

# Efficient and Equitable Taxation

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EC313 - Public Economics: Taxation

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# Goals of This Section

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- Lay out model for optimal taxation
- Derive the Ramsey Rule for optimal commodity taxation
- Discuss optimal user fees
- Discuss optimal income taxation
- Discuss political economy and time inconsistency
- Discuss other criteria for tax design

# Optimal Taxation

# Introduction

- As we have discussed, government raise taxes mainly to raise revenue
- But taxes involve costs
  - The direct cost of the revenue raised
  - Additional costs from distortions in behaviour (excess burden)
- Given that revenue needs to be raised, how should taxes be designed to minimize costs?
- Optimal taxation addresses this question

# Model

- Question of optimal taxation is answered with a model
- A representative citizen consumes two goods,  $X$  and  $Y$ 
  - Prices of these goods are  $P_X$  and  $P_Y$
- Person can also work and earn wage  $w$ 
  - Person has  $T$  hours available for work ( $h$ ) or leisure ( $L$ )
  - These are the only two uses of time, so  $T = h + L$
- Their budget constraint in this context is

$$w(T - L) = P_X X + P_Y Y$$

- Constraint says that income is split between spending on goods  $X$  and  $Y$

# Model

- If you rearrange the budget constraint, you can write it as

$$wT = P_X X + P_Y Y + wL$$

- Shows that “full income” is split between spending on goods  $X$  and  $Y$  and spending on leisure
  - Full income is  $wT$  because if all time was worked, income would be  $wT$
  - Price of leisure is  $w$  because each hour of leisure foregoes  $w$  in earnings
- Suppose we tax goods  $X$ ,  $Y$ , and  $L$  at rate  $t$
- Then the budget constraint becomes

$$wT = (1 + t)P_X X + (1 + t)P_Y Y + (1 + t)wL$$

# Model

- Rearranging gives

$$wT = (1 + t)(P_X X + P_Y Y + wL)$$

$$\frac{wT}{1 + t} = P_X X + P_Y Y + wL$$

- In this setup, a tax on all goods and leisure is equivalent to a reduction in full income by a factor of  $\frac{1}{1+t}$ 
  - We saw this in one of the practice questions



# Model

- If we could tax all goods and leisure at the same rate, we would not distort behaviour
  - People would still choose the same combination of goods and leisure
- Why?
  - Because relative prices are unchanged
  - The commodity taxes are equivalent to a lump-sum tax that reduces income
- But in practice, we cannot tax all goods and leisure at the same rate
  - We cannot tax leisure directly
  - Can only tax  $X$  and  $Y$  in this setup
- Taxing some goods ( $X$  and  $Y$ ) but not others ( $L$ ) distorts relative prices and leads to excess burden

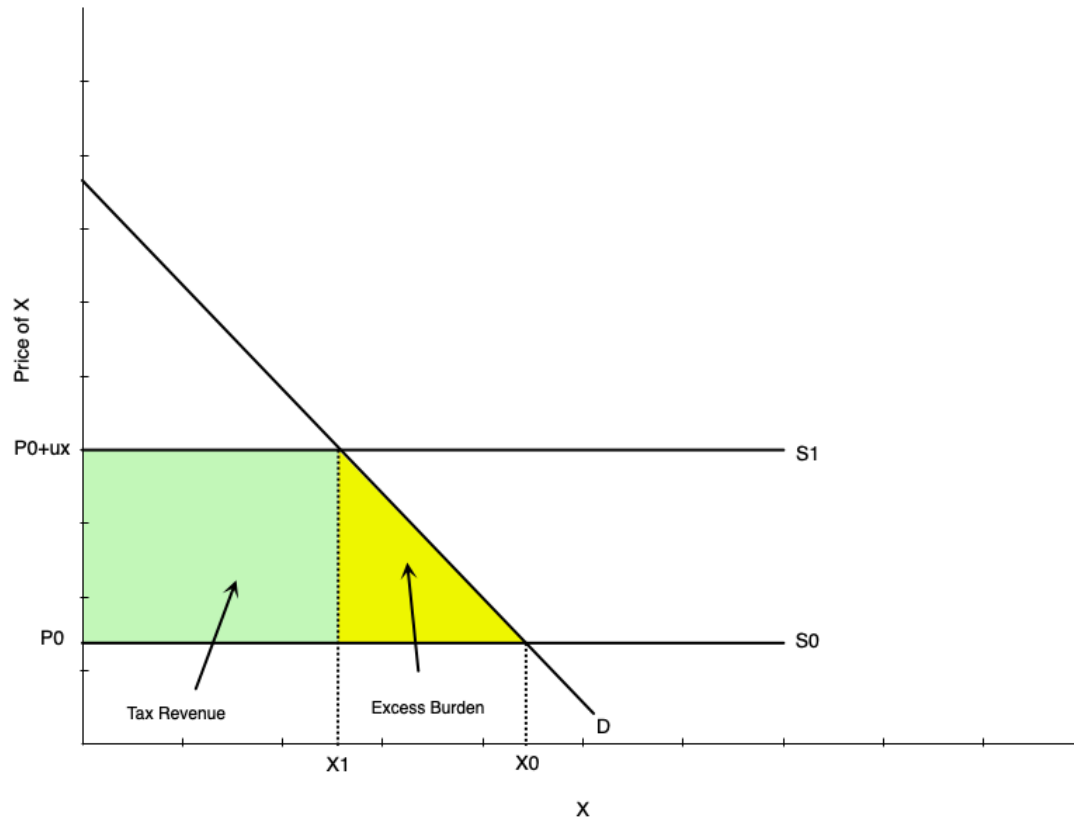
# Ramsey Rule

- Question then becomes: If we cannot tax all goods at the same rate, how should we tax them?
- One option is **neutral taxation**: taxing  $X$  and  $Y$  at the same rate
  - This is simple and easy to administer
  - But it still distorts behaviour and creates excess burden
- The optimal method is the **Ramsey Rule**: tax goods so that the percent reduction in quantity demanded is the same for all goods
  - Equivalently, tax goods so that the marginal excess burden of the last dollar of revenue raised is the same for all goods
- The next few slides establish this result

# Ramsey Rule

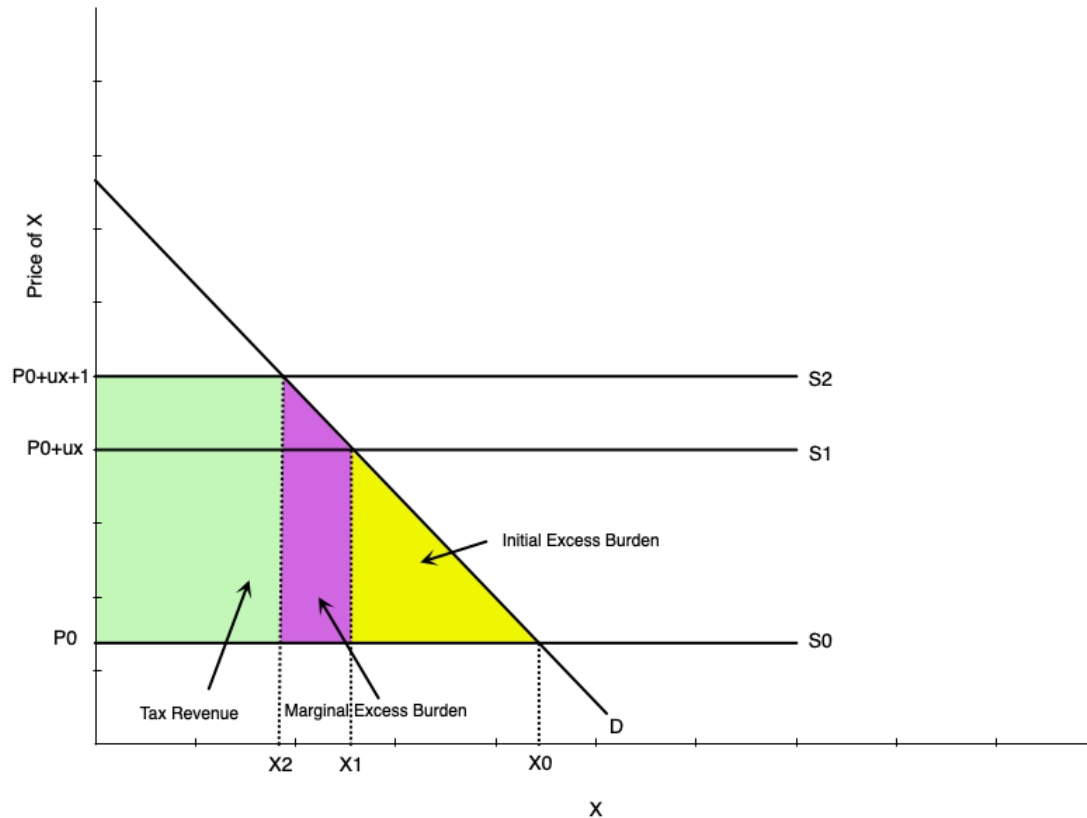
- Consider again two goods,  $X$  and  $Y$ , with prices  $P_X$  and  $P_Y$
- Pretend that goods are neither substitutes nor complements
  - Simplifies the analysis because changes in their prices do not affect each other's demand
  - Results still hold for substitutes and complements, but analysis is more complex
- Also pretend the supply curve is horizontal
  - Consumers face the entire economic incidence of the tax
  - Again simplifies the analysis because we do not have to consider supply side effects
- Consider a unit tax  $u_X$  on good  $X$

# Ramsey Rule



- Consumer initially consumes  $X_0$  at price  $P_0$
- After tax  $u_X$  is imposed, price rises to  $P_0 + u_X$
- Consumer reduces consumption to  $X_1$
- As we have learned this involves an excess burden
  - Equals area of yellow triangle
  - Represents the loss in consumer surplus

# Ramsey Rule



- Now imagine raising the tax further from  $u_X$  to  $u_X + 1$
- The additional excess burden created is the **marginal excess burden**
  - Equals area of purple triangle
- Part of that excess burden was tax revenue (purple rectangle)
- Other part is additional loss in consumer surplus (purple triangle)

# Ramsey Rule

- The marginal excess burden of the tax on good  $X$  is the purple area
  - Equals area of the purple triangle plus the purple rectangle
- To compute that mathematically, set  $\Delta x = X_1 - X_2$
- The total area is then

$$\left(\frac{1}{2}\Delta x \times 1\right) + (\Delta x u_X)$$

- As noted in the textbook, if we pretend that  $\frac{1}{2}\Delta x$  is very small, we can ignore it
  - So marginal excess burden is approximately  $\Delta x u_X$

# Ramsey Rule

- Going one step further, note that the slope of the demand curve is

$$\frac{u_X}{\Delta X} = \frac{1}{\Delta x}$$

- $\Delta X = X_0 - X_1$ , the initial change in quantity demanded from the initial tax
- Both represent the rise over the run
- Rearranging gives

$$\Delta x u_X = \Delta X$$

- The marginal excess burden of the tax on good  $X$  is approximately  $\Delta X$ 
  - The change in quantity demanded from the initial tax

# Ramsey Rule

- What about the change in revenue?
- Initially revenue was larger green rectangle plus purple rectangle
- After, it is larger green rectangle plus smaller green rectangle
- The marginal tax revenue is therefore the smaller green rectangle minus the purple rectangle

$$(X_2 \times 1) - (\Delta x u_X)$$

- We previously saw that  $\Delta x u_X = \Delta X$
- We also know that  $X_2 = X_1 - \Delta x$
- Subbing in gives us

$$X_1 - \Delta x - \Delta X$$



# Ramsey Rule

- IF we pretend that  $\Delta x$  is very small, we can ignore it
  - So marginal tax revenue is approximately  $X_1 - \Delta X$
- The marginal excess burden *per dollar of additional revenue* is

$$\frac{\Delta X}{X_1 - \Delta X}$$

- If we repeat this exact exercise for good  $Y$ , we find that the marginal excess burden per dollar of additional revenue is

$$\frac{\Delta Y}{Y_1 - \Delta Y}$$

# Ramsey Rule

- If we set them equal

$$\frac{\Delta X}{X_1 - \Delta X} = \frac{\Delta Y}{Y_1 - \Delta Y}$$

- Which simplifies to the **Ramsey Rule**

$$\frac{\Delta X}{X_1} = \frac{\Delta Y}{Y_1}$$

- If you multiply both sides by 100 to express this as a percentage, it says percent change in quantity demanded of  $X$  equals percent change in quantity demanded of  $Y$

# Ramsey Rule with Elasticities

- Economists enjoy expressing things in terms of elasticities
- The price elasticity of demand is

$$\eta_X = \frac{\Delta X}{\Delta P_X} \times \frac{P_X}{X}$$

- Rearrange that to get

$$\frac{\Delta X}{X} = \eta_X \times \frac{\Delta P_X}{P_X}$$

# Ramsey Rule with Elasticities

- In this case, consider instead an ad valorem tax that makes the price rise to  $P_X(1 + t_X)$
- The numerator of this fraction is then  $\Delta P_X = P_X(1 + t_X) - P_X = P_X t_X$
- Substituting into the equation on the previous slide gives

$$\frac{\Delta X}{X} = \eta_X \times t_X$$

# Ramsey Rule with Elasticities

- the Ramsay Rule then becomes

$$\eta_X t_X = \eta_Y t_Y$$

- Equivalently

$$\frac{t_X}{t_Y} = \frac{\eta_Y}{\eta_X}$$

# Ramsey Rule with Elasticities

- The **inverse elasticity rule** says that the optimal tax rate on a good is inversely proportional to its price elasticity of demand
  - If good  $Y$  has a high elasticity relative to good  $X$ , then  $t_X$  should be higher than  $t_Y$
- Why?
  - Because a tax on a good with a high elasticity will cause a large reduction in quantity demanded
  - This creates a large excess burden
  - So to minimize excess burden, tax goods with high elasticities less

# Corlett-Hague Rule

- Recall that we could not tax leisure directly
  - And therefore could not apply an efficient (lump-sum equivalent) tax
  - This created the excess burden
- Corlett-Hague suggest approximating a leisure tax by taxing goods that are complements (used together with) to leisure
  - Examples: tax sporting equipment, household appliances, recreational vehicles, etc.
- This indirectly lowers demand for leisure and acts like a tax on leisure
- Gets us a bit closer to the most efficient outcome

# Equity in Taxation

- Ramsey rule implies taxing goods that are inelastic more heavily
- Makes sense when those goods are socially undesirable (e.g., cigarettes, alcohol)
- But what if those goods are necessities (e.g., food, clothing, housing, medicine)?
- Problematic for a few reasons, but one is that it lacks **vertical equity**
  - Vertical equity: people with greater ability to pay should pay more in taxes
- You can modify the Ramsey rule to account for vertical equity
  - A modified rule may tax necessities less heavily
  - Essentially allows for larger excess burden to achieve greater equity



# Summary

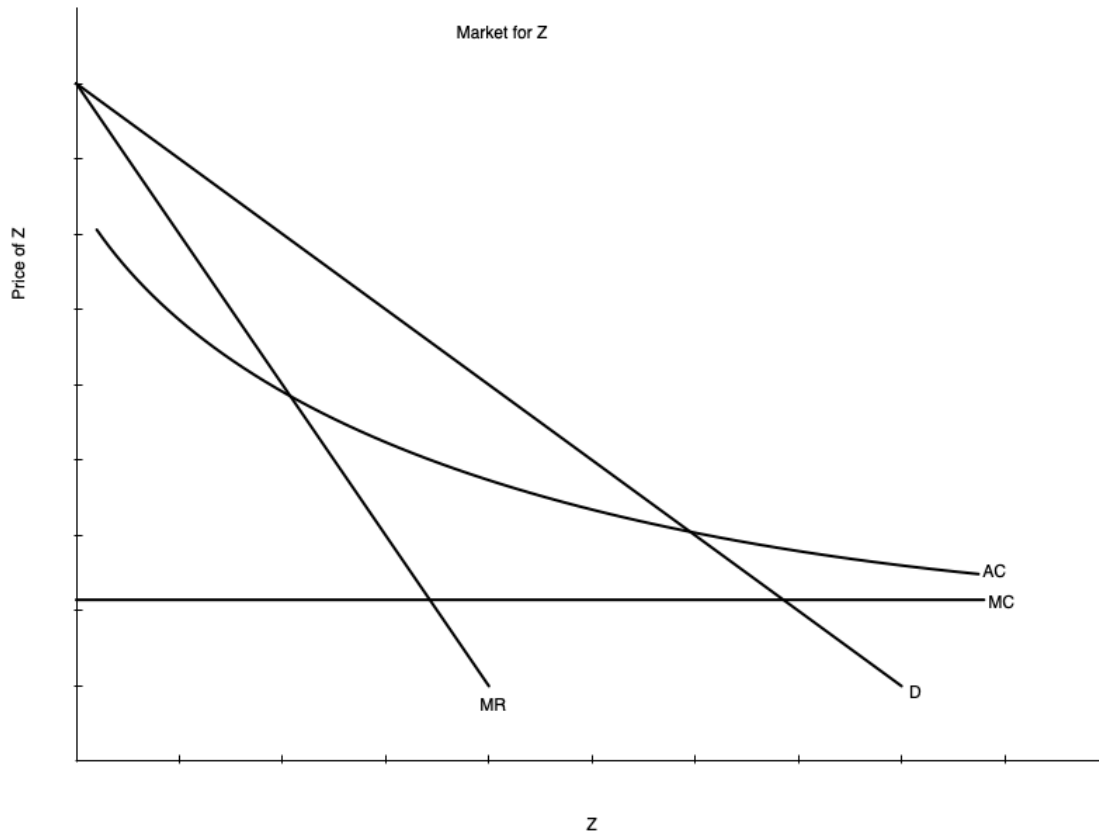
- In the real world we cannot achieve the efficient outcome of a lump-sum tax to raise government revenue
- Distortionary taxes change behaviour and create an excess burden
- We can design the tax system to minimize the excess burden
- The Ramsey rule gives that optimal design
- But tax efficiency is not the sole consideration
  - Equity is also important
  - The optimal rule can change to account for equity considerations

# Optimal User Fees

# Introduction

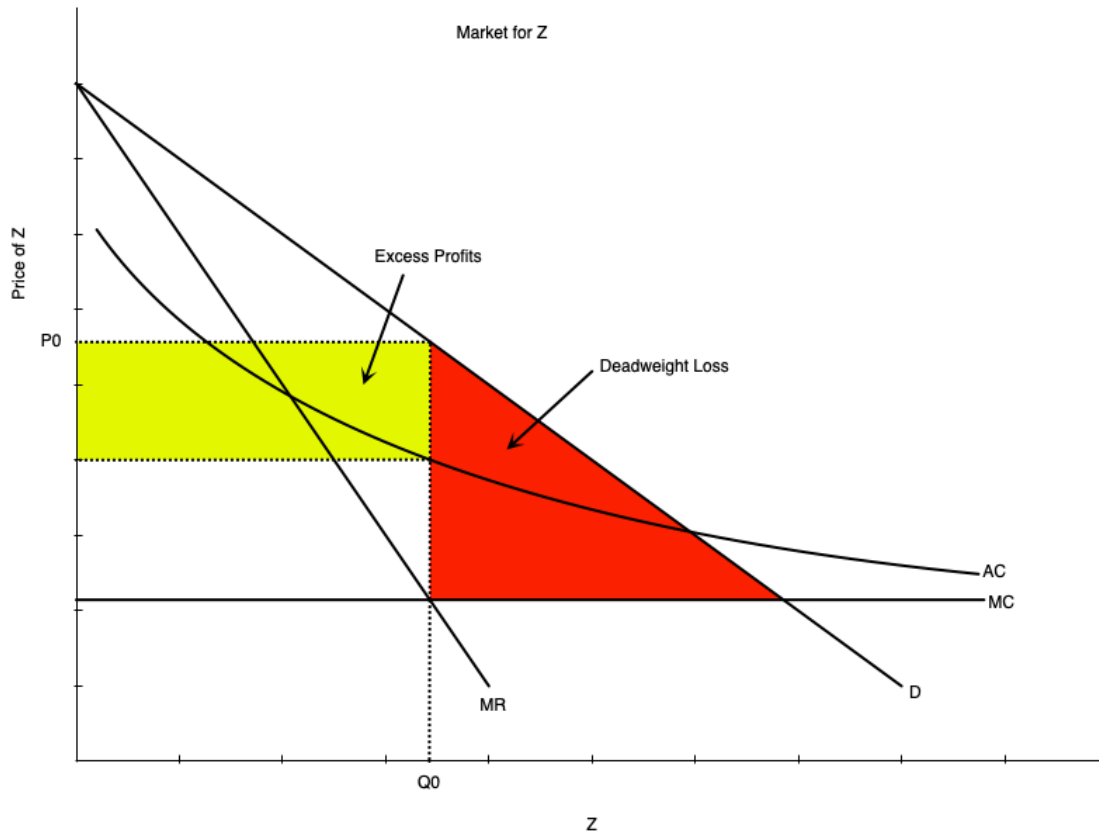
- Governments often charge user fees for public services
  - **User fee:** a fee charged by the government for the use of a good or service
  - Examples: park entrance fees, toll roads, public transit fares, etc.
- Like a tax, but not a tax
- Determining the optimal user fee is similar to determining the optimal tax

# Government Production



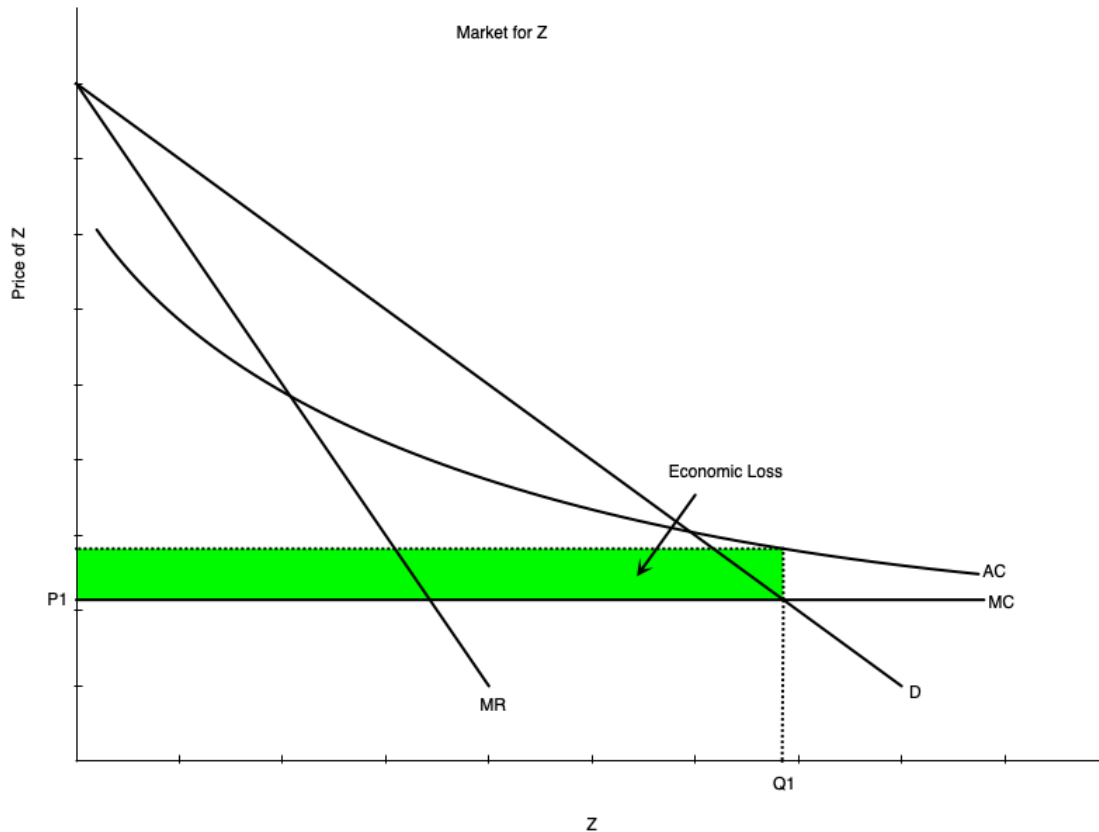
- Governments sometimes produce goods/services when there is a natural monopoly
  - **Natural Monopoly:** a market where a single firm can produce the entire output at lower cost than multiple firms
  - Happens with continuously decreasing average costs
  - Examples: water, electricity, public transit, etc.
- Natural monopoly depicted to the left
- Marginal cost fixed for ease of analysis
  - Could also draw it downward sloping as in the text

# Government Production



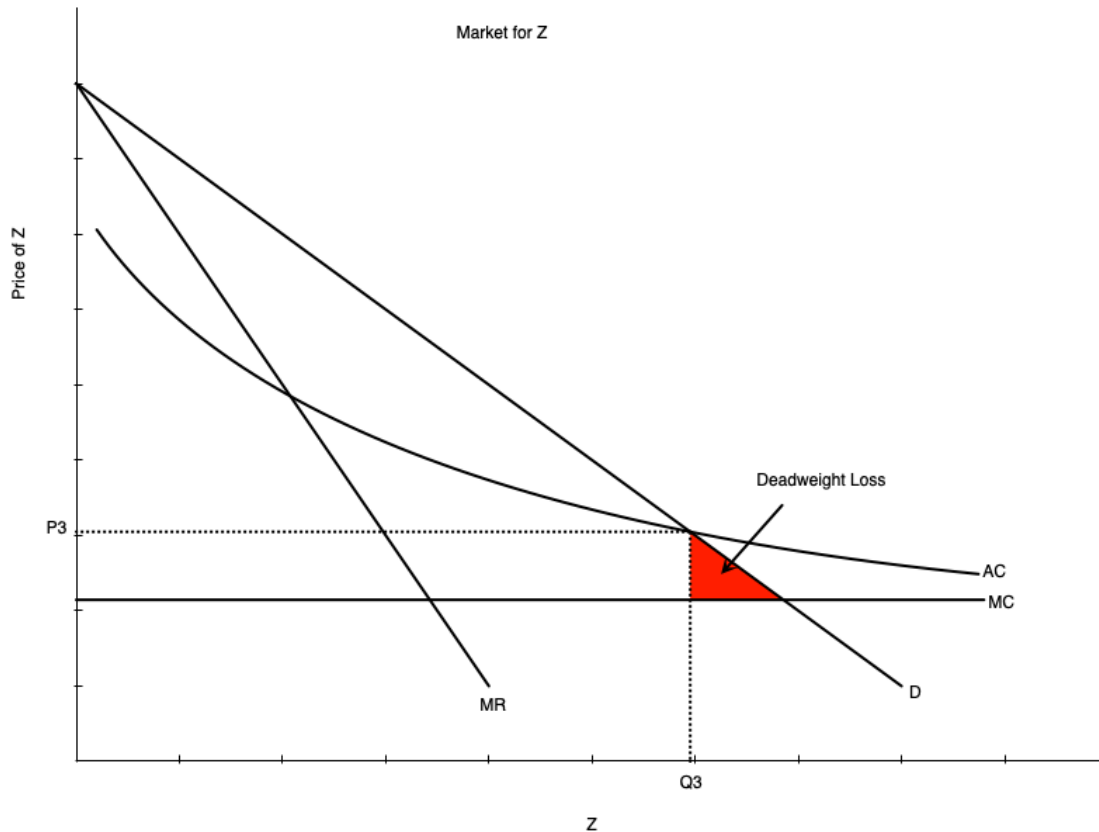
- An unregulated monopolist would produce  $Q_0$  where  $MR = MC$
- Would charge price  $P_0$
- The monopoly earns excess profits equal to yellow area
- Also associated with deadweight loss (excess burden) equal to red area
- The monopolist does not produce the efficient ( $P=MC$ ) level of Z

# Government Production



- The government could take over and produce the good/service
  - Or regulate the private monopolist
- There are different options for production
- One is to produce the efficient level  $Q_1$  where  $P = MC$
- But price  $P_1$  is below average cost  $AC$
- The government would incur a loss equal to green area

# Government Production



- Another option is average cost pricing
- Produce  $Q_2$  where  $P = AC$
- Government incurs no excess profits or losses
- Price  $P_2$  is above  $MC$ , so there is still some deadweight loss (excess burden) equal to red area

# Government Production

- A third option is to set  $P=MC$  and charge a lump sum tax to cover the loss
  - Hard to do in practice because generally cannot levy lump sum taxes
  - Consumers not using the good would pay for it
- A fourth option is a two-part tariff
  - Set  $P=MC$  and charge a fixed fee to cover the loss
  - Only users of the good pay the fee
  - Divide the fee by the number of users to get the fee per user



# Government Production

- Lastly, use the Ramsey Rule
  - If government produces multiple goods/services, set user fees for each
  - Optimally set them so that the percent reduction in quantity demanded is the same for all goods/services
  - We saw that this involves setting higher fees goods/services with inelastic demand
  - The user fee effectively acts like a tax on the good/service

# Optimal Income Tax

# Introduction

- So far we have discussed optimal commodity taxes and user fees
- Income taxes are another major source of government revenue
- Income taxes are different because they tax a person's ability to pay
  - Ability to pay is determined by income
  - Income taxes therefore affect both labour supply and consumption
- You can design an income tax optimally

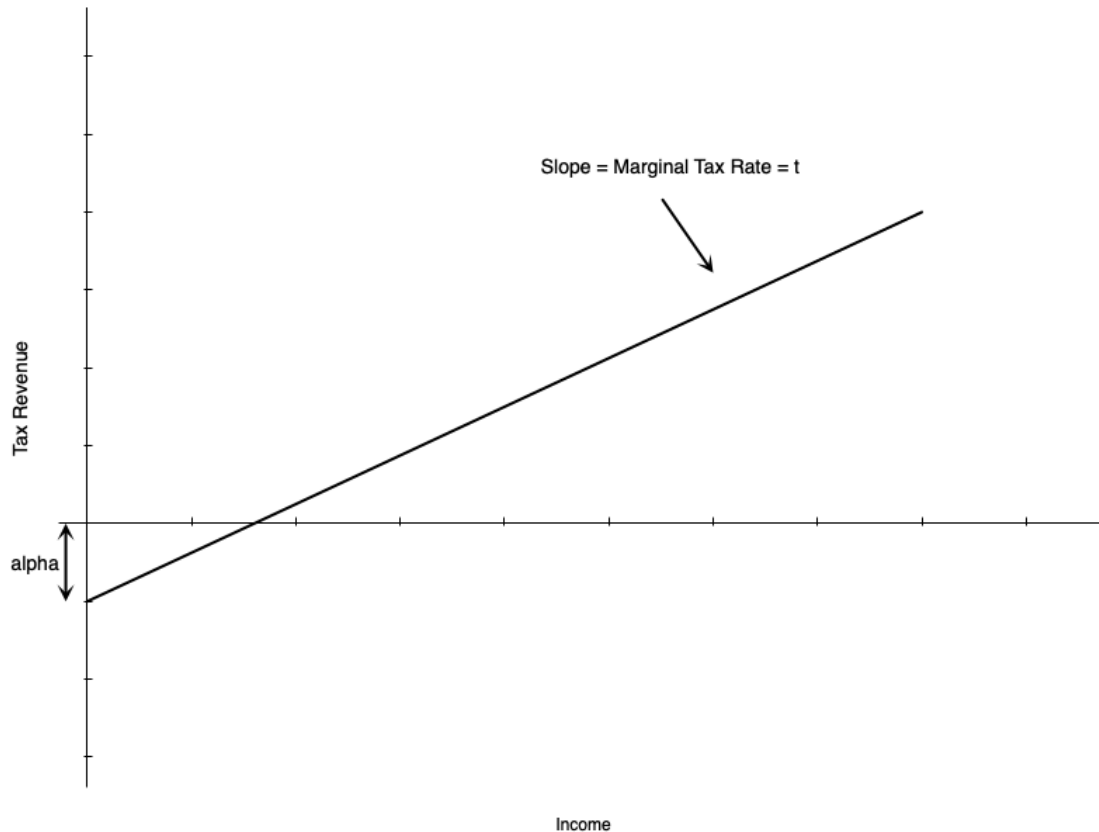
# Old School Optimal Tax

- A very old approach to optimal income taxation is **Edgeworth's Model**
- A model based on the following assumptions
  - Maximize **social welfare**: the combination of individual utilities
  - In this model, the social welfare is **utilitarian**: the sum of individual utilities
  - Each person's utility is the same, depends only on income
  - Has diminishing marginal utility of income
  - Total amount of income in society is fixed
- This model predicts that after-tax income is the same for everyone
  - Involves taxing rich and giving to poor until equality is reached
- This is not a realistic model, but it is a starting point

# Newer Approach to Optimal Tax

- Major problem with Edgeworth's model is that it ignores behavioural responses
  - People will supply less labour if taxed heavily
- We know from previous sections that taxes affect labour supply
- Modern models take this into account
  - Taxes raise revenue
  - But also create excess burden by distorting labour supply
  - Optimal tax balances these two effects
- These models compute optimal tax in a world where people choose labour and leisure

# Newer Approach to Optimal Tax

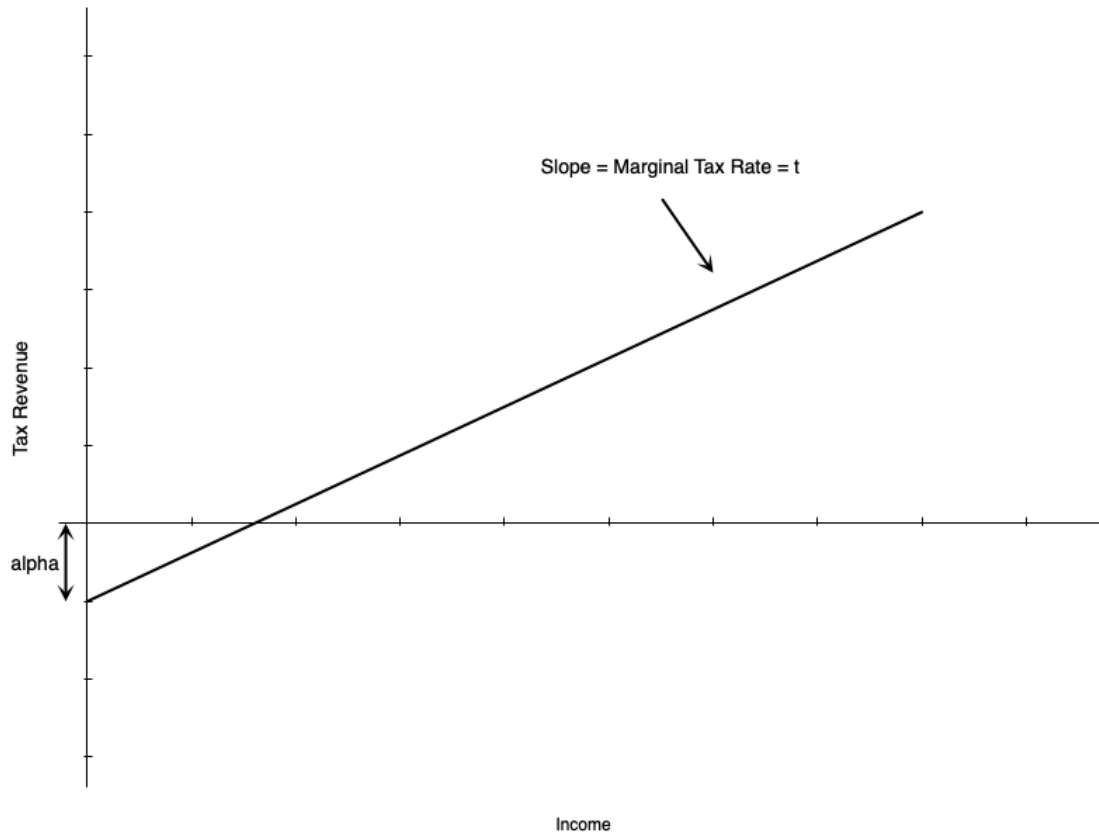


- A simplified version of this model is depicted to the left
- In this setup revenue is

$$Revenue = -\alpha + t \times Income$$

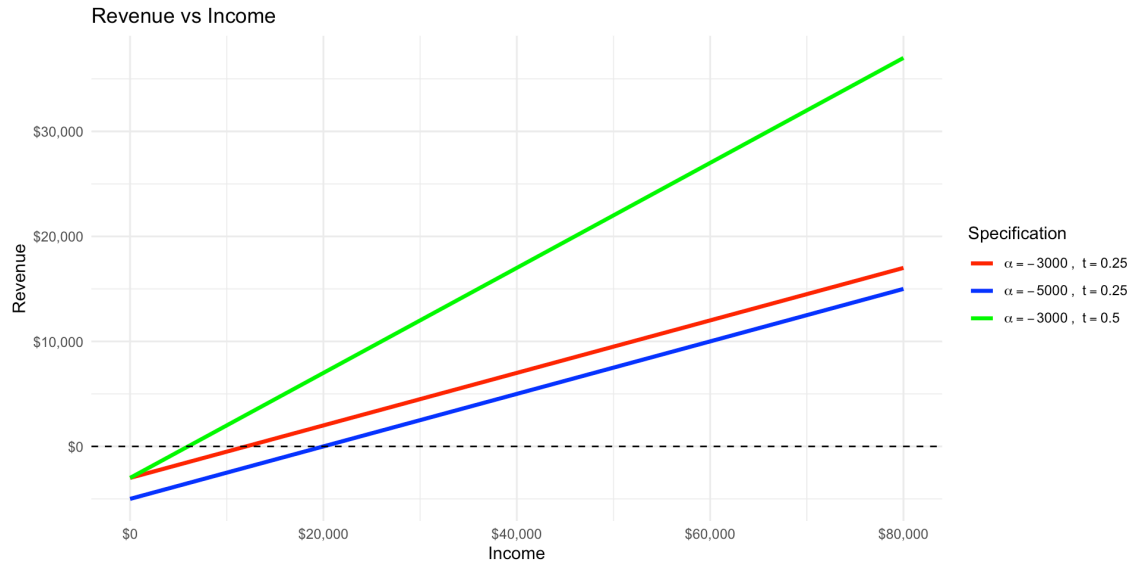
- The term  $-\alpha$  is a fixed payment to everyone
  - If  $\alpha > 0$ , it is a subsidy
  - If  $\alpha < 0$ , it is a lump-sum tax
- The term  $t$  is the marginal income tax

# Newer Approach to Optimal Tax



- A constant marginal tax is a **flat tax**
  - Comes from a linear tax schedule
- Studies find that the optimal income tax is non-linear
  - Marginal tax rate rises with income
  - Implies a progressive tax system
  - This exists in many countries, including Canada and the US
- However, they also find that the optimal non-linear tax is approximated a flat tax
  - So we can still gain insights from the flat tax

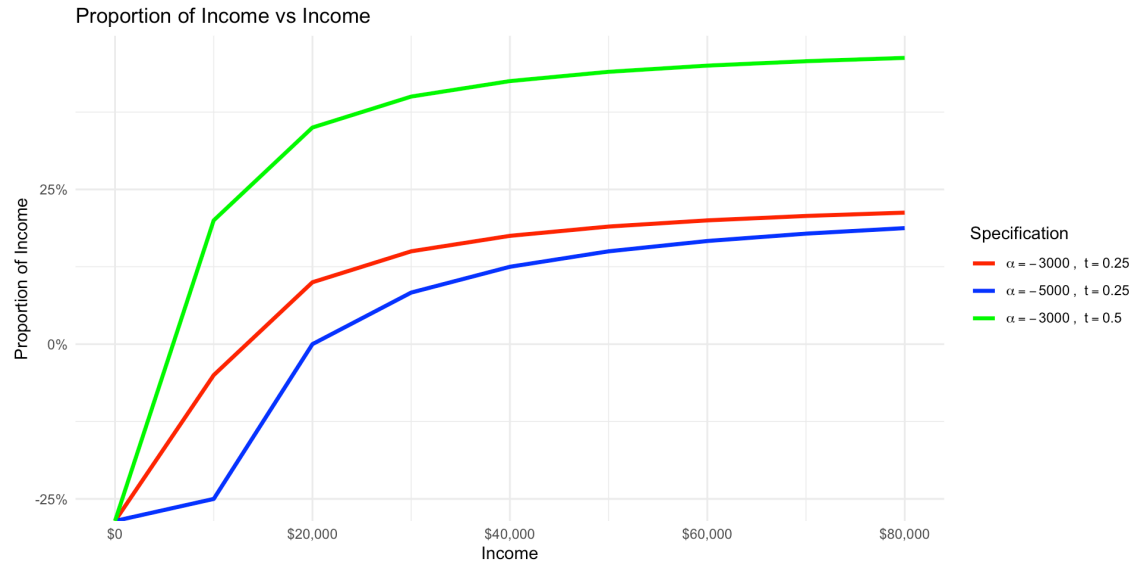
# Newer Approach to Optimal Tax



- An example with different flat taxes is shown to the left
- Graph plots total revenue
- There are three curves
  - $\alpha = -3000, t = 0.25$
  - $\alpha = -5000, t = 0.25$
  - $\alpha = -3000, t = 0.5$
- $\alpha$  changes the intercept
- $t$  changes the slope



# Newer Approach to Optimal Tax



- Now plot the proportion of income paid in tax
- Here you can see the progressivity of the tax
- The tax with the highest marginal tax rate ( $t = 0.5$ ) and the lowest fixed payment ( $\alpha = -3000$ ) is the most progressive
- The higher the marginal tax rate, the more progressive the tax
- The lower the fixed payment, the more progressive the tax

# Newer Approach to Optimal Tax

- The specific optimal tax rates depend on the model assumptions
  - Labour supply elasticity
  - Social welfare function
  - Ability to pay
  - Income distribution
- With the utilitarian social welfare function, optimal is around 50%
  - Optimal grant  $\alpha$  is about 60% of average worker income
- Using a more egalitarian social welfare function, optimal tax is higher
  - Egalitarian social welfare function puts more weight on poorer individuals
  - Optimal rate around 80%
- Finally, some studies find that if tax rates are allowed to be different, rich should pay lower marginal rate

# Politics and Time Inconsistency

# Introduction

- Optimal tax theory provides purely theoretical guidance on how to design taxes
- In practice, tax policy is determined by politics
  - Politicians care about getting elected
  - Voters care about their own taxes and benefits
- So the optimal tax is not a realistic outcome
- It is possible that in the real world, implementing the optimal tax is not even desirable

# Time Inconsistency Problem

- Imagine a government wants to tax society to raise revenue
- There are three possible goods that people can consume
  - $X$ ,  $Y$ , and Leisure  $L$
- Labour has a fixed supply so income is also fixed
- The government is allowed to tax  $X$ , but not  $Y$
- A tax economist suggests lowering the tax on  $X$  and taxing  $X$  and  $Y$  at the same rate
  - Efficient because relative prices are unchanged, no change in labour
  - No excess burden

# Time Inconsistency Problem

- Citizens are cynical
  - They think the government will implement the tax on  $Y$  but not lower it on  $X$
  - View the government as trying to maximize revenue at their expense
  - In certain situations they might be right
- If they are correct, taxing only  $X$  is more efficient than the alternative of taxing both
- This is an example of the **time inconsistency problem**: optimal tax policy is not credible over time
  - The government cannot commit to future policies
- To fully implement optimal tax, the government must credibly commit to future policies
  - This is difficult in practice
  - So optimal tax may not be achievable or even desirable

# Other Criteria for Tax Design

# Horizontal Equity

- **Horizontal Equity:** people in “equal positions” should be treated equally
  - When it comes to taxation, equal positions might mean equal ability to pay, out of income, consumption, or wealth
- Horizontal equity is desirable because it is viewed as fair
  - People in equal positions should pay the same tax
  - If not, people may view the tax system as unfair and try to avoid taxes
- Defining horizontal equity in terms of income or wage is problematic
  - Two people who earn the same wage but work different hours have different abilities to pay
  - Taxing wage instead is a problem because it is based on human capital investments



# Horizontal Equity

- Could instead define horizontal equity in terms of utility
  - **Utility definition of horizontal equity:** people with the same pre-tax utility should have the same after-tax utility, and taxes should not alter the utility ordering
- Several problems with this approach
  - Utility is unobservable
  - If taxing only income, it can penalize people who
    - Consume goods that are income intensive
    - Work in jobs where the pay is purely monetary instead of non-income benefits (e.g. nice work environment, flexible work)
  - Depends on utility ordering prior to any new taxes being imposed (i.e. biased towards status quo)
- Basic idea is that while horizontal equity is desirable, it is hard to define and implement in practice



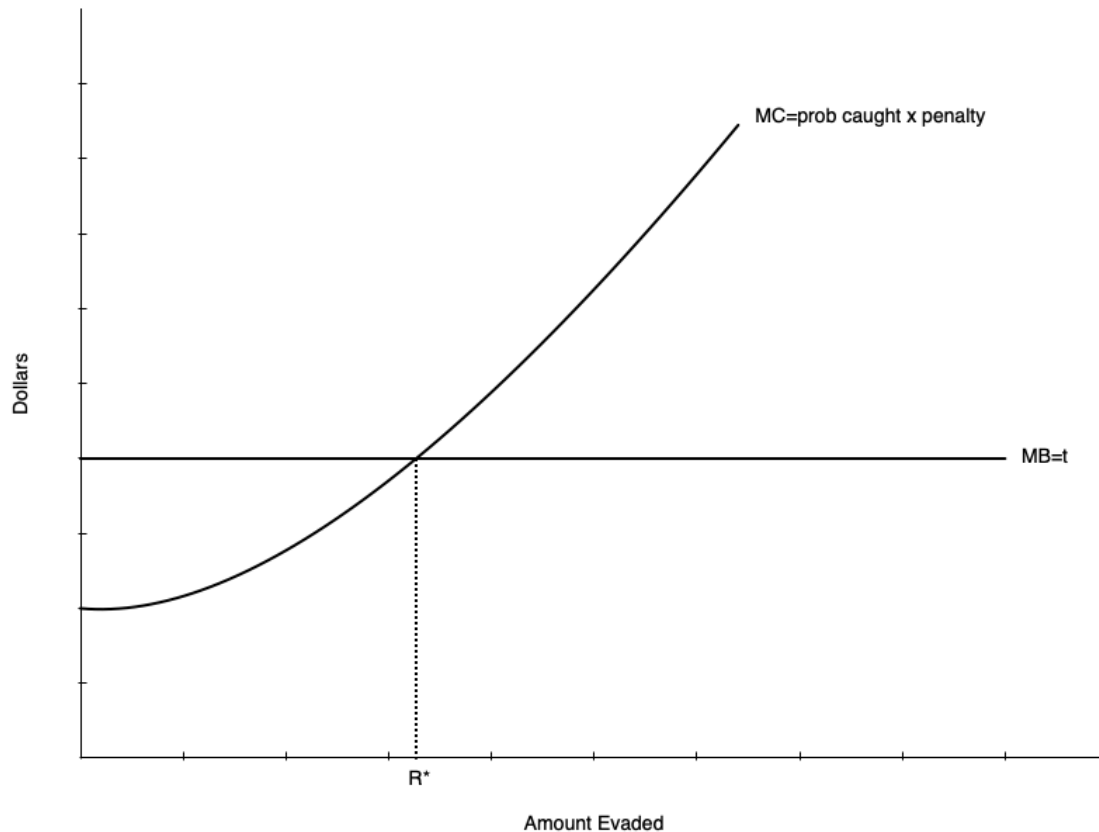
# Administrative Costs

- Taxes are costly to administer
  - Estimates suggest direct costs of about \$1 to collect \$100 of tax revenue in Canada
  - Average taxpayer pays about \$200-\$250 in tax preparation costs per year
  - Adding up costs over firms, individuals, government, it costs about 2% of GDP to collect taxes in Canada
- Tax design has to balance the benefits of a tax against its administrative costs
  - A tax that raises a lot of revenue but is very costly to administer may not be desirable
  - A tax that is easy to administer but raises little revenue may also not be desirable
- Some changes in taxation have lead to smaller costs
  - Online filing
  - Automatic download of tax forms to online portals
  - Potential for future: automatic filing

# Compliance

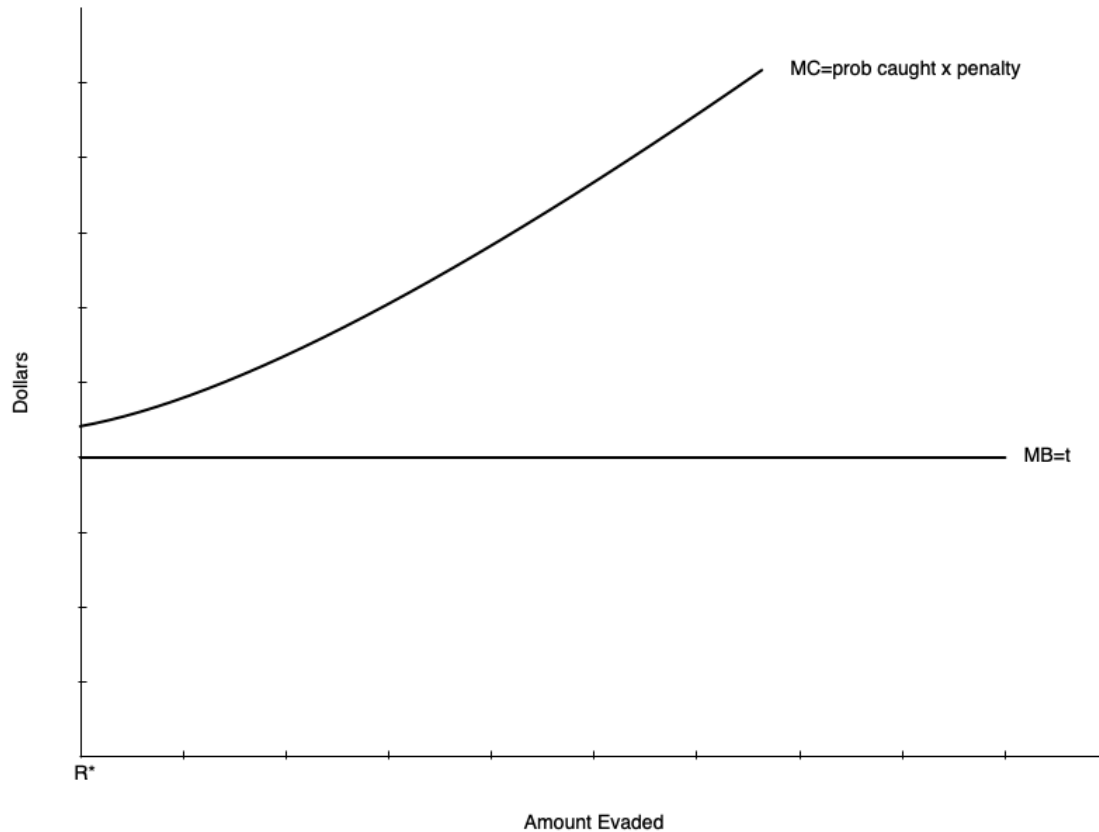
- Canada and other countries operate on a self-assessment system
  - Taxpayers are responsible for reporting their income and calculating their taxes
  - Government audits a small percentage of taxpayers to ensure compliance
- Some people engage in tax avoidance
  - Legal ways: changing behaviour to minimize taxes
  - Illegal ways: tax evasion, not reporting income, fraud
- It is not always easy to catch tax evasion
  - Government has to balance the costs of enforcement against the benefits
  - If enforcement is too costly, it may not be worth it
- There is an economic theory that determines “optimal” tax evasion

# Compliance



- Imagine that costs of evasion rise with the amount evaded
  - Penalty rises with amount evaded
  - There is some probability that you get caught
  - Could be psychic costs to cheating
  - Risk aversion might play a role
- Benefit is constant
  - Gain is  $t$  per \$1 evaded
- As with many economic decisions, optimal evasion occurs where marginal benefit equals marginal cost
- This person would evade  $R^*$

# Compliance



- If costs are too high, no evasion occurs
  - Probability of getting caught might be high
  - Penalty might be high
  - Large psychic costs or very risk averse

# Compliance

- Also the normative question about tax evasion: do we care?
- Some economies have large underground or informal economies
  - Developing countries often have significant employment in informal economies
- While it is incorrect to call them tax evaders, people in informal employment do not pay taxes
- It might be desirable to have the underground/informal economy even if they pay no tax
  - They are often poor and would not pay much tax
  - They often provide goods/services that are beneficial to society
  - Trying to eliminate the informal economy might do more harm than good

# Summary

# Summary

- Taxes create excess burden by distorting behaviour
- The Ramsey Rule gives the optimal way to tax goods to minimize excess burden
- User fees can be optimized in a similar manner
- Optimal income tax is progressive, but the specific rates depend on model assumptions
- Politics and time inconsistency can prevent optimal tax from being implemented
- Other criteria for tax design include horizontal equity, administrative costs, and compliance



# References

# References

- Rosen, Harvey S., and Lindsay M. Tedds, and Trevor Tombe, and Jean-Francois Wen, and Tracy Snoddon. Public Finance in Canada. 6th Canadian edition. McGraw-Hill Ryerson, 2023.
- Gruber, Jonathan. Public Finance and Public Policy. 7th edition. Worth Publishers, 2022.