Economics 701

Problem Set 4

1. Consider the utility function $U(x, y) = \min\{3x, y\}$. Derive the demand curves for good x and y (consult your textbook page 126, Case 3)

2. For each of the following utility functions, use the method of Lagrange to derive the demand functions for good x and for good y.
a. $U(x,y) = xy^{\frac{1}{2}}$ (Cobb–Douglas)
b. $U(x,y) = 3x^{\frac{1}{3}} + 3y^{\frac{1}{3}}$ (CES)

3. For each case above (2a & 2b),
a. Use the demand functions you derived to show, for each good, the proportion of income spent on the good. Are the proportions constant?
b. Show that the demand functions are homogeneous of degree zero (in both cases, you need only show this for the demand function for good x).

4. Consider the utility function from question 2a) above, $U(x,y) = xy^{\frac{1}{2}}$, and for which you have already derived demand functions.
a. Derive $V(p_x, p_y, I)$, the indirect utility function.
b. Use the method of Lagrange to solve the consumer's dual problem to derive $E(p_x, p_y, U)$, the expenditure function.
c. Show that the expenditure function in part b) is the inverse of the indirect utility function from part a).
d. Suppose initially $p_x = 0.50$, $p_y = 1.00$ and $I = 30$. Use the demand functions from question 2a) to determine the optimal quantities of x and y.
e. Now suppose the price of x rises to 1.00 while everything else remains the same. Use the demand functions from question 2a) again to find the new optimal quantities of x and y.
f. At the new prices, how much would the original consumption bundle (i.e., the bundle in part d) cost? If we wanted to give this consumer a lump sum cash subsidy to make the original bundle affordable at the new prices, how much would we have to give the consumer?
g. What is the consumer's level of utility from the original consumption bundle in part d?
h. What is the size of the lump sum cash grant we would have to give the consumer to keep him at the original level of utility, given the new prices? (hint: use the expenditure function)
i. Compare the size of the cash grants in parts f and h. (Observe that this is an illustration of the potential consequences of the well-known substitution bias inherent in certain price indicies such as the consumer price index. It is argued that cost-of-living adjustments pegged to the CPI overcompensate the consumer for price increases, as the CPI is based on a fixed consumption bundle and compensation sufficient to maintain affordability of a fixed bundle overcompensates the consumer, as it does not account for the fact that consumers make welfare-improving substitutions in consumption when

relative prices change. Ideally, the appropriate compensation is to maintain the original level of utility, not the original consumption bundle).

5. (From question 4.8 in text): Two of the simplest utility functions are:
   1. Fixed proportions: $U(x, y) = \min \{x, y\}$
   2. Perfect substitutes: $U(x, y) = x + y$
For each of these utility functions, compute the following:
- Demand functions for x and y
- Indirect utility function
- Expenditure function