Economics 701

Problem Set 12

1. A monopolist faces market demand given by: $P = 12 - Q$. The firm has short–run total cost given by $STC(Q) = \frac{1}{3}Q^3 + 2$.
a. What output and price will the monopolist choose to maximize profit? What is the value of profit?

For the following questions it might be helpful to use a graph, showing the firm's $MC(Q)$ curve (derive it!) and demand.

b. What is the competitive (efficient) level of output? Compute the deadweight loss of monopoly (note: because of nonlinearity, you will need to use integration).
c. Suppose the firm is able to perfectly price discriminate. At what output will the firm produce?
d. Compute the values of producer surplus and firm profit at the perfect price discrimination outcome in part c.
e. What is the value of total social welfare at the perfect price discrimination outcome and how does it compare to total social welfare at the efficient output level in part b?

2. The gross profit of a risk–neutral firm owner (principal) is related to the effort of a risk–averse manager (agent) according to: $\Pi_g = \gamma e + \epsilon$, where e represents the manager's effort level, $\gamma > 0$ is a constant and ϵ is a random term distributed $N(0, \sigma^2)$. The owner compensates the manager with a linear salary contract of the form $S = a + b\Pi_g$, a,$b \geq 0$. The manager has mean–variance utility given by: $U = E(I) - rVar(I)$, where I is the manager's net income, and $r \geq 0$ measures the degree of risk aversion. The manager's net income is given by $S - C(e)$, where $C(e)$ is the "cost" of effort given by $C(e) = ce^2$, $c > 0$. The owner chooses a and b to maximize expected net profit, $E[\Pi_n]$ (the expected value of gross profit minus salary paid) subject to the individual rationality (participation) and the incentive compatibility constraints. Assume that the reservation utility of the agent is zero.
a. Show that the individual rationality (IR) constraint is: $a + b\gamma e - rb^2\sigma^2 - ce^2 \geq 0$
b. Show that the incentive compatibility (IC) constraint is $e^*(b) = \frac{b\gamma}{2c}$. What happens to the agent's optimal effort level as the power of the incentive scheme is increased?
c. Show that the optimal value for b is $b^* = \frac{\gamma^2}{[\gamma^2 + 4rc\sigma^2]}$
d. What happens to the value of the optimal value of b as the agent becomes more risk averse?