SOLUTIONS - PROBLEM SET 9

① $\pi = R(q) - C(q) \quad R(q) = P \times q = (A - bq)q = Aq - bq^2$

$\pi = Aq - bq^2 - Kq^2 \quad \frac{d\pi}{dq} = A - 2bq - 2Kq = 0$ F.O.C.

$\Rightarrow q(2b+2K) = A \Rightarrow q^* = \frac{A}{2(b+K)} \quad P^* = A - b\left(\frac{A}{2(b+K)}\right)$

S.O.C. $d^2\pi/dq^2 = -(2b+2K) < 0$ for $b, K > 0 \Rightarrow$ max

② a) $e_{Q,P} = \frac{dQ}{dP}\frac{P}{Q} = -bAP^{-(b+1)}\frac{P}{Q} = -bA\,P^{-(b+1)}\frac{P}{AP^{-b}} = -b$

b) $q = AP^{-b} \Rightarrow P^{-b} = q/A \Rightarrow P = (q/A)^{-1/b}$; $R(q) = (q/A)^{-1/b} q = A^{1/b} q^{\frac{b-1}{b}}$

$MR = \frac{dR}{dq} = A^{1/b}\left(\frac{b-1}{b}\right) q^{-1/b} = (q/A)^{-1/b}(1 - 1/b) = P\left(1 - \frac{1}{|e_{Q,P}|}\right)$

c) $MR = P\left(1 - \frac{1}{|e_{Q,P}|}\right) \Rightarrow P - MR = P\left(\frac{1}{|e_{Q,P}|}\right)$ and since here, $|e_{Q,P}| = b$

we have $P - MR = (\frac{1}{b})P$

d) $MR = MC \Rightarrow P - MC = (\frac{1}{b})P \Rightarrow \frac{P - MC}{P} = \frac{1}{b}$

③ a) Firm's short-run production function, given $K_1 = 64$, is:

$q = 2(64)^{\frac{1}{2}}L^{\frac{1}{4}} = 16L^{1/4} \Rightarrow L^{1/4} = q/16 \Rightarrow L = q^4/16^4$

$STC = w(q/16)^4 + v\,64$

b) $MC = \frac{\partial STC}{\partial q} = 4w\,q^3/16^4$ The competitive firm chooses its output where $P = MC$, or $P = 4w\,q^3/16^4$ = short-run supply, or alternatively, $q^3 = \frac{16^4 P}{4w} = \frac{16^4 P}{16^{1/2} w} = \frac{16^{7/2} P}{w} = \frac{4^7 P}{w}$

$\Rightarrow q = 4^{7/3}\left(\frac{P}{w}\right)^{1/3}$. $AVC = \left(\frac{w}{16^4}\right)q^3$ and has a minimum of 0 (at $q = 0$), so the shut-down price is 0.

c) $\pi = P16L^{1/4} - wL - v64$

$\frac{\partial \pi}{\partial L} = \frac{1}{4}P16L^{-3/4} - w = 0$

$$\Rightarrow L^{-3/4} = \frac{w}{4P} \Rightarrow L = \left(\frac{w}{4P}\right)^{-4/3} = \left(\frac{4P}{w}\right)^{4/3}$$

$$\pi = P16\left[\left(\frac{4P}{w}\right)^{4/3}\right]^{1/4} - w\left(\frac{4P}{w}\right)^{4/3} - v64$$

$$\Rightarrow \pi = P16\left(\frac{4P}{w}\right)^{1/3} - w^{-1/3}(4P)^{4/3} - 64v = w^{-1/3}P^{4/3}\left(16 \cdot 4^{1/3} - 4^{4/3}\right) - 64v$$

$$= w^{-1/3}P^{4/3}\left(4^2 \cdot 4^{1/3} - 4^{4/3}\right) - 64v = w^{-1/3}P^{4/3}\left(4^{7/3} - 4^{4/3}\right) - 64v$$

$$= w^{-1/3}P^{4/3}4^{4/3}\left(4^{3/3} - 1\right) - 64v = 3w^{-1/3}P^{4/3}4^{4/3} - 64v$$

d) $\frac{\partial \pi}{\partial P} = 4w^{-1/3}P^{1/3}4^{4/3} = q = 4^{7/3}\left(\frac{P}{w}\right)^{1/3}$ ← same as in part b)

e) $\frac{\partial \pi}{\partial w} = -4^{4/3}w^{-4/3}P^{4/3} \Rightarrow L = \left(\frac{4P}{w}\right)^{4/3}$ ← note same as found in part c)

f) i) $L = \left(\frac{4 \times 16}{1}\right)^{4/3} \approx 256$

ii) $q = 4^{7/3}\left(\frac{16}{1}\right)^{1/3} \approx 64$

iii) $\pi = 3(1)^{-1/3}(16)^{4/3}4^{4/3} - 64(2) \approx 640$

g) i) $L = \left(\frac{4 \times 16}{2}\right)^{4/3} \approx 101.6$

ii) $q = 4^{7/3}\left(\frac{16}{2}\right)^{1/3} \approx 50.8$

iii) $\pi = 3(2)^{-1/3}(16)^{4/3}4^{4/3} - 64(2) \approx 481.6$

The wage increase reduces the quantity of labor demanded (256 → 101.6). The firm reduces output (64 → 50.8) and profits fall (640 → 481.6). Because here, capital is fixed at $K_1 = 64$, there is no substitution effect: the change in quantity demanded of labor has resulted entirely from an output effect.