

Example

- $aba^{-1}c \in F_3$
- $rk_F(aba^{-1}c) \leq rk_F(aba^{-1}) + rk_F(c) \leq rk_F(b) + rk_F(c) = 2$
- But how should we apply them? $a \cdots b \cdots a^{-1} \cdots b^{-1}$
- Claim: rank is computed by applying these relationships optimally.

Optimal Splits and Conjugates

- $w = s_1 \dots s_m$ is a string in the free generators
- $$C(w) = \begin{cases} \rho(s_2 \dots s_{m-1}) & s_1 = s_m^{-1} \\ \infty & \text{otherwise} \end{cases}$$
- $$\rho(w) = \begin{cases} m & m \leq 1 \\ \min(C(w), \min_{1 < i < m} (\rho(s_1, \dots, s_{i-1}) + \rho(s_i, \dots, s_m))) & \text{otherwise.} \end{cases}$$