factorial(n-x)*factorial(x)))
#print((0.5^x))
#print(((1-0.5)^(n-x)))

#answer = (factorial(n)/(factorial(n-x)*factorial(x))) * (0.5^x) * ((1-0.5)^(n-x))

answer = (topHalf/bottomHalf) * (middle) * (end)

print(answer)


#for (i in 1:n){
#  print((factorial(i)/(factorial(i-x)*factorial(x))) * (0.5^x) * ((1-0.5)^(i-x)))
#}


print('###########################################')
print('##### Wilcoxon signed rank test Sums #####')
print('###########################################')


# controlVar,treatmentVar

diff <- c(controlVar - treatmentVar) #calculating the vector containing the differences
diff <- diff[ diff!=0 ] #delete all differences equal to zero
diff.rank <- rank(abs(diff)) #check the ranks of the differences, taken in absolute
diff.rank.sign <- diff.rank * sign(diff) #check the sign to the ranks, recalling the signs of the
values of the differences
```

```
ranks.pos <- sum(diff.rank.sign[diff.rank.sign > 0]) #calculating the sum of ranks assigned to the differences as a positive, ie greater than zero
ranks.neg <- -sum(diff.rank.sign[diff.rank.sign < 0]) #calculating the sum of ranks assigned to the differences as a negative, ie less than zero
print(ranks.pos) #it is the value V of the wilcoxon signed rank test
print(ranks.neg)
```

Output

```
Attaching package: ‘dplyr’

The following objects are masked from ‘package:stats’:

    filter, lag

The following objects are masked from ‘package:base’:

    intersect, setdiff, setequal, union

Loading required package: carData

Attaching package: ‘car’

The following object is masked from ‘package:dplyr’:

    recode


Attaching package: ‘BSDA’

The following objects are masked from ‘package:carData’:

    Vocab, Wool

The following object is masked from ‘package:datasets’:

    Orange


Attaching package: ‘psych’

The following object is masked from ‘package:car’:

    logit

Loading required package: xts
Loading required package: zoo

Attaching package: ‘zoo’

The following objects are masked from ‘package:base’:

    as.Date, as.Date.numeric

Registered S3 method overwritten by 'xts':
  method     from
  as.zoo.xts zoo

Attaching package: ‘xts’

The following objects are masked from ‘package:dplyr’:

    first, last


Attaching package: ‘PerformanceAnalytics’

The following object is masked from ‘package:graphics’:

    legend

[1] "######################"
[1] "##### TWINS.txt #####"
```

```
[1] "###################"
[1] "*** Control Group ***"
 [1] 92.8 86.9 75.3 81.4 62.5 48.1 92.2 74.3 84.8 75.8 70.7 70.6 86.1 86.5
78.2 61.4 89.1 60.6 74.2 81.2
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  48.10   70.67   77.00   76.64   86.20   92.80 
[1] "Standard Deviation = 11.8158357167071"
[1] 0.2063618
[1] -0.7303988

        One-sample Kolmogorov-Smirnov test

data:  controlVar
D = 0.11836, p-value = 0.9113
alternative hypothesis: two-sided

[1] "*** Treatment Group ***"
 [1] 96.2 89.1 56.3 93.4 49.5 84.1 96.3 97.3 80.6 54.8 66.7 81.6 81.4 86.3
81.2 80.1 90.1 78.6 67.2 82.3
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  49.50   75.75   81.50   79.66   89.35   97.30 
[1] "Standard Deviation = 14.006031595446"
[1] "Kurtosis = -0.0384599590017527"
[1] "Skewness = -0.851196945339758"

        One-sample Kolmogorov-Smirnov test

data:  treatmentVar
D = 0.21998, p-value = 0.2488
alternative hypothesis: two-sided

[1] "*** Treatment vs Control Group ***"

        Wilcoxon signed rank test

data:  controlVar and treatmentVar
V = 87, p-value = 0.5217
alternative hypothesis: true location shift is not equal to 0
95 percent confidence interval:
 -9.5  3.6
sample estimates:
(pseudo)median 
          -2.2 


        Dependent-samples Sign-Test

data:  controlVar and treatmentVar
S = 8, p-value = 0.5034
alternative hypothesis: true median difference is not equal to 0
95 percent confidence interval:
 -10.196353   4.176706
sample estimates:
median of x-y 
        -1.65 

Achieved and Interpolated Confidence Intervals: 

                  Conf.Level   L.E.pt U.E.pt
Lower Achieved CI     0.8847  -4.1000 4.0000
Interpolated CI       0.9500 -10.1964 4.1767
Upper Achieved CI     0.9586 -11.0000 4.2000


        Two-sample Kolmogorov-Smirnov test

data:  controlVar and treatmentVar
```

```
D = 0.3, p-value = 0.3291
alternative hypothesis: two-sided


        Paired t-test

data:  controlVar and treatmentVar
t = -0.96763, df = 19, p-value = 0.3454
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -9.552353  3.512353
sample estimates:
mean of the differences
                  -3.02

[1] "####################"
[1] "##### GOATS.txt #####"
[1] "####################"
[1] "*** Control Group ***"
 [1]  85.1  83.3  87.5 104.5  78.4  65.8  82.8  91.2  93.1  52.3  71.6  86.5
74.9  94.1  76.2
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  52.30   75.55   83.30   81.82   89.35  104.50 
[1] "Standard Deviation = 12.7396008011016"
[1] 1.038877
[1] -0.6118158

        One-sample Kolmogorov-Smirnov test

data:  goatControl
D = 0.13066, p-value = 0.9308
alternative hypothesis: two-sided

[1] "*** Treatment Group ***"
 [1]  99.3 103.7  82.7  81.3  95.3  95.2  76.4  92.4 102.6  79.8  83.9  94.4
87.8  81.7 104.6
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  76.40   82.20   92.40   90.74   97.30  104.60 
[1] "Standard Deviation = 9.4379779916796"
[1] "Kurtosis = -1.4292481122916"
[1] "Skewness = 0.0655437646387487"

        One-sample Kolmogorov-Smirnov test

data:  goatTreat
D = 0.16569, p-value = 0.7458
alternative hypothesis: two-sided

[1] "***Treatment vs Control Group***"

        Wilcoxon rank sum test

data:  Judgement by Treatment
W = 66, p-value = 0.05553
alternative hypothesis: true location shift is not equal to 0


        Two-sample Kolmogorov-Smirnov test

data:  goatControl and goatTreat
D = 0.4, p-value = 0.1844
alternative hypothesis: two-sided


        Two Sample t-test

data:  goatControl and goatTreat
```

```
t = -2.179, df = 28, p-value = 0.03791
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -17.305517  -0.534483
sample estimates:
mean of x mean of y
    81.82     90.74


        Wilcoxon rank sum test

data:  Judgement by Treatment
W = 66, p-value = 0.05553
alternative hypothesis: true location shift is not equal to 0

[1] "***Levene Test of Homogeneity of Variance***"
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  1  0.2452 0.6243
      28
[1] "#######################################"
[1] "##### Hand Based Sign Calculation #####"
[1] "#######################################"
 [1]  85.1  83.3  87.5 104.5  78.4  65.8  82.8  91.2  93.1  52.3  71.6  86.5
74.9  94.1  76.2
 [1]  99.3 103.7  82.7  81.3  95.3  95.2  76.4  92.4 102.6  79.8  83.9  94.4
87.8  81.7 104.6
[1] 2.432902e+18
[1] 2.432902e+18
[1] 9.536743e-07
[1] 1
[1] 9.536743e-07
[1] "##########################################"
[1] "##### Wilcoxon signed rank test Sums #####"
[1] "##########################################"
[1] 87
[1] 123
 [1]  85.1  83.3  87.5 104.5  78.4  65.8  82.8  91.2  93.1  52.3  71.6  86.5
74.9  94.1  76.2
 [1]  99.3 103.7  82.7  81.3  95.3  95.2  76.4  92.4 102.6  79.8  83.9  94.4
87.8  81.7 104.6
Warning message:
In ks.test(controlVar, treatmentVar) :
  cannot compute exact p-value with ties
```

Factorial Table

TABLE B.9 Factorials.

n	$n!$
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40,320
9	362,880
10	3,628,800
11	39,916,800
12	479,001,600
13	6,227,020,800
14	87,178,291,200
15	1,307,674,368,000
16	20,922,789,888,000
17	355,687,428,096,000
18	6,402,373,705,728,000
19	121,645,100,408,832,000
20	2,432,902,008,176,640,000
21	51,090,942,171,709,440,000
22	1,124,000,727,777,607,680,000
23	25,852,016,738,884,976,640,000
24	620,448,401,733,239,439,360,000
25	15,511,210,043,330,985,984,000,000