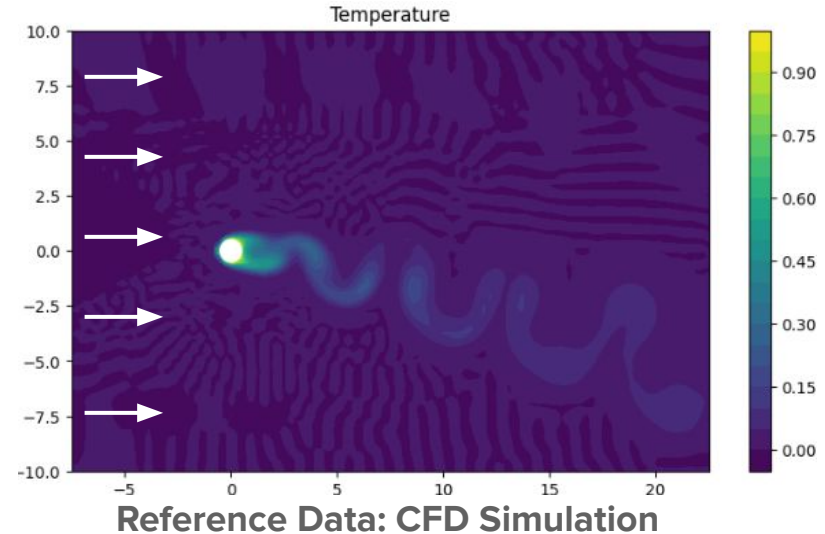


Inverse Heat Transfer using PINNs

Justin Miller

Project Outline:

- **Goal:** Design PINN to infer thermal BCs of hot cylinder and predict flow nearby
- **Reference Data:** 2D Inc. Flow over a constant (unknown) temperature cylinder undergoing mixed convection
- 46,200 points, 214 timesteps



Problem Outline: Success Metrics

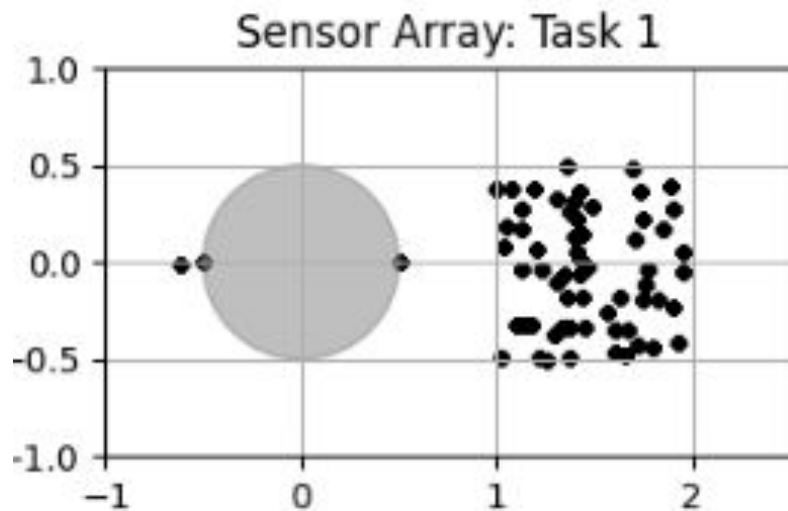
- Model Error
 - PDE Losses
 - Data Losses
 - Trainable Variable: Cylinder Temperature
- Profiles near the cylinder
 - Temperature
 - Velocity
 - Temperature Gradient

$$\begin{aligned}\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta &= \frac{1}{\text{Pe}} \nabla^2 \theta, \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \text{Ri} \theta, \\ \nabla \cdot \mathbf{u} &= 0,\end{aligned}$$

Task 1: Modeling with robust Sensor Grid

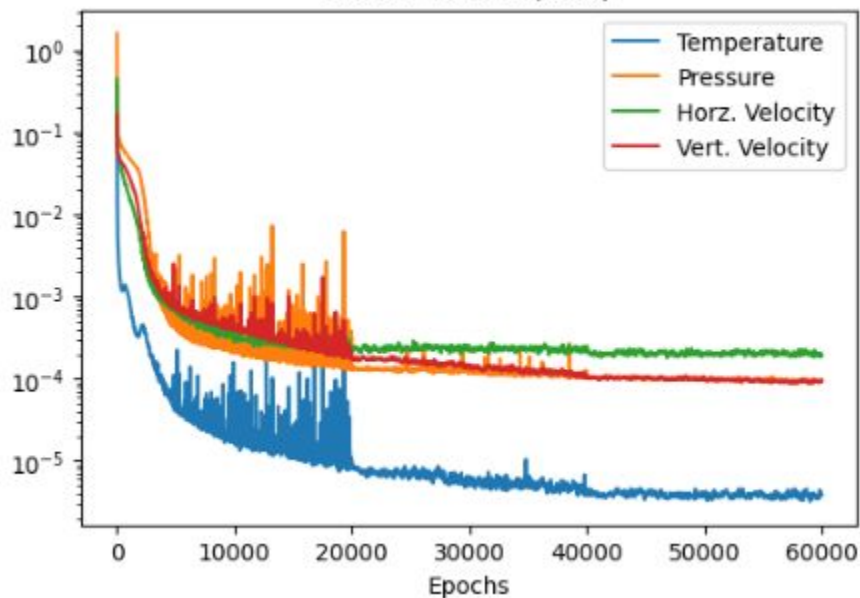
- Choosing “sensor” points from CFD data (mimics experiment)
- Two leading sensors, one trailing sensor
- 64 (randomly chosen) sensors in the grid downwind

- DeepXDE: FNN 6 layers, 50 width, Tanh
- 60k epochs, Adam, LR Decay

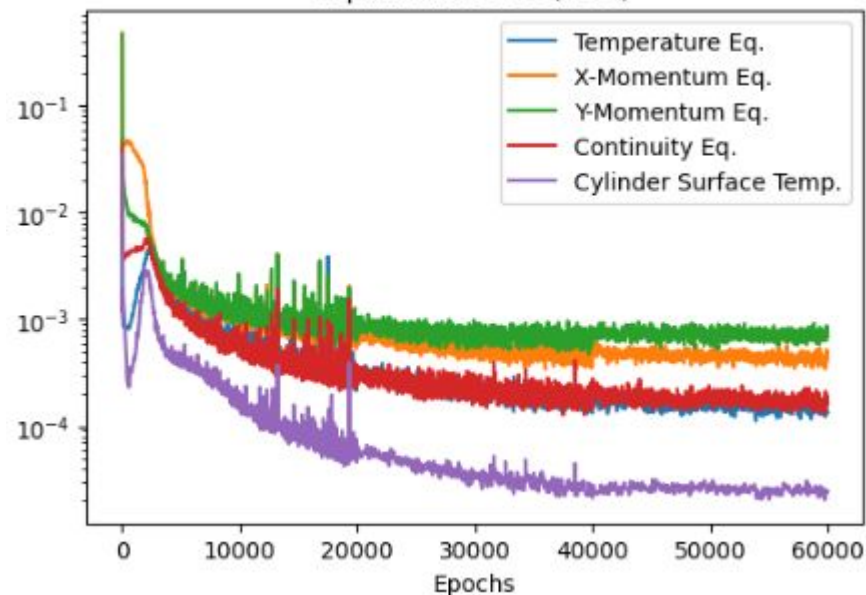


Task 1 Training:

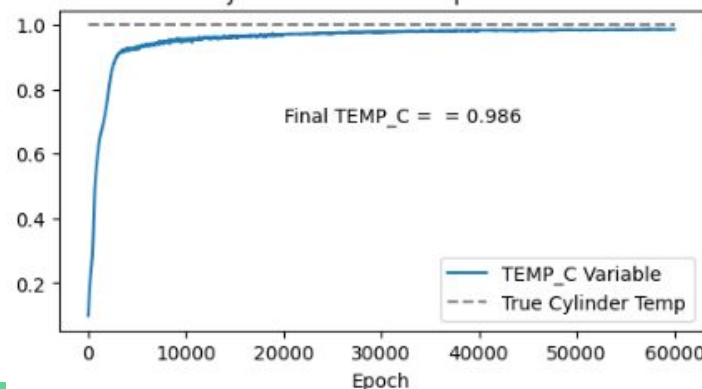
Sensor Losses (MSE)



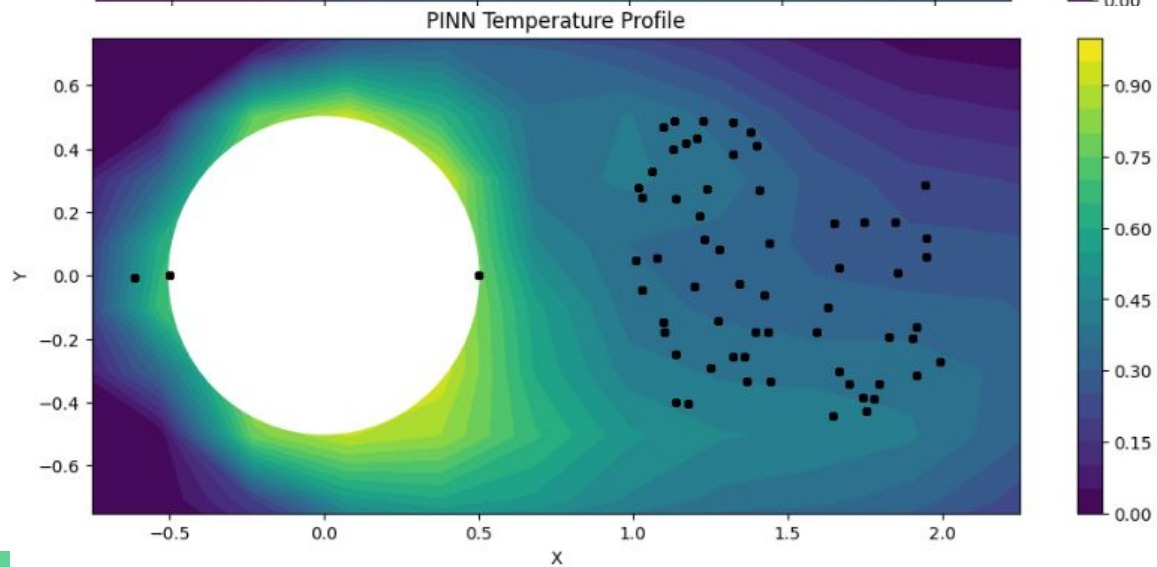
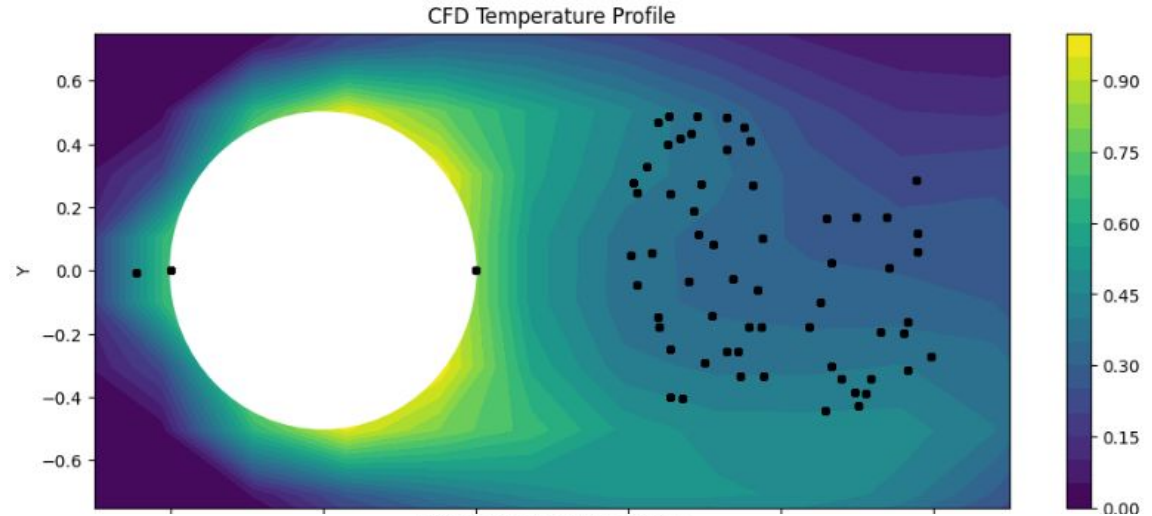
Equation Losses (MSE)



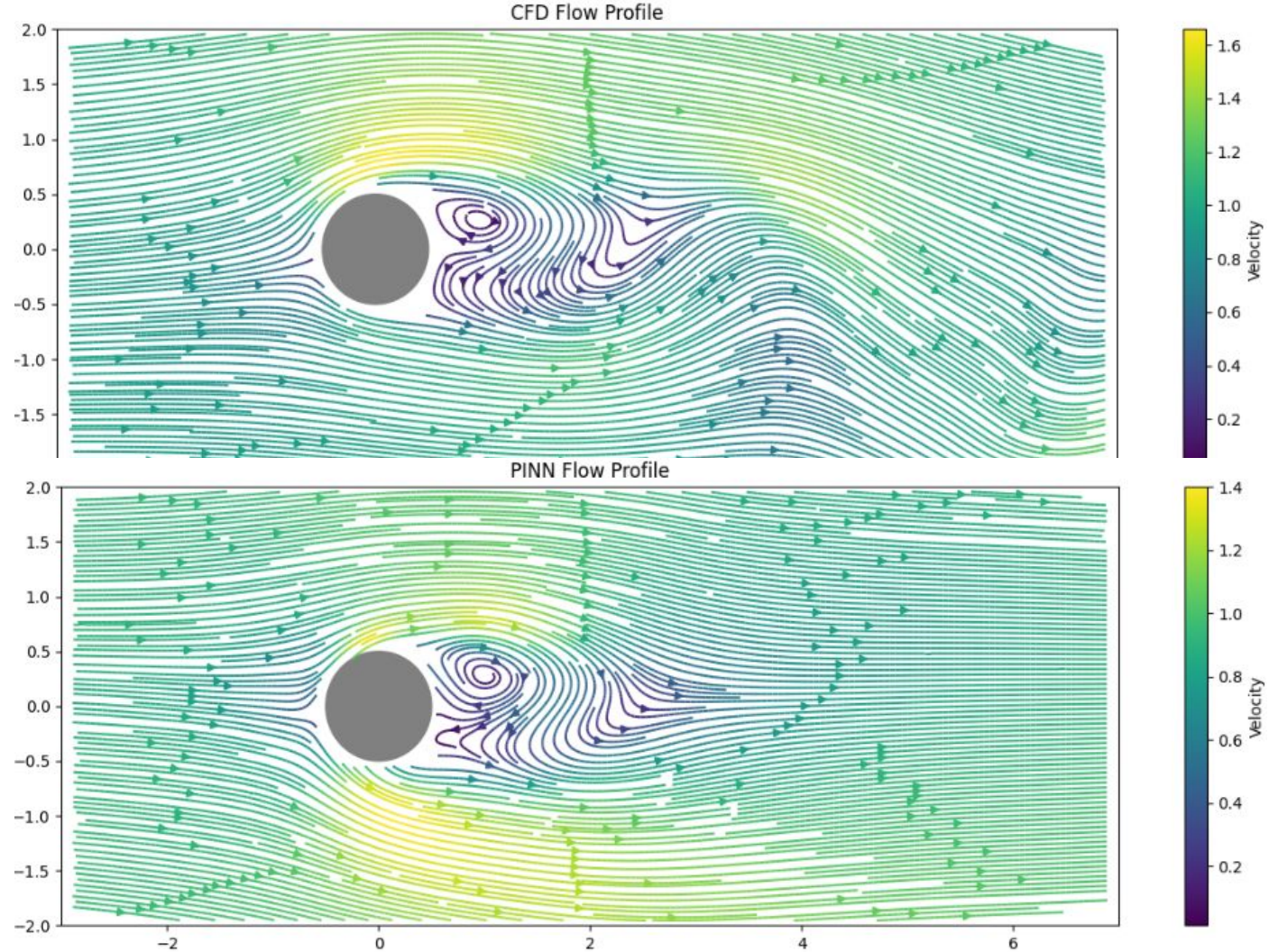
Cylinder Surface Temperature



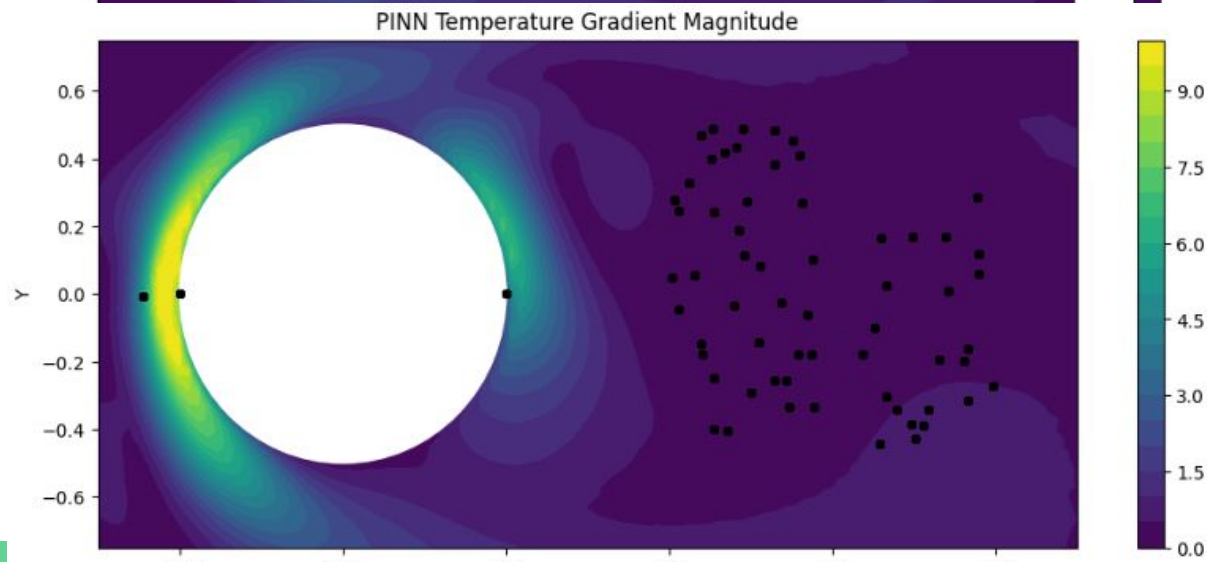
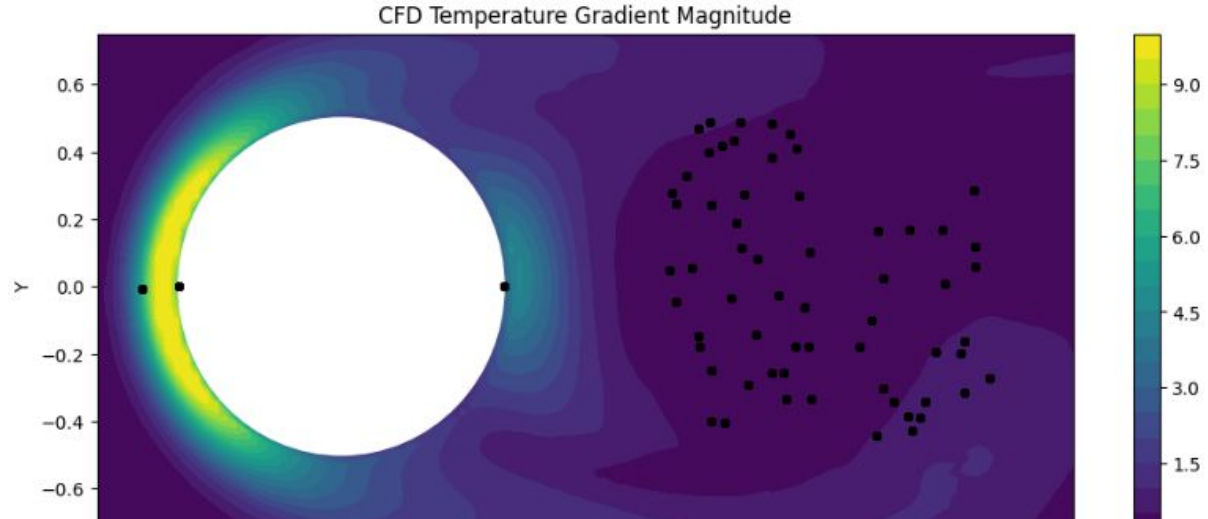
Task 1 Results:



Task 1 Results:

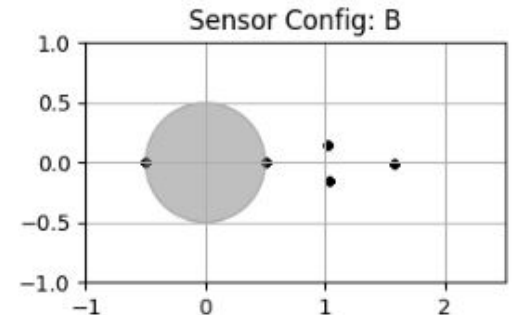
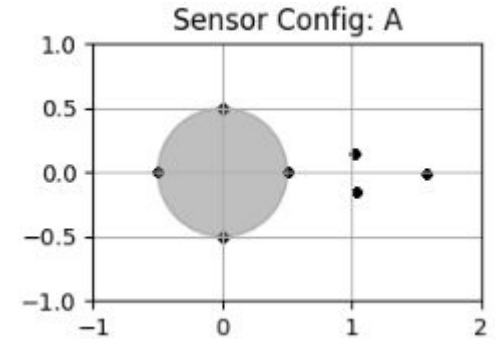
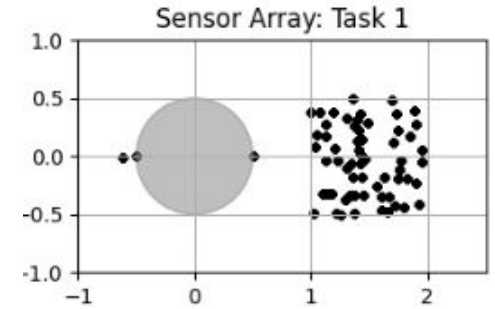


Task 1 Results:



Task 2: Minimum Sensor Requirements:

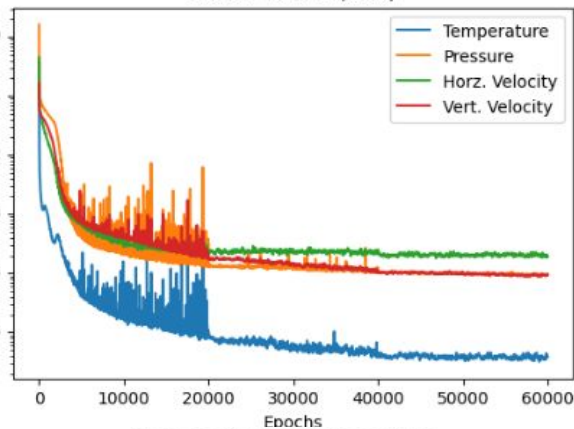
- Same model architecture and process as in task 1
- Reduced configurations were selected
- Two sample comparisons were tested and compared to initial run



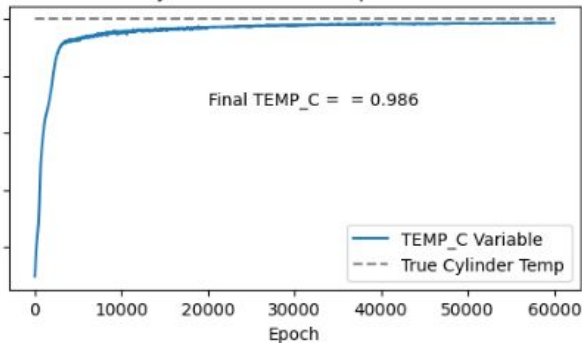
Task 2 Training:

Full Grid (reference)

Sensor Losses (MSE)

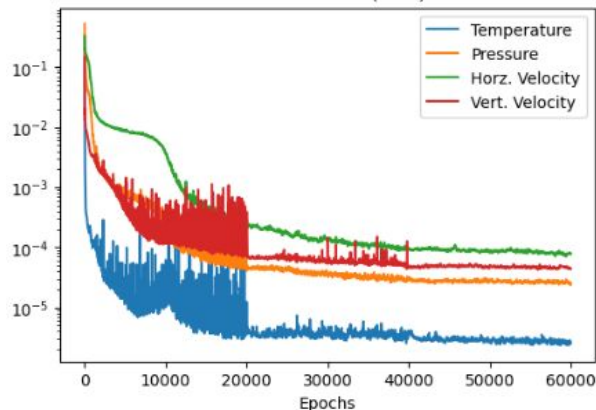


Cylinder Surface Temperature

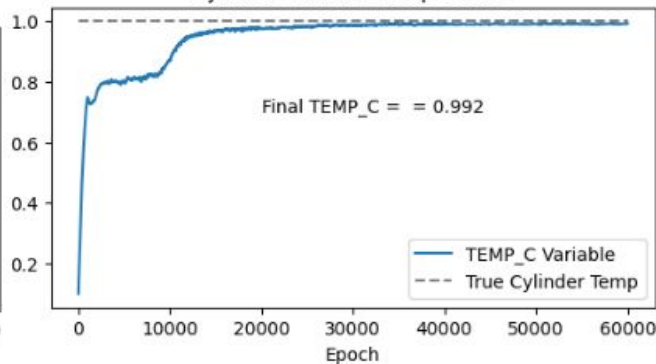


Sensor Config A

Sensor Losses (MSE)

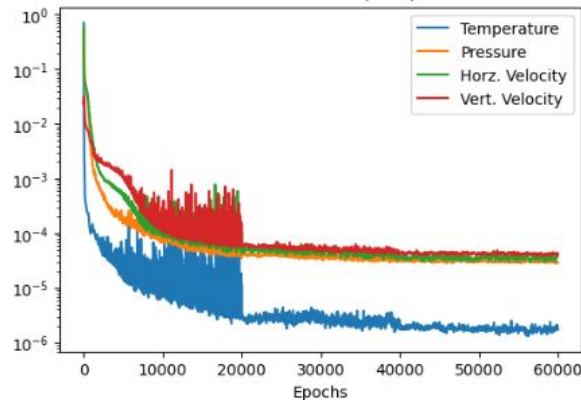


Cylinder Surface Temperature

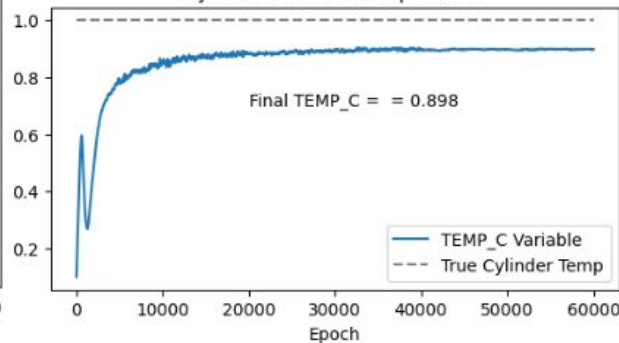


Sensor Config B:

Sensor Losses (MSE)

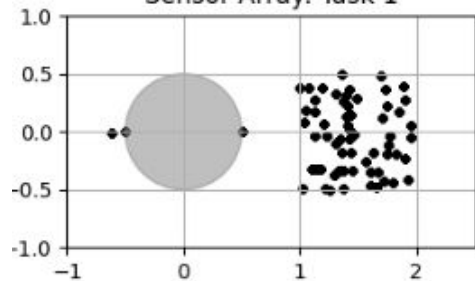


Cylinder Surface Temperature

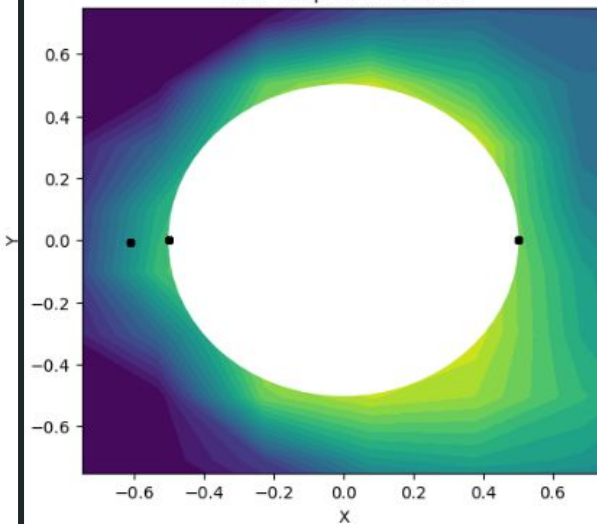


Robust Sensors

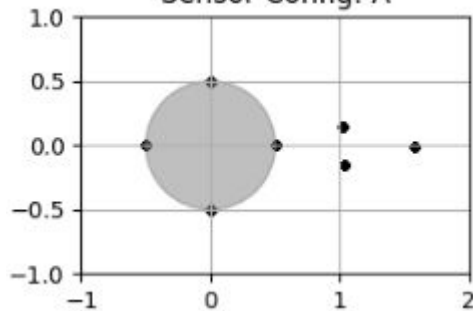
Sensor Array: Task 1



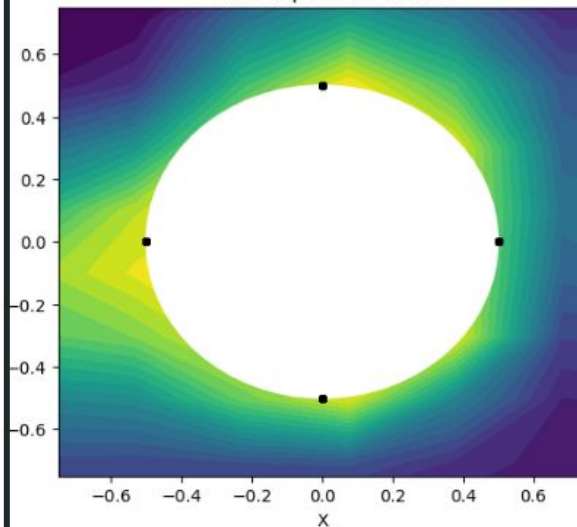
PINN Temperature Profile



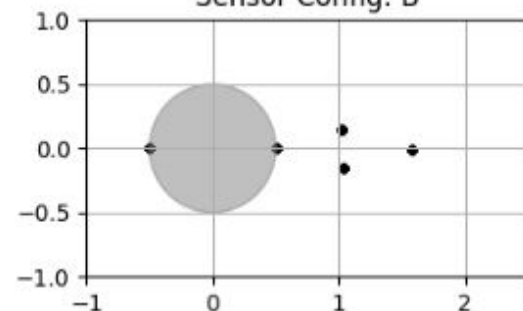
Sensor Config: A



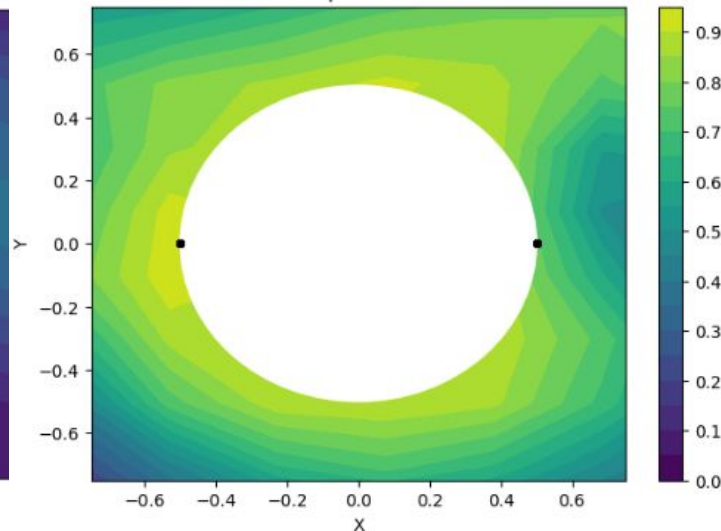
PINN Temperature Profile



Sensor Config: B

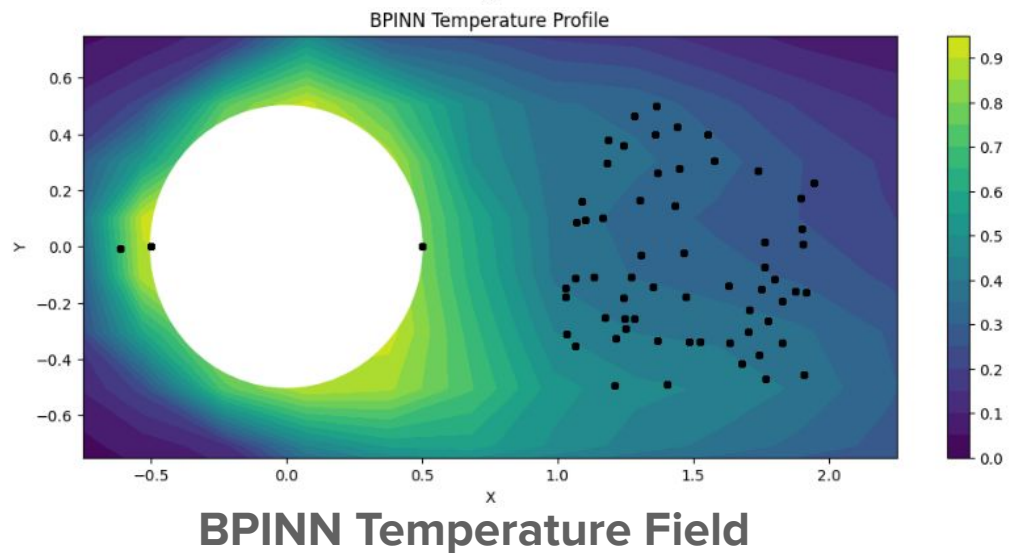
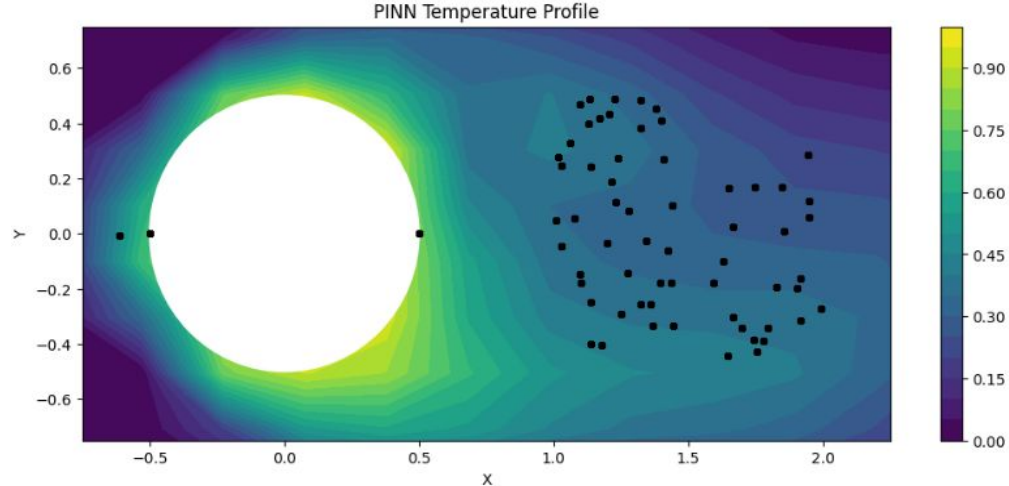


PINN Temperature Profile



Task 3: Noisy sensors

- Tasks 1 & 2 Repeated using **Bayesian PINNs** and 5% noise
- Torchbnn used with deepxde



Summary Comparisons: Overall Error, Surface Temp Error

	Temp. L2 Relative Error	TEMP_C var.
Full Grid	0.00027	0.986
Config A.	0.00081	0.992
Config B.	0.00109	0.898
Full Grid (BPINN)	0.00034	0.979
Config A. (BPINN)	0.00077	0.966
Config B. (BPINN)	0.00073	0.878

Questions?

References

- [1] S. Cai, Z. Wang, S. Wang, P. Perdikaris, and G. E. Karniadakis. Physics-informed neural networks for heat transfer problems. *Journal of Heat Transfer*, 143(6):060801, 2021.
- [2] A. F. Psaros, X. Meng, Z. Zou, L. Guo, and G. E. Karniadakis. Uncertainty quantification in scientific machine learning: Methods, metrics, and comparisons. *arXiv preprint arXiv:2201.07766*, 2022.
- [3] L. Yang, X. Meng, and G. E. Karniadakis. B-PINNs: Bayesian physics-informed neural networks for forward and inverse pde problems with noisy data. *Journal of Computational Physics*, 425:109913, 2021.