Problem Set 3

1. Which of the following utility functions are equivalent to $U(x,y) = xy$? For those that are, what is the monotonic transformation $h(f)$ that provides the equivalence?
a. $U(x,y) = 7xy + 2$
b. $U(x,y) = lnx + lny + 1$
c. $U(x,y) = x^2y$

2. For every function in 3a–c that you found equivalent to $U(x,y) = xy$, show that they all (including $U(x,y) = xy$) have the same marginal rates of substitution at bundle (2,1), but differing marginal utilities for good x at that same bundle. Discuss how this illustrates the difference in the ordinality and cardinality properties of the MRS and marginal utility.

3. Consider the utility function given by: $U(x,y) = \alpha\frac{x^\delta}{\delta} + \beta\frac{y^\delta}{\delta}$ (CES)
a. Compute the marginal rate of substitution (MRS_{xy}) to show that the utility function is homothetic (can it be expressed as a function of $\frac{y}{x}$?).
b. What is the MRS_{xy} when $\delta = 1$? What is the elasticity of substitution when $\delta = 1$ and what type of preference does this describe?
c. If $\delta = 1$ and $\frac{P_x}{P_y} < \frac{\alpha}{\beta}$, what fraction of income will be spent on good x? good y? (you do not need to perform any calculations here). Explain.
d. For the remaining questions d – g consider a consumer with CES utility where $\delta = 1/3$ and $\alpha = \beta = 1$. Suppose in the past year there has been a 5 percent increase in the price ratio, $\frac{P_x}{P_y}$. By how much did the consumer's relative consumption, $\frac{y}{x}$, change during the year? Explain. (hint: use the elasticity of substitution).
e. If the price of good x is \$4 and the price of good y is \$1, how much of goods x and y will the consumer purchase, given her income is \$60? Show all work.
f. Graph the optimum in part e) using a budget constraint and indifference curve (be sure to calculate the intercepts of the budget constraint and indicate the value of its slope).
g. What is the value of the consumer's marginal utility of income at the optimum?

4. Consider the utility function $U(x, y) = \min\{3x, y\}$
a. In what fixed proportion does the consumer consume the goods?
b. If the price of good x is \$10 and the price of good y is \$5, how much of goods x and y will the consumer purchase when his income is \$200?
c. Graph the optimum in part b) using a budget constraint and indifference curve (be sure to calculate the intercepts of the budget constraint and indicate the value of its slope).

(questions 5-7 are on page 2)

5. For Ben, 5 units of good y is always a perfect substitute for two units of good x.
a. Write down a specific utility function for Ben that represents his preferences.
b. If the price of good x is \$9 and the price of good y is \$3, how much of goods x and y does he purchase when his income is \$360?
c. Graph the optimum in part b) using a budget constraint and indifference curve (be sure to calculate the intercepts of the budget constraint and indicate the value of its slope).

6. (from question 3.7 in text):

3.7

a. A consumer is willing to trade 3 units of x for 1 unit of y when she has 6 units of x and 5 units of y. She is also willing to trade in 6 units of x for 2 units of y when she has 12 units of x and 3 units of y. She is indifferent between bundle (6, 5) and bundle (12, 3). What is the utility function for goods x and y? *Hint*: What is the shape of the indifference curve?

b. A consumer is willing to trade 4 units of x for 1 unit of y when she is consuming bundle (8, 1). She is also willing to trade in 1 unit of x for 2 units of y when she is consuming bundle (4, 4). She is indifferent between these two bundles. Assuming that the utility function is Cobb–Douglas of the form $U(x, y) = x^{\alpha}y^{\beta}$, where α and β are positive constants, what is the utility function for this consumer?

c. Was there a redundancy of information in part (b)? If yes, how much is the minimum amount of information required in that question to derive the utility function?

7. (from question 3.11 in text): Independent Marginal Utilities. Two goods have independent marginal utilities if:

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial y \partial x} = 0$$

This means that the quantity of one good does not affect the marginal utility of the other good. Show that if we assume diminishing marginal utility for each good, then any utility function with independent marginal utilities will have a diminishing MRS_{xy}. (Hint: express the MRS_{xy} in terms of partials of the utility function – when does it diminish as the amount of good x increases?)