$$\exp(\int_{[0,1]^2} f(x,y)dxdy)$$
$$= \exp(\lim_{n\to\infty} \sum_{i=1}^{n}\sum_{j=1}^{n} f(x_i*, y_i*)\Delta x \Delta y)$$
$$= \lim_{n\to\infty} \exp(\sum_{i=1}^{n}\sum_{j=1}^{n} f(i/n, j/n)1/n^2)$$
$$= \lim_{n\to\infty} \prod_{i=1}^{n}\prod_{j=1}^{n} \exp(f(i/n, j/n)1/n^2)$$

Let $x = f(i/n, j/n)$ Then

$$= \lim_{n\to\infty} \prod_{i=1}^{n}\prod_{j=1}^{n} (1 + \frac{x}{n^2} + \frac{x^2}{2!n^4} + \ldots)$$