Conclusion

- Check all 9 cases.
- Conclude there exists an expression tree T' for w' where $L(T') \leq L(T)$.

$\Rightarrow \rho(w') \leq \rho(w)$

ρ with dynamic programming

Algorithm:

- $w = s_1 \cdots s_k \in F_n$
- Allocate an upper triangular matrix $\{m_{i,j}\} \in M(k \times k, \mathbb{Z})$
- $m_{i,j} = \rho(s_i \cdots s_j)$
- Fill in diagonal with 1.
- Fill in entries for subwords of length 2 by computing ρ replacing recursive calls with $m_{i,j}$
- Continue up to k.

Example:

$$bcab^{-1}aba^{-1}c^{-1} \in F_3$$

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 3 & 4 & 3 & \mathbf{2} \\ 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 0 & 0 & 1 & 2 & 3 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$