# SOLUTIONS TO PROBLEM SET 5

① a) Find indirect utility function:

$$u(x,y) = x^{.3}y^{.7} \qquad x^* = \frac{.3I}{P_x} \quad y^* = \frac{.7I}{P_y}$$

$$V(P_x, P_y, I) = \left(\frac{.3I}{P_x}\right)^{.3}\left(\frac{.7I}{P_y}\right)^{.7}$$

$$= \frac{(.3)^{.3}(.7)^{.7}I}{P_x^{.3}P_y^{.7}} = .54I / P_x^{.3}P_y^{.7}$$

$E(P_x, P_y, u)$ : From indirect utility,

$$u = \frac{.54E}{P_x^{.3}P_y^{.7}}$$

$$\Rightarrow E(P_x, P_y, u) = \frac{u P_x^{.3}P_y^{.7}}{.54}$$

b) $x^c(P_x, P_y, u) = \dfrac{\partial E(P_x, P_y, u)}{\partial P_x}$ (Shephard's lemma)

$$\frac{\partial E}{\partial P_x} = \frac{.3uP_x^{-.7}P_y^{.7}}{.54} = .55u\left(P_y/P_x\right)^{.7}$$

c) Using uncompensated demand, we have

$$\frac{\partial x^*}{\partial P_x} = \boxed{-.3I/P_x^2} \quad \text{(direct calculation)}$$

Substitution effect: $\dfrac{\partial x^c}{\partial P_x} = -(.7)(.55)uP_y^{.7}P_x^{-1.7}$

$$= -.385\, u P_y^{.7}P_x^{-1.7}$$

Sub indirect utility for u:

$$-.385\left(\frac{.54I}{P_x^{.3}P_y^{.7}}\right)P_y^{.7}P_x^{-1.7}$$

$$= -.208\, I\, P_x^{-2}$$

Income effect $= -x\dfrac{\partial x}{\partial I}$

$$= -\left(\tfrac{.3I}{P_x}\right)\left(\tfrac{.3}{P_x}\right) = -.09\, I\, P_x^{-2}$$

So, $\dfrac{\partial x}{\partial P_x} = \underbrace{-.208\, I\, P_x^{-2}}_{\text{substitution effect}} \underbrace{-.09\, I\, P_x^{-2}}_{\text{income effect}}$ } Slutsky equation

$$= \boxed{\frac{-.298\, I}{P_x^2}}$$

→ same as direct calculation above, save for rounding differences

(2) Solution from page 729 of the text:

a) $X = \dfrac{I - P_x}{2P_x}$ $\quad Y = \dfrac{I + P_x}{2P_y}$

changes in $I$: For both goods an increase in $I$ increases the quantity demanded at every price ⇒ ↑ $I$ shifts both demands outward (both $x$ and $y$ are normal goods since $\frac{\partial x}{\partial I}$ and $\frac{\partial y}{\partial I} > 0$). Changes in $P_y$ do not effect changes in $x$, but changes in $P_y$ do affect $Y$.

b) $V = \dfrac{(I + P_x)^2}{4P_xP_y}$ and so $E = \sqrt{4P_xP_yV} - P_x$

c) The compensated demand function for $x$ depends on $P_y$, whereas the uncompensated function does not.

③ a) $\frac{\partial E}{\partial P_y} = \frac{.7u P_x^{.3} P_y^{-.3}}{.54} = 1.30u\left(P_x/P_y\right)^{.3}$

$= y^c(P_x, P_y, u)$ - by Shephard's Lemma

b) Objective function: $P_x X + P_y Y$

subbing: $P_x[.55u(P_y/P_x)^{.7}] + P_y[1.30u(P_x/P_y)^{.3}]$

$= .55u P_y^{.7} P_x^{.3} + 1.3u P_y^{.7} P_x^{.3}$

$= 1.85u P_x^{.3} P_y^{.7} = E(P_x, P_y, u)$

→ Same as what we derived in part a of Question ①

c) Sub $V(P_x, P_y, I)$ for $u$ and $I$ for $E(P_x, P_y, u)$ in the expenditure function and invert:

$1.85 V P_x^{.3} P_y^{.7} = I \Rightarrow V = \frac{I}{1.85 P_x^{.3} P_y^{.7}}$

→ same as in part a of Question ①

d) $\partial V/\partial P_x = \frac{I}{1.85 P_y^{.7}}\left(\frac{-(.3P_x^{-.7})}{P_x^{.6}}\right) = \frac{-I(.3)}{1.85 P_y^{.7} P_x^{1.3}}$

$\partial V/\partial I = \frac{1}{1.85 P_x^{.3} P_y^{.7}}$

$X = \underbrace{\frac{-\partial V/\partial P_x}{\partial V/\partial I}}_{\text{Roy's Identity}} = \frac{.3I}{1.85 P_y^{.7} P_x^{1.3}} \cdot \frac{1.85 P_x^{.3} P_y^{.7}}{1} = \frac{.3I}{P_x}$ ← Same as what was given in question

④ The consumer's utility maximization problem generates the following Lagrangean:

$$\mathcal{L} = u(x,y) + \lambda(I - P_x X - P_y Y)$$

where maximal utility is given by $u^*(P_x, P_y, I) = V(P_x, P_y, I)$

By the envelope theorem $\partial \mathcal{L}^*/\partial P_x^* = \partial V/\partial P_x = -\lambda X$ and $\frac{\partial \mathcal{L}^*}{\partial I} = \frac{\partial V}{\partial I} = \lambda$

so $-\partial V/\partial P_x \Big/ \frac{\partial V}{\partial I} = \frac{-(-\lambda X)}{\lambda} = X$