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- A **positive band** is a conjugate of a positive standard generator.

- A **negative band** is the inverse of a positive band.

- A **band presentation** for $x \in B_n$ is a tuple of bands $(\alpha_1 \sigma_{j_1}^{\epsilon_1} \alpha_1^{-1}, \ldots, \alpha_k \sigma_{j_k}^{\epsilon_k} \alpha_k^{-1})$ of any length such that $x = \prod_{i=1}^{k} \alpha_i \sigma_{j_i}^{\epsilon_i} \alpha_i^{-1}$.

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Quotients give lower bounds

- For a surjective homomorphism $\phi\colon B_n \to H$:
  - A **band** in H is a conjugate of $\phi(\sigma_i)$
  - $rk_H(x)$ is then defined similarly and $rk_H(\phi(x)) \leq rk(x)$.
  - Because $\phi(x) = \prod_{i=1}^{k} \phi(\alpha_i)\phi(\sigma_{j_i})^{\epsilon_i}\phi(\alpha_i)^{-1}$ gives a band presentation in H.