SOLUTIONS - Problem Set 6

① a) $X^* = \frac{.3(10)}{1} = 3 \qquad Y^* = \frac{.7(10)}{1} = 7$

b) i) Find the original level of utility (use the indirect utility function): $V(1,1,10) = \frac{.54\,(10)}{1 \times 1} = 5.4$

Use the expenditure function to determine the minimum expenditure to attain this utility level:

$$E(1,1,5.4) = \frac{(5.4)(1)(1)}{.54} = 10$$

which is, not surprisingly, the consumer's current income

Now, with the higher price of X, the minimum expenditure required to attain a utility level of 5.4 is:

$$E(6,1,5.4) = \frac{5.4\,(6)^{.3}(1)}{.54} = 17.12$$

So the CV $= E(6,1,5.4) - E(1,1,5.4)$

$= 17.12 - 10 = \boxed{7.12}$

In order to attain the original utility at the higher price of good X, the consumer would need compensation of 7.12

ii) The compensated demand curve is given by:

$.55(5.4)(1/P_x)^{.7} = 2.97 P_x^{-.7}$

$$CV = \int_1^6 2.97 P_x^{-.7}\, dP_x = 2.97\left(\tfrac{1}{.3}\right) P_x^{.3}\Big]_1^6$$

$$= 9.9\left[1.71 - 1\right] = \boxed{7.02}$$

(the same as difference in expenditures, save for rounding)

c) i) Use the indirect utility function to find the ending utility after the price increase: $V(6,1,10) = \frac{.54(10)}{6^{.3} \times 1} = 3.15$

$$E(6,1,3.15) = \frac{3.15(6)^{.3}(1)^{.7}}{.54} = 10$$, the minimum

expenditure necessary to attain the ending utility level after the price increase, which is again, equal to the consumer's income.

Now find the minimum expenditures required to attain the ending utility at the lower price of good X:

$$E(1,1,3.15) = \frac{3.15(1)^{.3}(1)^{.7}}{.54} = 5.83$$

So, the $EV = E(6,1,3.15) - E(1,1,3.15) = 10 - 5.83 = \boxed{4.17}$

(Starting at the ending utility with the higher price of good X, the consumer would be willing to pay at most 4.17 to have the lower price of good X).

ii) The compensated demand curve is given by:

$.55(3.15)(1/P_x)^{.7} = 1.73 P_x^{-.7}$

$$EV = \int_1^6 1.73 P_x^{-.7}\, dP_x = 1.73(1/.3) P_x^{.3}\Big]_1^6 = 5.77[1.71 - 1] = \boxed{4.09}$$

(same as difference in expenditures, save for rounding)

d) $X^* = \frac{.3I}{P_x} = \frac{.3(10)}{P_x} = \frac{3}{P_x}$

$$\int_1^6 3P_x^{-1} = 3\ln(P_x)\Big]_1^6 = 3[\ln(6) - \ln(1)] = \boxed{5.37},$$

which is between the CV and the EV.

(2) a) $e_{x,P_x} = \frac{\partial x}{\partial P_x}\frac{P_x}{x}$

$\frac{\partial x}{\partial P_x} = \frac{-\alpha I}{P_x^2}$, so $e_{x,P_x} = \left(\frac{-\alpha I}{P_x^2}\right)\left(\frac{P_x}{\underbrace{(\alpha I/P_x)}_{x}}\right) = -1$

$e_{x,P_y} = \frac{\partial x}{\partial P_y}\frac{P_y}{x}$, and since $\frac{\partial x}{\partial P_y} = 0$, $e_{x,P_y} = 0$

$e_{x,I} = \frac{\partial x}{\partial I}\frac{I}{x}$, so $e_{x,I} = \underbrace{\frac{\alpha}{P_x}}_{\frac{\partial x}{\partial I}}\left[\frac{I}{\underbrace{(\alpha I/P_x)}_{x}}\right] = 1$

b) $\frac{\partial x}{\partial P_x} = \frac{-10P_yI}{(5+P_x)^2}$, so $e_{x,P_x} = \frac{-10P_yI}{(5+P_x)^2}\left[\frac{P_x}{\underbrace{\left(\frac{10P_yI}{5+P_x}\right)}_{x}}\right] = \frac{-P_x}{(5+P_x)}$

$\frac{\partial x}{\partial P_y} = \frac{10I}{(5+P_x)}$, so $e_{x,P_y} = \frac{10I}{(5+P_x)}\left[\frac{P_y}{\underbrace{\left(\frac{10P_yI}{(5+P_x)}\right)}_{x}}\right] = 1$

$\frac{\partial x}{\partial I} = \frac{10P_y}{(5+P_x)}$, so $e_{x,I} = \frac{10P_y}{(5+P_x)}\left[\frac{I}{\underbrace{\left(\frac{10P_yI}{(5+P_x)}\right)}_{x}}\right] = 1$

(3) $\frac{\partial s_x}{\partial I} = \frac{IP_x\frac{\partial x}{\partial I} - P_x x}{I^2}$

Write I/s_x as $I/P_x x = I^2/P_x x$

Then $e_{s_x,I} = \frac{\partial s_x}{\partial I}\frac{I}{s_x} = \frac{IP_x\frac{\partial x}{\partial I} - P_x x}{I^2}\cdot\frac{I^2}{P_x x}$

$= \frac{I}{x}\frac{\partial x}{\partial I} - 1 = e_{x,I} - 1$

(4) a) Case 1: Her income in $t=0$ is $1\cdot 3 + 2\cdot 1 = 5$
Her income in $t=1$ is $2\cdot 1 + 2\cdot 2 = 6$
In $t=0$, bundle X_1 would cost: $1\cdot 1 + 2\cdot 2 = 5$
In $t=1$, bundle X_0 would cost: $2\cdot 3 + 2\cdot 1 = 8$

Since in $t=0$ bundle X_1 was affordable, but she chose bundle X_0 instead, she has revealed a preference for X_0 over X_1. Furthermore, although she chooses X_1 in $t=1$, this does not violate WARP, since in $t=1$, bundle X_0 is no longer affordable.

b) Case 2: Her income in $t=0$ is $1\cdot 4 + 4\cdot 4 = 20$
Her income in $t=1$ is $3\cdot 6 + 2\cdot 3 = 24$
In $t=0$, bundle X_1 would cost: $1\cdot 6 + 4\cdot 3 = 18$
In $t=1$, bundle X_0 would cost: $3\cdot 4 + 2\cdot 4 = 20$

Since in $t=0$ bundle X_1 was affordable but she chose X_0 instead, she reveals a preference for X_0 over X_1. This case violates WARP, however, since in $t=1$ bundle X_0 is affordable still, but she chooses X_1 instead.

(5) a) $P = \frac{P_t \cdot X_t}{P_b \cdot X_t}$

b) Suppose $P \geq I_t/I_b$, or $\frac{P_t \cdot X_t}{P_b \cdot X_t} \geq \frac{P_t \cdot X_t}{P_b \cdot X_b} \Rightarrow P_b \cdot X_b \geq P_b \cdot X_t$

This implies that in the base year X_t was affordable. Since X_t was affordable but X_b was chosen in the base year $\Rightarrow$ X_b is revealed prefered to X_t. Therefore, the consumer is worse off in time t (consuming X_t) than in the base year. So, if $P \geq I_t/I_b \Rightarrow$ consumer is unambiguously worse off.

Suppose $P < I_t/I_b$, or $\frac{P_t \cdot X_t}{P_b \cdot X_t} < \frac{P_t \cdot X_t}{P_b \cdot X_b} \Rightarrow P_b \cdot X_b < P_b \cdot X_t$

Here X_t was not affordable in the base year, so the fact the consumer chose X_b over X_t in the base year does not reveal a preference. Furthermore, we know nothing about whether or not X_b is affordable in time t. Therefore we do not have revealed preference at any time $\Rightarrow$ if $P < I_t/I_b \Rightarrow$ change in consumer's welfare is ambiguous.