Economics 701

# Problem Set 1

1. Let $f(x) = x^4 - 4x^3 + 4x^2 + 4$. Find the critical values of $x$ (note: there are three of them). For each one, use the second–order condition to indicate whether the critical value yields a local maximum or minimum.

2. For each multivariate function below, calculate $f_1$, $f_2$, $f_{11}$, $f_{22}$, $f_{12}$, $f_{21}$. Verify in each case that Young's theorem holds.

a. $f(x_1, x_2) = 2x_1x_2^3$
b. $f(x_1, x_2) = 14x_1 + 2x_1x_2 - x_1^2 - 2x_2^2$
c. $f(x_1, x_2) = -4x_1 - 6x_2 + x_1^2 - x_1x_2 + 2x_2^2$
d. $f(x_1, x_2) = x_1 ln x_2$
e. $f(x_1, x_2) = x_1 e^{(x_1 + x_2^2)}$

3. Consider the function in question 2b) above. Find the critical values of $x_1$ and $x_2$ that maximize/minimize the function. Use the second–order conditions to determine whether or not these values yield a local maximum or minimum.

4. Repeat question 3, but use the function in question 2c) above.

5. Consider the function $y = f(x_1, x_2) = 8x_1^2x_2^2 - 2x_1^3x_2^3$ .
a. Derive the total differential of the function.
b. Use the total differentail to estimate the change in $y$ if initally, $x_1 = x_2 = 1$, and $x_1$ increases by 0.5 and $x_2$ simultaneously increases by 0.2. Compute the *actual* change in $y$.

6. Consider the implicit function: $g(x, y) = x^2 - 3xy + y^3 - 7 = 0$. Use the implicit function theorem to caluclate $\frac{dy}{dx}$ and evaluate this derivative at $x = 4$, $y = 3$.

7. Consider the function $y = f(x, a) = -x^2 + 2ax + 4a$, where $a$ is a strictly positive parameter.
a. Use the first order condtion for a maximum to derive the critical value of $x$, which will be a function of $a$, $x^*(a)$.

In parts b) and c) below, you will calculate the effect of a unit increase in $a$ on the maximum value of $f(x, a)$. In part b), you will calculate the effect by substitution and taking the derivative. In part c, you will apply the envelope theorem.

b. Substitute $x^*(a)$ from part a) into $f(x, a)$ to obtain $y^* = f(x^*(a), a)$. Calculate $\frac{dy^*}{da}$.
c. Now use the envelope theorem to compute $\frac{dy^*}{da}$.
d. Suppose $a = 0.5$ initially. Use differential approximation to estimate the change in the maximum value of $f(x, a)$ if $a$ were to increase by $0.2$.
e. Continuing with part d), calculate the actual change in the maximized value of $f(x, a)$.