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Drag Coefficients of Reynolds-Matched Disks

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Abstract

At low wind speeds, scale models in a wind tunnel will experience identical forces and respective coefficients if the Reynolds number of the external flow is kept constant. The purpose of this experiment is to evaluate the practical application of Reynolds-matching to determine if there is a statistically significant difference in the drag coefficient of different sized disks in Reynolds-matched flow. Laser-cut disks with diameters of 5, 7.25, and 8 in were mounted to a load cell (Omega LC61SP-600g) and supported by a test stand inside the Forsyth wind tunnel. An anemometer (HT 9830) was used to measure the wind speed and air temperature to verify Reynolds-matching.

The drag coefficients for the small, medium, and large disks were found to be 1.869 ± 0.060, 2.715 ± 0.069, and 2.625 ± 0.038 respectively (95% CI). The small disk's drag coefficient is significantly smaller than the two larger disks, and the medium and large disk also had statistically significant drag coefficients ($p=0.024$). Although the statistical results do not agree with the expected theory, accumulation of experimental error can account for the observed differences.

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3 Acknowledgements

Thank you to Professor Gouldstone and Professor Caracoglia for handling the Forsyth wind tunnel logistics and ensuring the wind tunnel was operable. Also thank you to Professor Abedi for his help in driving home a solid understanding of fluid mechanics principles. Lastly, this project would not have been possible without the resources and support of Professor Smyser.

4 Problem Statement

Scale models in a wind tunnel can be used to measure the drag coefficient of their full-sized, proportional equivalents if the Reynolds numbers are matched. This means that the drag coefficient and drag force can be calculated for a full scale object by doing this experiment using a model of said object. The goal of this experiment is to verify Reynolds-matching by determining if there is a statistically significant difference in the drag coefficient of different sized disks if the Reynolds number is kept consistent. The results from this experiment will be applied in future projects with AerospaceNU, where scale model rockets will be used to predict the performance of their full-scale equivalents.

5 Introduction

5.1 Background

Scale models in a wind tunnel can be used to measure the drag coefficient of their full-sized equivalents if the Reynolds numbers are matched. These models can be any object of any shape, which can range from a complex model rocket to a simple cube. To verify this condition, this lab seeks to determine if there is a statistically significant difference in the drag coefficient of different sized disks if the Reynolds number is kept consistent.

Throughout this experiment, the model that was used to test this theory was a disk. Three different sized disks were used, which have diameters of 5 in, 7.25 in, and 8.5 in. In order to keep these disks steady in the wind tunnel, they were attached to a rig which was put into the wind tunnel facing upstream. To keep the rig in place, it was taped and weighed down.

This experiment is relevant to history because before the age of computers this procedure was done for many real world applications. Since no CAD modeling or testing could be done, manufacturers had to make scale models and test them to see if their design could work.

5.2 Sensors

There were two focal sensors used in this experiment. These sensors were an Omega "Single Point Load Cell with IP65 Rating" and an "HT 9830" hot wire anemometer. The load cell was attached to the back of each disk and recorded the force the disk experienced from the wind. It has a range from 0-6 N and an accuracy of ±0.028%. Observing these numbers makes it clear that this load cell is perfect for the precise results desired in this experiment, but much caution was used to ensure that the maximum force wasn't reached. The load cell will directly output a four-pin signal, on the order of millivolts from its internal circuits. This was amplified by an HX711 breakout board and converted into a digital signal using a 24 bit analog to digital converter. This digital signal was transmitted to an Arduino board that then outputted to the serial monitor on a connected computer. This method of data transfer eliminates the need for the Arduino to perform an analog to digital conversion, which would otherwise limit the whole system's accuracy. The 24 bit converter on the HX711 means that the resolution error will be negligible relative to the error of the load cell itself.

The anemometer was used to record the wind speed in the tunnel. It has a large range of 0.02-40 m/s with an accuracy of 3% of the reading. The resolution of this anemometer is 0.1 m/s. The anemometer operates based on measuring the rate of heat transfer of a probe and using convection math to calculate the velocity.

5.3 Relevant Theory

In order to understand the results and analysis of this experiment, there are a few equations that are important to know. The drag force is defined by the following equation [1]:

$$F_D = \frac{1}{2} C_D \rho U^2 A \qquad \text{Eq. (1)}$$

Where F_D is the drag force [N], C_D is the dimensionless drag coefficient, ρ is the fluid density [kg/m^3], U is the fluid velocity [m/s], and A is the cross-sectional area of the object [m^2]. All parameters must be known to determine the drag force. The density of air is approximately 1.20 kg/m^3, and varies with temperature [2-3]. The cross-sectional area of a disk is given by:

$$A = \pi r^2 \qquad \text{Eq. (2)}$$

Where r is the radius of the disk [m]. The air speed can be controlled in the wind tunnel, and the maximum drag force will occur at the highest air speeds. Finally, the drag coefficient depends on the expected Reynolds number of the flow. The Reynolds number is a dimensionless parameter given by [1]:

$$Re_d = \frac{Ud}{\nu} \qquad \text{Eq. (3)}$$

Where ν is the kinematic viscosity of the fluid [m^2/s], which is approximately 1.50×10^{-5} m^2/s for air and varies with temperature [2-3]. By combining and rearranging Eq's 1, 2, and 3, we obtain:

$$F_D = \frac{1}{8} C_D \rho {Re_d}^2 \nu^2 \pi = f(C_d, Re_d) \qquad \text{Eq. (4)}$$

The drag force is a function of the drag coefficient and the Reynolds number, since the air properties are constant. If the Reynolds number is kept constant for three differing disks, the drag force can be measured, and the drag coefficient can be calculated for each disk. In theory, the drag coefficient should remain the same as the geometry is scaled up [1].

In prior work by another AerospaceNU group, the maximum air speed for the wind tunnel was approximately 20 m/s [4]. Using this value, the approximate values for air density and viscosity, and Eq. (4), the drag force on the disk was predicted to be approximately 2 N. This value was used to select the 600-g full scale load cell.

6 Procedure

6.1 Setup

The experimental procedure required two main sensors. The first is the Omega LC61SP-600g Single Point Load Cell with IP65 Rating. The load cell is connected to a HX711 Sparkfun Load Cell Amplifier which amplifies the output voltage to a level readable by an Arduino. The second sensor is a HT 9830 Hot Wire Anemometer. The tool is used to collect both temperature in °C and wind speed in m/s. Both readings have an accuracy of 3% reading which is satisfactory for our needs. The wind speed range is from 0.02 - 40 m/s and the Temperature range is from -32 to 547 Celsius.

Figure 1. Load Cell Mounted on the Test Stand

The experimental setup consisted of a low-speed closed loop wind tunnel and an in-house test rig. The wind tunnel has a maximum safe speed of just under 10 m/s and a cross sectional area of roughly 5 square feet. The test rig was a simple setup consisting of side supports and a vertically oriented platform for the load cell to attach to. The test samples are to be attached onto the other end of the load cell facing the wind. As mentioned in *5.1: Background* these samples consisted of three flat disks with diameters of 5, 7.25, and 8.5 inches. All of these samples were laser cut from ¼ inch wood.

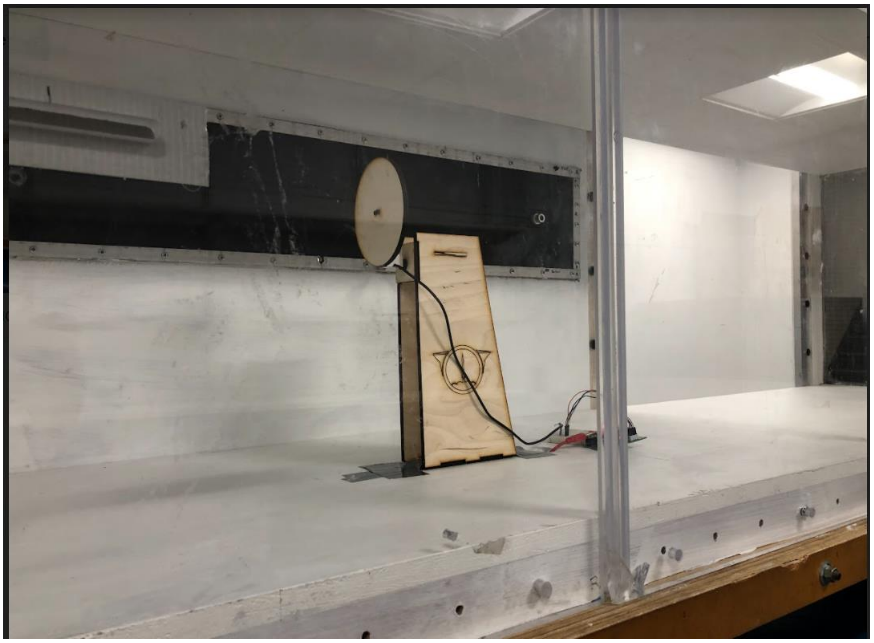
Figure 2. Test Stand in the Wind Tunnel

6.2 Experimental Steps

Before opening the wind tunnel, the first step was to calibrate the load cell. To do so, 5 known masses ranging from 0 to 360 grams were placed on the load cell for each disk. The resulting calibration plots compared known forces to the raw serialized output from the sensor.

To start, the first disk was attached to the test frame (Figure 1) and the entire rig was secured facing the wind. The load cell was connected to the amplifier and Arduino, both secured in the wind tunnel directly behind the test rig. The USB connection wire was run through a hole in the tunnel to a laptop for data acquisition (Figure 2). The anemometer was then placed sufficiently upstream of the test rig.

Once the setup was complete and all components were secure, the wind tunnel was turned on and tuned to the desired wind speed based off of the anemometer readings. The wind speed, temperature, and load cell output was recorded and the wind tunnel was turned off to rest for 10 seconds. It was then restarted and more readings were taken using the same process.

These steps were repeated for each disk with slightly different parameters. For the small disk, the wind tunnel was set at its maximum fan speed. For the other disks, the target velocity was calculated with disk diameters and Equation (3) to match the Reynolds number for each flow at the disk. In the end, the large disk was tested with 5 total readings using a target wind speed of 4.45 m/s. The medium disk was tested with 5 total readings using a target wind speed of 5.24 m/s. The small disk was tested with 10 total readings using a target wind speed of 7.56 m/s. Note that the number of tests for the small disk was increased due to differing results that will be explored further in the *Discussion.* Also note that the actual wind speeds varied slightly from the target wind speed during each test due to inconsistencies in the wind tunnel.

There were some significant safety concerns during this experiment. As with any procedure involving high-speeds, it was important to ensure that all components were secured and all hatches closed before operating the wind tunnel, even though the speeds were not that high. Another safety precaution was for the electricity hazard. The wind tunnel has high voltage components which are important to be aware of at all times. As such, we were especially careful when turning on the power supply and ensured that no food or water was taken into the lab room.

7 Results

7.1 Calibration

Each disk setup was individually calibrated to find a static sensitivity to relate the Arduino reading to a force value. The calibration curves for the small, medium, and large disks are displayed in Figure 3 below.

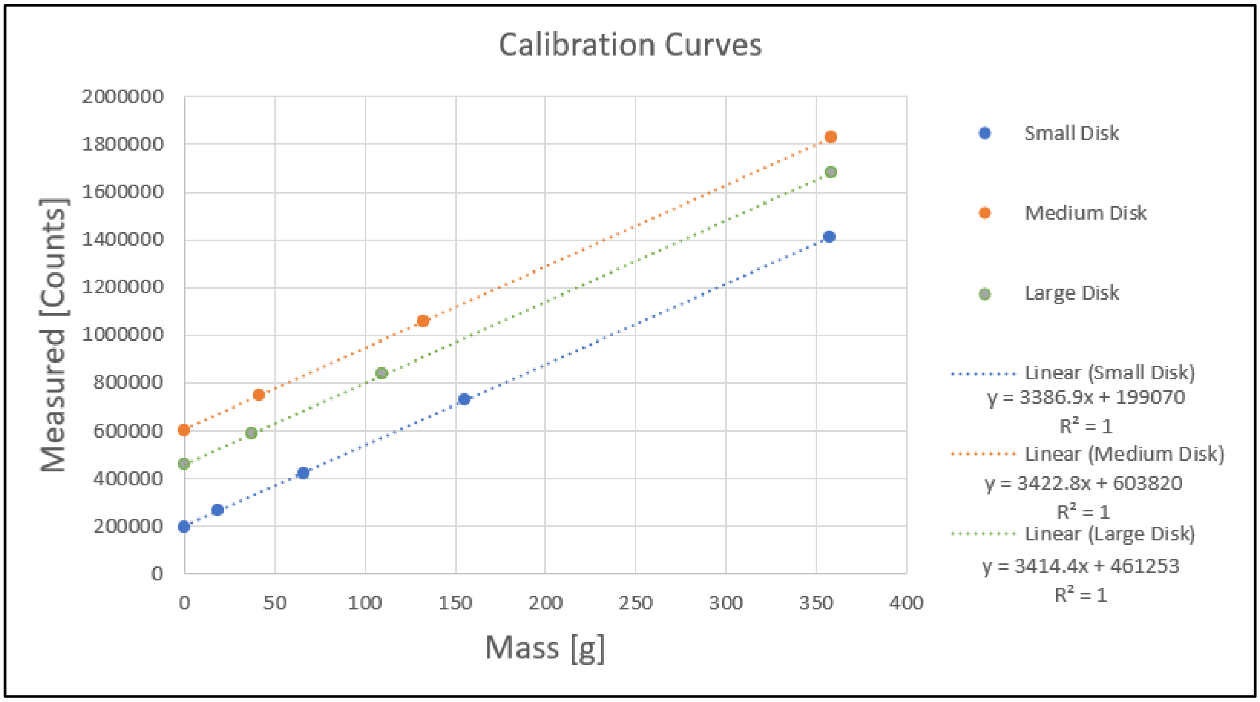

Figure 3: Calibration Curves

The static sensitivities for the small, medium, and large disks are 3386.9 counts/g, 3422.8 counts/g, and 3414.4 counts/g respectively. Since R^2 is 1 for each fit, the static sensitivities are assumed to be exact for each disk setup. Additionally, the calibration was only done with increasing weight order. This closely matches the conditions in the wind tunnel, where the force will slowly increase to the final value as the wind speed increases to steady-state.

7.2 Data Collection and Calculations

Each disk was tested after calibration. A baseline load in counts was measured while the wind tunnel was off. Once the wind tunnel speed settled, a final load (in counts), the wind speed, and air temperature were measured and recorded.

To convert the count reading to a force, the following equation was used:

$$F = \frac{(L_f - L_i)}{K} \times \frac{g}{1000} \quad \text{Eq. (5)}$$

where F_D is the drag force [N], L_f and L_i are the respective final and initial load cell readings [counts], K is the static sensitivity [counts/g], and g is gravitational acceleration [9.81 m/s^2]. The Reynolds number was calculated using:

$$Re = \frac{\rho V D}{\mu} \quad \text{Eq. (6)}$$

where Re is the dimensionless Reynolds number, ρ is the air density [kg/m^3], V is the air velocity [m/s], D is the disk diameter [m], and μ is the dynamic viscosity [kg/(m*s)].

Finally, the drag coefficient is calculated by:

$$C_D = \frac{8F_D}{\rho V^2 \pi D^2} \quad \text{Eq. (7)}$$

where C_D is the dimensionless drag constant, and all other variables are defined above.

The remaining air density and dynamic viscosity are assumed to depend only on the temperature in the wind tunnel and are determined using the University of Waterloo's "Fluid Properties Calculator" [3].

All data, including measured and calculated values, can be found in Appendix 1. Note that the small disk had 10 collected data points, while the other two disks only had 5. This change was made since the variance in C_D was very small for the first disk. It was determined that fewer data points could be collected for the remaining disks due to limited available time without compromising the results.

7.3 Summarized Results

The attained Reynolds numbers for each disk is summarized in Figure 4 below.

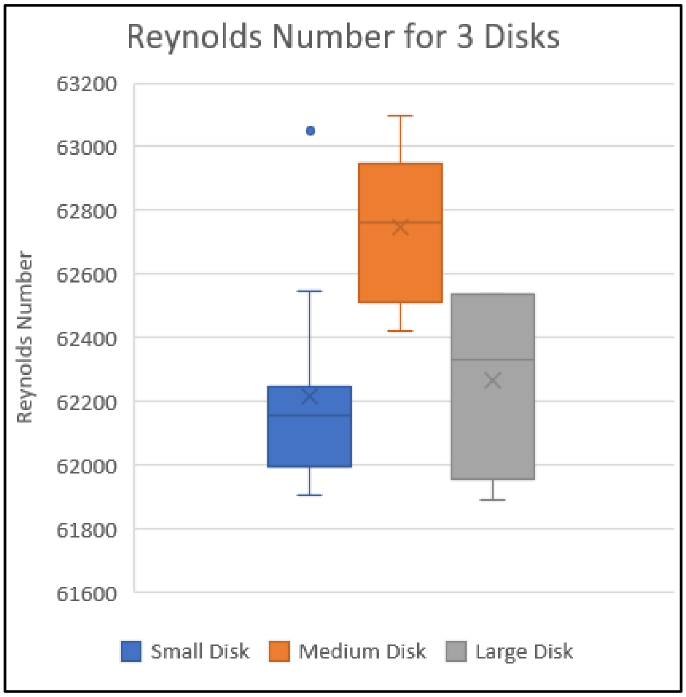

Figure 4. Reynolds Number Results

From this plot, it is apparent that the Reynolds numbers for the small and large disks (Disk 1 and Disk 3) are well matched, while the medium disk had larger Reynolds numbers. The percent difference between the means of Disk 1 and Disk 2 is 0.85%, which is assumed to be small in comparison to other experimental errors.

The average measured drag force and average drag coefficients are summarized in Table 1 below.

Table 1. Force and C_D Summary

	Small Disk	Medium Disk	Large Disk
Mean Force (N)	0.812	1.201	1.160
Mean C_D	1.874	2.706	2.660
C_D St. Dev	0.030	0.034	0.019

The drag coefficient data is displayed as a bar chart in Figure 5 below. The error bars represent a 95% confidence interval.

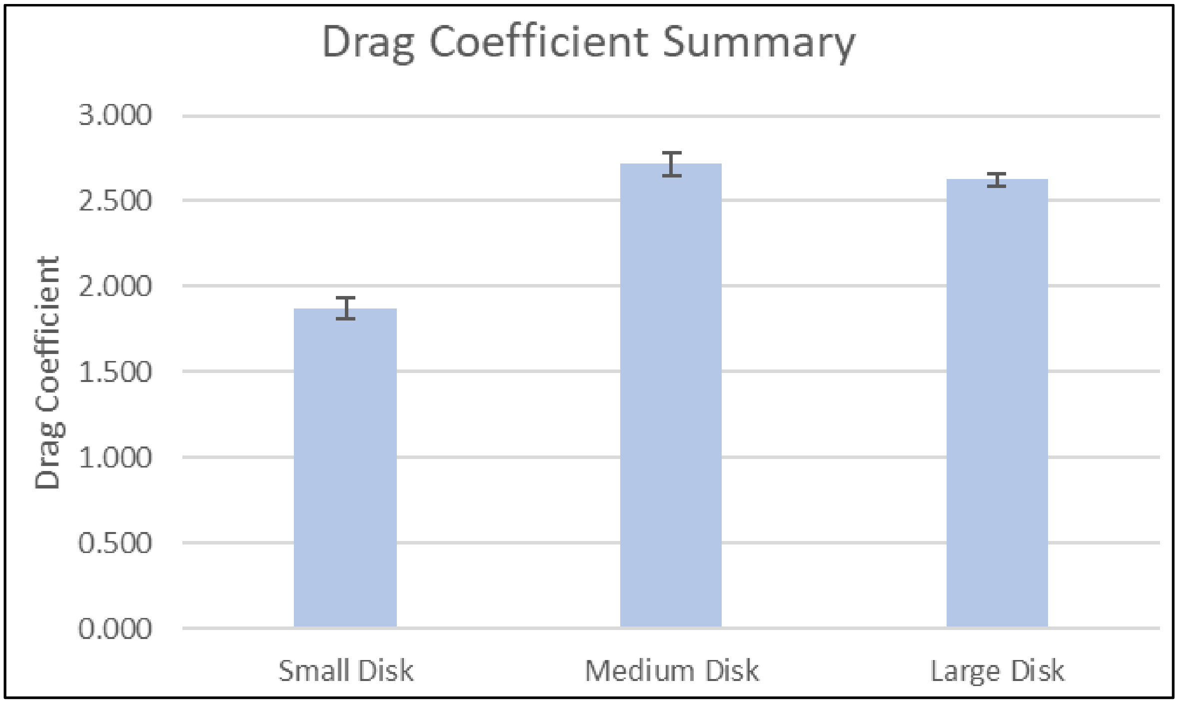

Figure 5. Drag Coefficient Summary

From the raw data and summary, it is apparent that the calculated drag coefficients for the smallest disk are much lower than the drag coefficients for the medium and large disk. The medium and large disk, however, appear to be similar. Statistical analysis is required to evaluate whether or not the disks had significantly different drag coefficients and is described in the following section.

8 Discussion

The main analysis for this project is statistical analysis using t-tests between the samples of data for each disk. ANOVA analysis was performed on the measurements of drag coefficient for the three disks and yielded a p value on the order of 10^{-24}, which indicates that there is high significant difference between samples. Since that doesn't indicate if any of the samples might have been statistically the same (not statistically different), individual t-tests were performed on the samples that were suspected as being matched.

To confirm the disks were Reynolds-matched, a t-test was performed on the Reynolds numbers for disks one and three, with a p value of 0.22. This shows that the Reynolds numbers for those two disks were not statistically different, a positive result that indicates the matching procedure worked as intended. The matching procedure for disk two was slightly off because of laser cutter tolerances, so that disk matched neither one or three.

To determine if the drag coefficients were matched, a t-test was performed on the drag coefficient data for disks two and three, since they qualitatively seemed closest. Disk one's drag coefficient was substantially different, for reasons that will be speculated later, so statistical analysis is not necessary. The results from this t-test are summarized in Table 2 on the following page.

Table 2. C_D 2-Sample t-test

	Disk 2	*Disk 3*
Mean	2.6774	2.6252
Variance	0.0011738	0.0003552
Observations	5	5
Hypothesized Mean Difference	0	
df	6	
t Stat	2.98505105	
P(T<=t) one-tail	0.01223813	
t Critical one-tail	1.94318028	
P(T<=t) two-tail	0.02447626	
t Critical two-tail	2.44691185	

The t-test between disks two and three yielded a p-value of 0.024, which is less than the general level of significance, 0.05. This means that the two samples are statistically different, indicating that the drag coefficients do not match to within the margins of error.

Error calculations, or a summation of all component errors affecting the drag coefficient, were not performed. This is because the dominant term in both of those analyses would have been the design stage uncertainty of the anemometer, 3% of the reading - the error of the load cell was 0.028% in comparison. However, this 3% spec for the anemometer seems conservative from how consistent it is run to run, and the larger source of error in this experiment was based on the setup.

Since the test stand and disks were taken out of the wind tunnel, swapped, calibrated and reattached each time, the aerodynamic load could have been transferred slightly differently to the load cell each time. Additionally, the alignment of the test stand within the wind tunnel is suspected as a source of error. If the test stand was oriented slightly angled, the disk would be slightly off of perpendicular to the air flow, which could have influenced the airflow patterns and drag. The third major source of error was the test stand itself, and how it was affecting the natural airflow in the disk’s wake region. The wake region directly behind a body is generally characterized by low pressure, swirling, turbulent flow, and this could affect the drag coefficient. Particularly for the small disk, number one, this is the suspected reason why its drag coefficient is so far from disks two and three. Because disk one was small, the airflow disturbance due to the test stand had a more substantial effect.

However, the drag coefficients of disks two and three did match within the 3% margin of error of the anemometer. So as for what conclusions can be drawn from the data, it seems reasonable to say that the underlying fluids principles of dimensional analysis held true to within the accuracy of the setup. Future work will focus on reducing that setup inaccuracy and hope to make stronger comparisons between measurements and theory, but in the context of this project's scope, the measurements are close to what they should be.

9 Conclusions

In this experiment, the group used a low-speed wind tunnel to measure the drag coefficients of three differently sized disks. In theory, if the Reynolds numbers of the airflow are matched by varying velocity the Drag Coefficients on each of these disks should be the same. After running 5 or more tests on each of the samples, the drag coefficients were statistically different with mean values of 1.87, 2.73 and 2.62 for the small, medium and large disks, respectively. Even after excluding the small disk as an outlier, the difference in means was determined statistically significant using a T-Test with a 95% confidence level.

This experience revealed the extreme challenges presented by fluid-flow experiments, specifically in the setup and test stand. The group learned a lot about the difficulties to neutralize interference in the results created by both inconsistencies in wind and the physical barriers created by the test stand. The results showed how poorly this type of study can go if we make incorrect assumptions about what we can ignore. Regardless, the group did address the initial goal of the problem statement to explore the Drag Coefficients and created usable test architecture for AerospaceNU.

10 References

[1] F. M. White, “Flow Past Immersed Bodies,” in Fluid mechanics, 8th ed., New York: McGraw-Hill Education, 2016, pp. 449–500.

[2] F. M. White, “Physical Properties of Fluids,” in Fluid Mechanics, 8th ed., New York etc.: McGraw-Hill, 2016, pp. 808–812.

[3] *Fluid Properties Calculator*, 1997. [Online]. Available: http://www.mhtl.uwaterloo.ca/old/onlinetools/airprop/airprop.html.

[4] R. Hart, I. Kramer, M. Molzon, and J. Wachala, “Skin Friction Drag on Rocket Bodies,” Apr-2021.

Appendix A: Full Data Tables

Small Disk (5 inch Diameter)

Wind Tunnel Setting (Hz)	Baseline Load (Counts)	Load (Counts)	Wind Speed (m/s)	Air Temp. (°C)	Dynamic Viscosity (kg/m.s)	Air Density (kg/m^3)	Reynolds Number	Force (N)	C_D
30	-53150	235500	7.59	21.9	1.830E-5	1.197	63050.5	0.836	1.914
30	-53235	236500	7.55	22.5	1.832E-5	1.195	62545.1	0.839	1.942
29.5	-53250	225500	7.55	23.4	1.837E-5	1.191	62166.1	0.807	1.868
29.5	-53530	223700	7.56	23.4	1.837E-5	1.191	62248.4	0.803	1.853
29.5	-53650	224900	7.55	23.7	1.838E-5	1.190	62080.1	0.807	1.867
29.5	-53700	223200	7.57	23.8	1.839E-5	1.189	62158.4	0.802	1.846
29.5	-53750	225000	7.55	23.9	1.839E-5	1.189	61994.2	0.807	1.868
29.5	-53820	225100	7.57	24.1	1.840E-5	1.188	62072.4	0.808	1.859
29.5	-53850	224300	7.56	24.3	1.841E-5	1.187	61904.5	0.806	1.859
29.5	-53930	225050	7.57	24.4	1.842E-5	1.187	61952.8	0.808	1.860

Medium Disk (7.25 inch Diameter)

Wind Tunnel Setting (Hz)	Baseline Load (Counts)	Load (Counts)	Wind Speed (m/s)	Air Temp. (°C)	Dynamic Viscosity (kg/m.s)	Air Density (kg/m^3)	Reynolds Number	Force (N)	C_D
23.4	-100	416815	5.25	24.1	1.840E-5	1.188	62420.8	1.208	2.748
23.4	-120	416192	5.26	24.1	1.840E-5	1.188	62539.7	1.206	2.734
23.4	-100	414395	5.29	24.1	1.840E-5	1.188	62896.4	1.201	2.691
23.4	-130	413606	5.31	24.2	1.841E-5	1.188	63099.9	1.198	2.666
23.4	-140	412293	5.28	24.1	1.840E-5	1.188	62777.5	1.195	2.688

Large Disk (8.5 inch Diameter)

Wind Tunnel Setting (Hz)	Baseline Load (Counts)	Load (Counts)	Wind Speed (m/s)	Air Temp. (°C)	Dynamic Viscosity (kg/m.s)	Air Density (kg/m^3)	Reynolds Number	Force (N)	C_D
19	-1420	395413	4.44	24	1.840E-5	1.188	61892.0	1.149	2.661
19	-1420	399947	4.44	23.8	1.839E-5	1.189	61977.7	1.163	2.691
19	-1450	400475	4.47	23.8	1.839E-5	1.189	62396.5	1.164	2.659
19	-1500	400255	4.48	23.8	1.839E-5	1.189	62536.1	1.164	2.646
19	-1490	399692	4.48	23.8	1.839E-5	1.189	62536.1	1.162	2.642