Solutions - Problem Set 7

① a) Since the individual is indifferent between 24 for certain and the lottery, it must be the case that 24 is the certainty equivalent.

b) Since 24 is the certainty equivalent, we have that $u(24)$ = Expected Utility of the lottery

or, $u(24) = u(10)\cdot 2/3 + u(90)\cdot 1/3$

$= 0 \cdot 2/3 + 1 \cdot 1/3$

$= 1/3$

c) EU Lottery A: $u(24)\cdot 3/4 + u(10)\cdot 1/4$

$= 1/3 \cdot 3/4 + 0 \cdot 1/4$

$= 1/4$

Expected utilities

EU Lottery B: $u(10)\cdot 2/5 + u(24)\cdot 1/2 + u(90)\cdot 1/10$

$= 0 \cdot 2/5 + 1/3 \cdot 1/2 + 1 \cdot 1/10$

$= 4/15$

Since EU Lottery B > EU Lottery A $\Rightarrow$ individual prefers lottery B

② a) $u' = \frac{1}{w}$ $u'' = \frac{-1}{w^2}$ $r(w) = \frac{-u''}{u'} = \frac{-\left(-\frac{1}{w^2}\right)}{\frac{1}{w}} = \frac{1}{w}$

$rr(w) = w\,r(w) = w\left(\frac{1}{w}\right) = 1$

- risk averse, since $u'' < 0$; DARA since $r'(w) < 0$
CRRA, since $rr'(w) = 0$

b) $u' = w^{-4}$ $u'' = -4w^{-5}$ $r(w) = \frac{-(-4w^{-5})}{w^{-4}} = 4/w$

$rr(w) = w(4/w) = 4$

risk averse, since $u'' < 0$; DARA since $r' < 0$
CRRA, since $rr' = 0$

c) $u' = 2e^{-2w}$ $u'' = -4e^{-2w}$ $r(w) = \frac{-(-4e^{-2w})}{2e^{-2w}} = 2$

$rr(w) = 2w$

risk averse, since $u'' < 0$; CARA since $r' = 0$
IRRA since $rr' > 0$

d) $u' = 2$ $u'' = 0$ $r(w) = -\frac{0}{2} = 0$ $rr(w) = 0 \times w = 0$

- risk neutral, since $u'' = 0$, also CARA & CRRA

③ a) $u(300K) = \ln(300K) = 12.61$

$u(100K) = \ln(100K) = 11.51$

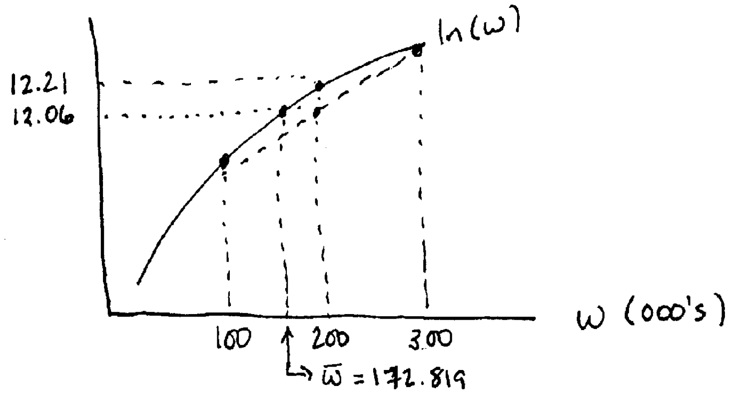
$E(w) = \frac{1}{2}(300K) + \frac{1}{2}(100K) = 200K$

$Eu = \frac{1}{2}(12.61) + \frac{1}{2}(11.51) = 12.06$

$u[E(w)] = \ln(200K) = 12.21$

Certainty equivalent, $\bar{w}$, is defined as that wealth where $u(\bar{w}) = Eu$: $\ln(\bar{w}) = 12.06 \Rightarrow \bar{w} = 172,819$

b)



c) $300,000 - 172,819 = \underline{\underline{127,181}}$ (paying an insurance premium of this amount leaves them with non-risky wealth equal to the certainty equivalent)

d) Fair insurance would have the premium equal to the expected loss, which is $\frac{1}{2}(200K) + \frac{1}{2}(0) = 100K$ here. This is less than the value in part c; the individual is risk averse and is therefore willing to pay in excess of the amount of fair insurance to eliminate risk.

e) The risk premium $= E(w) - \bar{w}$, or $200K - 172.819K = 27.181K$. From part d) we saw the expected loss was 100K. This individual is willing to pay 27.18K above the expected loss (from part c) we saw that individual would pay up to 127,181) for the insurance policy.

f) From 2a) we know they have DARA. Given a risky situation involving given absolute dollar amounts of changes to wealth, the individual will be willing to pay less to avoid the risk of equivalent gambles as wealth increases. Therefore, the maximum would be less than that in part c).

4) a) With probability 1/2, her wealth will be:

$(1-\alpha)100(1.02) + \alpha 100(1.20) = 102 + 18\alpha$

And with probability 1/2, her wealth will be:

$100(1-\alpha)(1.02) + \alpha 100(.90) = 102 - 12\alpha$

$$E[u(w)] = -\frac{1}{2}\frac{[102+18\alpha]^{-3}}{3} + -\frac{1}{2}\frac{[102-12\alpha]^{-3}}{3}$$ ← Her expected utility of wealth, which she will maximize with respect to α

$$\frac{dE[u(w)]}{d\alpha} = \frac{1}{2}[102+18\alpha]^{-4}(18) + \frac{1}{2}[102-12\alpha]^{-4}(-12) = 0$$

$$\Rightarrow 9[102+18\alpha]^{-4} = 6[102-12\alpha]^{-4}$$

$$\Rightarrow 1.5 = \left[\frac{102+18\alpha}{102-12\alpha}\right]^{4} \Rightarrow (1.5)^{1/4} = \frac{102+18\alpha}{102-12\alpha}$$

$$\Rightarrow 1.107(102-12\alpha) = 102+18\alpha \Rightarrow 112.914 - 13.284\alpha = 102 + 18\alpha$$

$$\Rightarrow 10.914 = 31.284\alpha \Rightarrow \alpha^* = .349$$

b) Since we know from part b, Question 2 that the individual has CRRA, we know that as wealth increases the fraction of wealth they will allocate toward a gamble w/ a given prospect of receiving certain fractional increments or losses to wealth is constant. Therefore the optimal α will be the same.