Mathematical Theorems

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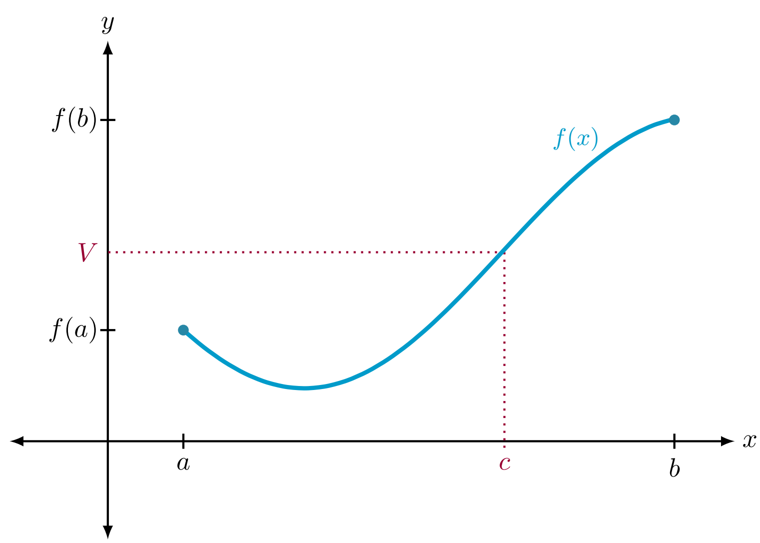
1 Functions

Theorem 1.1 (Intermediate Value Theorem). *Suppose f is a continuous function on the closed interval $[a, b]$. If V is any number between $f(a)$ and $f(b)$, then there exists at least one number c in the open interval (a, b) such that $f(c) = V$. [1]*



[2]

2 Polynomials

Theorem 2.1 (Extreme Value Theorem). *If $f : K \to \mathbb{R}$ is continuous on a compact set $K \subseteq \mathbb{R}$, then, f attains a maximum and minimum value. In other words, there exist $x_0, x_1 \in K$ such that $f(x_0) \leq f(x) \leq f(x_1)$ for all $x \in K$. [3]*

Proposition 2.1 (Local Extrema of Polynomials). *If $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ is a polynomial of degree n, then the graph of P has at most $n - 1$ local extrema. [1]*

Theorem 2.2 (**Remainder Theorem**). *If the polynomial* $P(x)$ *is divided by* $x - c$*, then the remainder is the value* $P(c)$ *[1]*

Theorem 2.3 (**Factor Theorem**). $P(c) = 0$ *if and only if* $(x - c)$ *is a factor of* $P(x)$*. [1]*

Theorem 2.4 (**Rational Zeros Theorem**). *If the polynomial* $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ *has integer coefficients (where* $a_n \neq 0$ *and* $a_0 \neq 0$*), then every rational zero of* P *is of the form* $\frac{p}{q}$*, where* p *and* q *are integers and*

- p *is a factor of the constant coefficient* a_0
- q *is a factor of the leading coefficient* a_n

[1]

Theorem 2.5 (**Upper and Lower Bound Theorem**). *Let* P *be a polynomial with real coefficients.*

1. *If we divide* $P(x)$ *by* $(x - b)$ *(with* $b \neq 0$*) using synthetic division, and if the row that contains the quotient and remainder has no negative entry, then* b *is an upper bound for the real zeros of* P*.*
2. *If we divide* $P(x)$ *by* $(x - a)$ *(with* $a \neq 0$*) using synthetic division, and if the row that contains the quotient and remainder has entries that are alternately nonpositive and nonnegative, then* a *is a lower bound for the real zeros of* P*.*

[1]

References

[1] James Stewart, Lothar Redlin, and Saleem Watson. *Precalculus Mathematics for Calculus.* Cengage Learning, 2015.

[2] L. K. Williams. Tikz gallery. Accessed on: 2023-12-11.

[3] Stephen Abbott. *Understanding Analysis.* Springer, New York, 2015.