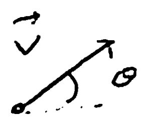
Justin meiners

The Optimal Angle for Launching Projectiles

A proof that $\frac{\pi}{4}$ provides the optimal distance for launching a projectile.



v: the initial velocity
θ: the launch angle

$v_x = v\cos\theta$ (vector components)
$v_y = v\sin\theta$

Time in air: $\Delta x(t) = v_0 t + \frac{1}{2}at^2$

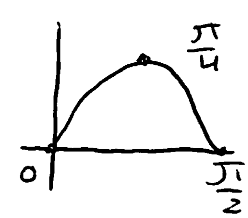
$\Delta y(t) = 0 = v\sin\theta t + \frac{1}{2}gt^2$ (Total vertical displacent is zero.)

$$t = \frac{-v\sin\theta \pm \sqrt{v^2\sin^2\theta}}{g}$$

$$= \frac{-v\sin\theta - v\sin\theta}{g} = \frac{-2v\sin\theta}{g}$$

Distance as function of angle: $\Delta x(t) = v\cos\theta\left(\frac{-2v\sin\theta}{g}\right)$

$$= \frac{-2v^2\sin\theta\cos\theta}{g}$$



Local maxima: $\Delta x'(t) = -\frac{2v^2}{g}\left(\cos^2\theta - \sin^2\theta\right)$

$\Delta x'(t) = 0 \Rightarrow \cos^2\theta = \sin^2\theta$ (set derivative to 0)

and $\theta \in [0, \frac{\pi}{2}]$ so $\boxed{\theta = \frac{\pi}{4}}$

Endpoints: $\Delta x(0) = -\frac{2v^2}{g}(0)1 = 0$ $\quad \Delta x(\frac{\pi}{2}) = -\frac{2v^2}{g}(1)0 = 0$