Economics 701

Problem Set 5

Note: Both Problems 5.4 and 5.5 from the text are reproduced on page 2 of this problem set.

1. Problem 5.4, page 174 in text

2. Problem 5.5, page 174, in text.

3. In question 1 above you started with the uncompensated demand to ultimately derive the Hicksian, or compensated demand. In this question you will start from the Hicksian demand to derive the uncompensated Marshallian demand, using the same numerical example in question 1 above.
a. In part b of question 1 you derived the compensated demand for good X. Apply Shephard's Lemma once again to the expenditure function to derive the compensated demand for good Y.
b. Substitute the compensated demands into the objective function (expenditure minimization) to derive the expenditure function, which should be exactly the same as what you derived in part a of question 1.
c. From the expenditure function, derive the indirect utility function, which again, should be exactly the same as what you derived in part a of question 1.
d. Now apply Roy's identity to derive the uncompensated (Marhallian) demand for good X. If you have done things correctly, it should be the same as what you were given for question 1.

4. Prove Roy's Identity. (Hint: consider the consumer's utility maximization problem in a simple two good case and apply the envelope theorem)

5.4

As in Example 5.1, assume that utility is given by

$$\text{utility} = U(x, y) = x^{0.3}y^{0.7}.$$

a. Use the uncompensated demand functions given in Example 5.1 to compute the indirect utility function and the expenditure function for this case.

b. Use the expenditure function calculated in part (a) together with Shephard's lemma to compute the compensated demand function for good x.

c. Use the results from part (b) together with the uncompensated demand function for good x to show that the Slutsky equation holds for this case.

5.5

Suppose the utility function for goods x and y is given by

$$\text{utility} = U(x, y) = xy + y.$$

a. Calculate the uncompensated (Marshallian) demand functions for x and y and describe how the demand curves for x and y are shifted by changes in I or the price of the other good.

b. Calculate the expenditure function for x and y.

c. Use the expenditure function calculated in part (b) to compute the compensated demand functions for goods x and y. Describe how the compensated demand curves for x and y are shifted by changes in income or by changes in the price of the other good.

The uncompensated demand functions are:

$$X^* = \frac{0.3I}{P_X}$$

$$Y^* = \frac{0.7I}{P_Y}$$