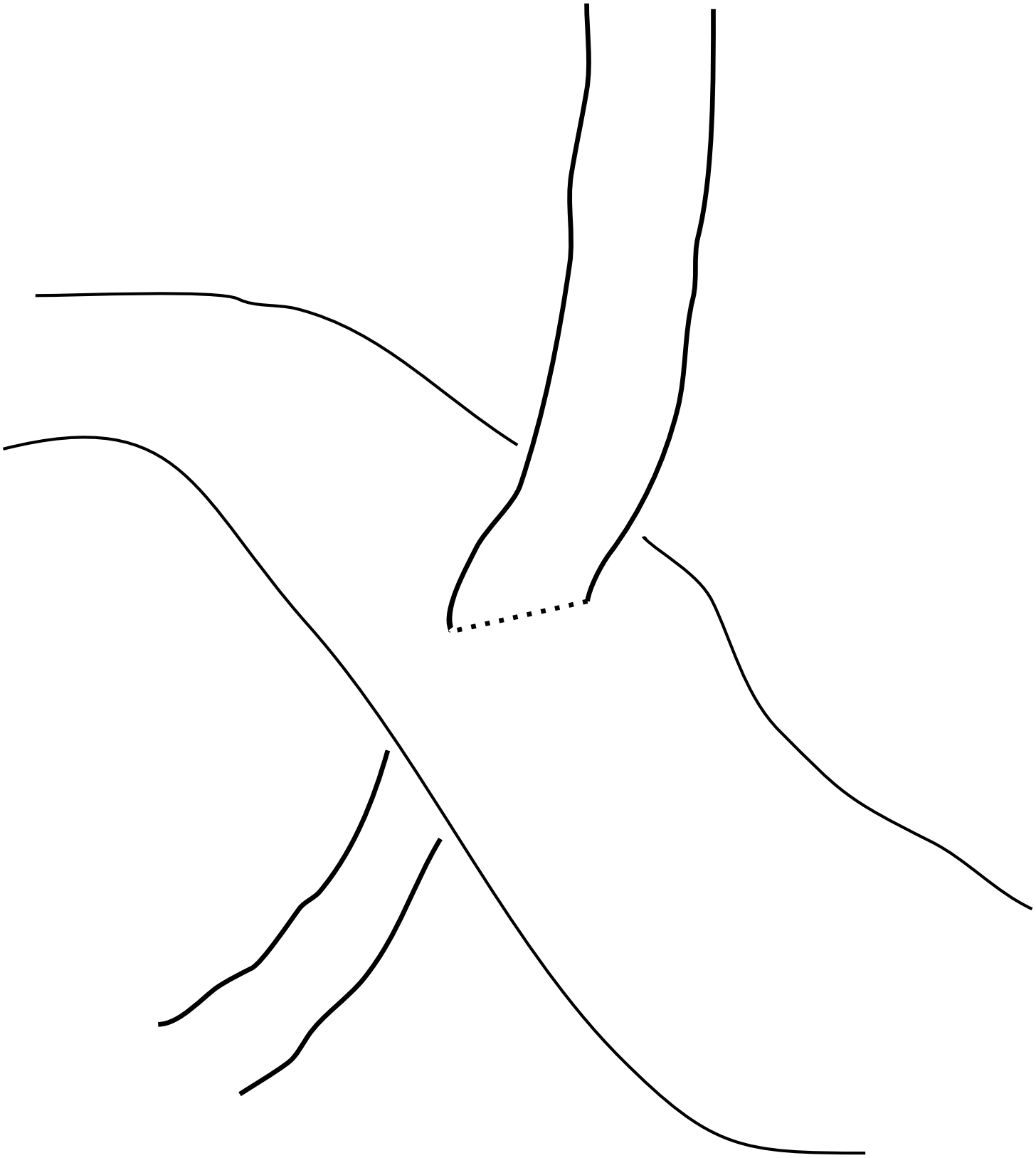
Slice and ribbon surfaces

- **Knot:** smooth embedding of the circle S^1 into the 3-sphere S^3 (up to isotopy).

- **Slice surface:** for a knot $K \subseteq S^3 = \partial D^4$ is an orientable surface smoothly embedded in D^4 whose boundary is K.

- **Ribbon surface:** for a knot $K \subseteq S^3$ is an orientable surface immersed in S^3 whose boundary is K and whose singularities are all of **ribbon type**.



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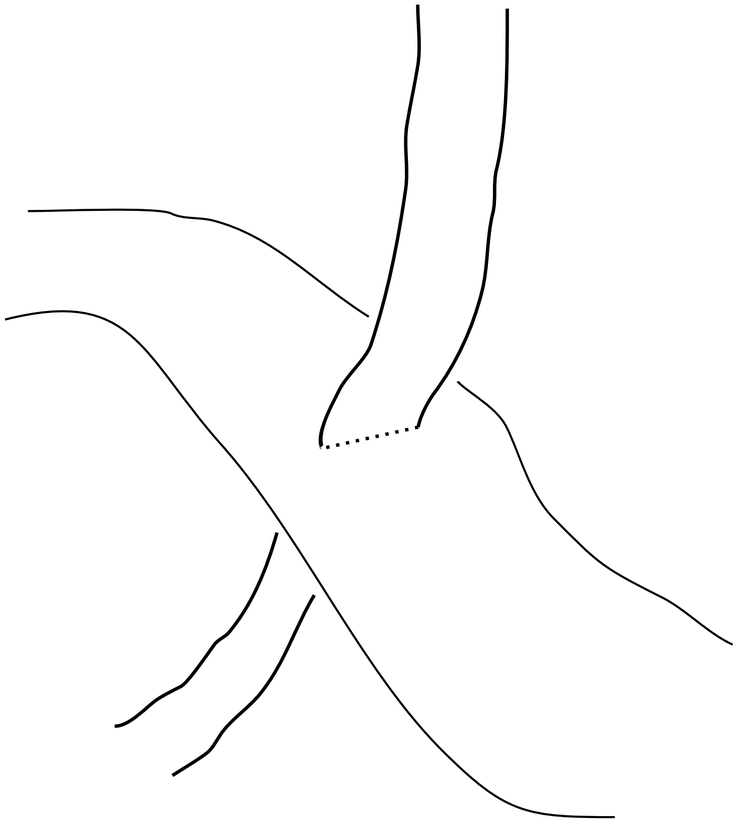

Slice and ribbon genus

- **Slice genus** $g_s(K)$: minimum genus of a slice surface for K.
- **Ribbon genus** $g_r(K)$: minimum genus of a ribbon surface for K.
- From a ribbon surface we can construct a slice surface of the same genus $g_s \leq g_r$ [Grigsby, 2018].