Economics 701

Problem Set 2

1. Determine whether or not the following functions are quasi–concave. Explain.
a) e^x
b) $2x_1^{\frac{1}{2}} + 4x_2^{\frac{1}{2}}$, where $x_1, x_2 \; > \; 0$ (hint: is this function concave?)

2. Use the Method of Lagrange to solve for x_1^* and x_2^* in the following problem:
maximize $f(x_1, x_2) = x_1 x_2$ subject to $x_1 + 4x_2 = 16$.

What is f^*, the maximized value (subject to the constraint) of $f(x_1, x_2)$?

3. Solve for x_1^* and x_2^* in the dual to the problem in question 2. Here, the dual problem is:
minimze $x_1 + 4x_2$ subject to $f(x_1, x_2) = f^*$

where you will substitute for f^* the maximized value (subject to the constraint) of $f(x_1, x_2)$ from question 2. Are x_1^* and x_2^* here the same as in question 2?

4. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, a certain factory produces $Q(x, y) = 50x^{\frac{1}{2}}y^{\frac{1}{2}}$ units of output.

a. How should $80,000 be allocated between labor and equipment to yield the largest possible output?
b. Use the critical value of the Lagrange multiplier, λ, to estimate the change in maximum output if this allocation DEcreased by $1,000.
c. Compute the exact change in part b).

5. Write down the dual problem to that in question 4 part a) and solve for x^* and y^*.

6. Show that the following functions are homogeneous (specify of what degree) and verify that Euler's theorem holds:
a. $f(x_1, x_2) = x_1 x_2^2$
b. $f(x_1, x_2) = x_1 x_2 + x_2^2$

7. Show that the following functions are homothetic by demonstrating that the derivatives of the level curves are constant along a ray from the origin:
a. $y = l\text{n}(x_1) + l\text{n}(x_2)$
b. $y = (x_1 x_2)^2 - x_1 x_2$