Back to Assignment

Attempts | 6 | - | - | Average 6 / 7

1. Problems 2.1

Suppose $U(x,y) = 4x^2 + 3y^2$

Therefore, $\frac{\partial U}{\partial x} =$ $8x$ ✔ , while $\frac{\partial U}{\partial y} =$, $6y$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

For a function of several variables, $y = f(x_1, x_1, ..., x_n)$, the partial derivative of y with respect to x_1 (denoted by $\frac{\partial y}{\partial x_1}, \frac{\partial f}{\partial x_1}, f_{x_1}$ or f_1), is calculated by taking the derivative of the function with respect to x_1, while holding all the other $x's$ constant. The same is true of the partial derivative of $y = f(x_1, x_1, ..., x_n)$, the partial derivative of y with respect to any of the other $x's$.

In this case, you can calculate $\frac{\partial U}{\partial x}$ by taking the derivative of U with respect to x, while treating y as constant.

$$\begin{aligned}\frac{\partial U}{\partial x} &= \frac{d(4x^2+3y^2)}{dx} \\ &= \frac{d(4x^2)}{dx} + \frac{d(3y^2)}{dx} \\ &= \frac{d(4x^2)}{dx} + 0 \textit{ because } y \textit{ (and therefore } 3y^2\textit{) is treated as a constant} \\ &= 8x\end{aligned}$$

Similarly, you can calculate $\frac{\partial U}{\partial y}$ by taking the derivative of U with respect to y, while treating x as constant.

$$\begin{aligned}\frac{\partial U}{\partial y} &= \frac{d(4x^2+3y^2)}{dy} \\ &= \frac{d(4x^2)}{dy} + \frac{d(3y^2)}{dy} \\ &= 0 + \frac{d(3y^2)}{dx} \textit{ because } x \textit{ (and therefore } 4x^2\textit{) is treated as a constant} \\ &= 6y\end{aligned}$$

At $x = 1$, $\frac{\partial U}{\partial x} =$ 0 ✘ , while at $y = 2$, $\frac{\partial U}{\partial y} =$ 0 ✘ .

Points: 0 / 1

Explanation: Close Explanation

Because $\frac{\partial U}{\partial x} = 8x$, plugging $x = 1$ into the partial derivative yields $\frac{\partial U}{\partial x} = 8(1) = 8$.

Because $\frac{\partial U}{\partial y} = 6y$, plugging $y = 2$ into the partial derivative yields $\frac{\partial U}{\partial y} = 6(2) = 12$.

Which of the following represents the total differential for U?

- ○ $dU = 8x + 6y$
- ✔ ◉ $dU = 8x\,dx + 6y\,dy$
- ○ $dU = 4x^2dx + 3y^2\,dy$
- ○ $dU = 6x\,dx + 8y\,dy$

Points: 1 / 1

Explanation: Close Explanation

For a function of several variables, $y = f(x_1, x_1, ..., x_n)$, the total differential is defined as:

$$dy = \frac{df}{dx_1}\, dx_1 + \frac{df}{dx_2}\, dx_2 + ... + \frac{df}{dx_n}\, dx_n$$

or

$$dy = f_1\, dx_1 + f_2\, dx_2 + ... + f_n\, dx_n$$

This equation states that the total change in a function is simply the sum of changes brought about by varying each of the independent variables.

In this case, because $\frac{\partial U}{\partial x} = 8x$ and $\frac{\partial U}{\partial y} = 6y$, the total differential for U can be written as:

$$\begin{aligned} dU &= \frac{dU}{dx}\, dx + \frac{dU}{dy}\, dy \\ &= 8x\, dx + 6y\, dy \end{aligned}$$

If $dU = 0$, then $\frac{\partial y}{\partial x} =$ $-\frac{4x}{3y}$ ✔.

Points: 1 / 1

Explanation: Close Explanation

If $dU = 0$, then from the total differential we have:

$$\begin{aligned} 0 &= 8x\, dx + 6y\, dy \\ -6y\, dy &= 8x\, dx \\ \frac{\partial y}{\partial x} &= -\frac{8x}{6y} \\ \frac{\partial y}{\partial x} &= -\frac{4x}{3y} \end{aligned}$$

Therefore, because $\frac{\partial y}{\partial x} = -\frac{4x}{3y}$ with the condition that there is no change in U (because $dU = 0$), $-\frac{4x}{3y}$ represents the implied trade-off between x and y while holding U constant.

In general for an implicit function of two variables x and y, this implied trade-off is simply the negative ratio of the partial derivatives of the implicit function. That is $\frac{\partial y}{\partial x} = -\frac{f_x}{f_y}$.

When $x = 1$ and $y = 2$, $U = 16$.

This can be shown by plugging $x = 1$ and $y = 2$ into $U(x,y)$ ✔

Points: 1 / 1

Explanation: Close Explanation

$U = 16$ when $x = 1$ and $y = 2$. This can be shown by plugging $x = 1$ and $y = 2$ into the function for U ,$U(x,y) = 4x^2 + 3y^2$, to obtain the following:

$$
\begin{aligned}
U(1,2) &= 4(1^2) + 3(2^2) \\
&= 4(1) + 3(4) \\
&= 4 + 12 \\
&= 16
\end{aligned}
$$

For movements away from $x = 1$ and $y = 2$, x and y must change in a ratio (x to y) of $\frac{-2}{3}$ ✔, in order for U to remain constant at 16.(**Hint**: In other words, if there is a one unit increase in x, how much must y change to keep U constant at 16.)

Points: 1 / 1

Explanation: Close Explanation

If U is to remain unchanged at 16, then $dU = 0$. Therefore, from the total differential, we have:

$$
\begin{aligned}
dU &= \frac{dU}{dx}\,dx + \frac{dU}{dy}\,dy \\
0 &= 8x\,dx + 6y\,dy \\
-6y\,dy &= 8x\,dx \\
\frac{\partial y}{\partial x} &= -\frac{8x}{6y} \\
\frac{\partial y}{\partial x} &= -\frac{4x}{3y}
\end{aligned}
$$

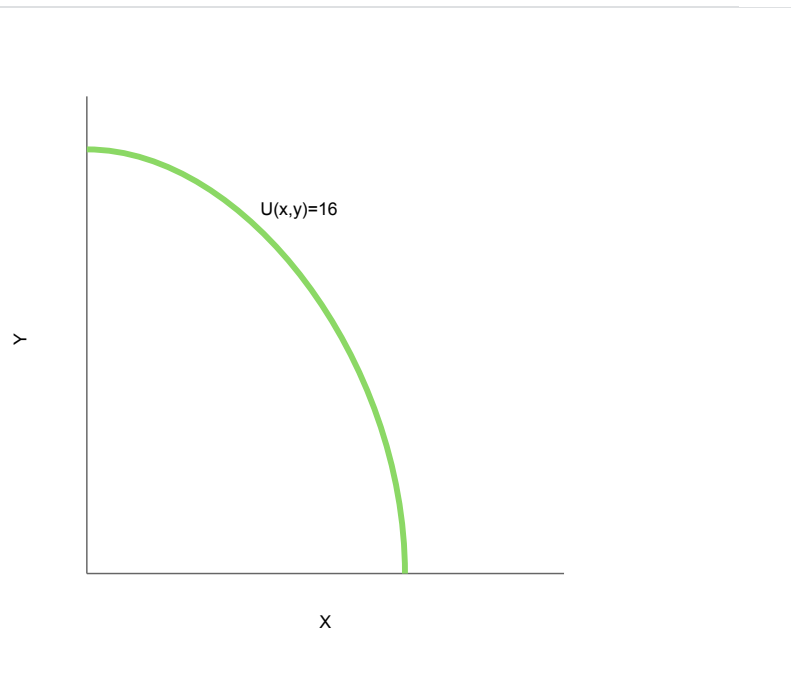
You can also use the fact that $\frac{\partial y}{\partial x} = -\frac{f_x}{f_y} = -\frac{4x}{3y}$.

Then, plugging in $x = 1$ and $y = 2$:

$$
\begin{aligned}
\frac{\partial y}{\partial x} &= -\frac{4 \cdot 1}{3 \cdot 2} \\
&= \frac{-4}{6} \\
&= \frac{-2}{3}
\end{aligned}
$$

Therefore, for movement away from the point $(1,2)$, x and y must change in a ratio of $\frac{\partial y}{\partial x} = \frac{-2}{3}$ in order to keep U constant at 16.

The following curve approximates the contour line for $U = 16$ in the first quadrant.



Which of the following represents the slope of this contour line?

- ○ $-\frac{4y}{3x}$
- ○ $\frac{4}{3}$
- ✓ ◉ $-\frac{4x}{3y}$
- ○ $\frac{4x}{3y}$

Points: 1 / 1

Explanation: Close Explanation

The $U = 16$ contour line is an ellipse centered at the origin, and its slope at any point is given by $\frac{\partial y}{\partial x}$ with no change in U ($dU = 0$). From the total differential, we have:

$$\begin{aligned} dU &= \frac{dU}{dx}\,dx + \frac{dU}{dy}\,dy \\ 0 &= 8x\,dx + 6y\,dy \\ -6y\,dy &= 8x\,dx \\ \frac{\partial y}{\partial x} &= -\frac{8x}{6y} \\ \frac{\partial y}{\partial x} &= -\frac{4x}{3y} \end{aligned}$$

You can also use the fact that $\frac{\partial y}{\partial x} = -\frac{f_x}{f_y} = -\frac{4x}{3y}$.

Try Another Version Continue

Back to Assignment

Attempts | 3 | - | - | Average 3 / 3

2. Problems 2.2

Suppose a firm's total revenues depend on the amount produced, q, according to the function:

$$R = 70q - q^2$$

Total costs also depend on q:

$$C = q^2 + 30q + 100$$

In order to maximize profits $(R\text{-}C)$, this firm should produce at a level of $q^* =$ 10 ✔ . This will lead to a profit-level of $\pi^* =$ 100 ✔ .

Points: 1 / 1

Explanation: Close Explanation

Profits are given by $\pi = R\text{-}C$:

$$\begin{aligned} \pi &= R\text{-}C \\ &= 70q - q^2 - (q^2 + 30q + 100) \\ &= 70q - q^2 - q^2 - 30q - 100 \\ &= -2q^2 + 40q - 100 \end{aligned}$$

The maximum value is for profits occurs where the derivative of profits with respect to q equals 0. The derivate of profits with respect to q is $\frac{d\pi}{dq} = -4q + 40$. Therefore:

$$\begin{aligned} -4q + 40 &= 0 \\ -4q &= -40 \\ q &= 10 \end{aligned}$$

Therefore the level of output that maximizes profits is $q^* = 10$. Plugging $q^* = 10$ into the profit function yields the maximum level of profit achieved (π^*):

$$\begin{aligned} \pi &= -2q^2 + 40q - 100 \\ \pi^* &= -2q^{*2} + 40q^* - 100 \\ \pi^* &= -2 \cdot 10^2 + (40 \cdot 10) - 100 \\ \pi^* &= -200 + 400 - 100 \\ \pi^* &= 100 \end{aligned}$$

At the profit maximizing level of output, $q = q^*$, the second derivative of profits with respect to q is $\frac{d^2\pi}{dq^2} =$ -4 ✔ , which meets the second ✔ order condition for a maximum.

Points: 1 / 1

Explanation: Close Explanation

For functions of a single variable, the first order condition for a maximum is that the first derivative is equal to 0. The first order condition alone, however, does not guarantee the function is at a maximum. The second order condition, which requires that the second derivative is negative, is also needed. In this case, this means that $\frac{d\pi}{dq} = -4q + 40 = 0$ does not guarantee a maximum, since the second order condition must be checked. Taking the second derivative of profits with respect to q yields $\frac{d^2\pi}{dq^2} = -4$. Since the second derivative of this function is always negative, it is negative at $q^* = 10$, and the second order condition is satisfied.

Therefore, since at $q^* = 10$, $\frac{d\pi}{dq} = -4q + 40 = 0$ and $\frac{d^2\pi}{dq^2} = -4 < 0$, the point where $q^* = 10$ and $\pi^* = 100$ must be a maximum.

True or False: The profit-maximizing level of output, $q = q^*$, equates marginal revenue with marginal cost.

- ✔ (●) True
- ( ) False

Points: 1 / 1

Explanation: Close Explanation

Marginal revenue is the change in revenue resulting from a change in quantity of output q. This is equal to the derivative of the total revenue function R with respect to q, which yields $\frac{dR}{dq} = 70 - 2q$. At $q = q^* = 10$, $\frac{dR}{dq} = 50$.

Marginal cost is the change in total cost resulting from a change in quantity of output q. This is equal to the derivative of the cost function C with respect to q, which yields $\frac{dC}{dq} = 2q + 30$. At $q = q^* = 10$, $\frac{dC}{dq} = 50$.

Marginal revenue equal to marginal cost implies that any more output will leave profit unchanged. If marginal revenue was greater than marginal cost, then profit could be increased by increasing output. If marginal revenue were lower than marginal cost, then reducing output would increase profit. Only when marginal cost and marginal revenue are identical is profit maximized.

Therefore, the profit maximizing quantity of output $q = q^*$ does equate marginal revenue with marginal cost at a level of $MR = MC = 50$.

Try Another Version Continue

Back to Assignment

Attempts | 5.5 | - | - | Average 5.5 / 6

3. Problems 2.3

Suppose that $f(x,y)=xy$, with the constraint that x and y are constrained to sum to 1. That is, $x + y = 1$.

Given this constraint, which of the following functions of x is equivalent to the original function $f(x,y)=xy$?

- ✔ ◉ $\tilde{f}(x) = x - x^2$
- ○ $\tilde{f}(x) = 1 - x$
- ○ $\tilde{f}(x) = x^2$
- ○ $\tilde{f}(x) = x + x^2$

Points: 1 / 1

Explanation: Close Explanation

By subtracting x from each side of the constraint, the constraint that $x + y = 1$ can be re-written as $y = 1 - x$. Substituting this constraint into $f(x,y)$ yields a $\tilde{f}(x) = f(x,1\text{-}x) = x(1\text{-}x) = x - x^2$. Substituting the constraint allows you to re-write $f(x,y)=xy$ as a function of one variable, x. With a simple function of one variable, like $\tilde{f}(x)$, it can be easier to solve a maximum by setting the first derivative equal to 0 (while also checking the second order condition that the second derivative is less than 0).

Using the first order condition that $\widetilde{f'}(x) = 0$, the value of x that maximizes $\tilde{f}(x)$ (and $f(x,y)$) is $x^* =$ **0.5** ✔ . Given this value of x^*, the corresponding value of y that maximizes $f(x,y)$ is $y^* =$ **0.5** ✔ . This means that the maximum value of $f(x,y)$ is $f(x^*,y^*) =$ **0.25** ✔ . The second derivative of $\tilde{f}(x)$, $\widetilde{f''}(x) =$, evaluated at x^*, is equal to $\widetilde{f''}(x^*) =$ **-2** ✔ , which satisfies the second order condition for a maximum.

Points: 1 / 1

Explanation: Close Explanation

The first order condition for a maximum (for a function of one variable) requires that the first derivative is equal to 0. In this case, the first derivative is $\widetilde{f'}(x) = 1 - 2x$. Setting this first derivative equal to 0 yields:

$$\begin{aligned} \widetilde{f'}(x) &= 1 - 2x \\ 0 &= 1 - 2x \\ 2x &= 1 \\ x &= \tfrac{1}{2} \text{ or } 0.5 \end{aligned}$$

Therefore, the value of x that maximizes $\tilde{f}(x)$ is $x^* = 0.5$. Plugging $x^* = 0.5$ into the constraint that $x + y = 1$ implies that $y = 1 - 0.5 = 0.5$. So $y^* = 0.5$ along with $x^* = 0.5$, maximizes $f(x,y)$. Plugging x^* and y^* into $f(x,y)$ yields the maximum value of $f(x^*,y^*) = x^*y^* = 0.5 \times 0.5 = 0.25$.

The second order condition for a maximum requires that the second derivative be less than 0. In this case, the second derivative is $\widetilde{f''}(x^*) = -2$. Since this second derivative is always negative, the second order condition is satisfied and $\tilde{f}(x^*) = 0.25$ does indeed represent a maximum.

The langrange method can also be used to solve this constrained maximization problem.

The langrangian for this constrained maximization problem is $L = xy + \lambda(1\text{-}x\text{-}y)$ ✔

Points: 1 / 1

Explanation: Close Explanation

When maximizing a function $y = f(x_1, x_2, ..., x_n)$ subject to a constraint of the independent variables $g(x_1, x_2, ..., x_n) = 0$, the langrangian multiplier method starts by setting up the following expressoin:

$$L = f(x_1, x_2, ..., x_k) + \lambda g(x_1, x_2, ..., x_n)$$

Where λ is an additional variable called the langranian multiplier. Notice when the constraint is satisfied the langranian L and the function f have the same value since $g(x_1, x_2, ..., x_n) = 0$.

Which of the following are the first order conditions for a critical point for the langranian function L? *Check all that apply.*

- ✔ ☑ $\frac{dL}{dy} = x - \lambda = 0$
- ✔ ☑ $\frac{dL}{d\lambda} = 1 - x - y = 0$
- ✔ ☑ $\frac{dL}{dx} = y - \lambda = 0$
- ✔ ☐ $\frac{dL}{dx} = 1 + x = 0$
- ✔ ☐ $\frac{dL}{dx} = x\lambda = 0$

Points: 1 / 1

Explanation: Close Explanation

When maximizing a function $y = f(x_1, x_2, ..., x_n)$ subject to a constraint of the independent variables $g(x_1, x_2, ..., x_n) = 0$, using the langranian method requires that teh first order conditions for a critical point are satisfied:

$$\frac{dL}{dx_1} = f_1 + \lambda g_1 = 0$$

$$\frac{dL}{dx_1} = f_2 + \lambda g_2 = 0$$

...

$$\frac{dL}{dx_1} = f_n + \lambda g_n = 0$$

$$\frac{dL}{d\lambda} = g(x_1, x_2, ..., x_n) = 0$$

In other words, the first order condtions require that each partial derivative of the langranian L with respect to each of the variables $x_1, x_2, ..., x_n$ and λ, must all be equal to 0.

In this case, taking the derivative of the langranian L with respect to x, y and λ, and setting equal to 0 yields the following first order conditions:

I. $\frac{dL}{dx} = y - \lambda = 0$
II. $\frac{dL}{dy} = x - \lambda = 0$
III. $\frac{dL}{d\lambda} = 1 - x - y = 0$

Solving the first order conditions for this Lagranian yields $x^* =$ 1 ✗ and $y^* =$ 1 ✗. This implies that the value of the langrange multiplier $\lambda =$ 0.5 ✔. Finally, the maximum value of $f(x^*, y^*) = x^*y^* =$ 0.25 ✔.

Points: 0.5 / 1

Explanation: Close Explanation

From the previous first order conditions, I. implies that $y = \lambda$ while II. implies that $x = \lambda$. Taken together, this means that $x = y$. Plugging $x = y$ into condition III. yields:

$$\begin{aligned} 1 - x - y &= 0 \\ 1 - x - x &= 0 \\ 1 - 2x &= 0 \\ 1 &= 2x \\ x &= \tfrac{1}{2} \; or \; 0.5 \end{aligned}$$

So $x^* = 0.5$. Since $x = y$, $y^* = 0.5$. Since condition I. implies that $y = \lambda$, $\lambda = 0.5$. Plugging in $x^* = 0.5$ and $y^* = 0.5$ into f yields $f(x^*,y^*) = x^*y^* = 0.5 \times 0.5 = 0.25$, the maximum value.

In order for the second order condition for a critical point to be satisfied, $f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2$ <0 ✔ at the critical point.

Points: 1 / 1

Explanation: Close Explanation

In order for the second order condition for a critical point to be satisfied, the second total differential $d^2\,y$ must be less than 0. Therefore, $f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2$ must be less than 0.

In this case, the second partial derivatives of f with respect to x and y are both 0. That is $f_{11} = 0$ and $f_{22} = 0$. The first partial derivatives of f are $f_1 = y$ and $f_2 = x$. The partial derivative f_{12}, taking the partial first with respect to y, and then with respect to x, yields $f_{12} = 1$. Plugging in the partial derivatives yields:

$$\begin{aligned} f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2 &= 0 - 21 \times x \times Y + 0 \\ &= -2\,xy \\ &= -2 \times 0.5 \times 0.5, \; since\ x^*=0.5\ and\ y^*=0.5\ at\ the\ critical\ point \\ &= -2 \times 0.25 \\ &= -0.5 \end{aligned}$$

Therefore, at the critical point of $y^* = 0.5$ and $x^* = 0.5$, the second order condition is satisfied.

Try Another Version Continue

Back to Assignment

Attempts | 3 | - | - | Average 3 / 4

4. Problems 2.4

Suppose you would like to maximize $f(x,y)=xy$, subject to the constraint that x and y are constrained to sum to 1. That is, $x + y = 1$.

Instead of working through this maximization problem, however, you could also work through the dual problem to this, which is:

Minimize $x + y$

subject to $xy = 0.25$

Hint: Assume x and y take only positive values.

The Lagrangian for this constrained minimization in the dual problem is $L = x + y + \lambda(0.25\text{-}xy)$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

When finding the critical points of a function $y = f(x_1, x_2, \ldots, x_n)$ subject to a constraint of the independent variables $g(x_1, x_2, \ldots, x_n) = 0$, the Lagrangian multiplier method starts by setting up the following expression:

$$L = f(x_1, x_2, \ldots, x_k) + \lambda g(x_1, x_2, \ldots, x_n)$$

Where λ is an additional variable called the Lagrangian multiplier. Notice when the constraint is satisfied the Lagrangnian L and the function f have the same value because $g(x_1, x_2, \ldots, x_n) = 0$.

In this case, because $f(x,y) = x + y$ and $g(x,y) = 0.25 - xy$, the Lagrangian is $L = x + y + \lambda\,(0.25 - xy)$.

Which of the following are the first order conditions for a critical point for the Lagrangian function L? *Check all that apply.*

- ✔ ☐ $\frac{dL}{dy} = y\lambda = 0$
- ✔ ☑ $\frac{dL}{dx} = 1 - y\lambda = 0$
- ✔ ☑ $\frac{dL}{d\lambda} = 0.25 - xy = 0$
- ✔ ☐ $\frac{dL}{dx} = 1 + x = 0$
- ✔ ☑ $\frac{dL}{dy} = 1 - x\lambda = 0$

Points: 1 / 1

Explanation: Close Explanation

When finding the critical points of a function $y = f(x_1, x_2, \ldots, x_n)$ subject to a constraint of the independent variables $g(x_1, x_2, \ldots, x_n) = 0$, using the Lagranian method requires that the first order conditions for a critical point are satisfied:

$$\frac{dL}{dx_1} = f_1 + \lambda g_1 = 0$$

$$\frac{dL}{dx_1} = f_2 + \lambda g_2 = 0$$

...

$$\frac{dL}{dx_1} = f_n + \lambda g_n = 0$$

$$\frac{dL}{d\lambda} = g(x_1, x_2, ..., x_n) = 0$$

In other words, the first order conditions require that each partial derivative of the Lagrangian L with respect to each of the variables $x_1, x_2, ..., x_n$ and λ must all be equal to 0.

In this case, taking the derivative of the Lagrangian L with respect to x, y, and λ, and setting equal to 0 yields the following first order conditions:

I. $\frac{dL}{dx} = 1 - y\lambda = 0$
II. $\frac{dL}{dy} = 1 - x\lambda = 0$
III. $\frac{dL}{d\lambda} = 0.25 - xy = 0$

Solving the first order conditions for this Lagrangian yields $x^* =$ **0.5** ✔ and $y^* =$ **0.5** ✔. This implies that the value of the Lagrangian multiplier is $\lambda =$ **2** ✔. Finally, this implies the minimum value of $f(x^*,y^*) = x^* + y^* =$ **1** ✔.

Points: 1 / 1

Explanation: Close Explanation

From the previous first order conditions, I. implies that $y = \frac{1}{\lambda}$ while II. implies that $x = \frac{1}{\lambda}$. Taken together, this means that $x = y$. Plugging $x = y$ into condition III. yields:

$$\begin{aligned} 0.25 - xy &= 0 \\ 0.25 - x^2 &= 0 \\ 0.25 &= x^2 \\ x &= \tfrac{1}{2} \text{ or } 0.5 \end{aligned}$$

So $x^* = 0.5$. Because $x = y$, $y^* = 0.5$. These are the same as the critical values for x and y found in the constrained maximization problem.

Because condition I. implies $y = \frac{1}{\lambda}$, then it follows that $\lambda = 2$. Plugging $x^* = 0.5$ and $y^* = 0.5$ into f yields $f(x^*,y^*) = x^* + y^* = 0.5 + 0.5 = 1$, the minimum value.

The Lagrangian multiplier λ for this constrained minimization dual problem is **equal to** ✘ the Lagrangian multiplier in the corresponding constrained maximization, or "primal," problem.

Points: 0 / 1

Explanation: Close Explanation

The Lagrangian multiplier for the corresponding constrained maximization problem is 0.5, which is the reciprocal of the Lagrangian multiplier $\lambda = 2$ derived from the constrained minimization problem. The is often the case with dual problems such as this.

Back to Assignment

Attempts | 4 | - | - | Average 4 / 4

5. Problems 2.5

The height of a ball that is thrown straight up with a certain force is a function of the time (t) from which it released given by $f(t) = -0.5\,gt^2 + 40t$ (where g is a positive constant determined by gravity).

The time at which the ball is at its maximum height depends on the parameter g, and is given by $t^* =$ $\frac{40}{g}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

The first order condition to find the value of t at which the height of the ball is maximized is given by $\frac{df}{dt} = -gt + 40 = 0$. This implies that:

$$\begin{aligned} -gt + 40 &= 0 \\ -gt &= -40 \\ gt &= 40 \\ t &= \frac{40}{g} \end{aligned}$$

The second derivative of f with respect to t is $\frac{d^2f}{dt^2} = -g$, which is always less than 0 since g is a positive constant. Therefore, the second order condition is satisfied at $t^* = \frac{40}{g}$.

Substituting t^* back into the function $f(t)$ yields $f(t^*) =$ $\frac{800}{g}$ ✔ , the derivative (with respect to g) of which is equal to $\frac{df(t^*)}{dg} =$ $-\frac{800}{g^2}$ ✔ . This shows how the maximum height of the ball changes as the parameter g changes.

Points: 1 / 1

Explanation: Close Explanation

Substituting t^* back into the function $f(t)$ yields $f(t^*) = -0.5g\left(\frac{40}{g}\right)^2 + 40\left(\frac{40}{g}\right)$

$$\begin{aligned} f(t^*) &= -0.5g\left(\frac{40}{g}\right)^2 + 40\left(\frac{40}{g}\right) \\ &= -0.5\,g\left(\frac{1{,}600}{g^2}\right) + \frac{1{,}600}{g} \\ &= -0.5\left(\frac{1{,}600}{g}\right) + \frac{1{,}600}{g} \\ &= -\frac{800}{g} + \frac{1{,}600}{g} \\ &= \frac{800}{g} \end{aligned}$$

Taking the derivative of $f(t^*)$ with respect to g yields $\frac{df(t^*)}{dg} = -\frac{800}{g^2}$.

Holding t constant at t^*, the time at which the ball reaches its maximum height, yields the function $f(t^*) = -0.5gt^{*2} + 40t^*$. Taking the derivative of this function with respect to g yields $\frac{df(t^*)}{dg} =$ $-0.5(t^*)^2$ ✔ . After plugging in the formula for t^* into $\frac{df(t^*)}{dg}$ yields **the same** ✔ result for $\frac{df(t^*)}{dg}$ than what you previously calculated.

Points: 1 / 1

Explanation: Close Explanation

Differentiating the original function at t^*, $f(t^*) = -0.5gt^{*2} + 40t^*$, with respect to g yields $\frac{df(t^*)}{dg} = -0.5(t^*)^2$. Plugging in $t^* = \frac{40}{g}$ yields the following:

$$\begin{aligned}\frac{df(t^*)}{dg} &= -0.5(t^*)^2 \\ &= -0.5\left(\frac{40}{g}\right)^2 \\ &= -0.5\left(\frac{1{,}600}{g^2}\right) \\ &= -\frac{800}{g^2}\end{aligned}$$

This is the same result for $\frac{df(t^*)}{dg}$ that you calculated in the previous part, when you first substited $t^* = \frac{40}{g}$ into $f(t)$ to get $f(t^*) = \frac{800}{g}$, and then took the derivative with respect to g. This is an example of an application the envelope theorem, by differentiating $f(t^*) = -0.5gt^{*2} + 40t^*$ with respect to g first, and then plugging in $t^* = \frac{40}{g}$ is a shortcut that can save on time and scratch work.

If $g = 32$, as it does on Earth, the height at which the ball reaches its maximum height is $t^* =$ $\frac{5}{4}$ ✔ and the maximum height reached by the ball is 25 ✔ . If g increases by a small amount, up to $g = 32.1$, then the maximum height reached by the ball changes to approximately 24.92 ✔ . This result **could have** ✔ been predicted by the envelope theorem.

Points: 1 / 1

Explanation: Close Explanation

If $g = 32$, then$t^* = \frac{40}{32} - \frac{5}{4}$. Plugging $t^* = \frac{40}{32} - \frac{5}{4}$ into $f(t)$ yields $f(t^*) = \frac{800}{32} = 25$. When g increases to $g = 32.1$, the maximum height reached by the ball decreases to $f(t^*) = \frac{800}{32.1} = 24.92$. This represents a reduction of 0.08 in height.

The envelope theorem predicts a similar such result. At $g = 32$, the envelope theorem implied that $\frac{df(t^*)}{dg} = -\frac{800}{g^2}$. Therefore $df(t^*) = -\frac{800}{g^2}dg$. Then, for a change in g of 0.1 from 32 to 32.1, we have:

$$\begin{aligned}df(t^*) &= -\frac{800}{g^2}dg \\ &= -\frac{800}{32^2}(0.1) \\ &= -\frac{25}{32}(0.1) \\ &\approx -0.08\end{aligned}$$

In other words, the envelope theorem also predicts a reduction of 0.08 (or a total change of -0.08) when g increases from 32 to 32.1.

Try Another Version Continue

Back to Assignment

Attempts	4.5	-	-

Average 4.5 / 5

6. Problems 2.7

Consider the following constrained maximization problem:

$$maximize\ y = x_1 + 5\ lnx_2$$

$$subject\ to\ k - x_1 - x_2 = 0$$

where k is a constant that can be assigned any specific value.

The Lagrangian for this problem is $L = x_1 + 5\ lnx_2 + \lambda\,(k - x_1 - x_2)$ ✔ . Given this Lagrangian, the first order conditions are $\frac{dL}{dx_1} =$ $1 - \lambda = 0$ ✔ , $\frac{dL}{dx_2} =$ $\frac{5}{x_2} - \lambda = 0$ ✔ , and $\frac{dL}{d\lambda} =$ $k - x_1 - x_2 = 0$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Setting up the Lagrangian involves original function $x_1 + 5\ lnx_2$ and adding the constraint, multiplied by the Lagrangian multiplier λ. This yields $L = x_1 + 5\ lnx_2 + \lambda\,(k - x_1 - x_2)$. Taking the partial derivatives of the Lagrangian, with respect to x_1, x_2, and λ, and setting them to 0,yields the following first order conditions.

I. $\frac{dL}{dx_1} = 1 - \lambda = 0$
II. $\frac{dL}{dx_2} = \frac{5}{x_2} - \lambda = 0$
III. $\frac{dL}{d\lambda} = k - x_1 - x_2 = 0$

If $k = 10$, then solving the first order conditions results in a solution of x^*_1 = 5 ✔ and x^*_2 = 5 ✔ .

Points: 1 / 1

Explanation: Close Explanation

The first order conditions I. and II. imply that $\lambda = 1 = \frac{5}{x_2}$ and, therefore, $x_2 = 5$. Plugging $x_2 = 5$ into condition III. yields:

$$\begin{aligned} k - x_1 - x_2 &= 0 \\ k - x_1 - 5 &= 0 \end{aligned}$$

Plugging in $k = 10$ results in:

$$\begin{aligned} 10 - x_1 - 5 &= 0 \\ -x_1 &= 0 - 10 + 5 \\ -x_1 &= -5 \\ x_1 &= 5 \end{aligned}$$

Therefore, the values of x_1 and x_2 maximize y are $x^*_1 = x^*_2 = 5$.

If $k = 4$, then solving the first order conditions results in a solution of $x^*_1 =$ -1 ✓ and $x^*_2 =$ _ ✗ .

Points: 0.5 / 1

Explanation: Close Explanation

The first order conditions I. and II. imply that $\lambda = 1 = \frac{5}{x_2}$ and, therefore, $x_2 = 5$. Plugging $x_2 = 5$ into condition III. yields:

$$\begin{aligned} k - x_1 - x_2 &= 0 \\ k - x_1 - 5 &= 0 \end{aligned}$$

Plugging in $k = 4$ results in:

$$\begin{aligned} 4 - x_1 - 5 &= 0 \\ -x_1 &= 0 - 4 + 5 \\ -x_1 &= 1 \\ x_1 &= -1 \end{aligned}$$

Therefore, the values of x_1 and x_2 maximize y are $x^*_1 = -1$ and $x^*_2 = 5$.

If both x_1 and x_2 must be nonnegative, and $k = 4$, then the values of x_1 and x_2 that maximize y are $x^*_1 =$ 0 ✓ and $x^*_2 =$ 4 ✓ .

Points: 1 / 1

Explanation: Close Explanation

If $k = 4$, then the constraint becomes $4 - x_1 - x_2 = 0$, or $x_2 = 4 - x_1$. Plugging this into y for x_2 yields $y = x_1 + 5 \ \ln \ (4 - x_1)$. Taking the derivative with respect to x_1 yields $\frac{dy}{dx_1} = 1 - \frac{5}{4-x_1}$, which is negative for all values of x_1 between 0 and 4 (noninclusive of 4). This means that y decreases as x_1 increases over all possible values of x_1 on the interval ([0,4)). This means that $x^*_1 = 0$ because any positive value of x_1 will decrease y.

This implies that $4 - x^*_1 - x^*_2 = 4 - 0 - x^*_2 = 4 - x^*_2 = 0$, so $x^*_2 = 4$.

Finally, plugging x^*_2 and x^*_1 into y yields $y^* = 0 + 5 \ ln4 = 5 \ ln4$.

Note, you can also solve this by setting up a Lagrangian with the constraints:

$$\begin{aligned} 4 - x_1 - x_2 &= 0 \\ x_1^2 - a^2 &= 0 \\ x_2^2 - b^2 &= 0 \end{aligned}$$

With slack variables a and b. The resulting Lagrangian would then have three Lagrangian multipliers:

$$L = x_1 + 5 \ lnx_2 + \lambda_1 (4 - x_1 - x_2) + \lambda_2 \left(x_1^2 - a^2\right) + \lambda_3 \left(x_2^2 - b^2\right)$$

The first order conditions would be that each partial derivative of L with respect to x_1, x_2, λ_1, λ_2, λ_3, a, and b would all equal 0.

If $k = 20$, then solving the first order conditions results in a solution of x^*_1 = 15 ✔ and x^*_2 = 5 ✔ . In other words, once $x^*_2 =$ 5 ✔ , y grows faster with increases in x_1 **than with increases in** x_2 ✔ .

Points: 1 / 1

Explanation: Close Explanation

The first order conditions I. and II. imply that $\lambda = 1 = \frac{5}{x_2}$ and, therefore, $x_2 = 5$. Plugging $x_2 = 5$ into condition III. yields:

$$\begin{aligned} k - x_1 - x_2 &= 0 \\ k - x_1 - 5 &= 0 \end{aligned}$$

Plugging in $k = 20$ results in:

$$\begin{aligned} 20 - x_1 - 5 &= 0 \\ -x_1 &= 0 - 20 + 5 \\ -x_1 &= -15 \\ x_1 &= 15 \end{aligned}$$

Therefore, the values of x_1 and x_2 maximize y are $x^*_1 = 15$ and $x^*_2 = 5$. This yields a maximum value for $y^* = 15 + 5\ ln5$.

Further, compared to the case when $k = 10$, when $x^*_1 = x^*_2 = 5$, all increases in y^* are due to increases in x_1. To see this, examine the first partials of y with respect to x_1 and x_2:

$$\begin{aligned} \frac{dy}{dx_1} &= 1 \\ \frac{dy}{dx_2} &= \frac{5}{x_2} \end{aligned}$$

For values of x_2 that are less than 5, $\frac{dY}{dx_1} < \frac{dY}{dx_2}$ and y increases more with x_2. For x_2 greater than 5, however, $\frac{dY}{dx_1} > \frac{dY}{dx_2}$ and y increases more with x_1. Therefore, once x_2 is equal to 5, in order to maximize y, x_1 should be increased instead of x_2.

This result stems from the fact that the $ln\ x_2$ portion of y exhibits diminishing returns, as evidenced by a decreasing first partial derivative with respect to x_2. The portion of the function that depends on x_1 does not exhibit such diminishing returns as the function exhibits constant returns since the first derivative, with respect to x_1 is a positive to constant.

Try Another Version Continue

Back to Assignment

Attempts 5 - - Average 5 / 6

7. Problems 2.8

Suppose that a firm has a marginal cost function given by $MC(q) = q + 1$.

Which of the following represent the firm's total cost function?(**Hint**: K is a constant of integration)

- ○ $q^2 + \frac{q}{2} + K$
- ○ $q^2 + q + K$
- ✓ ◉ $\frac{q^2}{2} + q + K$
- ○ $q^2 + K$

Points: 1 / 1

Fixed costs are represented by K ✓ .

Points: 1 / 1

Explanation: Close Explanation

Marginal cost is the incremental change in total cost arising from a change in quantity. In other words, the marginal cost function is the derivative of the total cost function. Therefore, the total cost function is the anti-derivative of the marginal cost function. Given this, the fundamental theorem of calculus states that:

$$\int_0^q MC(x)dx = TC(q) - TC(0)$$

Where $TC(0) = K$ is the fixed cost. Given this relationship, we can derive the total cost function:

$$\begin{aligned} TC(q) &= \int_0^q MC(x)dx \\ &= \int_0^q (x\text{-}1)dx \\ &= \left(\frac{x^2}{2} + x\right)\Big|_{x=0}^{x=q} + K \\ &= \frac{q^2}{2} + q + K \end{aligned}$$

Since K is part of the total cost function, yet is a constant that does not depend on quantity, it represents the fixed cost. When the derivative of the total cost function is taken, with respect to quantity, the constant K will drop away. Without more information, there is no way of knowing what the fixed costs and thus, total costs, are explicitly.

As you may know from an earlier economics course, if a firm takes price (p) as given in its decisions then it will produce that output for which $p = MC(q)$. Suppose the price is 15 $p = 15$.

If the firm follows this profit maximizing rule, it will produce $q^* =$ 14 ✓ . If the firm is breaking even, with 0 profit, then the firm is incurring a fixed cost of $K =$ 98 ✓ .

Points: 1 / 1

Explanation: Close Explanation

The stated profit maximizing rule requires that the firm produce at quantity q^* such that $p = MC(q^*)$. If $p = 15$, this means that:

$$\begin{aligned} p &= MC(q^*) \\ 15 &= q^* + 1 \\ 14 &= q^* \end{aligned}$$

So $q^* = 14$.

Profit is equal to total revenue less total cost, $TR - TC$. This yields:

$$\begin{aligned} TR\text{-}TC &= pq - TC(q) \\ &= 15 \cdot 14 - \left(\frac{14^2}{2} + 14 + K\right) \\ &= 98 - K \end{aligned}$$

If the firm is earning no profit, then $0 = 98 - K$, which means that fixed costs are $K = 98$.

Assume the price p increases to 20, while fixed costs remain the same.

After the price increase, profits will increase to 70 ✗ .

Points: 0 / 1

Explanation: Close Explanation

The stated profit maximizing rule requires that the firm produce at quantity q^* such that $p = MC(q^*)$. If $p = 20$, this means that:

$$\begin{aligned} p &= MC(q^*) \\ 20 &= q^* + 1 \\ 19 &= q^* \end{aligned}$$

So $q^* = 19$.

Profit is equal to total revenue less total cost, $TR - TC$. This yields:

$$\begin{aligned} TR\text{-}TC &= pq - TC(q) \\ &= 20 \cdot 19 - \left(\frac{19^2}{2} + 19 + 98\right) \\ &= 180.5 - 98 \\ &= 82.5 \end{aligned}$$

Therefore, profits have increased from 0 to 82.5 as a result of the price increase.

Assuming the firm sets $p = MC(q)$ to maximize profits, which of the following formulas represents profits as a function of p only?

- ○ $p^2 - 98$
- ✓ ◉ $\frac{(p-1)^2}{2} - 98$
- $(p-1)^2 - 98$

○

○ $\frac{(p\text{-}1)^2}{2}$

Points: 1 / 1

Explanation: Close Explanation

Assuming $p = MC(q) = q + 1$, then $q = p - 1$ and we can write profits as a function of price:

$$\begin{aligned}
\pi(p) &= pq - TC(q) \\
&= p(p\text{-}1) - \left(\frac{(p\text{-}1)^2}{2} + (p\text{-}1) + 98\right) \\
&= p^2 - p - \frac{(p\text{-}1)^2}{2} - (p\text{-}1) - 98 \\
&= p^2 - p - \frac{(p\text{-}1)^2}{2} - p + 1 - 98 \\
&= p^2 - 2p + 1 - \frac{(p\text{-}1)^2}{2} - 98 \\
&= (p\text{-}1)^2 - \frac{(p\text{-}1)^2}{2} - 98 \\
&= \frac{(p\text{-}1)^2}{2} - 98
\end{aligned}$$

Integrating the inverse marginal cost function $MC^{-1}(p) = p - 1$ from $p = 15$ to $p = 20$ yields $\frac{p^2}{2} - p \Big|_{p=15}^{p=20}$.

Points: 1 / 1

Explanation: Close Explanation

The envelope theorem states that $\frac{d\pi}{dp} = q^*(p) = p\text{-}1$. That is, the derivative of the profit function with respect to p yields the firms supply function. Integrating over p shows the change in profits by the fundamental theorem of calculus.

$$\begin{aligned}
\pi(20) - \pi(15) &= \int_{15}^{20} \frac{d\pi}{dp} dp \\
&= \int_{15}^{20} (p\text{-}1) dp \\
&= \left(\frac{p^2}{2} - p\right)\Big|_{p=15}^{p=20} \\
&= 180 - 97.5 \\
&= 82.5
\end{aligned}$$

Note you can also use the result $\pi(p) = \frac{(p\text{-}1)^2}{2} - 98$, from the previous part of the problem, to show that $\pi(p\text{=}20) - \pi(p\text{=}15) = 82.5 - 0 = 82.5$.

Try Another Version Continue

Back to Assignment

Attempts 0.8 | 1 | Keep the Highest 1 / 1

1. Problems 3.1

Use the following table to indicate whether the marginal rate of substitution (MRS) of each utility function increases, decreases, or is constant as x increases.

Utility Function	MRS Decreases with x	Constant MRS	MRS Increases with x
$U(x,y) = 3x + y$	○	●	○
$U(x,y) = \sqrt{x \cdot y}$	●	○	○
$U(x,y) = \sqrt{x} + y$	●	○	○
$U(x,y) = \sqrt{x^2 - y^2}$	●	○	○
$U(x,y) = \frac{x \cdot y}{x+y}$	●	○	○

Points: 1 / 1

Explanation: Close Explanation

For $U(x,y) = 3x + y$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{3}{1}$. This means the marginal rate of substitution is constant as x increases. Therefore, the indifference curves do not obey the assumption of diminishing MRS and the indifference curves are not convex.

For $U(x,y) = \sqrt{x \cdot y}$, let $U^*(x,y) = ln|\sqrt{x \cdot y}| = 0.5ln|x + 0.5\, lny$. Because taking the natural log is order preserving, the MRS for both U and U^* will be the same. $MRS = \frac{dU^*/dx}{dU^*/dy} = \frac{0.5/x}{0.5/y} = \frac{y}{x}$. This means the marginal rate of substitution decreases as x increases. Therefore, the indifference curves obey the assumption of diminishing MRS, and the indifference curves are convex.

For $U(x,y) = \sqrt{x} + y$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{0.5x^{-0.5}}{1} = \frac{0.5}{\sqrt{x}}$. This means the marginal rate of substitution decreases as x increases. Therefore, the indifference curves obey the assumption of diminishing MRS, and the indifference curves are convex.

For $U(x,y) = \sqrt{x^2 + y^2}$, let $U^*(x,y) = U(x,y)^2 = x^2 - y^2$. Because this transformation is order preserving, the MRS for both U and U^* will be the same. $MRS = \frac{dU^*/dx}{dU^*/dy} = \frac{2x}{-2y} = -\frac{x}{y}$. This means the marginal rate of substitution decreases (becomes more negative) as x increases. Therefore, the indifference curves do obey the assumption of diminishing MRS, and the indifference curves are convex.

For $U(x,y) = \frac{x \cdot y}{x+y}$, $MRS = \frac{dU/dx}{dU/dy} = \frac{(y(x+y)-xy)/(x+y)^2}{(x(x+y)-xy)/(x+y)^2} = \frac{y^2}{x^2}$. This means the marginal rate of substitution decreases as x increases. Therefore, the indifference curves obey the assumption of diminishing MRS and the indifference curves are convex.

Try Another Version Continue

Back to Assignment

Attempts 0.6 | 1 | Keep the Highest 1 / 1

2. Problems 3.2

For a utility function for two goods, $U(x,y)$, to have a strictly diminishing MRS (i.e., to be strictly quasi-concave), the following condition must hold:

$$U_{xx}U_y^2 - 2U_{xy}U_xU_y + U_{yy}U_x^2 < 0$$

Use the following table to indicate whether the condition holds for each of the utility functions.

Utility Function	Condition Holds	Condition Does Not Hold
$U(x,y) = 3x + y$	○	●
$U(x,y) = \sqrt{x \cdot y}$	●	○
$U(x,y) = \sqrt{x} + y$	●	○
$U(x,y) = \sqrt{x^2 - y^2}$	○	●
$U(x,y) = \frac{x \cdot y}{x+y}$	●	○

Points: 1 / 1

Explanation: Close Explanation

For $U(x,y) = 3x + y$, since $U_{xx} = U_{yy} = U_{xy} = 0$, then $U_{xx}U_y^2 - 2U_{xy}U_xU_y + U_{yy}U_x^2 = 0$. Therefore, the condition does not hold; this function does not have strictly diminishing MRS and is not strictly quasi-concave. Therefore, the indifference curves do not obey the assumption of diminishing MRS, and the indifference curves are not convex.

For $U(x,y) = \sqrt{x \cdot y}$, both $U_{xx} < 0$ and $U_{yy} < 0$. Also, $U_{xy} > 0$, $U_x > 0$, and $U_y > 0$. Therefore, all terms in the condition are negative, and so $U_{xx}U_y^2 - 2U_{xy}U_xU_y + U_{yy}U_x^2 < 0$. Thus, the condition holds; this function does have strictly diminishing MRS and is not strictly quasi-concave. The indifference curves are convex.

For $U(x,y) = \sqrt{x} + y$, $U_{xx} < 0$ and $U_{yy} = U_{xy} = 0$. Thus, both $-2U_{xy}U_xU_y = 0$ and $U_{yy}U_y^2 = 0$, while $U_{xx}U_x^2 < 0$. This means that $U_{xx}U_y^2 - 2U_{xy}U_xU_y + U_{yy}U_x^2 < 0$. Thus, the condition holds; this function does have strictly diminishing MRS and is not strictly quasi-concave. The indifference curves are convex.

For $U(x,y) = \sqrt{x^2 - y^2}$, the signs of both partial derivatives U_{xx} and U_{yy} are ambiguous. Therefore, the condition does not always hold; this function does not have strictly diminishing MRS for all x, and therefore is not necessarily strictly quasi-concave.

For $U(x,y) = \frac{x \cdot y}{x+y}$, both $U_{xx} < 0$ and $U_{yy} < 0$. Also, $U_{xy} > 0$, $U_x > 0$, and $U_y > 0$. Therefore, all terms in the condition are negative, and so $U_{xx}U_y^2 - 2U_{xy}U_xU_y + U_{yy}U_x^2 < 0$. Thus, the condition holds; this function does have strictly diminishing MRS and is not strictly quasi-concave. The indifference curves are convex.

Try Another Version Continue

Back to Assignment

Attempts 4 | | | Keep the Highest 4 / 4

3. Problems 3.3

For $U(x,y) = xy$, $MRS =$ $\frac{y}{x}$ ✔, while $U_{xx} =$ 0 ✔ and $U_{yy} =$ 0 ✔. This means that this utility function has **diminishing** ✔ MRS, while exhibiting **constant** ✔ marginal utility in x and y.

Points: 1 / 1

Explanation: Close Explanation

For $U(x,y) = xy$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{y}{x}$, which decreases as x increases. However, since $U_{xx} = U_{yy} = 0$, this utility function exhibits constant marginal utility in x and y.

For $U(x,y) = x^2y^2$, $MRS =$ $\frac{y}{x}$ ✔, while $U_{xx} =$ $2y^2$ ✔ and $U_{yy} =$ $2x^2$ ✔. This means that this utility function has **diminishing** ✔ MRS, while exhibiting **increasing** ✔ marginal utility in x and y.

Points: 1 / 1

Explanation: Close Explanation

For $U(x,y) = x^2y^2$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{2\,xy^2}{2y^2} = \frac{y}{x}$, which decreases as x increases. However, since $U_{xx} = 2y^2$ and $U_{yy} = 2x^2$, both of which are positive, so this utility function exhibits increasing marginal utility in x and y.

For $U(x,y) = ln\,x + ln\,y$, $MRS =$ $\frac{y}{x}$ ✔, while $U_{xx} =$ $\frac{-1}{x^2}$ ✔ and $U_{yy} =$ $\frac{-1}{y^2}$ ✔. This means that this utility function has **diminishing** ✔ MRS, while exhibiting **decreasing** ✔ marginal utility in x and y.

Points: 1 / 1

Explanation: Close Explanation

For $U(x,y) = ln\,x + ln\,y$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{1/x}{1/y} = \frac{y}{x}$, which decreases as x increases. However, since $U_{xx} = \frac{-1}{x^2}$ and $U_{yy} = \frac{-1}{y^2}$, both of which are positive, so this utility function exhibits increasing marginal utility in x and y.

True or False: If a utility function U exhibits diminishing MRS and constant marginal utility, then any monotonic transformation of U also exhibits diminishing MRS and constant marginal utility.

- ○ True
- ✔ ◉ False

Points: 1 / 1

Explanation: Close Explanation

You will notice that for $U(x,y) = xy$, both $ln|U(x,y)| = ln\,xy = ln\,x + ln\,y$ and $U(x,y)^2 = (xy)^2 = x^2x^2$ are monotonic transformations of $U(x,y)$. However, while $U(x,y) = xy$ exhibits constant marginal utility in x and y, $ln\,x + ln\,y$ exhibits decreasing marginal utility and x^2y^2 exhibits increasing marginal utility. All three utility functions do exhibit diminishing MRS.

Thus, for a utility function U, the transformed utility function $U*$ will exhibit the same MRS properties, but the marginal utilities of $U*$ may exhibit different properties than U.

Try Another Version Continue

Back to Assignment

Attempts 6 | | | Keep the Highest 6 / 6

4. Problems 3.4

One way to show convexity of indifference curves is to show that for any two points (x_1, y_1) and (x_2, y_2) on an indifference curve that promises $U = k$, the utility associated with $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is at least as great as k. In other words, one way to show convexity is to show that the following conditions hold:

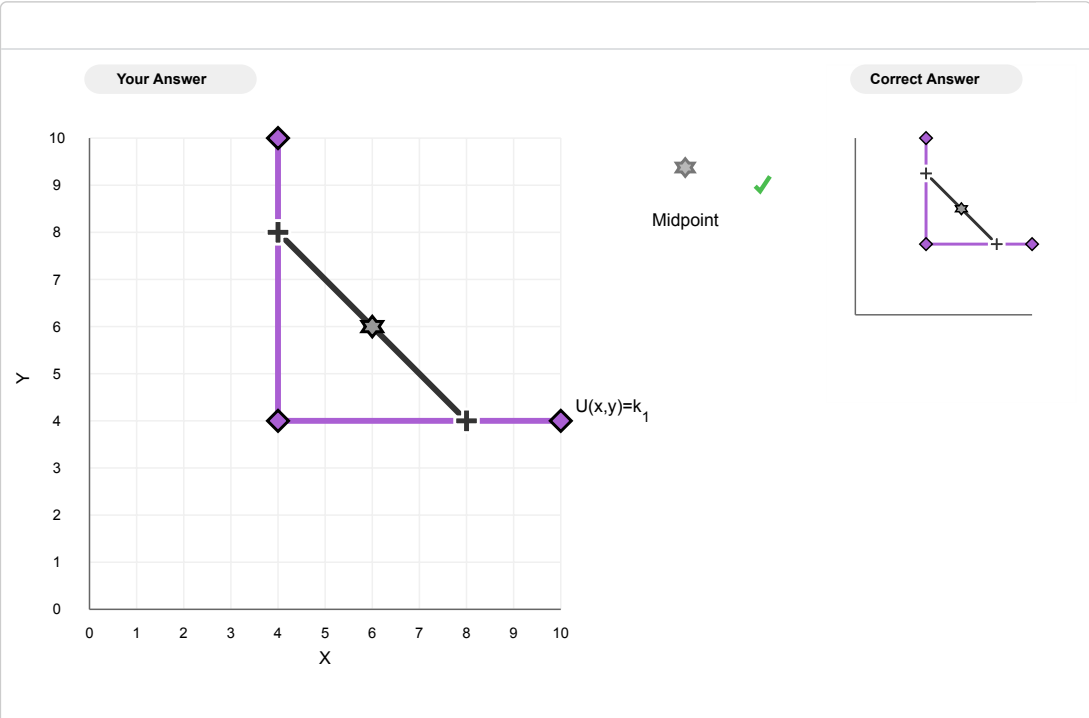
$$U\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \geq U(x_1, y_1)$$

and

$$U\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \geq U(x_2, y_2)$$

The following graph shows an indifference curve for the utility function $U(x,y) = min(x,y)$, where $U(x,y) = min(x,y) = k_1$. The black line connects two points $(x_1, y_1) = (4,8)$ and $(x_2, y_2) = (8,4)$ that are on the indifference curve.

Use the grey point (star symbol) to plot the midpoint of the black line. Then answer the questions below.



Points: 1 / 1

At the midpoint of the line connecting (x_1, y_1) and (x_2, y_2), the level of utility offered by this bundle $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is 6 ✓ . This is **higher than** ✓ the utility level of 4 ✓ that is offered by the bundles $(x_1, y_1) = (4,8)$ and $(x_2, y_2) = (8,4)$. Thus, for (x_1, y_1) and (x_2, y_2), the conditions **do** ✓ hold.

Points: 1 / 1

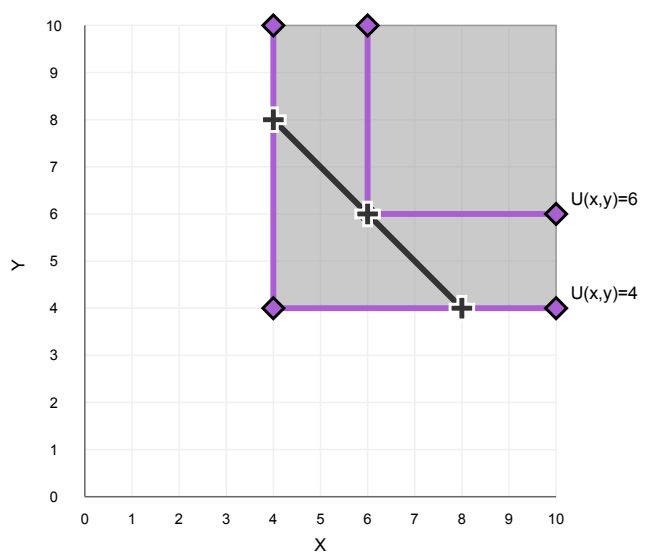
Explanation: Close Explanation

In this case, the indifference curves are convex and satisfy the conditions.

For example, suppose $(x_1, y_1) = (8,4)$ and $(x_2, y_2) = (4,8)$. Then both bundles offer the same level of utility $U(8,4) = U(4, 8) = 4$.

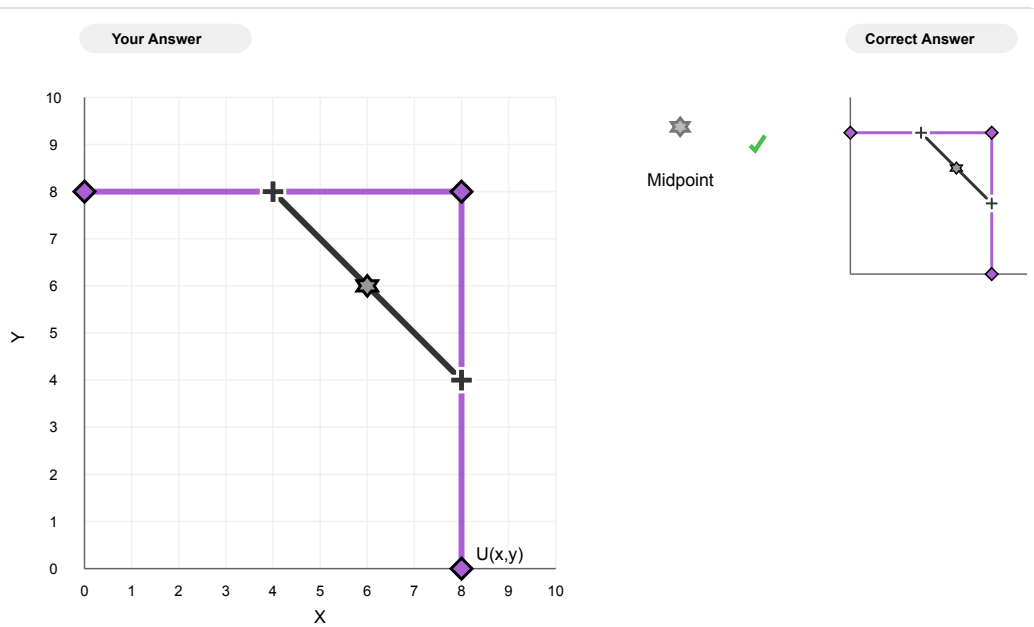
The midpoint of the line connecting these two bundles is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{8+4}{2}, \frac{4+8}{2}\right) = (6,6)$. Plugging these values into the utility function yields $U(6,6) = 6$, which is higher than the level of utility offered by the endpoints. Thus (6,6) is on a higher indifference curve than (4,8) and (8,4). Thus, the midpoint (6,6) is preferred to both (4,8) and (8,4), and the two conditions are satisfied.

In fact, the conditions are satisfied for any two points on the same indifference curve for this function, implying that this utility function has convex indifference curves.



The following graph shows an indifference curves for the utility function $U(x,y) = max(x,y)$, where $U(x,y) = min(x,y)$. The black line connects two points $(x_1, y_1) = (4,8)$ and $(x_2, y_2) = (8,4)$ that are on the indifference curve.

Use the grey point (star symbol) to plot the midpoint of the black line. Then answer the questions below.



Points: 1 / 1

At the midpoint of the line connecting (x_1, y_1) and (x_2, y_2), the level of utility offered by this bundle $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is 6 ✔ . This is **lower than** ✔ the utility level of 8 ✔ that is offered by the bundles $(x_1, y_1) = (4,8)$ and $(x_2, y_2) = (8,4)$. Thus, for (x_1, y_1) and (x_2, y_2), the conditions **do not** ✔ hold.

Points: 1 / 1

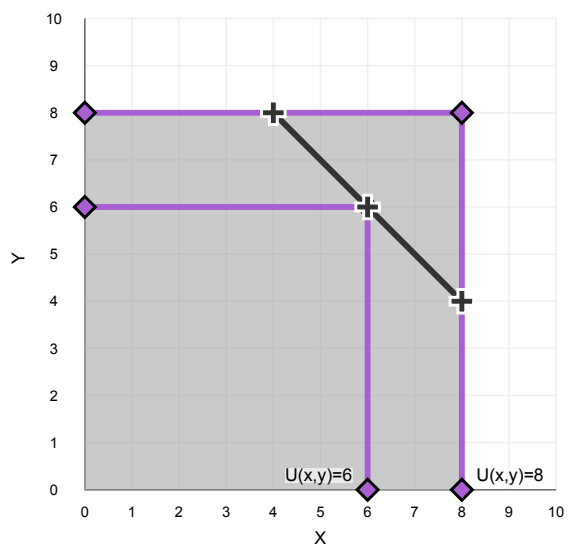
Explanation: Close Explanation

In this case, the indifference curves are convex and satisfy the conditions.

For example, suppose $(x_1, y_1) = (8,4)$ and $(x_2, y_2) = (4,8)$. Then both bundles offer the same level of utility $U(8,4) = U(4,8) = 8$.

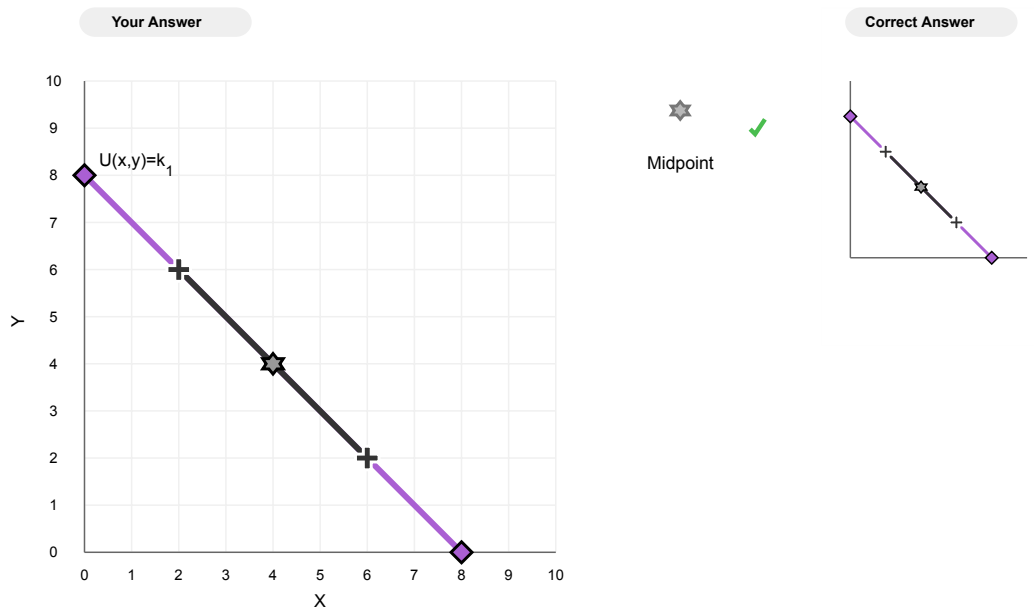
The midpoint of the line connecting these two bundles is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{8+4}{2}, \frac{4+8}{2}\right) = (6,6)$. Plugging these values into the utility function yields $U(6,6) = 6$, which is lower than the level of utility offered by the endpoints. Thus (6,6) is on a lower indifference curve than (4,8) and (8,4). Thus, the midpoint (6,6) is not preferred to either (4,8) or (8,4), and the two conditions are not satisfied.

In fact, one bundle is on the horizontal segment of the indifference curve, while the other is on the vertical segment of the indifference curve; the conditions are never satisfied because the resulting midpoint bundle will be below and to the left of initial indifference curve.



The following graph shows an indifference curves for the utility function $U(x,y) = x + y$, where $U(x,y) = min(x,y) = k_1$. The black line connects two points $(x_1, y_1) = (2,6)$ and $(x_2, y_2) = (6,2)$ that are on the indifference curve.

Use the grey point (star symbol) to plot the midpoint of the black line. Then answer the questions below.



Points: 1 / 1

At the midpoint of the line connecting (x_1, y_1) and (x_2, y_2), the level of utility offered by this bundle $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is 8 ✔ . This is **equal to** ✔ than the utility level of 8 ✔ that is offered by the bundles $(x_1, y_1) = (2,6)$ and $(x_2, y_2) = (6,2)$. Thus, for (x_1, y_1) and (x_2, y_2), the conditions **do** ✔ hold.

Points: 1 / 1

Explanation: Close Explanation

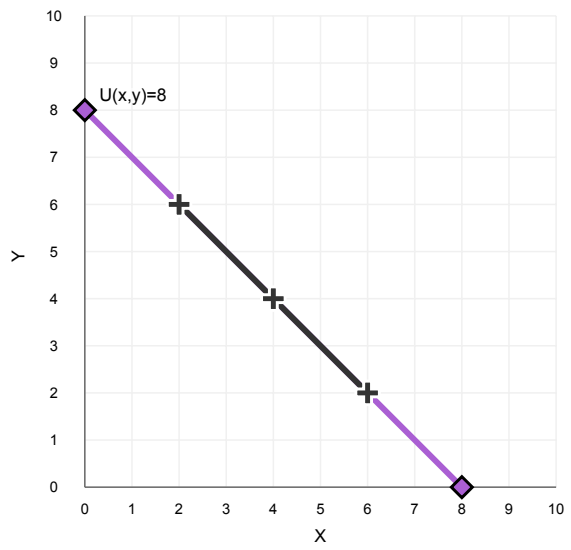
In this case, the indifference curves satisfy the conditions.

For example, suppose $(x_1, y_1) = (2,6)$ and $(x_2, y_2) = (6,2)$. Then both bundles offer the same level of utility $U(2,6) = U(6,2) = 8$.

The midpoint of the line connecting these two bundles is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+6}{2}, \frac{6+2}{2}\right) = (4,4)$. Plugging these values into the utility function yields $U(4,4) = 8$, which is the same as the level of utility offered by the endpoints. Thus (4,4) is on the same indifference curve as (2,6) and (6,2). Thus, this individual is indifferent between the midpoint (4,4) and (2,6) or (6,2). Because the inequalities in the two conditions are not strict, the two conditions are indeed satisfied in this case.

In fact, the conditions are satisfied for any two points on the same indifference curve for this function, implying that this utility function has convex indifference curves.

Notice that these indifference curves are not strictly convex because $U\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is always equal to $U(x_1, y_1) = k$ and $U(x_2, y_2) = k$, for any two points on the same indifference curve.



Try Another Version

Continue

Back to Assignment

Attempts 3 5 Keep the Highest 5 / 5

5. Problems 3.6

Many advertising slogans seem to be asserting something about people's preferences. How would you capture the following slogans with a mathematical utility function?

Let m represent quantity of margarine and b represent quantity of butter. Let x represent the quantity of some other good.

An advertising slogan that promises margarine is "just as good as butter" is equivalent to stating that $U(x,m) \geq U(x,b)$ *for* $p=b$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

In utility terms, the claim that margarine is "just as good as butter" implies that a given quantity of margarine offers at least as much utility as the same quantity of butter given similar consumption of other goods (x). This is equivalent to stating that $U(x,p) \geq U(x,b)$ *for* $p=b$.

Let c represent quantity of Coke. Let x represent the quantity of some other good.

An advertising slogan that promises that "things go better with Coke" is equivalent to stating that given $U(x,c)$, $U_{xc} = U_{cx} > 0$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

In utility terms, the claim that "everything goes better with Coke" implies coke enhances the marginal utility of other goods (x). In other words, the marginal utility of other goods U_x increases with more Coke. This means that the cross-partial U_{xc} , which is mathematically equivalent to U_{cx}, is greater than 0.

Let pr represent the quantity of Pringles chips. Let x represent the quantity of some other good.

An advertising slogan that claims that "You can't eat just one Pringles potato chip" is equivalent to stating that given $U(x,pr)$, $U(x,1) < U(x,0) < U(x,pr>1)$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

In utility terms, the claim that "You can't eat just one Pringle's potato chip" implies that eating just one Pringles chip (and stopping) yields a lower utility level than eating more than one chip and also yields a lower utility level than eating no chips at all. In this sense, you would strictly prefer multiple chips to no chips, and no chips to just one chip, so you would not ever want to eat just one chip. This means that given a similar level of consumption of other goods x, $U(x,1) < U(x,0) < U(x,pr>1)$.

Let kk represent the quantity of Krispy Kreme doughnuts and dd represent Dunkin' Donuts. Let x represent the quantity of some other good.

An advertising slogan that claims that "Krispy Kreme glazed doughnuts are just better than Dunkin' Donuts" is equivalent to stating that $U(x,kk) > U(x,dd)$ *for* $kk=dd$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

In utility terms, the claim that "Krispy Kreme glazed doughnuts are just better than Dunkin' Donuts" implies that a given quantity of Krispy Kreme doughnuts offers strictly more utility as the same quantity of Dunkin' Donuts given similar consumption of other goods (x). This is equivalent to stating that $U(x,kk) > U(x,dd)$ *for* $kk=dd$.

Let m represent the quantity of Miller beers, with $m_{responsible}$ is a responsible quantity of beers. Let x represent the quantity of some other good.

An advertising slogan that states that "Miller Brewing advises us to drink (beer) 'responsibly'." is equivalent to stating that

$U(x,m) < U(x, m_{responsible})$ for $m > m_{responsible}$ ✓ .

Points: 1 / 1

Explanation: Close Explanation

In utility terms, suggestion that "Miller Brewing advises us to drink (beer) 'responsibly' " implies that there is a responsible quantity of beers, $m_{responsible}$, such that consumption of more beers beyond this quantity would ($m > m_{responsible}$) lead to decreases in utility. This is equivalent to stating that $U(x,m) < U(x, m_{responsible})$ for $m > m_{responsible}$.

Try Another Version Continue

Back to Assignment

Attempts | 2 | 3 | | Keep the Highest 3 / 3

6. Problems 3.7

A consumer is willing to trade 3 units of x for 1 unit of y when she has 6 units of x and 5 units of y. She is also willing to trade in 6 units of x for 2 units of y when she has 12 units of x and 3 units of y. She is indifferent between bundle (6, 5) and bundle (12, 3).

A utility function that is consistent with this scenario is $U(x,y) = x + 3y$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Because the consumer is willing to trade 3 units of x for 1 unit of y, at (6,5), the marginal rate of substitution (MRS) at this point is $mrs = -\frac{dy}{dx}|_{U=U_1} = -\frac{1}{-3} = \frac{1}{3}$.

Because the consumer is willing to trade 6 units of x for 2 units of y, at (12,3), the marginal rate of substitution at this point is $mrs = -\frac{dy}{dx}|_{U=U_1} = -\frac{6}{-2} = \frac{1}{3}$.

So the MRS is the same at both points ($\frac{1}{3}$), and the slope is the same at both points ($\frac{1}{3}$). Because the points lie on the same indifference curve, and because the consumer is indifferent between the two bundles, the indifference curve has a constant slope of $\frac{-1}{3}$. This type of indifference curve corresponds to two goods that are perfect substitutes. A utility function for perfect substitutes with an MRS of $\frac{1}{3}$ and indifference curves with slopes of $\frac{-1}{3}$ would be $U(x,y) = x + 3y$.

A consumer is willing to trade 4 units of x for 1 unit of y when she is consuming bundle (8,1). She is also willing to trade in 1 unit of x for 2 units of y when she is consuming bundle (4,4). She is indifferent between these two bundles. Assuming that the utility function is Cobb-Douglas of the form $U(x,y) = x^{\alpha}y^{\beta}$), where α and β are positive constants.

A utility function that is consistent with this scenario is $U(x,y) = x^2y$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Because the consumer is willing to trade 4 units of x for 1 unit of y, at (8,1), the marginal rate of substitution (MRS) at this point is $mrs = -\frac{dy}{dx}|_{U=U_1} = -\frac{1}{-4} = \frac{1}{4}$.

Because the consumer is willing to trade 1 unit of x for 2 units of y, at (4,4), the marginal rate of substitution at this point is $mrs = -\frac{dy}{dx}|_{U=U_1} = -\frac{2}{-1} = 2.$

For a Cobb-Douglas utility function of the form $U(x,y) = x^{\alpha}y^{\beta}$, the marginal rate of substitution is $MRS = \frac{dU/dx}{dU/dy} = \frac{\alpha y}{\beta x}$. Therefore, at (8,1), we know that $\frac{\alpha}{8\beta} = \frac{1}{4}$ and at (4,4) that $\frac{4\alpha}{4\beta} = 2$. From $\frac{\alpha}{8\beta} = \frac{1}{4}$ we know that:

$$\begin{aligned} \frac{\alpha}{8\beta} &= \frac{1}{4} \\ \frac{\alpha}{\beta} &= \frac{8}{4} \\ \alpha &= 2\beta \end{aligned}$$

From $\frac{4\alpha}{4\beta} = 2$ we know that:

$$\begin{aligned} \frac{4\alpha}{4\beta} &= 2 \\ \frac{\alpha}{\beta} &= 2 \\ \alpha &= 2\beta \end{aligned}$$

So any combination of α and β where $\alpha = 2\beta$ would work. Let $\beta = 1$, then $\alpha = 2$ and $U(x,y) = x^2y$.

Which of the following pieces of information was not needed to formulate the Cobb-Douglas utility function?

- ○ The consumer was willing to trade in 1 unit of x for 2 units of y when she is consuming bundle (4,4).
- ○ The consumer was willing to trade 4 units of x for 1 unit of y when she is consuming bundle (8,1).
- ✓ ◉ The consumer was indifferent between these two bundles, (4,4) and (8,1).

Points: 1 / 1

Explanation: Close Explanation

There was a redundancy of information given in this problem. The fact that the two points were on the same indifference curve was not needed to solve for the Cobb-Douglas utility function. The MRS and the values of x and y at the points was all that was necessary to find the utility function given the function is Cobb-Douglas.

Try Another Version Continue

Back to Assignment

Attempts | 4.7 | 5.4 | **6** | Keep the Highest 6 / 6

1. Problems 4.1

Each day Paul, who is in third grade, eats lunch at school. He likes only Twinkies (t) and soda(s), and these provide him a utility of:

$$utility = U(t,s) = \sqrt{ts}$$

Suppose the price of twinkies is P_t each and the price of soda is P_s per cup. Also suppose Paul has I, given to him by his mother, to spend on lunch at school.

For this general case, which of the following are first order conditions for a utility maximizing bundle of twinkies and soda for Paul? *Check all that apply.*

- ✓ ☐ $\frac{dL}{d\lambda} = I - P_t t = 0$
- ✓ ☑ $\frac{dL}{dt} = 0.5\sqrt{\frac{s}{t}} - \lambda P_t = 0$
- ✓ ☐ $\frac{dL}{dt} = 0.5\sqrt{st} - \lambda P_t = 0$
- ✓ ☐ $\frac{dL}{dt} = 0.5\frac{t^2}{s^2} - \lambda P_s = 0$
- ✓ ☑ $\frac{dL}{ds} = 0.5\sqrt{\frac{t}{s}} - \lambda P_s = 0$
- ✓ ☐ $\frac{dL}{ds} = 0.5\frac{s^2}{t^2} - \lambda P_s = 0$
- ✓ ☐ $\frac{dL}{d\lambda} = I + P_t t + P_s s = 0$
- ✓ ☐ $\frac{dL}{ds} = 0.5\sqrt{\frac{s}{t}} - \lambda P_s = 0$
- ✓ ☑ $\frac{dL}{d\lambda} = I - P_t t - P_s s = 0$

Points: 1 / 1

Explanation: Close Explanation

Let P_t represent the price of twinkies and P_s represent the price of soda. Also let I represent Paul's income from his mother for lunch. The budget constrain for this maximization problem is $I = P_t t + P_s s$.

The lagranian for this maximization problem is:

$$L = U(t,s) + \lambda(I - P_t t - P_s s) = \sqrt{ts} + \lambda(I - P_t t - P_s s) = t^{0.5}s^{0.5} + \lambda(I - P_t t - P_s s)$$

The first order conditions require that the first partial derivatives of L (with respect to s, t, and λ) all equal 0.

In calculating $\frac{dL}{dt}$:

$$\begin{aligned}\frac{dL}{dt} &= 0.5t^{-0.5}s^{0.5} - \lambda P_t \\ &= 0.5\frac{s^{0.5}}{t^{0.5}} - \lambda P_t \\ &= 0.5\sqrt{\frac{s}{t}} - \lambda P_t\end{aligned}$$

Thus $\frac{dL}{dt} = 0.5\sqrt{\frac{s}{t}} - \lambda P_t = 0$.

Similarly, for $\frac{dL}{ds}$:

$$\begin{aligned}\frac{dL}{ds} &= 0.5t^{-0.5}s^{0.5} - \lambda P_t \\ &= 0.5\frac{t^{0.5}}{s^{0.5}} - \lambda P_t \\ &= 0.5\sqrt{\frac{t}{s}} - \lambda P_s\end{aligned}$$

Thus $\frac{dL}{ds} = 0.5\sqrt{\frac{t}{s}} - \lambda P_s = 0$.

The 3rd first order condition is simply $\frac{dL}{d\lambda} = I - P_t t - P_s s = 0$. Thus the three first order conditions are:

$$\begin{aligned}&\text{I.} \quad \frac{dL}{dt} = 0.5\sqrt{\frac{s}{t}} - \lambda P_t = 0 \\ &\text{II.} \quad \frac{dL}{ds} = 0.5\sqrt{\frac{t}{s}} - \lambda P_s = 0 \\ &\text{III.} \quad \frac{dL}{d\lambda} = I - P_t t - P_s s = 0\end{aligned}$$

After solving these first order conditions, it is clear that $t^* =$ $\frac{I}{2P_t}$ ✔ and $s^* =$ $\frac{I}{2P_s}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Condition I. implies that $0.5\frac{s}{t}^{0.5} = 0.5s^{0.5}t^{-0.5} = \lambda P_t$, while condition II. implies that $0.5\frac{t}{s}^{0.5} = 0.5t^{0.5}s^{-0.5} = \lambda P_s$. Dividing I. by II. yields:

$$\begin{aligned}\frac{\lambda P_t}{\lambda P_s} &= \frac{0.5s^{0.5}t^{-0.5}}{0.5t^{0.5}s^{-0.5}} \\ \frac{P_t}{P_s} &= \frac{s}{t} \\ \frac{P_t}{P_s} \times t &= s\end{aligned}$$

Plugging $s = \frac{P_t}{P_s} \times t$ into the 3rd first order condition (for s) yields:

$$\begin{aligned}0 &= I - P_t t - P_s s \\ 0 &= I - P_t t - P_s \times \frac{P_t}{P_s} \times t \\ 0 &= I - P_t t - P_t \times t \\ 0 &= I - 2P_t t \\ 2P_t t &= I \\ t &= \frac{I}{2P_t}\end{aligned}$$

So $t^* = \frac{I}{2P_t}$. Since $t^* = \frac{I}{2P_t}$ and we know that $s = \frac{P_t}{P_s} \times t$, then $s^* = \frac{P_t}{P_s} \times \frac{I}{2P_t} = \frac{I}{2P_s}$. In short:

$$\begin{aligned}t^* &= \frac{I}{2P_t} \\ s^* &= \frac{I}{2P_s}\end{aligned}$$

Suppose twinkies cost $\$0.1$ each and soda costs $\$0.25$ per cup. Also suppose Paul has $\$1$, given to him by his mother, to spend on lunch at school.

At the given prices and income, Paul's utility maximizing combination of twinkies and soda is t^*= 5 ✔ and s^*= 2 ✔ .

Points: 1 / 1

Explanation: Close Explanation

1. $t^* = \frac{I}{2P_t}$
2. $s^* = \frac{I}{2P_s}$

Since $I = 1$ and $P_s = 0.25$, then $s^* = \frac{1}{2\times0.25} = \frac{1}{0.5} = 2$. Since $I = 1$ and $P_t = 0.1$, then $t^* = \frac{1}{2\times0.1} = \frac{1}{0.2} = 5$.

Which of the following functions represents Paul's indirect utility function?

- ✔ (●) $V = \sqrt{\frac{I^2}{4P_tP_s}}$
- ( ) $V = \frac{1}{4P_tP_s}$
- ( ) $V = \sqrt{\frac{1}{4P_tP_s}}$
- ( ) $V = 4P_tP_s \cdot I^2$

Points: 1 / 1

Explanation: Close Explanation

Paul's indirect utility function can be found by plugging in the general formulas for t^* and s^* back into his utility function. Plugging $t^* = \frac{I}{2P_t}$ and $s^* = \frac{I}{2P_s}$ back into $U(s^*,t^*)$ yields $V = U(s^*,t^*) = \sqrt{\frac{I}{2P_t} \cdot \frac{I}{2P_s}} = \sqrt{\frac{I^2}{4P_tP_s}}$. This indirect utility function V gives the maximum utility achievable to Paul, given the prices of twinkies and sodas, as well as his income.

Given this indirect utility function, Paul's expenditure function is $E =$ $2P_s^{0.5}P_t^{0.5}U$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Paul's expenditure function can be found by solving his indirect utility function for I. Letting U denote utility instead of V, we have:

$$\begin{aligned} V &= \sqrt{\frac{I^2}{4P_tP_s}} \\ U &= \frac{I}{2P_t^{0.5}P_s^{0.5}} \\ 2P_t^{0.5}P_s^{0.5}\,U &= I \end{aligned}$$

Now, assuming Paul spends all his income and $I = E$, then we have Paul's expenditure function as $E = 2P_t^{0.5}P_s^{0.5}\,U$. This function tells us how much Paul must spend to achieve a utility level of U, given the prices of the goods he purchases P_t and P_s.

Suppose the school tries to discourage Twinkie consumption by increasing the price to $0.40.

In order to keep him at the same level of utility as before (in other words, on the same indifference curve), Paul's mother will need to increase his lunch money to $2.00 ✔ .

Points: 1 / 1

Explanation: Close Explanation

Before the increase in the price of twinkies, Paul used his $1 to purchase 5 twinkies and 2 cups of soda. This yielded a total utility of $U(2,5) = \sqrt{2 \times 5} = \sqrt{10}$.

When the price of twinkies rises to \$0.40, then $P_t = 0.4$, while the price of soda remains the same at $P_s = 0.25$. In order to leave Paul on the same indifference curve, U must remain at $\sqrt{10}$ (the level it was at before the price change).

Plugging these values into Paul's expenditure function yields:

$$\begin{aligned} E &= 2P_t^{0.5}P_s^{0.5}\ U \\ &= 2\sqrt{0.4}\cdot\sqrt{0.25}\cdot\sqrt{10} \\ &= 2\sqrt{10\times 0.4\times 0.25} \\ &= 2\sqrt{1} \\ &= 2 \end{aligned}$$

Therefore, with these new prices, Paul will need $\$2$ to achieve the same level of utility that he had before the price increase of twinkies.

Try Another Version Continue

Back to Assignment

Attempts 6 | 7 | Keep the Highest 7 / 7

2. Problems 4.2

Suppose a young wine connoisseur enjoys two vintages in particular: a 2001 French Bordeaux (w_f) and 2005 California varietal wine (w_c). Her utility function is:

$$U(w_f\ ,\ w_c\ ) = w_f^{2/3}\ w_c^{1/3}$$

Let P_c represent the price of the California wine and P_f represent the price of the french wine. Also let I be the income of the consumer.

In general, the combination of French wine and Californian wine that maximizes this consumers utility is $w_f^* =$ $\frac{2I}{3P_f}$ ✔ and $w_c^* =$ $\frac{I}{3P_c}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

For the general cobb-douglas utility function $U(x,y) = x^\alpha y^\beta$, it can be shown that setting up the lagranian and solving the first order conditions yields $x* = \frac{\alpha I}{P_x}$ and $y* = \frac{\beta I}{P_y}$.

In this case, $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. Therefore the utility maximizing combination of wines is $w_c^* = \frac{I}{3P_c}$ and $w_f^* = \frac{2I}{3P_f}$.

Suppose the young connoisseur has $600 to spend to build a small wine cellar. Also suppose the price of the 2001 French Bordeaux (w_f) is $40 per bottle while the less expensive 2005 California varietal wine (w_c) is priced at $8 per bottle.

At the given prices and income, this consumer's utility maximizing combination of wine is $w_f^* =$ 10 ✔ and $w_c^* = =$ 25 ✔ .

Points: 1 / 1

Explanation: Close Explanation

We know the utility maximizing combination of wine is:

$$w_f^* = \frac{2I}{3P_f}$$
$$w_c^* = \frac{I}{3P_c}$$

Given that $I = 600$,$P_f = 40$, and $P_c = 8$, then $w_f^* = \frac{2 \cdot 600}{3 \cdot 40} = \frac{1{,}200}{120} = 10$ and $w_c^* = \frac{I}{3P_c} = \frac{600}{3 \cdot 8} = \frac{600}{24} = 25$.

Suppose the young connoisseur has $600 to spend to build a small wine cellar. Also suppose the price of the 2001 French Bordeaux (w_f) is decreases to $20 per bottle while the less expensive 2005 California varietal wine (w_c) is still priced at $8 per bottle.

At these new prices , this consumer's new utility maximizing combination of wine is $w_f^* =$ 20 ✔ and $w_c^* = =$ 25 ✔ .

Points: 1 / 1

Explanation: Close Explanation

We know the utility maximizing combination of wine is:

$$w_f^* = \frac{2I}{3P_f}$$
$$w_c^* = \frac{I}{3P_c}$$

Given that $I = 600$,$P_f = 20$, and $P_c = 8$, then $w_f^* = \frac{2 \cdot 600}{3 \cdot 20} = \frac{1{,}200}{60} = 20$ and $w_c^* = \frac{I}{3P_c} = \frac{600}{3 \cdot 8} = \frac{600}{24} = 25$. Note that the consumer's demand for the Californian wine remained constant, as he demand function for Californian wine does not depend on the price of the French wine.

Before the change in prices, this consumers utility was approximately **13.6** ✔ . After the decrease in the price of french wine, this consumers utility was approximately **21.5** ✔ .

Points: 1 / 1

Explanation: Close Explanation

Before the price changes $w_f^* = 10$ and $w_c^* = 25$. Plugging these values into her utility function yields $U^* = 10^{2/3} \cdot 25^{1/3} \approx 13.6$.

After the price changes $w_f^* = 20$ and $w_c^* = 25$. Plugging these values into her utility function yields $U^* = 20^{2/3} \cdot 25^{1/3} \approx 21.5$.

Which of the following functions the consumers indirect utility function?

✘ ○ $V = \left(\frac{2I}{3P_c}\right)^{2/3} \left(\frac{I}{3P_f}\right)^{1/3}$

○ $V = \left(\frac{2I}{3P_f}\right)^{2/3} + \left(\frac{I}{3P_c}\right)^{1/3}$

✔ ◉ $V = \left(\frac{2I}{3P_f}\right)^{2/3} \left(\frac{I}{3P_c}\right)^{1/3}$

○ $V = \left(\frac{3I}{2P_f}\right)^{2/3} \left(\frac{3I}{P_c}\right)^{1/3}$

Points: 1 / 1

Explanation: Close Explanation

This consumers indirect utility function can be found by plugging in the general formulas for w_f^* and w_c^* back into his utility function. Plugging $w_f^* = \frac{2I}{3P_f}$ and $w_c^* = \frac{I}{3P_c}$ back into U yields $V = \left(\frac{2I}{3P_f}\right)^{2/3} \left(\frac{I}{3P_c}\right)^{1/3}$. This indirect utility function V gives the maximum utility achievable to this consumer, given the prices of wine, as well as her income.

Plug in the new prices of wine $P_f = 20$, and $P_c = 8$ into the indirect utility function V. Also plug in her level of utility achieved with the original prices (before the price decrease), into the indirect utility function for V. Finally, solve for I in the indirect utility function after plugging in these values.

At the new prices of wine, this consumer would only need income equal to $I \approx$ **378** ✔ in order to achieve just the same utility has she enjoyed with the original prices (without having extra income to purchase more wine and end up with a higher utility).

Points: 1 / 1

Explanation: Close Explanation

This consumers utility was about 13.6 before the price decrease of french wine. Plugging this, along with the new prices of wine $P_f = 20$, and $P_c = 8$ into her indirect utility function yields

$$
\begin{aligned}
V &= \left(\frac{2I}{3P_f}\right)^{2/3}\left(\frac{I}{3P_c}\right)^{1/3} \\
13.6 &\approx \left(\frac{2I}{3\times 20}\right)^{2/3}\left(\frac{I}{3\times 8}\right)^{1/3} \\
13.6 &\approx \left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{3}\right)^{1/3} I\, 20^{-2/3} 8^{-1/3} \\
\frac{13.6}{\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{3}\right)^{1/3} 20^{-2/3} 8^{-1/3}} &\approx I \\
377.97 &\approx I
\end{aligned}
$$

This means that this consumer would only need an income of about 378 to have the exact same utility as before the price decrease. In other words, this price change was worth about $600 - 378 = 222$

Plug in the old prices of wine $P_f = 40$, and $P_c = 8$ into the indirect utility function V. Also plug in her new level of utility achieved after the price decrease of french wine, into the indirect utility function for V. Finally, solve for I in the indirect utility function after plugging in these values.

At the old prices of wine, this consumer would have needed an income equal to $I \approx$ **952** ✔ in order to achieve just the same utility has she enjoyed after the price decrease of french wine.

Points: 1 / 1

Explanation: Close Explanation

This consumers utility is about 21.5 before the price decrease of french wine. Plugging this, along with the old prices of wine $P_f = 40$, and $P_c = 8$ into her indirect utility function yields

$$
\begin{aligned}
V &= \left(\frac{2I}{3P_f}\right)^{2/3}\left(\frac{I}{3P_c}\right)^{1/3} \\
13.6 &\approx \left(\frac{2I}{3\times 40}\right)^{2/3}\left(\frac{I}{3\times 8}\right)^{1/3} \\
13.6 &\approx \left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{3}\right)^{1/3} I\, 40^{-2/3} 8^{-1/3} \\
\frac{13.6}{\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{3}\right)^{1/3} 40^{-2/3} 8^{-1/3}} &\approx I \\
952.44 &\approx I
\end{aligned}
$$

This means that this consumer would need an income of about 952 to have the exact same utility increase that would result from the price decrease of french wine. In other words, this price change was worth about $952 - 600 = 352$

Try Another Version Continue

Back to Assignment

Attempts 3 | | | Keep the Highest 3 / 3

3. Problems 4.3

On a given evening, J. P. enjoys the consumption of cigars (c) and brandy (c) according to the utility function:

$$U(c,b) = 20c - c^2 + 18b - 3b^2$$

Cost is no object to J.P.

Which of the following are the first order conditions for this unconstrained maximization problem? *Check all that apply.*

- ✓ ☐ $\frac{dU}{dc} = 20c - c^2 = 0$
- ✓ ☑ $\frac{dU}{db} = 18 - 6b = 0$
- ✓ ☑ $\frac{dU}{dc} = 20 - 2c = 0$
- ✓ ☐ $\frac{dU}{db} = 18b - 3b^2 = 0$
- ✓ ☐ $\frac{dU}{db} = 18 - 3b^2 = 0$
- ✓ ☐ $\frac{dU}{dc} = 20 - c^2 = 0$

Points: 1 / 1

Explanation: Close Explanation

The first order conditions for a maximum require that the first partial derivatives of U, with respect to (c) and (c), are equal to 0. Since there is no constraint, there is no need to construct a lagrangian or introduce a lagranian multiplier.

Differentiating U with respect to c yields $\frac{dU}{dc} = 20 - 2c$. This means that one of the first order conditions is $\frac{dU}{dc} = 20 - 2c = 0$.

Differentiating U with respect to b yields $\frac{dU}{db} = 18 - 6b$. This means that one of the first order conditions is $\frac{dU}{db} = 18 - 6b = 0$.

The combination of cigars and brandy that maximizes P.J.'s utility is $c^* =$ 10 ✓ and $b^* =$ 3 ✓ . This leads to a total utility level of $U^* =$ 127 ✓

Points: 1 / 1

Explanation: Close Explanation

The first order condition $\frac{dU}{dc} = 20 - 2c = 0$, implies that $20 = 2c$ and thus $c^* = 10$.

The first order condition $\frac{dU}{db} = 18 - 6b = 0$, implies that $18 = 6b$ and thus $b^* = 3$.

Plugging $c^* = 10$ and $b^* = 3$ back into the utility function yields $U(c^*,b^*) = 20 \cdot 10 - 10^2 + 18 \cdot 3 - 3 \cdot 3^2 = 200 - 100 + 54 - 27 = 127$.

Lately, however, J. P. has been advised by his doctors that he should limit the sum of glasses of brandy and cigars consumed to 5.

With this new constraint, the combination of cigars and brandy that maximizes P.J.'s utility is $c^* =$ 4 ✓ and $b^* =$ 1 ✓ . This leads to a total utility level of $U^* =$ 79 ✓ .

Points: 1 / 1

Explanation: Close Explanation

With this constraint imposed by the doctor, which is mathematically equivalent to the constraint that $5 = b + c$, this constrained maximization problem can be solved using the lagrangian method. This leads to constructing the following lagranian:

$$L = 20c - c^2 + 18b - 3b^2 + \lambda\,(5 - c - b)$$

The first order conditions for a maximum require that the first partial derivatives of L, with respect to c ,b, and λ are equal to 0. This leads to the following first order conditions:

I. $\frac{dU}{dc} = 20 - 2c - \lambda = 0$
II. $\frac{dU}{db} = 18 - 6b - \lambda = 0$
III. $\frac{dU}{d\lambda} = 5 - c - b = 0$

Condition I. implies that $20 - 2c = -\lambda$, while condition II. implies that $18 - 6b = -\lambda$. Setting these equal yields $18 - 6b = 20 - 2c$. Solving for c yields $c = 3b + 1$.

Plugging $c = 3b + 1$ into condition III yields $5 - (3b + 1) - b = 0$. Therefore $4 - 4b = 0$ and thus, $b^* = 1$.

Plugging $b^* = 1$ into the constraint yields $c^* = 4$.

The combination of $c^* = 4$ and $b^* = 1$ yields $U(c^*,b^*) = 20 \cdot 4 - 4^2 + 18 \cdot 1 - 3 \cdot 1^2 = 80 - 16 + 18 - 3 = 79$.

Try Another Version Continue

Back to Assignment

Attempts 1.5 | 3 | Keep the Highest 3 / 3

4. Problems 4.4

Mr. Odde Ball enjoys commodities x and y according to the utility function:

$$U(x,y) = \sqrt{x^2 + y^2}$$

Suppose that $p_x = 3$, $p_y = 4$ and Mr.Ball has $50 of income to spend.

Hint: It may be easier to maximize U^2 rather than U.

The combination of x and y that maximize his utility is $x^* =$ 6 ✔ and y^* 8 ✔ . This combination of x and y would yield a utility level of $U^* =$ 10 ✔ .

Points: 1 / 1

Explanation: Close Explanation

Because squaring U, to get U^2 is a monotonic transformation, the combination of x and y that maxmizes U^2 will also maximize U. In this case, algebraically, it is easier to maximize $U^2 = x^2 + y^2$ than it is to maximize $U(x,y) = \sqrt{x^2 + y^2}$.

Given the budget constraint that $50 = 3x + 4y$, the lagranian to maximize $U^2 = x^2 + y^2$ is:

$$L = x^2 + y^2 + \lambda(50 - 3x + 4y)$$

The first order conditions require that the first partial derivatives of L with respect to x,y, and λ are all equal to zero. These first order conditions are:

$$\begin{aligned}
&\text{I.} \quad \frac{dL}{dx} = 2x - 3\lambda = 0 \\
&\text{II.} \quad \frac{dL}{dy} = 2y - 4\lambda = 0 \\
&\text{III.} \quad \frac{dL}{dy} = 50 - 3x - 4y = 0
\end{aligned}$$

Conditions I. and II. imply that $\frac{x}{y} = \frac{3}{4}$. This means that $y = \frac{4}{3}x$. Plugging this into condition III. for y yields:

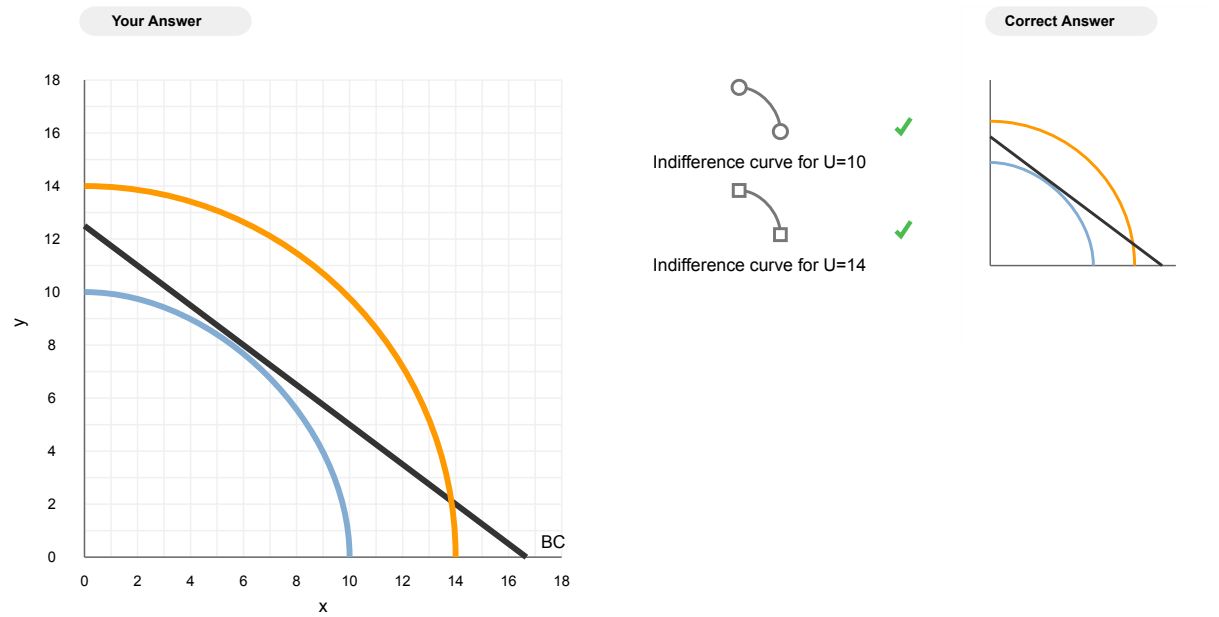
$$\begin{aligned}
0 &= 50 - 3x - 4\tfrac{4}{3}x \\
3x + 4\tfrac{4}{3}x &= 50 \\
\tfrac{9}{3}x + \tfrac{16}{3}x &= 50 \\
\tfrac{25}{3}x &= 50 \\
25x &= 150 \\
x &= 6
\end{aligned}$$

Thus, $x^* = 6$ and $y^* = \frac{4}{3} \times 6 = 8$. Plugging $x^* = 6$ and $y^* = 8$ into the original utility function yields $U(x^*,y^*) = \sqrt{x^{*2} + y^{*2}} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

The following graph shows the budget constrain for Mr.Ball, labeled as BC.

Use the blue curve (circle symbols) to plot the indifference curve for Mr.Ball when $U = 10$ and use the orange curve (orange symbols) to plot the indifference curve for Mr.Ball when $U = 14$.

Hint: It may be easier to place the endpoints on the x and y intercepts in order to plot the indifference curves.



Points: 1 / 1

Explanation: Close Explanation

You can plot the indifference curve for $U = 14$ by placing the endpoints on *(0,14)* and *(14,0)*. These two combinations are both on the same indifference curve because both bundles will lead to the same level of utility $U = \sqrt{14^2 + 0^2} = \sqrt{0^2 + 14^2} = 14$.

You can plot the indifference curve for $U = 10$ by placing the endpoints on *(0,10)* and *(10,0)*. These two combinations are both on the same indifference curve because both bundles will lead to the same level of utility $U = \sqrt{10^2 + 0^2} = \sqrt{0^2 + 10^2} = 10$.

The shape of the indifference curves should resemble a quarter of a circle, in the first quadrant of the graph.

Given the graph, Mr.Ball is **minimizing** ✓ his utility, subject to his budget constraint.

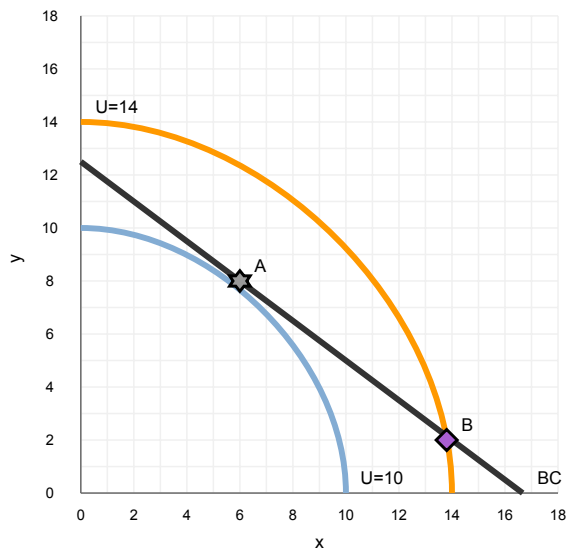
Points: 1 / 1

Explanation: Close Explanation

Given Mr.Ball's indifference curves, using the first order conditions of the lagranian method to find the point *(x*,y*)* where the indifference curve is tangent to the budget constraint will actually yield a utility *minimizing* bundle.

For example, at point A the indifference curve is just tangent to the budget constraint. As was calculated in the previous part of this question, this point occurs at *(6,8)* and yields a level of utility of 10. However, Mr.Ball can increase his utility by consuming at any other point on his budget constrain and enjoy higher utility. Mr Ball can consume more x and less y, like at point B, and end up on a higher indifference curve where $U = 14$.

In fact, any other combination on budget constrain yields a higher level of utility than point A. Therefore, combination A actually minimizes utility. This result stems from the shape of the indifference curves, which do not exhibit diminishing marginal rate of substitution.



Try Another Version

Continue

Back to Assignment

Attempts 2 5 Keep the Highest 5 / 5

5. Problems 4.5

Mr. A derives utility from martinis (m) in proportion to the number he drinks:

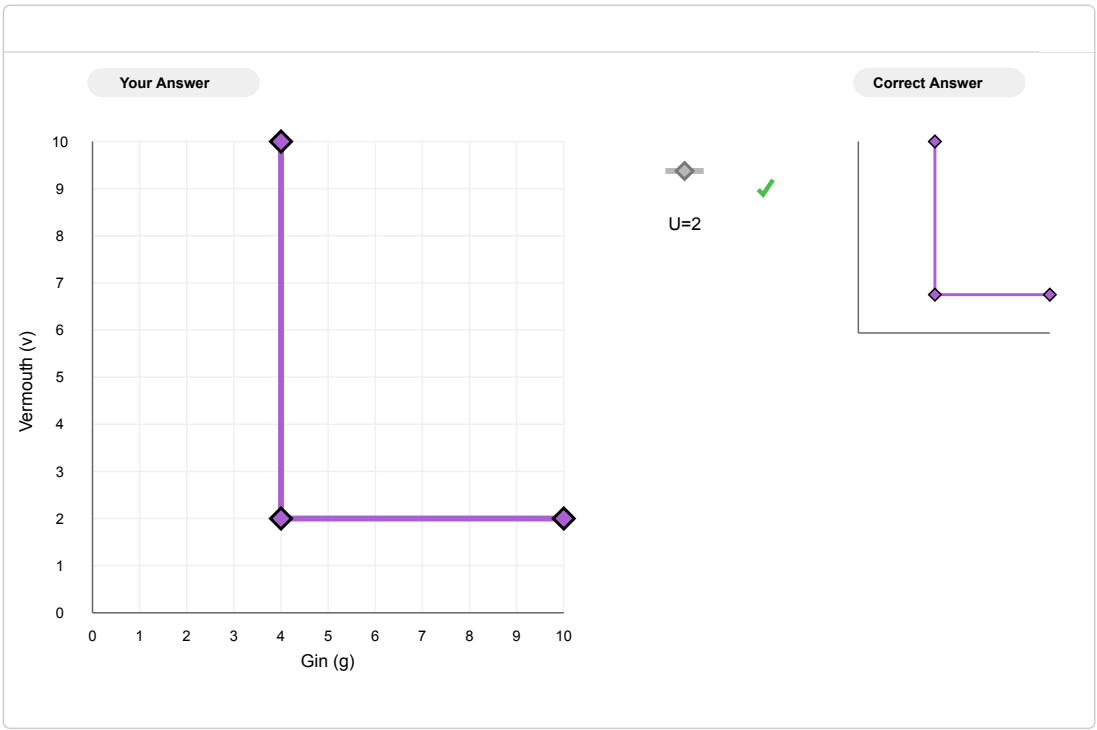
$$U(m) = m$$

Mr.A is particular about his martinis, however: He only enjoys them made in the exact proportion of two parts gin (g) to one part vermouth (v). Hence, we can rewrite Mr.A's utility function as:

$$U(m) = U(g,v) = min(\frac{g}{2}, v)$$

The following graph measures gin (g) along the horizontal axis and vermouth (v) along the vertical axis.

On the following graph, use the purple points (diamon symbols) to plot Mr.A's indifference curve for $U = 2$. The points will connect automatically. Plot the points from left to right in the order that you wish them to connect.



Points: 1 / 1

The shape of the indifference curves, along with the utility function, imply that vermouth and gin are **perfect complements** ✔ .

Points: 1 / 1

Explanation: Close Explanation

Because Mr.A always consumes gin and vermouth in fixed proportions (2 parts gin to one part vermouth), gin and vermouth are considered to be perfect complements to Mr.A.

In general, for perfect complements with utility function $U(x,y) = min(\alpha x, \beta y)$ the indifference curves have a "L" shape with the kink at the point where $\alpha x = \beta y$. In this case, the indifference curve for $U = 2$ has an "L" shape with the kink at the point at which $\frac{g}{2} = v = 2$. This means the

kink is at the point where $g = 4$ and $v = 2$, or *(4,2)* on the graph. For perfect complements, consumers will always choose to consume at this kink in one of the indifference curves, regardless of the relative prices of the complementary goods.

Let p_v be the price of vermouth and p_g be the price of gin. Also let I represent Mr.A's income.

Mr.A's demand curve for gin is $g =$ $\frac{2I}{2p_g+p_v}$ ✔, while his demand curve for vermouth is $v =$ $\frac{I}{2p_g+p_v}$ ✔.

Points: 1 / 1

Explanation: Close Explanation

Given that vermouth and gin are perfect complements, there is no need to use a lagranian to derive the demand curve for each good. Instead, we can use the fact that Mr.A will always consume at the kink of one of his indifference curves. These kinks occur at the points where $\frac{g}{2} = v$. This relationship is equivalent to $g = 2v$. This relationship, along with the budget constraint of $I = p_g g + p_v v$, can be used to derive the demand curves for gin and vermouth.

Plugging $g = 2v$ into the budget constraint for g yields the following:

$$\begin{aligned} I &= p_g g + p_v v \\ I &= p_g \cdot 2v + p_v \\ I &= v\left(2p_g + p_v\right) \\ \frac{I}{2p_g+p_v} &= v \end{aligned}$$

So the demand curve for vermouth is $v = \frac{I}{2p_g+p_v}$. Since $g = 2v$, the demand curve for gin is simply $g = 2v = 2\frac{I}{2p_g+p_v} = \frac{2I}{2p_g+p_v}$

Based on the demand functions for gin and vermouth, which of the following function best describes Mr.A's indirect utility function?

- ✔ (●) $V = \frac{I}{2p_g+p_v}$
- ( ) $V = \frac{2I}{2p_g+p_v}$
- ( ) $V = \frac{I^2}{2p_g+p_v}$
- ( ) $V = 2p_g + p_v$

Points: 1 / 1

Explanation: Close Explanation

The indirect utility function can be found by plugging the demand functions for gin and vermouth into the utility function U. This yields $U = min(g/2,v) = min(\frac{I}{2p_g+p_v}, \frac{I}{2p_g+p_v}) = \frac{I}{2p_g+p_v}$. Let $U = V$ with V representing the maximum utility attainable to Mr.A given the current prices and his income. Then $V = \frac{I}{2p_g+p_v}$ represents Mr.A's indirect utility.

Let E denote the expenditures required, given the current prices and income, to attain the utility level V.

Mr.A's expenditure function is can be stated as $E =$ $(2p_g + p_v)V$ ✔.

Points: 1 / 1

Explanation: Close Explanation

The expenditure function can be derived by assuming $I = E$, and solving for E from the indirect utility function. Given Mr.A's indirect utility function of $V = \frac{I}{2p_g+p_v}$ we have:

$$V = \frac{I}{2p_g + p_v}$$

$$V = \frac{E}{2p_g + p_v}$$

$$(2p_g + p_v)V \quad E$$

Thus, the expenditure function is $E = (2p_g + p_v)V$.

Try Another Version

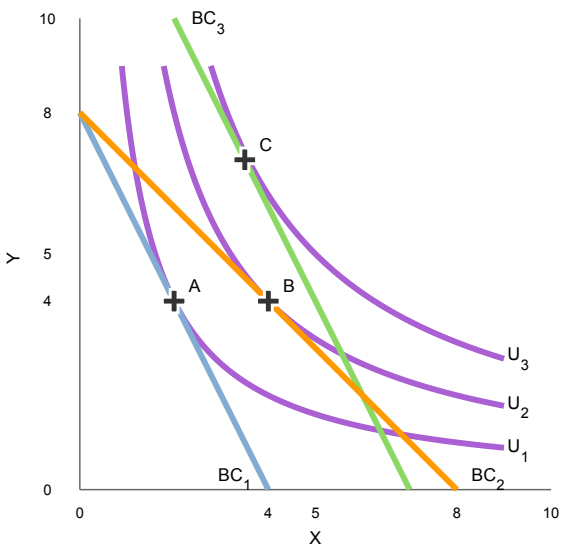
Continue

Back to Assignment

Attempts 3 4 Keep the Highest 4 / 4

6. Problems 4.7

The following graph shows three indifference curves and budget constraints for a consumer. The consumer is initially consuming at point A, on the indifference curve U_1 and is constrained by the budget constraint BC_1 (indicated by the blue line).



Suppose the government provides this consumer a subsidy on good x, which effectively lowers the price of x. This is represented by a **rotation** ✔ of BC_1 out away from the origin. The result is this consumer faces a budget constraint that is similar to BC_2 ✔, which leads to him having a utility level of U_2 ✔.

Points: 1 / 1

Suppose , instead, the government provides this consumer an income grant that can be used on either good x or y, leaving the prices of the actual goods constant. This is represented by a **parallel shift** ✔ of BC_1 out away from the origin. The result is this consumer faces a budget constraint that is similar to BC_3 ✔, which leads to him having a utility level of U_3 ✔. This income grant would leave the consumer on a **higher** ✔ indifference curve than the subsidy on good x.

Points: 1 / 1

Explanation: Close Explanation

Initially this person has an income of I and is choosing to consume the combination of (x, y) represented by point A. A subsidy on good x would lower its price, rotating the budget constraint out away from the origin to a budget constraint like BC_2 and the utility-maximizing choice would shift to combination (x, y) represented by point B. This would cost the government $s{\cdot}x$ (where s is the per unit subsidy on x).

Alternatively, an income grant that shifted the budget constraint outward to BC_3, would also cost the same amount of revenue to government than the subsidy did. The utility provided by the income grant U_3 exceeds that provided by the subsidy on x alone U_2. Hence the utility gain of the subsidy is smaller. A similar argument can be used to illustrate the utility burden of a lump some income tax is smaller than a tax on a specific good.

Let $E(p_x, p_x, U) = 2p_x^{0.5}p_y^{0.5}\ U$ be the expenditure function for a cobb-douglas utility function. Also let $p_x = 1$, and $p_y = 4$.

In order to achieve a utility level of $U = 2$, this consumer would an expenditure of $8 ✓ , while a utility level of $U = 3$ would require an an expenditure of $12 ✓ . In other words, a consumer would need an extra $4 ✓ on purchasing power in order to increase their utility from $U = 2$ to $U = 3$.

Points: 1 / 1

Explanation: Close Explanation

From the given expenditure function, plugging in $p_x = 1$, $p_y = 4$ and $U = 2$ yields $E(p_x, p_x, U) = E(1,4,2) = 21^{0.5} \cdot 4^{0.5} \cdot 2 = 8$. So achieving $U = 2$ would require $8 in expenditures.

From the given expenditure function, plugging in $p_x = 1$, $p_y = 4$ and $U = 3$ yields $E(p_x, p_x, U) = E(1,4,2) = 21^{0.5} \cdot 4^{0.5} \cdot 3 = 12$. So achieving $U = 3$ would require $12 in expenditures.

Therefore, increasing utility from $U = 2$ to $U = 3$ would require $4 more dollars in purchasing power.

Again let $E(p_x, p_x, U) = 2p_x^{0.5} p_y^{0.5}\, U$ be the expenditure function for a cobb-douglas utility function. Also let $p_x = 1$, and $p_y = 4$.

In order to increase the consumers utility while keeping expenditure constant at the level that generated $U = 2$, the government could also subsidize the purchase of x. In order to increase the consumers utility from $U = 2$ to the point where $U = 3$, the government could pay $\frac{5}{9}$ ✓ of the original price of x. At this reduced price, the consumer will choose to purchase 9 ✓ units of good x, resulting in a total cost to the government of $ 5 ✓ . This is a **higher** ✓ cost to the government than the lump sum income grant that would be required to achieve the same gain in utility.

Points: 1 / 1

Explanation: Close Explanation

From the previous part of the question, we know that $E = 8$ is required to achieve utility of U 2.

With no change in income, the government must subsidize x until $E(p_x, p_x, U) = 8 = 2p_x^{0.5} \cdot 4^{0.5} \cdot 3 = 8$ (since the price of y is still 4 and the goal is to increase utility to 3). This implies that:

$$
\begin{aligned}
8 &= 2p_x^{0.5} \cdot 4^{0.5} \cdot 3 \\
4 &= p_x^{0.5} \cdot 6 \\
\tfrac{2}{3} &= p_x^{0.5} \\
\tfrac{4}{9} &= p_x
\end{aligned}
$$

Therefore x needs to be subsidized until $p_x = \frac{4}{9}$. Given that the original price of x was 1, this means the government would need to pay the other $\frac{5}{9}$ of the price of x.

Recall that for this type of cobb-douglas utility function, the demand curve for x is simply $\frac{0.5I}{p_x}$. At this new, lower price, the consumer would purchase $\frac{8}{2 \times \frac{4}{9}} = \frac{8}{\frac{8}{9}} = 9$. Since the government pays $\frac{5}{9}$ for each unit of good x, the total cost of the subsidy is $9 \times \frac{5}{9} = 5$. This is a higher cost than the $4 lump sum income grant from the previous problem.

Try Another Version Continue

Back to Assignment

Attempts 5 | 7 | Keep the Highest 7 / 7

7. Problems 4.8

Consider the fixed proportions case where $U(x,y) = min[x,y]$

In the fixed proportions case, the demand function for x is $x =$ $\frac{I}{p_x+p_y}$ ✔ and the demand function for y is $y =$ $\frac{I}{p_x+p_y}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Since this is the fixed proportions, or perfect complements, case, utility maximization requires that $x = y$. That is, utility maximization always occurs at the kink of the indifference curves. Plugging this into the budget constraint means that :$I = p_x x$

$$\begin{aligned} I &= p_x x + p_y y \\ I &= p_x x + p_y x \\ I &= x\left(p_x + p_y\right) \\ \frac{I}{p_x+p_y} &= x \end{aligned}$$

Therefore $x = \frac{I}{p_x+p_y}$. Since $x = y$, then it must also be the case that $y = \frac{I}{p_x+p_y}$.

Plugging these demand functions back into the utility function yields the indirect utility function which equals $V =$ $\frac{I}{p_x+p_y}$ ✔

Points: 1 / 1

Explanation: Close Explanation

The indirect utility function can be found by plugging the demand functions for x and y back into the utility function. Since the utility function takes the form of $U(x,y) = min[x,y]$, and $x = y$, then the utility function is simply equal to the demand function (for either good). That is, the minimum of $x = \frac{I}{p_x+p_y}$ and $y = \frac{I}{p_x+p_y}$ is simply $\frac{I}{p_x+p_y}$. Therefore, $V = \frac{I}{p_x+p_y}$ is also the indirect utility function.

Starting with the indirect utility function, then solving for I and letting $E = I$ yields the expenditure function which takes the form of $E =$ $(p_x + p_y)V$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Beginning with $V = \frac{I}{p_x+p_y}$, we obtain $I = \left(p_x + p_y\right)V$. Given that expenditures equal income, $I = E$ the expenditure function becomes $E = \left(p_x + p_y\right)V$.

Consider the case of perfect substitutes where $U(x,y) = x + y$

In this perfect substitutes case, when $p_y > p_x$ the demand function for x is $x =$ $\frac{I}{p_x}$ ✔ and the demand function for y is $y =$ 0 ✔ .

Points: 1 / 1

When $p_x > p_y$ the demand function for y is $y =$ $\frac{I}{p_y}$ ✔ when and the demand function for y is $x =$ 0 ✔ .

Points: 1 / 1

Explanation: Close Explanation

In this perfect substitutes case, when $U = x + y$, utility maximization requires that the consumer purchase whichever good has the lower price. That is, when $p_y > p_x$, the consumer will spend all income on good x and no income on good y. Therefore, in this case, $x = \frac{I}{p_x}$ and $y = 0$.

On the other hand, when $p_x > p_y$, the consumer will spend all income on good y and no income on good x. Therefore, in this case, $y = \frac{I}{p_y}$ and $x = 0$.

Plugging these demand functions back into the utility function yields the indirect utility function which can be written in general terms as $V =$ $\frac{I}{min(p_x, p_y)}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

The indirect utility function can be found by plugging the demand functions for x and y back into the utility function.

When $p_y > p_x$, then $x = \frac{I}{p_x}$ and $y = 0$, and thus $V = \frac{I}{p_x} + 0 = \frac{I}{p_x}$.

When $p_x > p_y$, then $y = \frac{I}{p_y}$ and $x = 0$, and thus $V = 0 + \frac{I}{p_y} = \frac{I}{p_y}$.

In both cases, the indirect utility is found by dividing income by the lower of the two prices of the goods. Therefore, the indirect utility function can be written as $V = \frac{I}{min(p_x, p_y)}$

Starting with the indirect utility function, then solving for I and letting $E = I$ yields the expenditure function which takes the form of $E =$ $min(p_x, p_y)V$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Beginning with $V = \frac{I}{min(p_x, p_y)}$, we obtain $I = min(p_x, p_y)V$. Given that expenditures equal income, $I = E$ the expenditure function becomes $E = min(p_x, p_y)V$.

Try Another Version Continue

Back to Assignment

Attempts | 2.5 | 3 | | Keep the Highest 3 / 3

8. Problems 4.9

Suppose that we have a utility function involving two goods that is linear of the form $U(x,y) = ax + by$.

Suppose $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$.

Hint: It may be helpful to draw yourself a diagram. What is true about the slope of the indifference curves and the slope of the budget constraint?

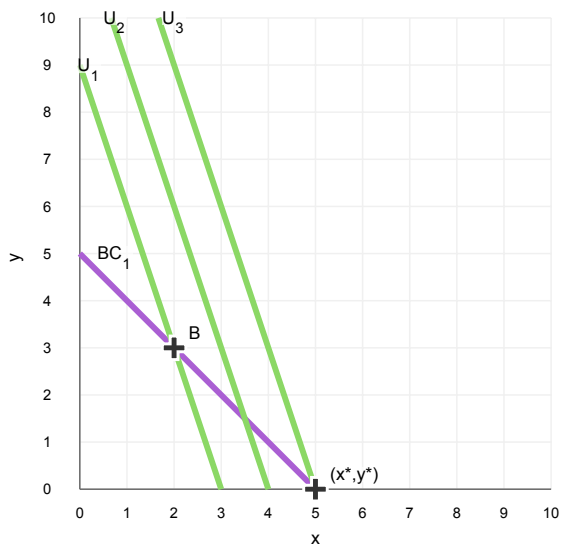
This means the slope of the indifference curves are **steeper** ✓ than the slope of the budget constraint. This implies that all of the consumers income is spent on good x ✓. Therefore, the demand function for x is $x^* =$ $\frac{I}{p_x}$ ✓ and the demand function for y is $y^* =$ 0 ✓.

Points: 1 / 1

Explanation: Close Explanation

If $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$, this means that $\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$. Since $\frac{MU_x}{MU_y}$ represents the magnitude of the slope of the indifference curves and $\frac{p_x}{p_y}$ represents the magnitude of the slope of the budget constraint, $\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$ implies that the indifference curves are steeper than the budget constraint.

Graphically you can see that this will lead to a corner solution in which the consumer spends all income on x and no income on y. Let the green lines represent the linear indifference curves associated with the utility function $U(x,y) = ax + by$, where $U_3 > U_2 > U_1$. Let BC_1 represent the budget constraint for a particular set of prices and income.



The highest indifference curve attainable is U_3, where all income is spent on x and no y is purchased. Any other combination of x and y will lead to a lower utility, on lower indifference curves such as point B.

Thus, when the slopes of the indifference curves are steeper than the slope of the budget constraint, the consumer will allocate all income to good x. Thus $x^* = \frac{I}{p_x}$ and $y^* = 0$.

Suppose $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$.

Hint: It may be helpful to draw yourself a diagram. What is true about the slope of the indifference curves and the slope of the budget constraint?

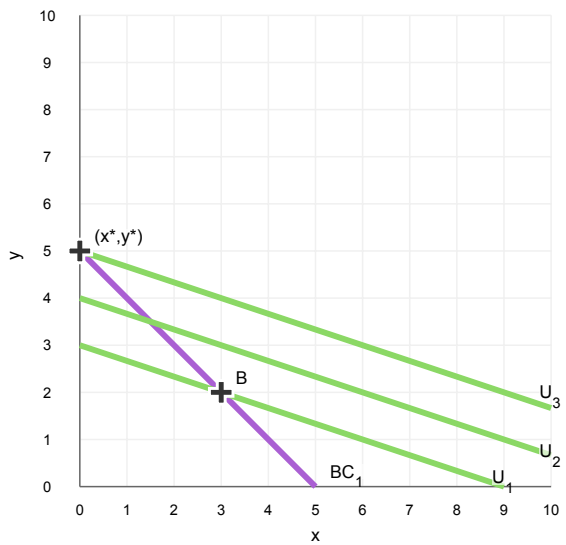
This means the slope of the indifference curves are **flatter** ✔ than the slope of the budget constraint. This implies that all of the consumers income is spent on good y ✔. Therefore, the demand function for x is $x^* =$ 0 ✔ and the demand function for y is $y^* =$ $\frac{I}{p_y}$ ✔.

Points: 1 / 1

Explanation: Close Explanation

If $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$, this means that $\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$. Since $\frac{MU_x}{MU_y}$ represents the magnitude of the slope of the indifference curves and $\frac{p_x}{p_y}$ represents the magnitude of the slope of the budget constraint, $\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$ implies that the indifference curves are flatter than the budget constraint.

Graphically you can see that this will lead to a corner solution in which the consumer spends all income on y and no income on x. Let the green lines represent the linear indifference curves associated with the utility function $U(x,y) = ax + by$, where $U_3 > U_2 > U_1$. Let BC_1 represent the budget constraint for a particular set of prices and income.



The highest indifference curve attainable is U_3, where all income is spent on y and no x is purchased. Any other combination of x and y will lead to a lower utility, on lower indifference curves such as point B.

Thus, when the slopes of the indifference curves are flatter than the slope of the budget constraint, the consumer will allocate all income to good y. Thus $y^* = \frac{I}{p_y}$ and $x^* = 0$.

When $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$, the expenditure function is $E =$ $\frac{p_x U}{a}$ ✔. However, when $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$, the expenditure function is $E =$ $\frac{p_y U}{b}$ ✔. .

Points: 1 / 1

Explanation: Close Explanation

To derive the expenditure function, the indirect utility function is needed (for each of the two cases).

When $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$, we know that $x^* = \frac{I}{p_x}$ and $y^* = 0$. Thus the indirect utility function is simply $V = ax^* + by^* = \frac{aI}{p_x} + 0 = \frac{aI}{p_x}$. Solving for I we have $I = \frac{Vp_x}{a}$. For the expenditure function, $E = I$ and the utility is now denoted as U Thus the expenditure function in this case is simply $E = \frac{p_x U}{a}$.

Similarly, when $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$, we know that $y^* = \frac{I}{p_y}$ and $x^* = 0$. Thus the indirect utility function is simply $V = ax^* + by^* = 0 + \frac{bI}{p_y} = \frac{bI}{p_y}$. Solving for I we have $I = \frac{Vp_y}{b}$. For the expenditure function, $E = I$ and the utility is now denoted as U Thus the expenditure function in this case is simply $E = \frac{p_y U}{b}$.

When $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$ the budget constraint and the indifference curves are parallel and thus, overlap completely for some level of utility. IN that case, the any point on the budget constraint is a utility maximizing choice, and thus the expenditure function is simply $E = \frac{p_x U}{a} = \frac{p_y U}{b}$.

Try Another Version Continue

Back to Assignment

Attempts 5.8 | 6 | **7** Keep the Highest 7 / 7

4. Problems 5.2

David gets $3 per week as an allowance to spend any way he pleases. Because he likes only peanut butter and jelly sandwiches, he spends the entire amount on peanut butter (b) (at $0.05 per ounce) and jelly (at $0.10 per ounce). Let b represent ounces of peanut butter and (j) represent ounces of jelly. The, price of peanut butter is $p_b = 0.05$, and the price of jelly is $p_j = 0.1$. Bread is provided free of charge by a concerned neighbor. David is a particular eater and makes his sandwiches with exactly 1 ounce of jelly and 2 ounces of peanut butter. He is set in his ways and will never change these proportions.

Which of the following best represents David's utility function?

- ○ $U(b,j) = b + 2j$
- ✗ ○ $U(b,j) = min(2b,j)$
- ✓ ◉ $U(b,j) = min(b,2j)$
- ○ $U(b,j) = bj^2$

Points: 1 / 1

Explanation: Close Explanation

Because David always consumes peanut butter and jelly in fixed proportions, peanut butter and jelly are perfect complements. Thus, David's utility function takes the form of $U(b,j) = min(b,2j)$ because each peanut butter and jelly sandwich contains 2 ounces of peanut butter and 1 ounce of jelly.

Given the current prices of peanut butter and jelly, if David has $3 to spend each week, he will consume $b^* =$ 30 ✓ ounces of peanut butter and $j^* =$ 15 ✓ ounces of jelly.

Points: 1 / 1

Explanation: Close Explanation

For all two-good fixed proportion utility functions, the utility maximizing bundle occurs when the two arguments of the function are equal to each other (at the "kink" of the indifference curves). In this case, this means that David will consume at a point where $b = 2j$ and will still be constrained by his budget constraint $I = p_b B + p_j j$. Plugging in $b = 2j$ to the budget constraint yields:

$$\begin{aligned} I &= p_b B + p_j j \\ I &= p_b 2j + p_j j \\ I &= j\left(2(p_b) + p_j\right) \\ \frac{I}{2(p_b)+p_j} &= j^* \end{aligned}$$

So $j^* = \frac{I}{2p_b+p_j}$. Because $b = 2j$, then $b^* = \frac{2I}{2p_b+p_j}$.

Plugging the current prices and income ($p_b = 0.05$, $p_j = 0.1$, and $I = 3$) into these demand functions yields:

$$j^* = \frac{3}{2\,((0.05))+(0.1)} = \frac{3}{0.2} = 15$$
$$b^* = \frac{6}{2\,((0.05))+(0.1)} = \frac{3}{0.2} = 30$$

If the price of jelly increases to $\$0.15$ per ounce, while income and the price of peanut butter remains the same, David will now consume $b^* =$ 24 ✓ ounces of peanut butter and $j^* =$ 12 ✓ ounces of jelly.

Points: 1 / 1

Explanation: Close Explanation

Given the general demand functions $j^* = \frac{I}{2(p_b)+p_j}$ and $b^* = \frac{2I}{2(p_b)+p_j}$, the new consumption bundle for David becomes:

$$j^* = \frac{3}{2\,((0.05))+(0.15)} = \frac{3}{0.2} = 12$$
$$b^* = \frac{6}{2\,((0.05))+(0.15)} = \frac{3}{0.2} = 24$$

In order to continue purchasing the quantities of jelly and peanut butter that he consumed **before** the price increase of jelly, David would need an increase in allowance of $0.75 ✓ .

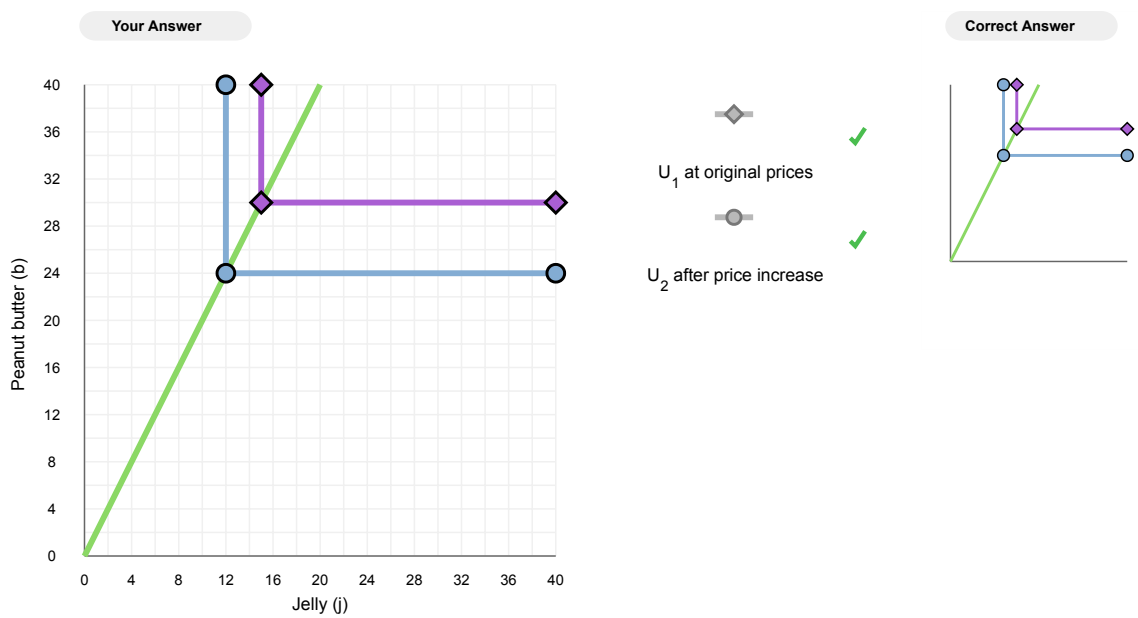
Points: 1 / 1

Explanation: Close Explanation

David originally consumed a bundle consisting of $j^* = 15$ and $b^* = 30$. In order to purchase this original bundle at the new prices, David would need a total income of $(15 \times 0.15) + (30 \times 0.05) = 3.75$. Because his original income was $3, he would need $0.75 more in allowance to be able to maintain the same level of utility he enjoyed before the price increase.

The following graph measures peanut butter on the vertical axis and jelly on the horizontal axis.

On the following graph, plot two of David's indifference curves. Use the purple diamond symbols to plot the indifference corresponding to U_1, the level of utility he attains at the original price levels. Use the blue circle symbols to plot the indifference curve corresponding to U_2, the level of utility he attains after the price increase of jelly. Plot the points from left to right in the order in which you would like them to be connected. Line segments will connect automatically.



Points: 1 / 1

Explanation: Close Explanation

Because peanut butter and jelly are perfect complements, David's indifference curve will have the familiar "L" shape. The corner, or kink, of the indifference curve occurs along the line $b = 2j$ because 1 ounce of peanut butter is always consumed with 2 ounces of jelly, which is shown as a green line on the graph.

Before the price change, David consumes $j^* = 15$ and $b^* = 30$, so the kink of the indifference curve for U_1 should be at the point (15,30). After the price change, David consumes $j^* = 12$ and $b^* = 24$, so the kink of the indifference curve for U_2 should be at the point (12,24).

You will notice that the kinks in both indifference curves lay perfectly on the green $b = 2j$ line.

Instead of looking at two goods (peanut butter and jelly), David's utility can be viewed as a function of only a single good: peanut butter and jelly sandwiches.

With this point of view, the price of sandwiches before the price increase was $p_s =$ $0.20 ✔ . At this price, David consumed 15 ✔ sandwiches. After the price of jelly increased, the price of sandwiches increased to $p_s =$ $0.25 ✔ . At this new price, David consumed 12 ✔ sandwiches. In general, David's demand function for sandwiches can be stated as $q_s = \frac{3}{p_s}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

Because David always uses 2 ounces of peanut butter and 1 ounce of jelly on his sandwiches, the price of a sandwich is $p_s = 2p_b + p_j$.

At the original prices, the price of a sandwich was $p_s = (2 \times 0.05) + 0.1 = 0.2$. At this price, he consumed 15 ounces of jelly and 30 ounces of peanut butter, which allows for 15 sandwiches.

After the price increase of jelly, the price of a sandwich changed to $p_s = (2 \times 0.05) + 0.15 = 0.25$. At this price, he consumed 12 ounces of jelly and 24 ounces of peanut butter, which allows for 12 sandwiches.

In general, because David buys only one good (sandwiches), his demand function for sandwiches is simply his income divided by the price of sandwiches. In this case, because his income is $3 per week, his demand function for sandwiches is $q_s = \frac{3}{p_s}$.

In this case there is no **substitution** ✔ effect as a result of the price increase of jelly; the change David's consumption is due entirely to the **income** ✔ effect.

Points: 1 / 1

Explanation: Close Explanation

Because David always consumes peanut butter and jelly in fixed proportions, there is no substitution effect when the price of one of the goods changes. In other words, when the price of jelly increased David did not respond by increasing his consumption of peanut butter even though peanut butter was then cheaper relative to jelly than before. Instead, the price increase caused him to purchase less jelly. As a result, given the lower amount of jelly and because he always consumes the two goods in fixed proportions, David needed less peanut butter to make his sandwiches. In this way, the increase in the price of jelly has purely an income effect because the decrease in David's overall purchasing power is what changed David's consumption (and not the relative prices of the two goods).

Try Another Version Continue

Back to Assignment

Attempts 1 3 Keep the Highest 3 / 3

1. Problems 5.3

A utility function is homothetic if any straight line through the origin cuts all indifference curves at points of equal slope: The MRS depends only on the ratio $\frac{y}{x}$. Assume the utility function $U(x,y)$ is homothetic.

For a generic budget constraint $I = p_x x + p_y y$, when I increases, the slope of the budget constraint **remains constant** ✔. Given that, in the two-good case, the conditions for utility maximization require that $MRS = \frac{p_y}{p_y}$ ✔, the MRS at the utility maximizing bundle must **remain constant** ✔ as income rises. Given that this utility function is homothetic, the ratio $\frac{y}{x}$ must **remain constant** ✔.

Points: 1 / 1

Explanation: Close Explanation

For a generic budget constraint $I = p_x x + p_y y$, changes in I are modeled as parallel shifts. An increase in I is represented by a parallel shift away from the origin. Because the shift is parallel, the slope of the budget constraint does not change and remains as $-\frac{p_y}{p_y}$, or an absolute value of $\frac{p_y}{p_y}$. That is, the slope of the budget constraint does not change as income changes, ceterus paribus.

In the two-good case, the first order conditions for a maximum require that the slope of the budget constraint must be equal to the marginal rate of substitution. This is equivalent to $MRS = \frac{p_y}{p_y}$. Because the MRS depends only on the ratio $\frac{y}{x}$, this ratio must also remain constant in order for the MRS to be equal to $\frac{p_x}{p_y}$.

True or False: Given this relationship between the ratio $\frac{y}{x}$ and income I, as well as the fact that all income is spent on only two goods, both x and y must be proportional to income I.

✔ (●) True

( ) False

Points: 1 / 1

Explanation: Close Explanation

First, because the consumer spends all income on only two goods, at least one good must be a normal good. That is, when income rises, consumption of at least one good must increase. In other words, when income rises it cannot be the case that the consumer consumes less of both goods. Let y be a normal good. Thus, $\frac{dy}{dI} > 0$

Thus, when I increases y must also increase. In order for $\frac{y}{x}$ to remain constant, however, x must also increase. The same is true for when I decreases; both y and x must decrease. Therefore, x is also a normal good and $\frac{dx}{dI} > 0$.

For example, if $\frac{y}{x} = \frac{4}{2} = 2$ and an increase in income increases y^* to 10, x^* must increase to 5 so that $\frac{y}{x} = \frac{10}{5} = 2$. In fact, $\frac{y}{x}$ must change by the same proportion in order to maintain this ratio. Thus, both x and y must be proportional to income I.

Because **both x and y are normal goods** ✔, the Giffin paradox **cannot** ✔ arise in this case.

Points: 1 / 1

Explanation: Close Explanation

Given that $\frac{dx}{dI} > 0$, x must always rise with an increase in income. Giffin's paradox occurs (for an inferior good), when the price of that good increases and the loss of purchasing power actually causes consumers to consume more of that good. This occurs for inferior goods and when the income effect is very strong. Because x and y are both normal goods, however, the Giffin paradox cannot arise in the case of homothetic preferences.

Try Another Version Continue

Back to Assignment

Attempts 4.8 | 6 | Keep the Highest 6 / 6

2. Problems 5.4

Assume utility is given by:

$$U(x,y) = x^{0.3}y^{0.7}$$

Because this is a Cobb-Douglas utility function, without using the Lagrangian method you know that the (uncompensated) demand function for x is $x^* =$ $\frac{0.3I}{p_x}$ ✔, and the (uncompensated) demand function for y is $y^* =$ $\frac{0.7I}{p_y}$ ✔.

Points: 1 / 1

Explanation: Close Explanation

For a general Cobb-Douglas utility function where $U(x,y) = x^{\alpha}y^{\beta}$ with income I and prices p_x and p_y, the uncompensated demand functions can be written as $x^* = \frac{\alpha I}{p_x}$ and $y^* = \frac{\beta I}{p_y}$. In other words, for this type of utility function, the consumer will spend α proportion of income on good x and β proportion of income on good y.

In this case $\alpha = 0.3$ and $\beta = 0.7$, so $x^* = \frac{0.3I}{p_x}$ and $y^* = \frac{0.7I}{p_y}$.

Which of the following functions best represents the indirect utility function for this utility function?

- ✔ (●) $V = \frac{0.3^{0.3}0.7^{0.7}\,I}{p_x^{0.3}p_y^{0.7}}$
- ( ) $V = \frac{0.3^{0.3}0.7^{0.7}}{p_x^{0.3}p_y^{0.7}}$
- ( ) $V = \frac{I}{p_x^{0.3}p_y^{0.7}}$
- ( ) $V = \frac{0.3^{0.7}0.7^{0.3}\,I}{p_x^{0.7}p_y^{0.3}}$

Points: 1 / 1

Explanation: Close Explanation

The indirect utility function is found by plugging the uncompensated demand functions x^* and y^* back into the utility function $U(x,y) = x^{0.3}y^{0.7}$. In this case, plugging $x^* = \frac{0.3I}{p_x}$ and $y^* = \frac{0.7I}{p_y}$ into the utility function yields:

$$\begin{aligned} U(x^*,y^*) &= x^{*0.3}y^{*0.7} \\ &= \left(\frac{0.3I}{p_x}\right)^{0.3}\left(\frac{0.7I}{p_y}\right)^{0.3} \\ &= \frac{0.3^{0.3}0.7^{0.7}I^{0.3}I^{0.7}}{p_x^{0.3}p_y^{0.7}} \\ V &= \frac{0.3^{0.3}0.7^{0.7}\,I}{p_x^{0.3}p_y^{0.7}} \end{aligned}$$

Therefore, the indirect utility function is $V = \frac{0.3^{0.3}0.7^{0.7}\,I}{p_x^{0.3}p_y^{0.7}}$.

Which of the following functions best represents the expenditure function for this utility function?

- ( ) $E = \frac{p_x^{0.3}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}}$

- ○ $E = Up_x^{0.3}p_y^{0.7}\,0.3^{0.3}0.7^{0.7}$
- ✔ ◉ $E = \frac{Up_x^{0.3}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}}$
- ○ $E = Up_x^{0.3}p_y^{0.7}$

Points: 1 / 1

Explanation: Close Explanation

The expenditure function can be found by beginning with the indirect utility function and solving for I.

In this case, beginning with $V = \frac{0.3^{0.3}0.7^{0.7}\,I}{p_x^{0.3}p_y^{0.7}}$, solving for I yields:

$$\begin{aligned} V &= \frac{0.3^{0.3}0.7^{0.7}\,I}{p_x^{0.3}p_y^{0.7}} \\ p_x^{0.3}p_y^{0.7}V &= 0.3^{0.3}0.7^{0.7}\,I \\ \frac{p_x^{0.3}p_y^{0.7}V}{0.3^{0.3}0.7^{0.7}} &= I \end{aligned}$$

Assuming the consumer spends all income I, then $I = E$. Also, let $V = U$ represent utility. Thus, the expenditure function is $E = \frac{Up_x^{0.3}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}}$.

Using the expenditure function, along with Shephar's Lemma, results in a compensated demand function for x of

$x^c =$ $\frac{0.3Up_y^{0.7}}{0.3^{0.3}0.7^{0.7}p_x^{0.7}}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

The compensated demand function for a good can always be found from the expenditure function by differentiation with respect to that good's price. In this case, finding the compensated demand curve for x involves taking the first partial derivative of $E = \frac{Up_x^{0.3}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}}$ with respect to p^x. Doing this (using the power rule) yields:

$$\begin{aligned} E &= \frac{Up_x^{0.3}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}} \\ \frac{dE}{p^x} &= \frac{0.3Up_x^{-0.7}p_y^{0.7}}{0.3^{0.3}0.7^{0.7}} \\ \frac{dE}{p^x} &= \frac{0.3Up_y^{0.7}}{0.3^{0.3}0.7^{0.7}p_x^{0.7}} \end{aligned}$$

Hint: You can find the elasticities without doing any calculations because each value is (1) either constant for all Cobb-Douglas utility functions of this form or (2) can be read directly from the Cobb-Douglas utility function itself.

For this Cobb-Douglas utility function, $\alpha + \beta = 1$, the own price elasticity of demand for x is $e_{x,p_x} =$ -1 ✔ , while the income elasticity of demand for x is $e_{x,I} =$ 1 ✔ , the cross price elasticity of demand for x with p_y is $e_{x,p_y} =$ 0 ✔ , and the Cournot aggregation is $s_x =$ -0.3 ✔ . Finally, the compensated demand elasticity is $e_{xc,p_x} =$ -0.7 ✔ .

Points: 1 / 1

Explanation: Close Explanation

For a general Cobb-Douglas utility function where $U(x,y) = x^\alpha y^\beta$ and $\alpha + \beta = 1$, the demand elasticities for x require no calculations because derivations using the formal definitions always yield the same result.

Own price elasticity of demand is defined as $\frac{dx}{dp_x} \cdot \frac{p_x}{x} = \frac{-\alpha I}{p_x^2} \cdot \frac{p_x}{\alpha I / p_x} = -1$.

Cross price elasticity of demand is defined as $\frac{dx}{dp_y} \cdot \frac{p_y}{x} = 0 \cdot \frac{p_y}{x} = 0$.

Income elasticity of demand is defined as $\frac{dx}{dI} \cdot \frac{I}{x} = \frac{\alpha}{p_x} \cdot \frac{I}{\alpha I/p_x} = 1$.

The compensated demand elasticity is $e_{xc,p_x} = e_{x,p_x} + s_x e_{x,I} = -1 + \alpha = \alpha - 1 = -\beta = -0.7$.

Cournot aggregation s_x is always equal to the exponent on x, which is α. Thus, in this case,Cournot aggregation is $s_x = 0.3$.

Given the values of the elasticities and Cournot aggregation, which of the following statements is true?

- ○ $e_{x,p_x} < e_{xc,p_x} - s_x e_{x,I}$
- ✓ ◉ $e_{x,p_x} = e_{xc,p_x} - s_x e_{x,I}$
- ○ $e_{x,p_x} > e_{xc,p_x} - s_x e_{x,I}$

Points: 1 / 1

Explanation: Close Explanation

The Slutsky equation in elasticity form states that $e_{x,p_x} = e_{xc,p_x} - s_x e_{x,I}$.

Plugging in the values for the elasticities confirms this equality because $-1 = -0.7 - (0.3) \times (1) = -1$. Thus, the Slutsky equation holds in this case.

Try Another Version Continue

Back to Assignment

Attempts 3 6 Keep the Highest 6 / 6

3. Problems 5.5

Suppose the utility function for goods x and y is given by:

$$U(x,y) = xy + y$$

Which of the following represent first order conditions for a utility maximizing combination of $x$and y? *Check all that apply.*

- ✔ ☐ $\frac{dL}{d\lambda} = I - p_x x - p_y y > 0$
- ✔ ☐ $\frac{dL}{dx} = x - \lambda p_x = 0$
- ✔ ☑ $\frac{dL}{dy} = x + 1 - \lambda p_y = 0$
- ✔ ☐ $\frac{dL}{dy} = y + 1 - \lambda p_y = 0$
- ✔ ☐ $\frac{dL}{dx} = \frac{1}{x} - \lambda p_x = 0$
- ✔ ☑ $\frac{dL}{d\lambda} = I - p_x x - p_y y = 0$
- ✔ ☑ $\frac{dL}{dx} = y - \lambda p_x = 0$
- ✔ ☐ $\frac{dL}{dy} = y - \lambda p_y = 0$

Points: 1 / 1

Explanation: Close Explanation

The Langranian for this problem is $L = xy + y + \lambda\left(I - p_x x - p_y y\right)$. The first order conditions require that the first partial derivatives with respect to x, y and λ are all equal to 0. Thus, taking these partials and setting equal to 0 yields:

I. $\frac{dL}{dx} = y - \lambda p_x = 0$
II. $\frac{dL}{dy} = x + 1 - \lambda p_y = 0$
III. $\frac{dL}{d\lambda} = I - p_x x - p_y y = 0$

Solving these first order conditions yields an uncompensated demand function for x of $x^* =$ $\frac{I-p_x}{2p_x}$ ✔ and an uncompensated demand function for y of $y^* =$ $\frac{I+p_x}{2p_y}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

You can use the first order conditions to derive the (uncompensated) demand curves for x and y.

Condition I. implies that $y = \lambda p_x$while condition II. is equivalent to $x + 1 = \lambda p_y$. Dividing condition I. by condition II. yields $\frac{y}{x+1} = \frac{p_x}{p_y}$ or $p_y y = (x + 1)p_x$.

Plugging this $p_y y = (x + 1)p_x = p_x x + p_x$ into condition III. yields:

$$
\begin{aligned}
I - p_x x - p_y y &= 0 \\
I - p_x x - (p_x x + p_x) &= 0 \\
I - p_x x - p_x x - p_x &= 0 \\
I - 2p_x x - p_x &= 0 \\
I - p_x &= 2p_x x \\
\frac{I-p_x}{2p_x} &= x
\end{aligned}
$$

Therefore, $x^* = \frac{I-p_x}{2p_x}$. Because $p_y y = p_x x + p_x$, we have :

$$
\begin{aligned}
p_y y &= p_x x + p_x \\
p_y y &= p_x \times \frac{I-p_x}{2p_x} + p_x \\
p_y y &= \frac{I-p_x}{2} + p_x \\
p_y y &= \frac{I+p_x}{2} \\
y &= \frac{I+p_x}{2p_y}
\end{aligned}
$$

Thus, $y^* = \frac{I+p_x}{2p_y}$.

Based on the uncompensated demand functions, changes in income affect **the demand for both x and y** ✔ . Changes in the price of x affect **both x^* and y^*** ✔ while changes in the price of y affect **y^* but not x^*** ✔ .

Points: 1 / 1

Explanation: Close Explanation

From the uncompensated demand formulas $y^* = \frac{I+p_x}{2p_y}$ and $x^* = \frac{I-p_x}{2p_x}$, the price of x appears in both demand functions, but the price of y appears only in the demand function for y. Thus, p_x affects both x^* and y^*, while p_y affects y^* but not x^*.

Because income I appears in both demand functions, changes in income also affect y^* but not x^*.

Which of the following best represents the expenditure function for this utility function?

- ✔ (●) $E = \sqrt{U4p_x p_y} - p_x$
- ( ) $E = \sqrt{U4p_x p_y}$
- ( ) $E = \sqrt{4p_x p_y} - p_x$
- ( ) $E = \sqrt{I4p_x p_y} - p_x$

Points: 1 / 1

Explanation: Close Explanation

To find the expenditure function, plug the uncompensated demand functions for x^* and y^* back into the utility function to find the indirect utility function V. This yields $V = U(x^*,y^*) = x^*y^* + y^* = \frac{I+p_x}{2p_y} \cdot \frac{I-p_x}{2p_x} + \frac{I+p_x}{2p_y}$. Simplifying yields:

$$
\begin{aligned}
V &= \frac{I+p_x}{2p_y} \cdot \frac{I-p_x}{2p_x} + \frac{I+p_x}{2p_y} \\
&= \frac{I^2-p_x^2}{4p_xp_y} + \frac{I+p_x}{2p_y} \times \frac{2p_x}{2p_x} \\
&= \frac{I^2-p_x^2}{4p_xp_y} + \frac{2Ip_x+2p_x^2}{4p_yp_x} \\
&= \frac{I^2+2Ip_x+p_x^2}{4p_xp_y} \\
V &= \frac{(I+p_x)^2}{4p_xp_y}
\end{aligned}
$$

Thus the indirect utility function is $V = \frac{(I+p_x)^2}{4p_xp_y}$.

To get the expenditure function, you must solve this equation for I, yielding:

$$
\begin{aligned}
V &= \frac{(I+p_x)^2}{4p_xp_y} \\
V4\,p_xp_y &= (I+p_x)^2 \\
\sqrt{V4\,p_xp_y} &= I+p_x \\
\sqrt{V4\,p_xp_y} - p_x &= I
\end{aligned}
$$

Assuming all income is spent, $I = E$ and letting $U = V$ represent utility, the expenditure function becomes $E = \sqrt{U4p_xp_y} - p_x$.

Using the expenditure function, as well as Shephard's Lemma, yields a compensated demand curve for x of $x^c =$ $U^{0.5}p_x^{-0.5}p_y^{0.5} - 1$ ✔ and a compensated demand curve for y of $y^c =$ $U^{0.5}p_x^{0.5}p_y^{-0.5}$ ✔ .

Points: 1 / 1

Explanation: Close Explanation

The uncompensated demand curves can be found by taking the first partial derivative of the expenditure function with respect to p_x and p_y. That is $\frac{dE}{dp_x} = x^c = U^{0.5}p_x^{-0.5}p_y^{0.5} - 1$ by the power rule. Similarly, $\frac{dE}{dp_y} = y^c = U^{0.5}p_x^{0.5}p_y^{-0.5}$.

True or False: A change in the price of y affects the compensated and uncompensated demand function for x in the same way.

- ○ True
- ✔ ◉ False

Points: 1 / 1

Explanation: Close Explanation

The uncompensated demand function for x is $x^* = \frac{I-p_x}{2p_x}$, and changes in the price of y did not affect the uncompensated demand function for x. The compensated demand function x^c , however, depends onp_y. So while the price of y does not affect x^*, it can still influence x^c.

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