ρ with dynamic programming

Algorithm:

- $w = s_1 \cdots s_k \in F_n$
- Allocate an upper triangular matrix $\{m_{i,j}\} \in M(k \times k, \mathbb{Z})$
- $m_{i,j} = \rho(s_i \cdots s_j)$
- Fill in diagonal with 1.
- Fill in entries for subwords of length 2 by computing ρ replacing recursive calls with $m_{i,j}$
- Continue up to k.

Example:

$bcab^{-1}aba^{-1}c^{-1} \in F_3$

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 3 & 4 & 3 & \mathbf{2} \\ 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 0 & 0 & 1 & 2 & 3 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Inductive braid relationships

- Same relationships hold for braids:
    - Splitting: $rk(xy) \leq rk(x) + rk(y)$ for $x, y \in B_n$,
    - Conjugacy: $rk(\alpha x \alpha^{-1}) = rk(x)$
- Find additional subwords we can improve.