Economics 701

Problem Set 6

1. This question is a continuation of Question 1 from Problem Set 5. Suppose $p_x = 1$, $p_y = 1$, and $I = 10$ inititally.
a. Use the uncompensated demands to find the quantities of X and Y this consumer will consume.
b. Now let p_x increase to 6. Compute the compensating variation (CV) from this price increase (the income necessary to maintain the original utility at the higher price) two ways: i) by computing the appropriate difference in expenditures and ii) by integration, using the appropriate compensated demand curve
c. Repeat question b for the equivalent variation (EV) from this price increase (the expenditures the consumer would be willing to pay to avoid the price increase).
d. Use the uncompensated demand curve to compute the loss of Marshallian consumer surplus from the price increase. How does this value compare to the CV and the EV?

2. For each of the following demand functions, calculate the own price, cross price, and income elasticities for good x:
a. $x = \frac{\alpha I}{p_x}$

b. $x = \frac{10 p_y I}{(5+p_x)}$

3. Consider a consumer's demand for good $X = X(p_x, p_x, I)$. Let $S_x = \frac{p_x X}{I} =$ good X's income (budget) share. Show that $e_{S_x,I} = e_{x,I} - 1$, where $e_{S_x,I}$ is the elasticity of good X's budget share with respect to income: $\frac{\partial S_x}{\partial I}\left(\frac{I}{S_x}\right)$, and $e_{x,I}$ is good X's income elasticity of demand.

4. A consumer spends her entire income on two goods. Each case below shows the price and quantity vectors in two different time periods, $t = 0$ and $t = 1$ (i.e., $\mathbf{p_0}$ gives the price of each of the two goods in $t = 0$ and $\mathbf{x_0}$ is the bundle of the corresponding quantities consumed in $t = 0$, etc.). For each case below, determine whether or not there is revealed preference between $\mathbf{x_0}$ and $\mathbf{x_1}$. Do either of the cases below violate WARP? Explain your answers.
a. Case 1: $\mathbf{p_0} = (1, 2)$ $\mathbf{x_0} = (3, 1)$; $\mathbf{p_1} = (2, 2)$ $\mathbf{x_1} = (1, 2)$
b. Case 2: $\mathbf{p_0} = (1, 4)$ $\mathbf{x_0} = (4, 4)$; $\mathbf{p_1} = (3, 2)$ $\mathbf{x_1} = (6, 3)$

5. Define:
$\mathbf{x}_{\boldsymbol{b}} =$ vector of quantities of n goods consumed in a given base year
$\mathbf{x}_{\boldsymbol{t}} =$ vector of quantities of n goods consumed in year t
$\mathbf{p}_{\boldsymbol{b}} =$ vector of base-year prices of the n goods
$\mathbf{p}_{\boldsymbol{t}} =$ vector of year t prices of the n goods

a. Write the expression for the Paasche price index, P, for year t.

b. Use revealed preference to determine under which of the following conditions we can determine unambigously whether consumers are better or worse off: i) $P \geq \frac{I_t}{I_b}$ and/or ii) $P < \frac{I_t}{I_b}$,where $\frac{I_t}{I_b}$ is the ratio of year t to base-year incomes (assumed equal to ezpenditures in each year). Explain your answers.