San Francisco State University
Department of Economics

Economics 701

Revealed Preference and Price Indices

Determining an individual's optimal consumption bundles requires that we have information about the individual's preferences and their budget constraint. The theory of revealed preference reverses this process and asks whether we can use an individual's optimal consumption bundle and their budget constraint to infer something about their preferences. Note that quantities consumed (i.e., optimal consumption bundles) and prices and incomes (i.e., budget constraints) are observable, whereas preferences are not. The question addressed by revealed preference then, is whether or not we can say something about an individual's unobservable preferences based on things that are observable.

The main question we'll be interested in here is whether or not we can tell if someone is made better or worse off when prices and/or their income changes. Some changes in an individual's budget constraint produce unambiguous changes in welfare. For example, if income increases while prices remain fixed, the budget constraint shifts outward in a parallel fashion, and there is no doubt that the individual will move onto a higher indifference curve and therefore be made better off. If the price of X falls while income and the price of Y remain fixed, we can also infer that the consumer will move to a higher indifference curve and be made better off (assuming the individual originally consumed a positive amount of X to begin with).

Some changes to the budget constraint, however, produce potentially ambiguous changes in welfare. Consider the graphs below. In each case, the original budget constraint, BC_0shifts to BC_1. This change could occur, for example, if the price of Y increased and the price of X decreased at the same time. Consider Person I and Person II. Suppose both individuals have identical budget constraints. Futhermore, suppose that as a result of the price/income changes, we observe that Person I changes his consumption bundle from A to B as represented by the left graph. Although we do not observe Person I's preferences/indifference curves, revealed preference allows us to deduce that Person I is made worse off by the price/income changes. Suppose that as a result of the price/income changes, we observe that Person II changes her consumption from C to D as represented by the right graph. In this case, we can apply revealed preference to conclude that she is made better off by the changes. Thus, while we will never observe an individual's indifference curves, we can observe the choices the individual makes both before and after a change to their budget constraint. This may help us to determine something about the change in their welfare.

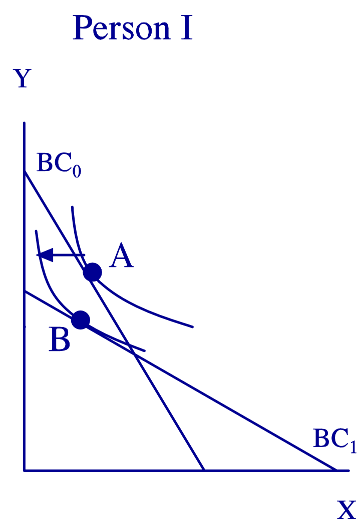
We say that an individual **reveals a preference for bundle A over bundle B if both A and B are affordable and the individual chooses A.** Affordability is crucial here. For example, if we observe an individual choosing A but B is NOT affordable, we cannot say the individual has revealed a preference for A. Notice that because affordability is crucial, revealed preference depends critically on the budget constraint, as compared to preferences, which are independent of the constraint.

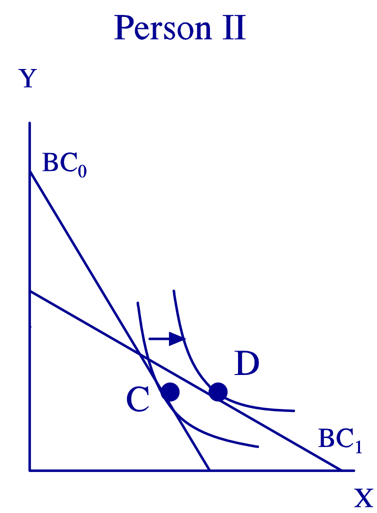
Revealed preference then also suggests that **if we ever observe an individual revealing a preference for bundle A over B, then if under a different budget constraint we observe the individual consuming B, the individual must be worse off than they were under the initial budget constraint**.

Consider the situation for Person I in the left graph above. The change in the budget constraint has caused the individual to change consumption from bundle A to bundle B. Since the individual originally consumed A when B was also affordable, Person I has revealed a preference for A over B. The change in consumption from A to B has thus made Person I worse off.

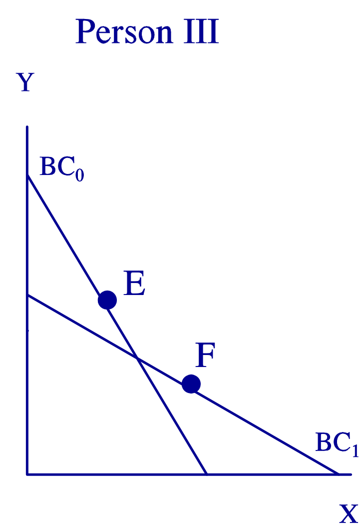
Consider the situation for Person II in the right graph above. In this case, the change in the budget constraint has caused the individual to change consumption from bundle C to bundle D. Since under the new constraint, C is also affordable but Person II chose D instead, Person II is revealing a preference for D over C and therefore must be better off.

Because revealed preference has allowed us to deduce the direction of change in each individual's welfare, we can then further deduce that their respective indifference curves must look like:

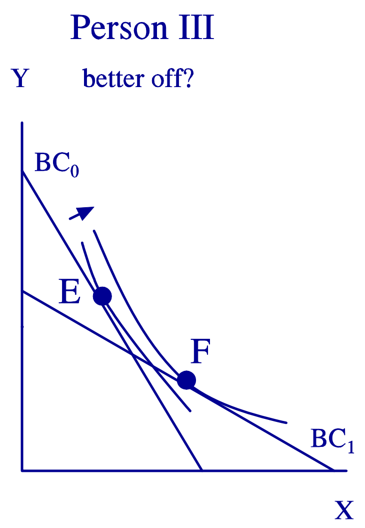


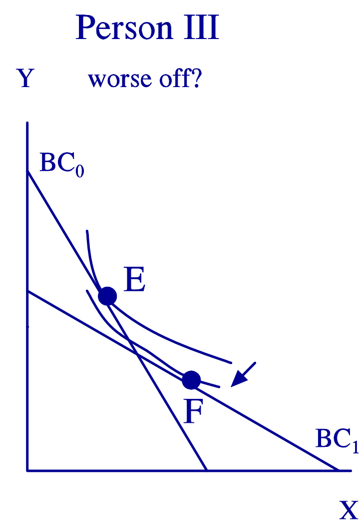


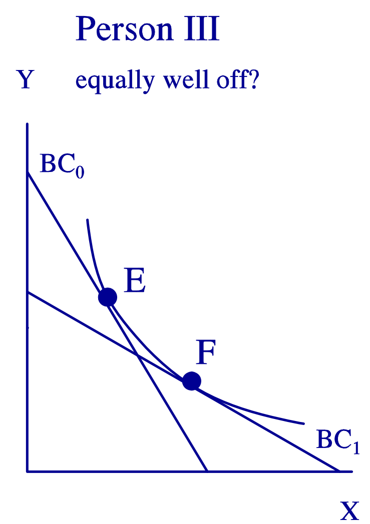
Unfortunately, we cannot always use revealed preference to determine the change in an individual's welfare. Consider the following graph:



Here, the change in the budget constraint is exactly as in the above graphs. However, we now observe the individual choosing E under the original constraint and F under the new constraint. Can we tell if they are better or worse off? The answer is no. In this case, the individual never reveals a preference for E over F or for F over E. When E is chosen, F is not affordable. Similarly, when F is chosen, E is not affordable. Therefore, we are unable to determine the change in this individual's welfare based on observables alone. The graphs below illustrate three different possibilities:







The lesson here is that sometimes it is impossible to tell from observed choices and budget constraints whether individuals are made better or worse off.

Weak Axiom of Revealed Preference (WARP)

Define:

$\mathbf{x_0}$ = vector of quantities of n goods consumed at time t = 0

$\mathbf{x_1}$ = vector of quantities of n goods consumed at time t = 1

$\mathbf{p_0}$ = vector of prices of the n goods at time t = 0

$\mathbf{p1}$ = vector of prices of the n goods at time t = 1

Suppose a consumer chooses bundle $\mathbf{x_0}$ at prices $\mathbf{p_0}$ and chooses bundle $\mathbf{x_1}$ at prices $\mathbf{p_1}$. Then WARP states: if $\mathbf{p_0} \cdot \mathbf{x_1} \leq \mathbf{p_0} \cdot \mathbf{x_0}$ then it must be the case that $\mathbf{p_1} \cdot \mathbf{x_0} > \mathbf{p_1} \cdot \mathbf{x_1}$ (note that we are using vector dot products). That is, if $\mathbf{x_1}$ was affordable in time t = 0 (and therefore the consumer revealed a preference for $\mathbf{x_0}$ over $\mathbf{x_1}$ in time t = 0), then it must be the case that $\mathbf{x_0}$ is no longer affordable in time t = 1. In words, this means that if a consumer ever reveals a preference for one bundle ($\mathbf{x_0}$) over another

($\mathbf{x_1}$), then if the consumer is ever observed to choose the other bundle ($\mathbf{x_1}$), it must be because $\mathbf{x_0}$ is no longer affordable.

Price Indices and Revealed Preference

Price indices are used to measure the "average" level of prices. During any given time span, the prices of some goods may be rising while the prices of other goods may be falling. A change in the price index tells us how prices, on average have changed.

A price index is formed by taking the ratio of the expenditures necessary at some time t to purchase some bundle of goods to the expenditure that was required to purchase the same bundle of goods during a given base year. There are two main types of indices, the Laspeyres index and the Paache index. These indices differ depending on what year's bundle of goods is being used in the calculation of the index. The Laspeyres index is calculated holding the base year's bundle fixed, while the Paasche index is calculated holding the year t bundle fixed. The Consumer Price Index (CPI) is a type of Laspeyres index, in which the bundle of goods being held fixed is the base year bundle. Presently, the base year is an average over 1982–1984.

Define:
$\mathbf{x_b} =$ vector of quantities of n goods consumed in a given base year
$\mathbf{x_t} =$ vector of quantities of n goods consumed in year t
$\mathbf{p_b} =$ vector of base-year prices of the n goods
$\mathbf{p_t} =$ vector of year t prices of the n goods

The Laspeyres index (e.g., Consumer Price Index) would be calculated as:

$$L = \frac{\mathbf{p_t} \cdot \mathbf{x_b}}{\mathbf{p_b} \cdot \mathbf{x_b}}$$

and the Paasche index (e.g. GDP deflator) would be calculated as:

$$P = \frac{\mathbf{p_t} \cdot \mathbf{x_t}}{\mathbf{p_b} \cdot \mathbf{x_t}}$$

When can we use such price indicies to determine changes in welfare? Consider the Laspeyres index. Let I_b and I_t represent incomes during the base year and year t, respectively. The idea is that if $L > \frac{I_t}{I_b}$, prices have risen on average more than our income between the base year and year t so we'd like to conclude that we are worse off. If, on the other hand, $L < \frac{I_t}{I_b}$, prices have risen on average less than our incomes and we'd like to conclude that we are better off.

Consider the conditon $L < \frac{I_t}{I_b}$. Subbing for L gives us $\frac{\mathbf{p_t} \cdot \mathbf{x_b}}{\mathbf{p_b} \cdot \mathbf{x_b}} < \frac{I_t}{I_b}$. But because $\mathbf{p_b} \cdot \mathbf{x_b}$ is simply equal to I_b (income in the base year, since expenditure is equal to income) the condition becomes: $\frac{\mathbf{p_t} \cdot \mathbf{x_b}}{I_b} < \frac{I_t}{I_b}$ or simply: $\mathbf{p_t} \cdot \mathbf{x_b} < I_t$. This implies that the base year bundle IS affordable in year t. Therefore we must be better off in year t, since we have revealed a preference for the year t bundle over the base year bundle (in year t both bundles were affordable, but we chose $\mathbf{x_t}$ over $\mathbf{x_b}$.

Now consider the conditon $L > \frac{I_t}{I_b}$. Applying the same analysis above leads us to the final condition of $\mathbf{p_t} \cdot \mathbf{x_b} > I_t$. This implies that the base year bundle is NOT affordable in year t. However, we cannot conclude that we are worse off necessarily because we cannot apply revealed preference. For example, consider Person 3's situation from above. As a result of price/income changes, they altered their consumption bundle from E to F. If we were to calculate the Laspeyres index for this person (where E is the base year bundle), we'd find $L > \frac{I_t}{I_b}$ or equivalently, $\mathbf{p_t} \cdot \mathbf{x_b} > I_t$, since the original bundle E is no longer affordable for the individual. However, as we also saw, this individual may indeed be better off.

As we know from revealed preference, there are some situations in which it is impossible to determine the change in an individual's welfare from observables alone. Price indices do not escape this fact. **Laspeyres indices (e.g. the Consumer Price Index) tends to understate our welfare and therefore could even suggest that we are worse off when in fact we are not. On the other hand, if the Laspeyres index indicates that we are better off, then we in fact are better off.**

The reason for the ambiguity lies in the basic construction of the Laspeyres price index. By assuming that our year t consumption bundle is the same as what we consumed in the base year, **the index misses the fact that in response to price changes, we make welfare improving substitutions in consumption.** Thus, the index tends to understate our welfare and could in fact suggest that we are worse off when in reality we are not.