Economics 701

# Problem Set 7

1. An individual faces three possible outcomes (in dollars), ranked from least desired to most desired: {10, 24, 90}. The Von Neumann–Morgenstern (VNM) utility of outcome 10 is 0 and that for 90 is 1; i.e., $U(10) = 0$ and $U(90) = 1$. Suppose the individual is indifferent between 24 for certain and a lottery which pays 10 with probability 2/3 and pays 90 with probability 1/3.
a. What is the value of the certainty equivalent of the above lottery?
b. What is the VNM utility of the outcome 24; i.e., what is $U(24)$?
c. Use the expected-utility hypothesis to determine which lottery below the individual would prefer:

- Lottery A: pays 24 with probability 3/4 and pays 10 with probability 1/4
- Lottery B: pays 10 with probability 2/5, pays 24 with probability 1/2, and pays 90 with probability 1/10

2. Compute the Arrow-Pratt measures of absolute risk aversion and relative risk aversion for the following and discuss whether they exhibit risk aversion, risk neutrality, etc. and also what they exhibit in regard to CARA, CRRA, DARA, etc.:
a. $U(w) = ln\,(w)$ b. $U(w) = \frac{-w^{-3}}{3}$ c. $U(w) = -e^{-2w}$ d. $U(w) = 2w$

3. Suppose a homeowner has logarithmic utility, $U(w) = ln\,(w)$. The homeowner's current wealth is \$300,000. Suppose there is a 50% probability of a flood, in which case the homeowner will suffer a loss of \$200,000, so that wealth will be only \$100,000.
a. Compute the folowing: the utilities of the two possibile outcomes, the expected value of wealth, the expected utility, the utility of the expected value, and the certainty equivalent wealth.
b. Graph the utility function and show on your graph the values you computed in part a.
c. What is the maximum amount the individual will pay for insurance that will cover the loss in the event of a flood?
d. Compute the value of the actuarily fair insurance premium (the value that will just cover the expected loss payout) and compare it to the value you found in part c. If the two values differ, explain why.
e. What is the value of the risk premium? Provide an interpretation in the context of the present exercise.
f. Without performing any calculations, if the homeowner's current wealth was \$500,000 instead, how woould the maximum amount they would be willing to pay for insurance compare to that which you computed in part c? Explain. (Hint: consider your answer to Question 2, part a)

4. Consider an investor with utility function given by that in Question 2, part b: $U(w) = \frac{-w^{-3}}{3}$. The investor has initial wealth of \$100. that she can allocate between two assets. One asset is risk free, with a certain return of 2 percent. The other asset carries risk, such that it yields a positive return of 20 percent with 50 percent probability and a negative return of $-10$ percent with 50 percent probability. Let $\alpha$ represent the share of the investor's wealth that she allocates to the risky asset.
a. Find the optimal value of $\alpha$ for the investor that maximizes her expected utility.
b. Without performing any calculations, how would the optimal share allocated to the risky asset in part a differ if the investor's initial wealth were \$1500 instead? Explain. (Hint: consider your answer to Question 2, part b).