Appendix

Proof of Proposition 1. Proceed by backward induction. In the second period, a leader of type θ maximizes

$$\max_{a_2(\theta)} \beta\Big(a_2(\theta) - \theta\frac{\lambda_M}{2}a_2(\theta)^2 - (1-\theta)\frac{\lambda_M+\lambda_S}{2}a_2(\theta)^2\Big).$$

The first-order condition is

$$\beta\Big(1 - \theta\lambda_M a_2(\theta) - (1-\theta)(\lambda_M+\lambda_S)a_2(\theta) = 0,$$

which has solution $a_2^*(1) = \frac{1}{\lambda_M}$ and $a_2^*(0) = \frac{1}{\lambda_M+\lambda_S}$. These choices are a maximum because the leader's utility function is globally concave, as the second-order condition is

$$-\theta\lambda_M - (1-\theta)(\lambda_M+\lambda_S) < 0.$$

Since $a_2^*(1) > a_2^*(0)$ and in particular $a_2^*(1)$ is the median voter's ideal point, the median voter wants to retain the incumbent leader when his posterior belief about the leader's honesty is greater than the prior. Moreover, since x_1 is FOSD-increasing in a_1, higher signals are on average more likely to signal honesty. Therefore the voter prefers to retain the incumbent whenever the signal x_1 is greater than some threshold $\hat{x}$. Let $\mu(x) = P(\theta = 1|x_1 = x)$ be the voter's posterior belief that the incumbent is honest given the realized policy outcome $x_1 = x$. As effort is unobserved, let the voter have conjecture about the incumbent's effort choice, $\hat{a}_1(\theta)$. Formally, posterior beliefs can be expressed as

$$\mu(x) = \frac{\gamma\phi(\sqrt{\zeta}(x-\hat{a}_1(1)))}{\gamma\phi(\sqrt{\zeta}(x-\hat{a}_1(1))) + (1-\gamma)\phi(\sqrt{\zeta}(x-\hat{a}_1(0)))}.$$

The voter retains the incumbent iff $\mu(x) \geq \gamma$, which is equivalent to

$$x \geq \frac{\hat{a}_1 + \hat{a}_0}{2}.$$

Given $x_1 = a_1 + \varepsilon_1$, the incumbent leader survives iff $a_1 + \varepsilon_1 \geq \frac{\hat{a}_1 + \hat{a}_0}{2}$. Since $\varepsilon_1 \sim N(0, \frac{1}{\zeta})$, the incumbent's reelection probability is equal to

$$\pi(a_1) = \Phi(\sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1) + \hat{a}_1(0)}{2})).$$

In the first period, the leader of type θ maximizes

$$\max_{a_1(\theta)} \beta\Big(a_1(\theta) - \theta\frac{\lambda_M}{2}a_1(\theta)^2 - (1-\theta)\frac{\lambda_M + \lambda_S}{2}a_1(\theta)^2\Big) + \pi(a_1(\theta))\Psi,$$

which leads to the first-order condition

$$\beta - \theta\lambda_M a_1(\theta) - (1-\theta)(\lambda_M + \lambda_S)a_1(\theta) + \sqrt{\zeta}\phi(\sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1) + \hat{a}_1(0)}{2}))\Psi = 0.$$

Since beliefs are correct in equilibrium, $a_1(\theta) = \hat{a}_1(\theta) = a_1^*(\theta)$, this simplifies to

$$\beta - \theta\lambda_M a_1(\theta) - (1-\theta)(\lambda_M + \lambda_S)a_1(\theta) + \sqrt{\zeta}\phi(\sqrt{\zeta}(\frac{a_1^*(1) + a_1^*(0)}{2}))\Psi = 0.$$

Substituting in $\theta = 1$ and $\theta = 0$ yields the two equations in the proposition.

To show that this solution is a maximum, we ensure that the leader's utility is concave. The second-order condition is

$$-\theta\lambda_M - (1-\theta)(\lambda_M + \lambda_S) + \zeta^{3/2}(a_1 - \frac{\hat{a}_1(1) + \hat{a}_1(0)}{2})\phi(\sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1) + \hat{a}_1(0)}{2}))\Psi.$$

Let $\eta = \sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1)+\hat{a}_1(0)}{2})$ so the second-order condition can be rewritten as

$$-\lambda_M - (1-\theta)\lambda_S + \zeta\eta\phi(\eta)\Psi.$$

The standard normal density tends to zero faster than any polynomial so $\eta\phi(\eta)$ is zero at $\eta = 0$ and approaches zero as $\eta \to \pm\infty$. The derivative of $\eta\phi(\eta)$ is $\phi(\eta) - \eta^2\phi(\eta)$ with critical points at $\eta = \pm 1$. Note that if $\eta = -1$ then the problem is globally concave. Hence the relevant constraint is at $\eta = 1$, where $\eta\phi(\eta) = \frac{1}{\sqrt{2\pi e}}$. Hence the leader's utility is concave iff

$$-\lambda_M - (1-\theta)\lambda_S + \frac{\zeta}{\sqrt{2\pi e}}\Psi < 0,$$

or $\zeta < \frac{\lambda_M + (1-\theta)\lambda_S\sqrt{2\pi e}}{\Psi}$. Hence a sufficient condition for both leaders to have concave utility functions is $\zeta < \frac{\lambda_M\sqrt{2\pi e}}{\Psi}$.

Furthermore, this equilibrium is unique because pooling cannot be an equilibrium. By way of contradiction, suppose that the voter believed $\hat{a}_1(\theta) = \hat{a}$ for any θ. Then $\mu(x) = \gamma$ for any x, and the voter is indifferent between retaining and replacing the incumbent. Hence, depending on how ties are broken, the incumbent's reelection probability is either zero or 1. This means that the incumbent leader's maximization problem is equivalent to

$$\max_{a_1(\theta)} \beta\Big(a_1(\theta) - \theta\frac{\lambda_M}{2}a_1(\theta)^2 - (1-\theta)\frac{\lambda_M+\lambda_S}{2}a_1(\theta)^2\Big),$$

the solution to which is $a_1(1) = \frac{1}{\lambda_M}$ and $a_1(0) = \frac{1}{\lambda_M+\lambda_S}$ such that $a_1(1) \neq a_1(0)$. □

Proof of Corollary 1. From Proposition 1, leader's effort choices satisfy

$$\beta + \sqrt{\zeta}\phi(\sqrt{\zeta}(\frac{a_1^*(1) - a_1^*(0)}{2}))\Psi = \beta\lambda_M a_1^*(1).$$
$$\beta + \sqrt{\zeta}\phi(\sqrt{\zeta}(\frac{a_1^*(0) - a_1^*(1)}{2}))\Psi = \beta(\lambda_M+\lambda_S)a_1^*(0).$$

The LHS of these equations are the same, which implies that $\beta\lambda_M a_1^*(1) = \beta(\lambda_M + \lambda_S)a_1^*(0)$, or $a_1^*(1) = \frac{\lambda_M+\lambda_S}{\lambda_M} a_1^*(0)$ so $a_1^*(1) > a_1^*(0)$.

To see that $a_1^*(1) > \frac{1}{\lambda_M}$, substitute $a_1 = \frac{1}{\lambda_M}$ into the honest leader's first-order condition to get

$$\sqrt{\zeta}\phi(\sqrt{\zeta}(\frac{1/\lambda_M - a_1^*(0)}{2}))\Psi > 0,$$

which holds for any $a_1^*(0)$. Since this first-order condition is positive, we must have $a_1^*(1) > \frac{1}{\lambda_M}$. Similarly, substituting $a_1 = \frac{1}{\lambda_M}$ into the captured leader's first-order condition yields

$$\beta(1 - \frac{\lambda_M + \lambda_S}{\lambda_M}) + \sqrt{\zeta}\phi(\sqrt{\zeta}(\frac{1/\lambda_M - a_1^*(1)}{2}))\Psi.$$

This expression can be either positive or negative. Note that the standard normal density takes a maximum value of $\frac{1}{\sqrt{2\pi}}$, and so a sufficient condition for the captured leader's equilibrium effort to be larger than $\frac{1}{\lambda_M}$ is

$$\beta(1 - \frac{\lambda_M + \lambda_S}{\lambda_M}) + \sqrt{\frac{\zeta}{2\pi}}\Psi > 0,$$

which occurs whenever $\lambda_S < \lambda_M + \sqrt{\frac{\zeta}{2\pi}}\frac{\Psi}{\beta}$. □

Proof of Corollary 2. Follows from x_1 FOSD-increasing in a_1 and $a_1^*(1) > a_1^*(0)$. □

Proof of Corollary 3. Define the Jacobian for type θ as

$$\mathbf{J}_\theta = \begin{bmatrix} \frac{\partial^2 v_\theta}{\partial a_\theta^2} & \frac{\partial^2 v_\theta}{\partial a_\theta \partial a_{\theta'}} \\ \frac{\partial^2 v_{\theta'}}{\partial a_\theta \partial a_{\theta'}} & \frac{\partial^2 v_{\theta'}}{\partial a_{\theta'}^2} \end{bmatrix}.$$

Observe that $\frac{\partial^2 v_1}{\partial a_1^2} < 0$ and $\frac{\partial^2 v_0}{\partial a_0^2} < 0$ at the equilibrium (a_1^*, a_0^*) because they are maxima. Further observe that $\frac{\partial^2 v_1}{\partial a_1 \partial a_0} > 0$ and $\frac{\partial^2 v_0}{\partial a_1 \partial a_0} < 0$, hence $|\mathbf{J}_\theta| > 0$. Given this structure, the direct and indirect effects have the same sign; without loss of generality I simply consider the direct effects.

By monotone comparative statics, taking the cross-partial of the leader's utility with respect to parameters yields

$$
\begin{aligned}
\frac{\partial^2 v_L(a_t;\theta)}{\partial a_1 \partial \Psi} &= \sqrt{\zeta}\phi(\sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1)+\hat{a}_1(0)}{2})) \geq 0. \\
\frac{\partial^2 v_L(a_t;\theta)}{\partial a_1 \partial \lambda_M} &= -a_1 \leq 0. \\
\frac{\partial^2 v_L(a_t;\theta)}{\partial a_1 \partial \lambda_S} &= -(1-\theta)a_1 \leq 0. \\
\frac{\partial^2 v_L(a_t;\theta)}{\partial a_1 \partial \zeta} &= \frac{\Psi}{2\sqrt{\zeta}}\phi(\sqrt{\zeta}(a_1 - \frac{\hat{a}_1(1)+\hat{a}_1(0)}{2}))\Big(1-\zeta(a_1 - \frac{\hat{a}_1(1)+\hat{a}_1(0)}{2})^2\Big).
\end{aligned}
$$

These inequalities imply that effort a^*_θ is increasing in Ψ, decreasing in λ_M, and a^*_0 is decreasing in λ_S. Furthermore, while the direct effect $\frac{\partial a^*_1}{\partial \lambda_S} = 0$, the indirect effect from a^*_0 is such that a^*_1 is decreasing in λ_S as well. Also observe that $\Big(1-\zeta(a_1 - \frac{\hat{a}_1(1)+\hat{a}_1(0)}{2})^2\Big) > 0$ is positive as $\zeta \to 0$ and decreasing in ζ so that the effect is inverse U-shaped. $\square$

Proof of Proposition 2. Proof is analogous to that of Proposition 1. The only difference is the derivation of the voter's policy cutoff, which is a function of conjectures about the leader's effort $\hat{a}_\theta$ as well as conjectures about the messages sent to the IO $\hat{p}_\theta$.

Denote $\mu(x,s)$ as the voter's posterior belief about the leader's type having observed IO report s and signal x of the leader's effort. Since the leader's true message m and true effort a are unobserved, the voter needs to have conjectures. Let $\hat{a}_\theta$ be the voter's conjecture about leader-type θ's effort, and let $\hat{p}_\theta = P(m=1|\theta)$ be the voter's conjecture about the probability that leader-type θ sent message $m=1$ to the IO. Then $\hat{m}_\theta = \hat{p}_\theta\phi(\sqrt{\tau}(s-1)) + (1-\hat{p}_\theta)\phi(\sqrt{\tau}s)$ is the total probability that that IO's report is realized as the value s given voter's conjectures. Then $\mu(x,s)$ can be expressed as

$$
\mu(x,s) = \frac{\gamma\phi(\sqrt{\zeta}(x-\hat{a}_1))\hat{m}_1}{\gamma\phi(\sqrt{\zeta}(x-\hat{a}_1))\hat{m}_1 + (1-\gamma)\phi(\sqrt{\zeta}(x-\hat{a}_0))\hat{m}_0},
$$

such that it is optimal to retain the incumbent leader whenever

$$x \geq \frac{\hat{a}_1 + \hat{a}_0}{2} + \frac{\log(\frac{\hat{m}_0}{\hat{m}_1})}{\zeta(\hat{a}_1 - \hat{a}_0)} \equiv \hat{x}(\hat{a}, \hat{p}).$$

Given this cutoff the rest of the proof is identical with an identical characterization of the optimal first period effort. □

Proof of Corollary 4. Follows from the fact that the IO's report s has the MLRP in $m(\theta)$. □

Proof of Corollary 5. Recall that the voter's cutoff is defined as

$$\hat{x}(\hat{a}, \hat{p}) = \frac{\hat{a}_1 + \hat{a}_0}{2} + \frac{\log(\frac{\hat{m}_0}{\hat{m}_1})}{\zeta(\hat{a}_1 - \hat{a}_0)}.$$

It is immediate that whenever $\hat{p}_1 = \hat{p}_0$ then $\hat{x}(\hat{a}, \hat{p}) = \frac{\hat{a}_1 + \hat{a}_0}{2}$, as in the model without the IO. Optimal effort is thus identical to that characterized in Proposition 1.

Suppose $\hat{m}_1 \neq \hat{m}_0$. The first-order condition for leader-type θ's effort is

$$\beta - \beta\theta\lambda_M a - \beta(1-\theta)(\lambda_M + \lambda_S)a + \sqrt{\zeta}\phi(\sqrt{\zeta}(a - \frac{\hat{a}_1 + \hat{a}_0}{2} - \frac{\log(\frac{\hat{m}_0}{\hat{m}_1})}{\zeta(\hat{a}_1 - \hat{a}_0)}))\Psi = 0.$$

Since the normal density is log-concave, it is single peaked. Hence $\phi(\sqrt{\zeta}(a - \hat{x}(\hat{a}, \hat{p})))$ is single peaked in s such that there is a s^{max} where $\frac{d}{ds}\phi(\sqrt{\zeta}(a - \hat{x}(\hat{a}, \hat{p}))) > 0$ for $s < s^{max}$ and $\frac{d}{ds}\phi(\sqrt{\zeta}(a - \hat{x}(\hat{a}, \hat{p}))) < 0$ for $s > s^{max}$. As such optimal effort is single peaked in s, $\frac{da^*_\theta}{ds}$ is nonmonotonic in s. Moreover, observe that $\lim_{s \to -\infty} \phi(\sqrt{\zeta}(a - \hat{x}(\hat{a}, \hat{p}))) = 0$ and $\lim_{s \to \infty} \phi(\sqrt{\zeta}(a - \hat{x}(\hat{a}, \hat{p}))) = 0$ such that as $s \to \pm\infty$, $a^*_\theta \to \frac{1}{\lambda_M + (1-\theta)\lambda_S}$.

Denote leader-type θ's optimal effort in the model without the IO as $\tilde{a}_\theta$. Therefore since a^*_θ is continuous in s and $\tilde{a}_\theta > \frac{1}{\lambda_M + (1-\theta)\lambda_S}$ there exists $\underline{s}_\theta$ such that $a^*_\theta = \tilde{a}_\theta$ when $\frac{da^*_\theta}{ds} > 0$ and $\overline{s}_\theta$ such that $a^*_\theta = \tilde{a}_\theta$ when $\frac{da^*_\theta}{ds} < 0$. □

Lemma 1. *If $\hat{p}_1 \neq \hat{p}_0$, the voter's threshold $\hat{x}(\hat{a}, \hat{p})$:*

- *increases in $\hat{p}_1$ if $s < \frac{1}{2}$ and decreases in $\hat{p}_1$ if $s > \frac{1}{2}$;*
- *decreases in $\hat{p}_0$ if $s < \frac{1}{2}$ and increases in $\hat{p}_0$ if $s > \frac{1}{2}$.*

Proof of Lemma 1. The voter's threshold is

$$\hat{x}(\hat{a}, \hat{p}) = \frac{\hat{a}_1 + \hat{a}_0}{2} + \frac{\log(\frac{\hat{m}_0}{\hat{m}_1})}{\zeta(\hat{a}_1 - \hat{a}_0)},$$

where $\hat{m}_\theta = \hat{p}_\theta \phi(\sqrt{\tau}(s-1)) + (1-\hat{p}_\theta)\phi(\sqrt{\tau}s)$. Observe that

$$\frac{\partial \hat{m}_\theta}{\partial \hat{p}_\theta} = \phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s),$$

which is negative if $s < \frac{1}{2}$ and positive if $s > \frac{1}{2}$.

Differentiating with respect to $\hat{p}_1$ yields

$$\frac{\partial \hat{x}(\hat{a}, \hat{p})}{\partial \hat{p}_1} = -\frac{1}{\zeta(\hat{a}_1 - \hat{a}_0)\hat{m}_1} \frac{\partial \hat{m}_1}{\partial \hat{p}_1},$$

such that $\frac{\partial \hat{x}(\hat{a},\hat{p})}{\partial \hat{p}_1} > 0$ if $s < \frac{1}{2}$ and $\frac{\partial \hat{x}(\hat{a},\hat{p})}{\partial \hat{p}_1} < 0$ if $s > \frac{1}{2}$.

Similarly, differentiating with respect to $\hat{p}_0$ yields

$$\frac{\partial \hat{x}(\hat{a}, \hat{p})}{\partial \hat{p}_0} = \frac{1}{\zeta(\hat{a}_1 - \hat{a}_0)\hat{m}_0} \frac{\partial \hat{m}_0}{\partial \hat{p}_0},$$

such that $\frac{\partial \hat{x}(\hat{a},\hat{p})}{\partial \hat{p}_0} < 0$ if $s < \frac{1}{2}$ and $\frac{\partial \hat{x}(\hat{a},\hat{p})}{\partial \hat{p}_0} > 0$ if $s > \frac{1}{2}$. □

Proof of Proposition 3. The leader maximizes

$$\max_{m \in \{0,1\}} \int_{-\infty}^{\infty} \Big[\beta\Big(a_\theta^* - \frac{\lambda_M + (1-\theta)\lambda_S}{2} a_\theta^{*^2}\Big) + \pi(a_\theta^*(s))\Psi\Big]\phi(\sqrt{\tau}(s-m))ds,$$

therefore choosing $m = 1$ over $m = 0$ whenever

$$\int_{-\infty}^{\infty}\Big[\beta\Big(a_\theta^* - \frac{\lambda_M + (1-\theta)\lambda_S}{2}a_\theta^{*2}\Big) + \pi(a_\theta^*(s))\Psi\Big]\phi(\sqrt{\tau}(s-1))ds \geq \int_{-\infty}^{\infty}\Big[\beta\Big(a_\theta^* - \frac{\lambda_M + (1-\theta)\lambda_S}{2}a_\theta^{*2}\Big) + \pi(a_\theta^*(s))\Psi\Big]\phi(\sqrt{\tau}s)ds,$$

which simplifies to

$$\int_{-\infty}^{\infty}\pi(a_\theta^*(s))\Big(\phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s)\Big)ds \geq 0.$$

Define $\Delta_\theta(\hat{p}_1, \hat{p}_0) = \int_{-\infty}^{\infty}\pi(a_\theta^*(s))\Big(\phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s)\Big)ds$ as the leader's difference in expected reelection probability from sending message $m = 1$ versus $m = 0$ when she is of type θ. If $\hat{p}_1 = \hat{p}_0$, then $\hat{x}(a^*, \hat{p}) = \frac{a_1^* + a_0^*}{2}$, and $\pi(a_\theta^*; s)$ is constant in s so $\Delta_\theta(\hat{p}_1, \hat{p}_0)$ is the difference of two densities integrated over their entire support, thus $\Delta_\theta(\hat{p}_1, \hat{p}_0) = 0$. If $\Delta_\theta(\hat{p}_1, \hat{p}_0) = 0$, it must be because $\hat{p}_1 = \hat{p}_0$. Observe that $\pi(a^*; s) = 0$ only if $s \to \pm\infty$, so for any finite s $\pi(a^*; s) > 0$. Moreover we are integrating over the entire space of s so it must be that $\pi(a^*; s)$ is constant in s and $\int_{-\infty}^{\infty}\Big(\phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s)\Big)ds = 0$, which occurs when $\hat{p}_1 = \hat{p}_0$. Hence $\Delta_\theta(\hat{p}_1, \hat{p}_0) = 0$ iff $\hat{p}_1 = \hat{p}_0$.

Now we show that $\hat{p}_1 = \hat{p}_0$ must occur at an interior $p^* \in (0, 1)$. For the honest type,

$$\frac{\partial\Delta_1(\hat{p}_1, \hat{p}_0)}{\partial\hat{p}_1} = \int_{-\infty}^{\infty}\sqrt{\zeta}\phi(\sqrt{\zeta}(a_1^* - \hat{x}(a^*, \hat{p})))\frac{1}{\zeta(a_1^* - a_0^*)\hat{m}_1}\Big(\phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s)\Big)^2 ds > 0,$$

so increasing the voter's belief that the honest type sends $m = 1$ increases the return from playing $m = 1$ versus $m = 0$. For the captured type,

$$\frac{\partial\Delta_0(\hat{p}_1, \hat{p}_0)}{\partial\hat{p}_0} = \int_{-\infty}^{\infty}-\sqrt{\zeta}\phi(\sqrt{\zeta}(a_0^* - \hat{x}(a^*, \hat{p})))\frac{1}{\zeta(a_1^* - a_0^*)\hat{m}_0}\Big(\phi(\sqrt{\tau}(s-1)) - \phi(\sqrt{\tau}s)\Big)^2 ds < 0.$$

From this we know that $\Delta_1(\hat{p}_1, \hat{p}_0) < 0$ if $\hat{p}_1 < \hat{p}_0$ and $\Delta_1(\hat{p}_1, \hat{p}_0) > 0$ if $\hat{p}_1 > \hat{p}_0$. Furthermore, $\Delta_0(\hat{p}_1, \hat{p}_0) > 0$ if $\hat{p}_1 > \hat{p}_0$ and $\Delta_1(\hat{p}_1, \hat{p}_0) < 0$ if $\hat{p}_1 < \hat{p}_0$. To see that $\hat{p}_1 = \hat{p}_0 = 1$

or $\hat{p}_1 = \hat{p}_0 = 0$ cannot be an equilibrium, observe that $\Delta_1(\hat{p}_1, 1) < 0$ for any $\hat{p}_1$, meaning the honest type would deviate to $m = 0$. Similarly, $\Delta_1(\hat{p}_1, 0) > 0$ for any $\hat{p}_0$, meaning the captured type would deviate to $m = 1$. Thus the only equilibrium is $p_1^* = p_0^* = p^* \in (0, 1)$. □

Proof of Proposition 4. The probability of a leader fulfilling their commitment is

$$\begin{aligned} P(x > s) &= P(a + \varepsilon > m + \nu) \\ &= P(a - m > \nu - \varepsilon). \end{aligned}$$

Since ν and ε are independent normal random variables, $\nu - \varepsilon \sim N(0, \frac{1}{\tau} + \frac{1}{\zeta})$. Given this and results from Proposition 3—specifically that leaders send $m = 1$ w.p. p^* and that p^* is independent of a—we can express the probability of fulfilling the commitment as

$$\mathcal{P}(a) = p^* \Phi\Big(\sqrt{\frac{\tau\zeta}{\tau + \zeta}}(a - 1)\Big) + (1 - p^*)\Phi\Big(\sqrt{\frac{\tau\zeta}{\tau + \zeta}}a\Big)$$

for any effort choice a chosen by the leader. Differentiating, we observe that $\mathcal{P}(a)$ is increasing in a,

$$\frac{d\mathcal{P}(a)}{da} = p^* \sqrt{\frac{\tau\zeta}{\tau + \zeta}}\phi\Big(\sqrt{\frac{\tau\zeta}{\tau + \zeta}}(a - 1)\Big) + (1 - p^*)\sqrt{\frac{\tau\zeta}{\tau + \zeta}}\phi\Big(\sqrt{\frac{\tau\zeta}{\tau + \zeta}}a\Big) > 0.$$

The result follows because honest leaders exert greater effort in equilibrium than captured leaders. □

A basic extension of the model allows for the inclusion of a valence shock. In addition to the utility specified in the main text, consider a case where the voter also has a predisposed bias toward the incumbent leader, which represents the value of the incumbent on all other dimensions besides the implementation of the public good. Denote bias as y_i, where $y_i \sim U[-\alpha,\ \alpha]$. The value of this bias is realized right before the voter makes his choice to retain the leader or not.

Proposition A.1. *The inclusion of the valence shock y_i produces qualitatively equivalent results to the domestic politics model in the main text.*

Proof of Proposition A.1. Second period behavior is unaffected. The voter i adopts a decision rule in which he retains the leader if and only if

$$P(\theta = 1|x) + y_i \geq \gamma.$$

Posterior beliefs $P(\theta = 1|x)$ are equivalent to those derived above in the proof of Proposition 1. Conditional on some value of his bias y_i, the voter is thus exactly indifferent between retaining the incumbent leader and replacing her when

$$x(y_i) = \frac{\hat{a}_1 + \hat{a}_0}{2} + \frac{\log\left(\frac{(1-\gamma)(\gamma - y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1 - \hat{a}_0)}.$$

The likelihood ratio $\frac{\phi(\sqrt{\zeta}(x_i - \hat{a}_1))}{\phi(\sqrt{\zeta}(x_i - \hat{a}_0)}$ is increasing in the signal x_i. Therefore, the voter in country i retains his leader if and only if $x_i \geq \hat{x}$. Also note that if $y_i > \gamma$ then $x(y_i) \to -\infty$ and if $y_i < -1 + \gamma$ then $x(y_i) \to \infty$. The threshold $\hat{x}$ that the voter uses to reelect the incumbent is

$$\hat{x}(y_i) = \begin{cases} \infty & y_i < -1 + \gamma \\ \frac{\hat{a}_1 + \hat{a}_0}{2} + \frac{\log\left(\frac{(1-\gamma)(\gamma - y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1 - \hat{a}_0)} & -1 + \gamma < y_i < \gamma \\ -\infty & y_i > \gamma. \end{cases}$$

Clearly, this means that if $y_i > \gamma$ the leader is retained with probability 1 and if $y_i < -1 + \gamma$ the leader is retained with probability zero. This means that the leader's effort can only affect the outcome of the election if bias is moderate, or when $-1 + \gamma < y_i < \gamma$.

Therefore, the probability of reelection can be decomposed into two terms. If $y_i > \gamma$, the leader survives with probability 1, which occurs with $P(y_i > \gamma) = \frac{\alpha-\gamma}{2\alpha}$. Second, if $-1+\gamma < y_i < \gamma$, the leader survives with probability $\Phi(\sqrt{\zeta}(a_i - \hat{x}(y_i)))$. Therefore, the total probability of survival in office is

$$\frac{1}{2\alpha}\int_{-1+\gamma}^{\gamma} \Phi(\sqrt{\zeta}(a_i - \hat{x}(y_i)))\ dy + \frac{\alpha-\gamma}{2\alpha}.$$

Leader i maximizes the following expected utility:

$$EU_i(a;\theta_i) = u(a;\theta_i) + \Big[\int_{-1+\gamma}^{\gamma} \Phi(\sqrt{\zeta}(a_i - \hat{x}(y_i)))\ dy + \alpha - \gamma\Big]\frac{\Psi}{2\alpha},$$

where $u(a;\theta_i)$ is the leader's policy utility as defined in the main text.

For type θ_i, the first-order condition is

$$\frac{\partial u(a;\theta_i)}{\partial a_i} + \frac{\sqrt{\zeta}\Psi}{2\alpha}\int_{-1+\gamma}^{\gamma} \phi\Big(\sqrt{\zeta}(a_i - \frac{\hat{a}_1+\hat{a}_0}{2} - \frac{\log\left(\frac{(1-\gamma)(\gamma-y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1-\hat{a}_0)})\Big)\ dy = 0.$$

Equilibrium requires that voters' conjectures are correct, so this simplifies to

$$\frac{\partial u(a;\theta_i)}{\partial a_i} + \frac{\sqrt{\zeta}\Psi}{2\alpha}\int_{-1+\gamma}^{\gamma} \phi\Big(\sqrt{\zeta}(\frac{a_1^*+a_0^*}{2} - \frac{\log\left(\frac{(1-\gamma)(\gamma-y_i)}{\gamma(1-\gamma+y_i)}\right)}{\beta(a_1^*-a_0^*)})\Big)\ dy = 0.$$

Because leaders/countries are symmetric, there are 2 equations in 2 unknowns. Solving these equations yield optimal effort levels (a_1^*, a_0^*). To confirm that the equilibrium policy choices are a maximum, I take the second-order condition. Define $\eta(a_i, y_i) = \sqrt{\zeta}(a_i - \frac{\hat{a}_1+\hat{a}_0}{2} - \frac{\log\left(\frac{(1-\gamma)(\gamma-y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1-\hat{a}_0)})$. Using the fact that $\frac{d}{da}\phi(\eta) = -\eta\phi(\eta)\frac{\partial\eta}{\partial a}$, the second-order condition is

$$-\frac{\partial^2 u(a;\theta)}{\partial a_i^2} - \frac{\zeta\Psi}{2\alpha}\int_{-1+\gamma}^{\gamma} \eta(a_i, y_i)\phi(\eta(a_i, y_i))\ dy.$$

Note that $\eta(a_1^*, y_i) = \frac{a_1^* - a_0^*}{2} - \frac{\log\left(\frac{(1-\gamma)(\gamma - y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1 - \hat{a}_0)}) > 0$. Therefore the function inside the integral in the second-order condition for the honest type is always positive, meaning the second-order condition $\frac{\partial^2 u(a;\theta)}{\partial a_i^2} - \frac{\zeta\Psi}{2\alpha}\int_{-1+\gamma}^{\gamma} \eta(a_1^*, y_i)\phi(\eta(a_1^*, y_i))\ dy < 0$ for the honest type.

Now consider the second-order condition for the captured type. Note that $\eta(a_0^*, y_i) = \frac{a_0^* - a_1^*}{2} - \frac{\log\left(\frac{(1-\gamma)(\gamma - y_i)}{\gamma(1-\gamma+y_i)}\right)}{\zeta(\hat{a}_1 - \hat{a}_0)})$ need not be positive. A sufficient condition to show that the equilibrium effort a_0^* is a maximum is to find a lower bound on the integral. Differentiating $\eta(a_0^*, y_i)\phi(\eta(a_0^*, y_i))$ with respect to y_i yields the critical points $y_i = \frac{\gamma - 1}{\frac{1}{\gamma e^{\frac{1}{2}\zeta(a_0^* - a_1^*)^2 + \sqrt{\zeta}(a_0^* - a_1^*)} - \gamma} + 1}$ and $y_i = \frac{1}{1 - \frac{\gamma e^{\frac{1}{2}\zeta(a_0^* - a_1^*)^2 + \sqrt{\zeta}(a_1^* - a_0^*)}}{\gamma - 1}} + \gamma - 1$. Evaluating $\eta(a_0^*, y_i)\phi(\eta(a_0^*, y_i))$ at the critical points yields values $-\frac{1}{\sqrt{2\pi e}}$ and $\frac{1}{\sqrt{2\pi e}}$. Further, since the integral is over an interval of length 1 with uniform density, the integral has a lower bound of $-\frac{1}{\sqrt{2\pi e}}$. Substituting this into the second-order condition yields the condition

$$\frac{\partial^2 u(a;\theta)}{\partial a_i^2} + \frac{\zeta\Psi}{2\alpha}\frac{1}{\sqrt{2\pi e}} \leq 0,$$

yielding the condition $\zeta \leq -\frac{2\alpha\sqrt{2\pi e}}{\Psi}\frac{\partial^2 u(a;\theta=0)}{\partial a_i^2}$.

Since the second-order condition is negative at the equilibrium effort choice, it is a maximum. Further, this is the only maximum by concavity of the utility function. Therefore, such an optimal policy must be unique. Indeed, this is the unique equilibrium because pooling equilibria cannot exist. Pooling can be ruled out by noticing that, in any pooling equilibrium, the probability of reelection is not a function of the choice variable (i.e, it is a constant). The solution to the problem in that case is the leader's ideal point, contradicting pooling. $\square$