Solutions - Problem Set 12

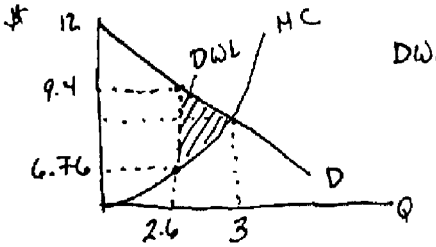
① a) $(12-Q)Q - [\frac{1}{3}Q^3+2] = \pi = 12Q - Q^2 - \frac{1}{3}Q^3 - 2$

$\frac{d\pi}{dQ} = 12 - 2Q - Q^2 = 0$ which has positive root

of $Q^M = \boxed{2.6}$ $P^M = 12 - 2.6 = \boxed{9.4}$ $\pi = 9.4 \times 2.6 - [\frac{1}{3}(2.6)^3 + 2] = \boxed{16.58}$

b) $MC = \frac{dSTC}{dQ} = Q^2$ $P = MC \Rightarrow 12 - Q = Q^2$

$\Rightarrow Q^2 + Q - 12 = 0$ which has positive root of $\boxed{Q^C = 3}$

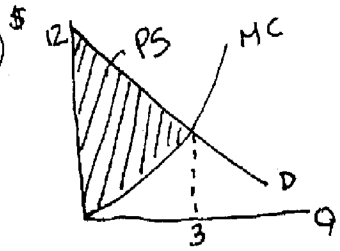


$DWL = \int_{2.6}^{3} [(12-Q) - Q^2]\,dQ$

$= 12Q - \frac{1}{2}Q^2 - \frac{1}{3}Q^3 \Big]_{2.6}^{3} = \boxed{.536}$

c) The firm will produce at the efficient output level, $Q = 3$.

d)



$PS = \int_{0}^{3} [(12-Q) - Q^2]\,dQ$

$= 12Q - \frac{1}{2}Q^2 - \frac{1}{3}Q^3 \Big]_{0}^{3} = \boxed{22.5}$

Since fixed costs here are 2,

and PS: π + Fixed cost, (in short-run)

$\boxed{\pi = 20.5}$

e) The value of social welfare at the efficient level of output (3) is the same as the producer surplus in part d: 22.5

2.

a) The IR (or participation) constraint ensures the agent will accept the contract. This means the utility from the contract must exceed reservation utility (which is zero here).

$$I = S - c(e) = a + b\pi_g - ce^2 = a + b\gamma e + b\epsilon - ce^2$$

$$E(I) = a + b\gamma e - ce^2, \text{ since } E(b\epsilon) = 0$$

$$Var(I) = b^2 Var(\epsilon) = b^2\sigma^2$$

So, $u(I) = a + b\gamma e - ce^2 - rb^2\sigma^2 \geq 0 \Rightarrow$ IR constraint

b) The ICC constraint ensures that the agent will choose the level of effort that the owner wants him/her to. The manager will rationally choose effort to maximize $u(I)$:

$\dfrac{du(I)}{de} = b\gamma - 2ce = 0 \Rightarrow e^*(b) = b\gamma/2c \rightarrow$ IC constraint
$\rightarrow$ optimal effort is increasing in b

c) $\pi_n = \pi_g - S = \pi_g - a - b\pi_g = (1-b)\pi_g - a = (1-b)(\gamma e + \epsilon) - a$

and $E[\pi_n] = (1-b)\gamma e - a$, since $E(\epsilon) = 0$

From the IR constraint, which we know will hold with equality, we have that $-b\gamma e - a = -ce^2 - rb^2\sigma^2$. Substituting into $E[\pi_n] \Rightarrow E[\pi_n] = \gamma e - ce^2 - rb^2\sigma^2$. Then, from the IC constraint we can sub $b\gamma/2c$ for e: $E[\pi_n] = \dfrac{b\gamma^2}{2c} - c\left(\dfrac{b\gamma}{2c}\right)^2 - rb^2\sigma^2$

$$\frac{dE[\pi_n]}{db} = \frac{\gamma^2}{2c} - 2c\left(\frac{b\gamma}{2c}\right)\left(\frac{\gamma}{2c}\right) - 2rb\sigma^2 = 0$$

$$\Rightarrow \gamma^2/2c = b\left(\gamma^2/2c + 2r\sigma^2\right) = b\left(\frac{\gamma^2 + 4rc\sigma^2}{2c}\right)$$

$$\Rightarrow b^* = \frac{\gamma^2}{\gamma^2 + 4rc\sigma^2}$$

d) As the agent becomes more risk averse $\Rightarrow r\uparrow \Rightarrow b^*\downarrow$