

Problem Set 9

1. Let demand be given by $P = A - bq$ and cost be given by: $C = kq^2$. Calculate explicitly the formula for the optimal output and its price to maximize profits.
2. Consider the constant elasticity demand curve: $q = AP^{-b}$, $A, b > 0$.
 - a. Calculate the price elasticity of demand.
 - b Show that the relationship $MR = P(1 - \frac{1}{|e_{x,P_x}|})$ holds.
 - c. Show that for the constant elasticity demand curve, the vertical distance between the demand curve (with price on the vertical axis) and marginal revenue is a constant fraction of the height of the demand curve.
 - d. What is the value of the Lerner Index for a firm that faces this demand?
3. Consider a competitive firm's production function given by: $q = 2K^{\frac{1}{2}}L^{\frac{1}{4}}$. In the short run, the firm's capital is fixed at $K_1 = 64$. The wage and rental rates are w and v respectively, and the price of the firm's output is p .
 - a. Derive the firm's short-run total cost function.
 - b. Derive the firm's supply function (what is the shut-down price?)
 - c. Derive the firm's short-run profit function $\Pi(p, w, v)$.
 - d. Use Hotelling's lemma to derive the firm's supply function and compare to that in part b.
 - e. Use an appropriate envelope result to derive the short-run demand for labor function, $L(P, w, v)$.
 - f. Suppose $p = 16$, $w = 1$, and $v = 2$. Use the functions you derived above to calculate i) the amount of labor the firm will hire to maximize short-run profits, ii) the quantity of output produced, and iii) the resulting profits.
 - g. Suppose there is an increase in the wage, so that now $p = 16$, $w = 2$, and $v = 2$. Repeat question f. Compare the values computed to that in part f) and explain, in words, the resulting change in the quantity of labor in terms of output and substitution effects.