Permutation templates

- A **k-template** in S_n is tuple of transpositions $((i_1 j_1), \ldots, (i_k, j_k))$ such that $\phi(x) = (i_1 j_1) \cdots (i_k j_k)$
- Example: $x = \bar{1}211$
  - 2-templates: $((13), (12)), ((23), (13)), ((12), (23))$
  - The presentation $(\bar{1}21)(1)$ satisfies the template: $(13), (12)$

Exponential sum templates

- A **k-template** in $\mathbb{Z}$ is a tuple in $(s_1, \ldots s_k) \in \{1, -1\}^k$ such that $\sum_{i=1}^{k} s_i = \gamma(x)$.
- Alternatively written as string of signs: $\{+, -\}^k$
- Example: $w = \bar{2}\bar{2}1\bar{2}1$ and $\gamma(w) = -1$
  - 3-templates: $+--$, $-+-$, $--+$
  - The presentation $(\bar{2}12)(2\bar{1}\bar{2})(\bar{1}\bar{2}1)$ satisfies the template $+--$