Proof of Correctness

- $\rho(w) \geq rk_F(w)$ (from splitting and conjugacy)

- $\rho(w') = rk_F(w)$ when w' is a minimal band presentation for w.

- $\rho(w') \leq \rho(w)$ when w' is w after free reduction.

  $\Rightarrow \rho(w) = rk(w)$ for reduced words.

Expression Trees

Expression Tree for w

- Each node is labeled with a string.

- Root is w

- If $|v| \leq 1$ then v is a leaf.

- Otherwise v has one or two children:

  - If one child v' then $v = sv's^{-1}$
    (**conjugate node**)

  - If two children w_1, w_2 then $v = w_1 w_2$
    (**leaf node**)

$$abca^{-1}b$$

$$abca^{-1} \qquad b$$

$$bc$$

$$b \qquad c$$