University of California Santa Barbara

Control under energy and time constraints

A dissertation submitted in partial satisfaction of the requirements for the degree

 $\begin{array}{c} {\rm Doctor~of~Philosophy} \\ {\rm in} \\ {\rm Electrical~and~Computer~Engineering} \end{array}$

by

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To Jen.

Note: Joao gave me the following advice:

"Regarding the thesis, my main suggestion if to write a nice introduction that talks about state of the art and summarizes the key results. This allows the reader to easily pick and choose the chapter that he/she may find more interesting to explore further."

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- J. Pearson, J. Hespanha, D. Liberzon. "Control with minimal cost-per-symbol encoding and quasi-optimality of event-based encoders." *IEEE Transactions on Automatic Control*, May 2017.

- J. Pearson, J. Hespanha, D. Liberzon. "Quasi-optimality of event-based encoders." *Proc. of the 54th Conference on Decision and Control*, Dec. 2015.
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Abstract

Control under energy and time constraints

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The performance of a control system is often limited by constraints on timing, bandwidth, and energy. This dissertation explores the trade-offs between constraints on these resources, the control system performance, and the system to be controlled.

In this work we solve every problem ever.

Contents

A	cknov	vledgements	V
Cı	ırricı	ılum Vitae	vi
Αl	bstra	ct	viii
Li	st of	Figures	x
Li	st of	Tables	xi
1	Intr	oduction	1
2	2.1	trol with Minimum Energy Per Symbol Problem Statement	6 7 7
3	3.1	si-optimality of Event-based control Definition of the event-based scheme	9 10 10
4	tem 4.1	emption-resistant control on a non-real-time operating sys- Real-time I/O coprocessor concept	11 11 12
\mathbf{A}	Pro	ofs of lemmas	13
Bi	bliog	ranhy	14

List of Figures

1.1	The limited-communication control setup. At sampling times, the	
	encoder measures the process state and selects symbols from a	
	finite alphabet to send to the decoder/controller. The decoder/-	
	controller constructs the actuation signal for the plant	5

List of Tables

Chapter 1

Introduction

Control systems are frequently hampered by constraints on resources like limits on bandwidth, energy, and processor speed.

This work explores the theoretical and practical trade-offs between these resources and control system performance. It consists of three chapters, each summarized next.

Control with Minimum Energy Per Symbol

(Chapter 2)

Chapter 2 considers the problem of stabilizing a continuous-time linear timeinvariant process subject to communication constraints. The basic setup is shown

in Figure 1.1, in which a finite capacity communication channel connects the process sensors to the controller/actuator. An encoder at the sensor sends a symbol through the channel once per sampling time, and the controller determines the actuation signal based on the incoming stream of symbols. The question arises: what is the smallest channel average bit-rate for which a given process can be stabilized?

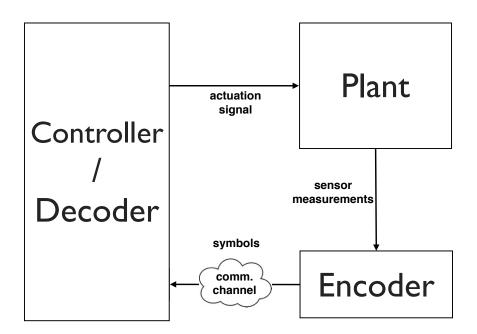


Figure 1.1: The limited-communication control setup. At sampling times, the encoder measures the process state and selects symbols from a finite alphabet to send to the decoder/controller. The decoder/controller constructs the actuation signal for the plant.

Prior work

Variants of the limited-communication environment in Figure 1.1 were also considered in [3, 4, 8, 11, 7, 6] and many other works.

...

Contributions

A starting point for the present work is the observation that an encoder can effectively save communication resources by occasionally not transmitting information — the absence of an explicitly transmitted symbol nevertheless conveys information. We formulate a framework to capture this by supposing that each symbol's transmission costs one unit of communication resources, except for one special free symbol that represents the absence of a transmission.

. . .

Quasi-optimality of Event-based control

(Chapter 3)

In Chapter 3, we use the framework from Chapter 2 to analyze a family of event-based controllers.

• • •

Prior work

Recent results in event-based control [2, 1, 5, 10] indicate that an encoder can conserve communication resources by transmitting only on a "need-to-know" basis.

...

Contributions

In contrast to the optimal encoders introduced in Chapter 2, the proposed eventbased encoders are easy to implement but not optimal. However, they are only slightly sub-optimal. Specifically, ...

Preemption-resistant control on a non-real-time operating system (Chapter 4)

The previous chapters indicate that networked control systems can benefit from precise timing by embedding information in the timing of messages. In Chapter 4, we turn our attention away from networked control systems and explore how precise timing can benefit a controller running (locally) on a non-real-time operating system.

• • •

Background and prior work

A program may execute nondeterministically for several reasons.

...

Contributions

The specific contribution of this chapter is a controller architecture that enables a controller to run on a non-real-time OS like Linux, yet maintain precise timing of the sensing and actuation despite OS preemption. This is achieved by performing the sensing and actuation on a dedicated "bare metal" microcontroller that in essence serves as a real-time I/O coprocessor; we refer to this as the "Real-Time Unit" (RTU).

...

Chapter 2

Control with Minimum Energy

Per Symbol

Parts of this chapter come from [9]:

2017 IEEE. Reprinted, with permission, from J. Pearson, J. Hespanha, D. Liberzon. Control with minimal cost-per-symbol encoding and quasi-optimality of event-based encoders. IEEE Trans. on Automat. Contr., 62(5):2286–2301, May 2017.

In this chapter we consider the problem of stabilizing a continuous-time linear time-invariant process subject to communication constraints. We develop a framework for exploring the notion that the absence of communication nevertheless conveys information, yet it consumes no communication resources. We model the absence of a communication by appending a special "free" symbol to the set of symbols offered by the communications channel. Transmitting a normal symbol costs one unit of communications resources, but transmitting the free symbol costs no resources. This yields the notion of an encoder's average cost per symbol — essentially the average fraction of non-free symbols sent by the encoder. We then develop a condition under which a stabilizing encoder with the smallest average cost per symbol may be designed.

This chapter is organized as follows.

...

2.1 Problem Statement

Consider a stabilizable linear time-invariant process

$$\dot{x} = Ax + Bu, \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^m,$$
 (2.1)

for which it is known that x(0) belongs to a known bounded set $\mathcal{X}_0 \subset \mathbb{R}^n$.

2.2 Conclusion

In this chapter, we considered the problem of bounding the state of a continuoustime linear process under communication constraints. We considered constraints on both the channel average bit-rate and the encoding scheme's average cost per symbol. Our main contribution was a necessary and sufficient condition on the process and constraints for which a bounding encoder/decoder/controller exists.

...

Chapter 3

Quasi-optimality of Event-based control

Parts of this chapter come from [9]:

2017 IEEE. Reprinted, with permission, from J. Pearson, J. Hespanha, D. Liberzon. Control with minimal cost-per-symbol encoding and quasi-optimality of event-based encoders. IEEE Trans. on Automat. Contr., 62(5):2286–2301, May 2017.

In the last chapter we constructed an N-of-M encoding scheme that stabilizes process (2.1) provided that the bit-rate and average cost condition (??) holds. This scheme may be difficult to implement in practice if the encoder/decoder pair use a large number of codewords. In this section we present an *event-based*

encoding scheme that is easy to implement and does not require storing a large set of codewords. Instead, it uses a library of only three symbols $\{-1,0,1\}$ and does not group them into codewords. The basic idea is to monitor in parallel each one-dimensional component of the error system, and as long as it stays inside a fixed interval, send the free symbol 0.

• • •

3.1 Definition of the event-based scheme

Unlike the scheme from Section ??, this scheme differs in what symbols are sent and how the state estimate \hat{x} is updated:

. . .

3.2 Conclusion

In this chapter examined an event-based controller based on the framework from Chapter 2. We proved its average bit-rate requirements were order-optimal with respect to the necessary and sufficient condition for stabilizability from Chapter 2. This supports the use of event-based controllers in limited-communication control schemes.

. . .

Chapter 4

Preemption-resistant control on a non-real-time operating system

In this chapter we consider the problem of stabilizing a system with unpredictable timing due to the controller running on a non-real-time operating system. We propose a method of implementing a discrete-time control algorithm on a non-real-time operating system so that the sensing and actuation occur at precise times, even if the OS preempts the control task.

4.1 Real-time I/O coprocessor concept

We first describe the architecture of our control and sensing/actuation scheme.

Figure ?? illustrates the basic idea, which is that the controller and sensor/actu-

ator execute on two separate processors.

...

4.2 Conclusion

In this chapter, we presented a controls architecture that pairs a real-time I/O coprocessor with a controller on a non-real-time operating system. The RTU enables sampling and actuation at precise times, even when the controller is preempted by the OS. This enables control designers to reap the benefits of an OS with minimal concern for the timing uncertainties associated with the OS task scheduler.

Appendix A

Proofs of lemmas

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