Economics 701

Problem Set 8

1. Consider the production function given by: $q = \frac{10KL}{K+L}$

a. What returns to scale does this production function exhibit?

b. Assuming K is fixed at $\overline{K}$, derive the functions for: i) the average product of labor and ii) the marginal physical product of labor.

c. Is the marginal product of labor diminishing? How does the marginal product of labor behave as more capital is employed?

d. Derive the marginal rate of technical substitution ($RTS_{L,K}$). Is the $RTS_{L,K}$ diminishing? Is the production function homothetic?

2. Consider again the production function from question 1 above.

a Suppose the wage rate $w = 1$, and the rental rate $v = 4$. Using the Method of Lagrange, find the cost-minimizing input combination to produce $q = 10$ units of output. What is the minimum cost?

b. Using an isocost curve and an isoquant, graph the optimum from part a. Be sure to calculate the intercepts of the isocost curve and indicate the value of its slope.

c. Now letting w and v be variable, derive the conditional input demand functions directly from the FOCs of the constrained cost minimization problem. Are these functions homogeneous in w and v? If so, of what degree?

d. Derive the long-run total cost function, the marginal cost function, and the average total cost function, all as functions of w, v, and q.

e. Show how the conditional input demand functions you derived in part c can be obtained by application of Shephard's Lemma.

f . With $w = 1$, and $v = 4$, find the long-run total, average and marginal cost *curves*.

g. Now, still assuming $w = 1$, and $v = 4$, let capital be fixed at $K_0 = 4$ in the short run. Derive the short run cost *curves*: total cost, average total cost, variable cost, average variable cost, average fixed cost, and marginal cost.

h. Continuing with part g, what is the short run total cost of producing $q = 10$ units of output? Compare to the minimum cost from part a and explain the reason for any difference.

i. Still assuming $w = 1$, and $v = 4$, what level of output is $K_0 = 4$ optimal for producing? Show that at this output level the long run and short run total costs are the same.