Quotients give lower bounds

- For a surjective homomorphism $\phi : B_n \to H$:

  - A **band** in H is a conjugate of $\phi(\sigma_i)$

  - $rk_H(x)$ is then defined similarly and $rk_H(\phi(x)) \leq rk(x)$.

  - Because $\phi(x) = \prod_{i=1}^{k} \phi(\alpha_i)\phi(\sigma_{j_i})^{\epsilon_i}\phi(\alpha_i)^{-1}$ gives a band presentation in H.

Example

- $x = 123\overline{1}$

- Length $\Rightarrow rk(x) \le 4$

- Exponential Sum: $\omega(x) = 2 \Rightarrow rk(x) \ge 2$

- Band presentation: $(123\overline{1}) = (12\overline{1})(13\overline{1}) \Rightarrow rk(x) = 2$