San Francisco State University — Economics 701
Department of Economics

The Principal Agent Problem: Application to the Owner–Manager Relationship

Firm owner (principal) wishes to hire a manager (agent). The owner's gross profit will depend on the effort level of the manager in the following way:

Gross profit to owner:

$$\Pi_g = \Pi(e) + \epsilon, \tag{1}$$

where e is the manager's effort level, $\Pi'(e) > 0$ and ϵ is a random term, $\epsilon \sim N(0, \sigma^2)$.

The owner will offer the manager a linear salary/wage contract of the form:

$$S(\Pi_g) = a + b\Pi_g \tag{2}$$

where a is a fixed component. indpendent of the level of profit, and b, $0 \le b \le 1$, is the "power" of the incentive scheme, where higher values of b strengthens the link between the manager's salary and firm profit, thereby providing greater incentive for the manager to apply more effort.

The owner is risk neutral and the manager is risk averse with utility given by:

$$U = E(I) - rVar(I), \tag{3}$$

where I is income, $E(I)$ is the expected value of income, $Var(I)$ is the variance of income, and r represents the degree of risk aversion. If $r = 0$, for example, the individual is risk neutral and maximizing utility is the same as maximizing expected income. As the degree of risk aversion, r, or the variability of income increases, utility declines. This type of utility function is known as Mean-Variance Utility.

Given the linear payment schedule, the manager's net income from choosing effort level e is:

$$I = S(\Pi_g) - C(e) = a + b\Pi_g - C(e) = a + b\Pi(e) + b\epsilon - C(e), \tag{4}$$

where $C(e)$ represents the manager's cost/disutility of effort, $C'(e) > 0$. The manager's utility is therefore given by:

$$U = \overset{\substack{E(I)\\ \downarrow}}{[a + b\Pi(e) - C(e)]} - \overset{\substack{rVar(I)\\ \downarrow}}{[rb^2\sigma^2]} \tag{5}$$

The owner's objective is to choose the wage contract that will induce the desired effort level and maximize profits.

CASE 1: EFFORT IS OBSERVABLE (First-Best Outcome)

In this case, there are no hidden actions, and the level of effort can be specified in the contract. The owner therefore needn't offer an incentive component in the wage contract (that is $b^* = 0$) and the wage will be a fixed salary of a^*entirely.

The only constraint is that the manager accept the contract. The wage must be chosen such that the manager's utility from the contract is at least as great as the manager's reservation utility - the highest utility from alternate opportunities (here assumed to be zero). This type of constraint is known as the **participation constraint, sometimes also known as an individual rationality (IR) constraint**. Here, the participation constraint is given by:

$$[a + b\Pi(e) - C(e)] - [rb^2\sigma^2] \geq 0 \tag{6}$$

But since in the case of observable effort, $b = 0$, the constraint becomes simply:

$$a - C(e) = 0 \tag{7}$$

where the owner will choose the lowest wage such that the constraint holds with equality. As implied by (7), the optimal fixed salary is:

$$a^* = \ C(e) \tag{8}$$

The wage contract will therefore specify a fixed salary equal to the manager's cost of effort. What effort level will the owner specify in the contract? The owner's expected net profit after paying the manager is given by:

$$E[\Pi_n] = E[\Pi_g] - a^* = \Pi(e) - a^* = \ \Pi(e) - C(e) \tag{9}$$

Maximizing expected net profits with respect to effort, e, gives the following first order condtion:

$$\Pi'(e) - C'(e) = 0 \quad \Rightarrow \quad \Pi'(e) = C'(e) \tag{10}$$

This is the first-best outcome. The level of effort is that for which the marginal benefit of additional effort is just equal to the marginal cost. In this outcome, the risk–averse agent is also fully insured against risk by receiving a fixed salary.

CASE 2: EFFORT IS UNOBSERVABLE (Second-Best Outcome)

In this case, effort is a hidden action. And since it is unobservable, it cannot be explicitly contracted for. If the owner paid the manager a fixed wage as in Case 1, the manager will always choose a low effort level, since effort is costly and there is no incentive to exert it. One way to align the manager's incentives with the owner's is to have a positive "power" component of the incentive scheme, i.e. have $b > 0$ which makes the manager's salary be partly dependent on profits, and hence effort. However, the tradeoff between incentives and risk reduction (insurance) is easily seen: higher values of b create greater incentives for effort, but they also shift more risk onto the risk-averse manager.

What does the optimal wage structure look like now? The owner's problem is esentially the same as in Case 1, however, there is an additional constraint now. This constraint is that the manager must be given the incentive to undertake the effort level desired by the owner. This type of constraint is known as an **incentive-compatibility (IC) constraint**.

Derivation of the incentive-compatibility constraint:

The manager's utility is given in (5) and will choose the effort level that maximize$s$ utility. The first-order condition for this is:

$$\frac{\partial U}{\partial e} = b\Pi'(e) - C'(e) = 0 \qquad \Rightarrow \qquad b\Pi'(e) = C'(e) \tag{11}$$

This will define the optimal level of effort as a function of b, which is the incentive compatibility constraint:

$$e^* = e^*(b) \tag{12}$$

It is easily shown that $e^{*\prime}(b) > 0$, or the optimal effort level is increasing in the power of the incentive scheme. The owner's problem is to choose a and b to maximize expected net profits subject to the participation and incentive compatibility constraints. Specifically, the owner's problem is:

$$\underset{a,b}{max} \;\; E[\Pi_n] = (1-b)E[\Pi_g] - a = (1-b)\Pi(e) - a \tag{13}$$

subject to: (6) (the participation constraint) and (12) (the incentive compatibility constraint).

Subbing the constraints into the objective function yields the following maximization problem for the principal:

$$\underset{b}{max} \;\; E[\Pi_n] = \Pi(e^*(b)) - C(e^*(b)) - rb^2\sigma^2 \tag{14}$$

with first-order condition:

$$[\Pi'(e^*(b)) - C'(e^*(b))]e^{*\prime}(b) - 2rb\sigma^2 = 0 \tag{15}$$

Once b is found from (15), a can be found from the participation constraint (6).

Note: as seen in Case 1, the efficient effort level is that for which $\Pi'(e) = C'(e)$. From (11), we note that unless $b = 1$, or the manager is made full residual claimant, the manager's effort level will be less than efficient, since $e^{*\prime}(b) > 0$. And clearly, from (15), as long as $r > 0$, it is never optimal for the owner to choose $b = 1$, since the agent would choose to equate $\Pi'(e^*(b))$ and $C'(e^*(b))$, leaving the left–hand side of (15) negative.