Solutions for Variable Ratio Coupler-Based Mach–Zehnder Interferometer

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Abstract

An analytical method for achieving specific wave modulations from the device designed in *Polarization-Independent, Multifunctional All-Fiber Comb Filter Using Variable Ratio Coupler-Based Mach–Zehnder Interferometer* will be derived from the Jones calculus representing the device in question. This will, as a consequence, show the mathematical reasoning for many characteristics of the device such as the polarization-independence, and the ability to achieve the effects of a comb filter.

1 Introduction

The purpose of the device outlined in *Polarization-Independent, Multifunctional All-Fiber Comb Filter Using Variable Ratio Coupler-Based Mach–Zehnder Interferometer* is to create a tunable comb filter. The optical system of the comb filter device proposed as represented by Jones calculus is

$$\mathbf{E}_{out} = \mathbf{C}_1\mathbf{F}\mathbf{C}_2\mathbf{R}\mathbf{C}_2\mathbf{F}\mathbf{C}_1\mathbf{E}_{in} \tag{1.1}$$

where

$$\mathbf{C}_i = \begin{bmatrix} \sqrt{1-c_i} & j\sqrt{c_i} \\ j\sqrt{c_i} & \sqrt{1-c_i} \end{bmatrix},\ \mathbf{F} = \begin{bmatrix} e^{j\phi} & 0 \\ 0 & 1 \end{bmatrix},\ \mathbf{R} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

where c_i is the coupling ratio for coupler $\mathbf{C}_i$, j is the imaginary unit $\sqrt{-1}$, and ϕ is the phase difference between the two sections of the $\mathbf{F}$ component caused by the difference in length. The phase difference ϕ can be expressed as a complete phase of the wavelength, $\phi = 2\pi\lambda$.

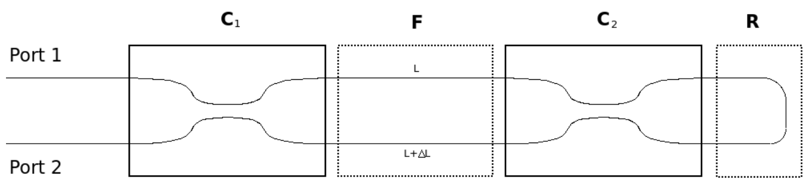

Figure 1.1: Device diagram

This device is tunable because the coupling ratios c_1 and c_2 can be changed to create a desired output.

Assuming a standard input where port 1, $E_{in\ 1}$, is used and port 2, $E_{in\ 2}$ is not

$$\mathbf{E}_{in} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $\mathbf{E}_{out}$ is calculated

$$E_{out\ 1} = 2j(e^{2j\phi}\sqrt{c_2(1-c_2)}(1-c_1)+e^{j\phi}\sqrt{c_1(1-c_1)}(1-2c_2)+(-1)\sqrt{c_2(1-c_2)}c_1 \tag{1.2}$$

$$E_{out\ 2} = 2(-1)(e^{2j\phi}+1)\sqrt{c_1c_2(1-c_2)(1-c_1)} + e^{j\phi}\sqrt{(1-2c_1)}(1-2c_2) \tag{1.3}$$

with transmission functions, $\mathbf{T} = |\mathbf{E}_{out}|^2$,

$$\begin{aligned} T_{1\ out} &= 4{c_1}^2c_2(1-c_2) \\ &+ 4c_2(1-c_1)^2(1-c_2) \\ &+ 4c_1(1-c_1)(1-2c_2)^2 \\ &- 8c_1c_2(1-c_1)(1-c_2)\cos(2\phi) \\ &+ 8(1-2c_2)(1-2c_1)\sqrt{c_1c_2(1-c_1)(1-c_2)}\cos(\phi) \end{aligned} \tag{1.4}$$

$$\begin{aligned} T_{2\ out} &= 8c_1c_2(1-c_1)(1-c_2) \\ &+ (1-2c_1)^2(1-2c_2)^2 \\ &- 8(1-2c_1)(1-2c_2)\sqrt{c_1c_2(1-c_1)(1-c_2)}\cos(\phi) \\ &+ 8c_1c_2(1-c_1)(1-c_2)\cos(2\phi) \end{aligned} \tag{1.5}$$

2 Analysis

For practical uses, only the output from port 1 will be considered. Looking at (1.4), it is clear that the first 3 terms

$$A = 4{c_1}^2c_2(1-c_2) + 4c_2(1-c_1)^2(1-c_2) + 4c_1(1-c_1)(1-2c_2)^2 \tag{2.1}$$

will result only in a change of amplitude on the output wave, while the two oscillating terms

$$O_1 = -8c_1c_2(1-c_1)(1-c_2)\cos(2\phi) \tag{2.2}$$

$$O_2 = 8(1-2c_2)(1-2c_1)\sqrt{c_1c_2(1-c_1)(1-c_2)}\cos(\phi) \tag{2.3}$$

will be the 2 waves that the comb filter superimposes. This effect is clearly displayed when plotting the terms individually against their transmission function (1.4).

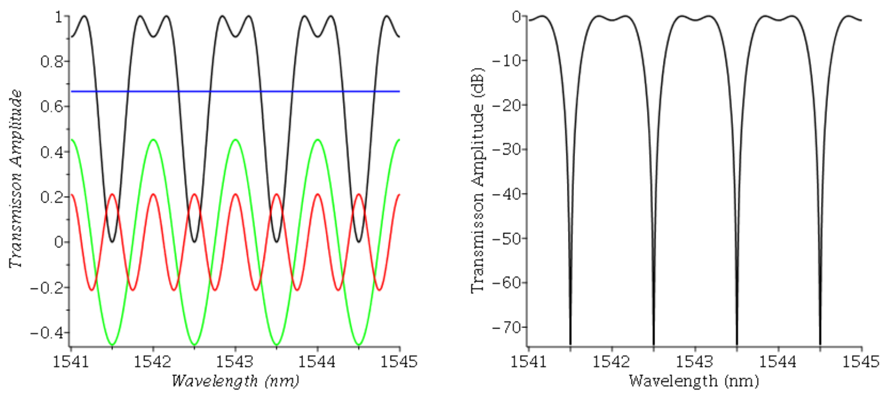

Figure 2.1: Plot of transmission function against each of it's components.
$c_1 = 0.20$, $c_2 = 0.21$
●(1.4) ●(2.1) ●(2.2) ●(2.3)

The shape of the transmission curve is thus determined by the magnitude of (2.2) and (2.3). To control the amplitudes of (2.2) and (2.3), it is important to make note of when their oscillation amplitudes are 0 and maximized. Solving for (2.2) having no amplitude

$$0 = -8c_1c_2(1-c_1)(1-c_2)\cos(2\phi)$$

$$\Rightarrow c_1 \; or \; c_2 = \{0, \; 0.5, \; 1\} \tag{2.4}$$

and (2.2) having a maximized amplitude

$$0 = \frac{d}{dc_1}\frac{d}{dc_2} - 8c_1c_2(1-c_1)(1-c_2)\cos(2\phi)$$

$$\Rightarrow c_1 \; and \; c_2 = \{\frac{2-\sqrt{2}}{4}, \; \frac{2+\sqrt{2}}{4}\} \approx \{0.854, \; 0.146\} \tag{2.5}$$

For (2.3) having no amplitude

$$0 = 8(1-2c_2)(1-2c_1)\sqrt{c_1c_2(1-c_1)(1-c_2)}$$

$$\Rightarrow c_1 \ or \ c_2 = \{0,\ 1\} \tag{2.6}$$

and (2.3) having a maximized amplitude

$$0 = \frac{d}{dc_1}\frac{d}{dc_2}8(1-2c_2)(1-2c_1)\sqrt{c_1c_2(1-c_1)(1-c_2)}$$

$$\Rightarrow c_1 \ and \ c_2 = 0.5 \tag{2.7}$$

These results have a few interesting consequences. When c_1 or $c_2 = 0.5$, (2.2) will be 0 and the superimposing of (2.3) onto it will cause the transmission function to produce a wave with the amplitude of (2.3) at ϕ, only negative. (2.3) can also be maximized since the parameters for cancelling (2.2) will still be valid, meaning the transmission function can have an amplitude that is as large as (2.3) can be. Of course any transmission amplitude can be achieved since only c_1 or c_2 has to be 0.5 for the reflection effect to occur, meaning (2.3) can be set to any amplitude using whichever c_1 or $c_2 \neq 0.5$.

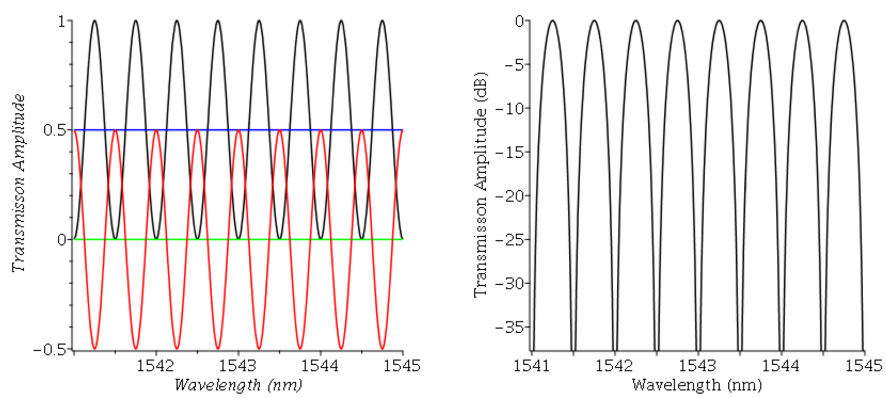

Figure 2.2: Plot of transmission function reflecting a maximized (2.3).
$c_1 = 0.50$, $c_2 = 0.50$
●(1.4) ●(2.1) ●(2.2) ●(2.3)

The transmission can also be turned off simply by setting c_1 and $c_2 = 0$ or 1.

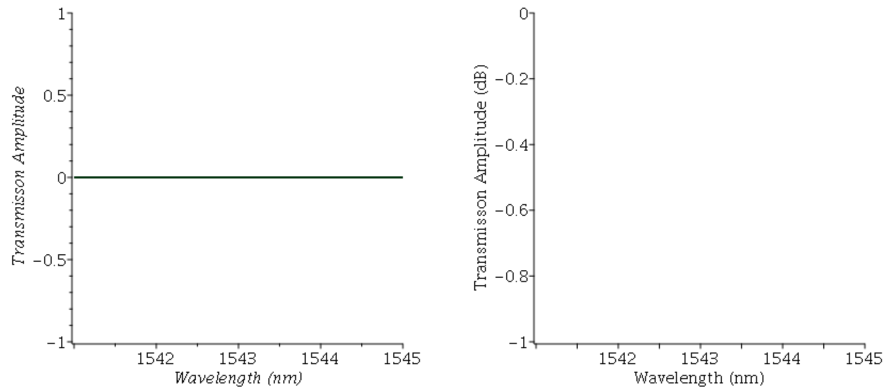

Figure 2.3: Plot of no transmission.
$c_1 = 1,\ c_2 = 0$

This effect is actually just setting the transmission to the the amplitude value found by (2.1). This means that the transmission can be set to any constant by changing c_1 or c_2 while leaving the other at 1. The variable causing (2.2) and (2.3) to be 0 can be either c_1 or $c_2 = 1$ set to 0 or 1.

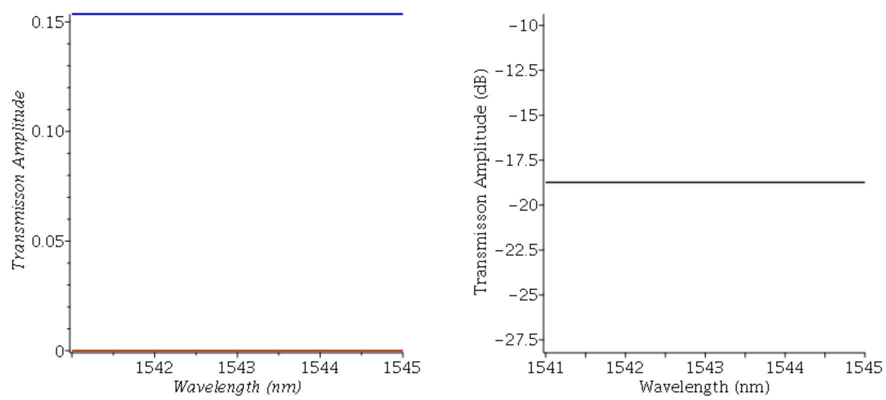

Figure 2.4: Transmission with a constant value.
$c_1 = 1,\ c_2 = 0.96$
●(1.4) ●(2.1) ●(2.2) ●(2.3)

In the above situation, the desired value, DC, can be found by solving (2.1) for c_2.

$$\begin{aligned} DC &= 4{c_1}^2 c_2(1-c_2) + 4c_2(1-c_1)^2(1-c_2) + 4c_1(1-c_1)(1-2c_2)^2 \\ &= 4(1)^2 c_2(1-c_2) + 4c_2(1-1)^2(1-c_2) + 4c_1(1-1)(1-2c_2)^2 \\ &= -4c_2(c_2-1) \end{aligned}$$

$$\Rightarrow c_2 = \frac{1 \pm \sqrt{1-DC}}{2} \tag{2.8}$$

Applying (2.8) for an amplitude of 0.15 as shown in figure 2.4 returns a solution of

$$c_2 = \frac{1 \pm \sqrt{1-0.15}}{2} \approx \{0.04,\ 0.96\}.$$

To produce a flat top wave, the difference in amplitude of (2.2) and (2.3) needs to be constant over some interval of ϕ. When they are superimposed, the subtraction of their values will be the same over the interval, making a constant output for the duration of the interval. If their difference is constant over an interval, their rate of change will be equal over the interval

$$\frac{d}{d\phi}O_1 = \frac{d}{d\phi}O_2$$

$$-16c_1c_2(1-c_1)(1-c_2)\sin(2\phi) = 8(1-2c_1)(1-2c_2)\sqrt{c_1c_2(1-c_1)(1-c_2)}\sin(\phi)$$

Solving for c_2 and setting $\phi = 0$ finds that

$$\Rightarrow c_2 = \frac{1}{2} \pm \sqrt{-c_1(c_1-1)}. \tag{2.9}$$

This shows that for any given c_1, the needed c_2 to produce a flat top transmission can be found.

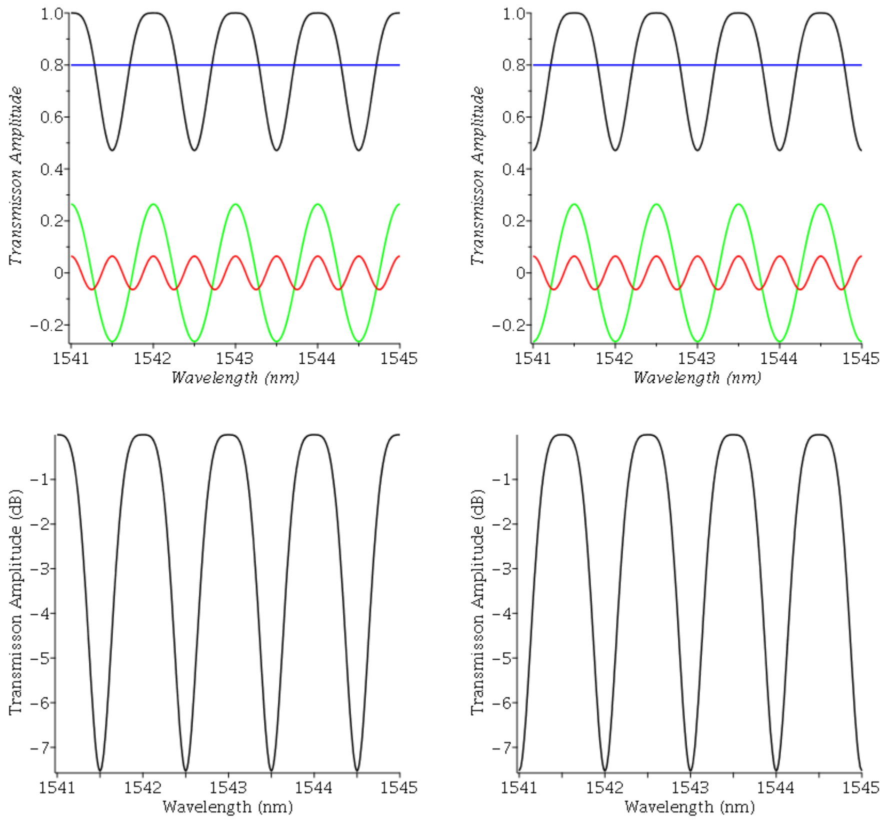

Figure 2.5: Transmission producing flat top waveform.
$c_1 = 0.30$, $c_2 = 0.04$ (Left), $c_2 = 0.96$ (Right)
●(1.4) ●(2.1) ●(2.2) ●(2.3)

Calculating c_2 as found in (2.5) using (2.9) given $c_1 = 0.30$.

$$c_2 = \frac{1}{2} \pm \sqrt{-0.30(0.30-1)} \approx \{0.042,\ 0.958\}$$

A transmission flat top with the greatest amplitude can be found by minimizing a coupler ratio in A, like c_2, and solving for c_1, then finding c_2 with (2.9).

$$0 = \frac{d}{dc_2} 4{c_1}^2 c_2(1-c_2) + 4c_2(1-c_1)^2(1-c_2) + 4c_1(1-c_1)(1-2c_2)^2$$

$$\Rightarrow c_1 = \frac{3 \pm \sqrt{3}}{6} \approx \{0.789,\ 0.211\}$$

$$c_2 = \frac{1}{2} \pm \sqrt{\{0.789,\ 0.211\}(\{0.789,\ 0.211\}-1)} \approx \{0.908,\ 0.092\}$$

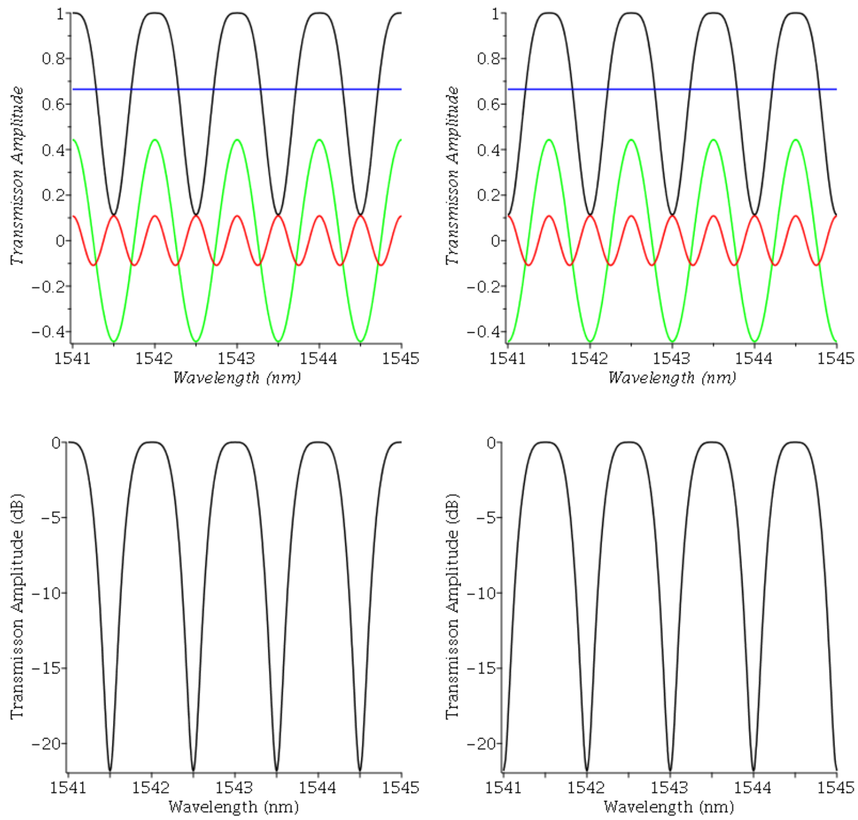

Figure 2.6: Transmission with flat top with largest amplitude.
$c_1 = 0.79$, $c_2 = 0.91$ (Left), $c_2 = 0.09$ (Right)
●(1.4) ●(2.1) ●(2.2) ●(2.3)

3 Conclusion

The key method of analysis is to break the transmission wave equation into separate oscillation and non oscillation terms, then determining the calculus based transformation required for the desired output. Using this approach various transmission forms were achieved such as flattop transmission. Equation (2.9) was derived to determine any c_1, c_2 permutation that achieves a flattop transmission, and later it was used with a minimization of a coupler component to determine a maximum amplitude flattop transmission.

References

[1] Zhi-Chao Luo, Wen-Jun Cao, Ai-Ping Luo, and Wen-Cheng XuLeslie Lamport, *Polarization-Independent, Multifunctional All-Fiber Comb Filter Using Variable Ratio Coupler-Based Mach–Zehnder Interferometer*, Journal of Lightwave Technology, vol. 30, no. 12, June 15, 2012

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