A-Star 2016 Winter Math Camp

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Outline

1 Introduction

Welcome to A-Star Winter Math Camp 2016! This is my fourth A-Star camp.

- I've attended once as a student before.
- I've taught the AMC class twice before in the summer of 2015 and 2016.
- Number Theory is my favourite subject to teach :).

1.1 Schedule

Time	Subject
9-10:30 AM	Number Theory
10:45AM-12:15PM	Algebra
1:45-3:15PM	Geometry
3:30-5:00PM	Counting

Table 1: A-Star Teaching Schedule

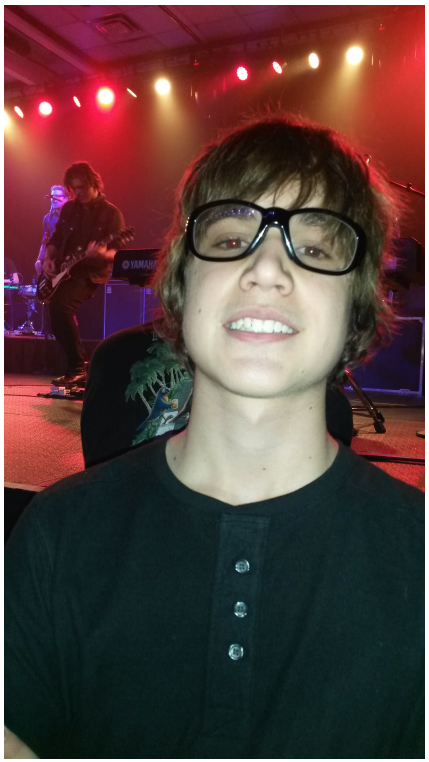
1.2 Icebreaker Activity

Three Truths and a Lie

Write down three truths and one lie about yourself on your piece of paper. I'll guess which one is the lie! Good luck guessing which one is my lie.

- I've seen over 100 different bands live in concert.
- I've programmed a human sized robot.
- My family has 2 cats.
- I've competed in and won a crib race.

Concerts: Truth



Robot: Truth

Cats: (Deceptive) Lie!

We have 5...

Crib Race??: Truth

Celebration!

2 Math Time

The topic for today is divisibility and prime factorization.

2.1 Divisibility Rules

- 2 - Last digit is even.
- 3 - Sum of the digits is divisible by 3.
- 4 - Number formed by last two digits is divisible by 4.
- 5 - Last digit is either 0 or 5.
- 6 - Divisibility rules for both 2 and 3 hold.
- 7 - Take the last digit of the number and double it. Subtract this from the rest of the number. Repeat the process if necessary. Check to see if the final number obtained is divisible by 7.

Lucky Seven

Choose **one** number below and determine if it is divisible by 7.

- 1729
- $2,718,281$
- $16,180,339$
- $31,415,926,535$

Taxicab Number

"It is a very interesting number; it is the smallest number expressible as the sum of two positive cubes in two different ways." - Srinivasa Ramanujan (1919)

$$\begin{aligned} 1729 &\rightarrow 172 - 2 \cdot 9 = 154 \\ 154 &\rightarrow 15 - 2 \cdot 4 = 7 \end{aligned}$$

Therefore, 1729 **is** divisible by 7.

Euler's Number

$$\begin{aligned} 2718281 &\rightarrow 271828 - 2 \cdot 1 = 271826 \\ 271826 &\rightarrow 27182 - 2 \cdot 6 = 27170 \\ 27170 &\rightarrow 2717 - 2 \cdot 0 = 2717 \\ 2717 &\rightarrow 271 - 2 \cdot 7 = 257 \\ 257 &\rightarrow 25 - 2 \cdot 7 = 11 \end{aligned}$$

Therefore, 2718281 is **not** divisible by 7.

More on Euler's number (e) during Algebra lectures!

The Golden Ratio - $\phi = \frac{1+\sqrt{5}}{2} = 1.6180339\cdots$

$$\begin{aligned}
16180339 &\rightarrow 1618033 - 2 \cdot 9 = 1618015 \\
1618015 &\rightarrow 161801 - 2 \cdot 5 = 161791 \\
161791 &\rightarrow 16179 - 2 \cdot 1 = 16177 \\
16177 &\rightarrow 1617 - 2 \cdot 7 = 1603 \\
1603 &\rightarrow 160 - 2 \cdot 3 = 154 \\
154 &\rightarrow 15 - 2 \cdot 4 = 7
\end{aligned}$$

Hence, 16180339 **is** divisible by 7.

Pi

$31,415,926,535$ is too big of a number. Therefore, I wrote a computer program!

Seven.ipynb

AMC Number Theory Day 1

In [8]:
```
#Author:  Justin Stevens
#A-Star Winter Math Camp, 2016
#Determines if a number is divisible by 7

def divis_sev(x):
    """Inputs an integer x and prints out a list of numbers generated by following the -2 last digit rule
        Returns True or False based on whether the integer is divisible by 7."""
    cur_num=x
    while cur_num>7:
        print(cur_num)
        trunc_num=cur_num//10  #Removes last digit from the number
        last_dig=cur_num%10   #Stores the last digit in last_dig
        cur_num=trunc_num-2*last_dig  #Applies the divisibility rule for 7
    if cur_num>0:
        print(cur_num)
    if cur_num%7==0:
        return True
    else:
        return False
```

In [13]:
```
divis_sev(31415926535)
```

```
31415926535
3141592643
314159258
31415909
3141572
314153
31409
3122
308
14
```

Out[13]: True

2.2 Explanation of the Magic

Let the number that we want to determine its divisibility by 7 be N. Let the last digit of N be x. Then, we can represent N as

$$N = 10a + x.$$

Note that we want to prove that if 7 divides N, then 7 also divides $a - 2x$.

To do so, we will multiply N by some integer.

Magic Continued

The magic integer is 5. If 7 divides N, then 7 should also divide $5N$. From the expression above for N, we have

$$5N = 50a + 5x.$$

Now, the question is, how do we get $a - 2x$ out of this?

Moving Around

We think to take the difference between $5N$ and $a - 2x$. Since we know that $5N$ is divisible by 7 if the difference is divisible by 7, then $a - 2x$ must also be divisible by 7.

Using the expression for $5N$ we found on the previous slide,

$$\begin{aligned} 5N - (a - 2x) &= 50a + 5x - (a - 2x) \\ &= 49a + 7x. \end{aligned}$$

This is clearly a multiple of 7, therefore, our proof is complete!

2.3 Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu

1. Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
2. Nam cursus est eget velit posuere pellentesque
3. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

2.4 Verbatim

How to include a theorem in this presentation:

```
\mybox{0.8\textwidth}{
\begin{theorem}[Murphy (1949)]
Anything that can go wrong, will go wrong.
\end{theorem}
}
```

3 Displaying Information

3.1 Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 2: Table caption

3.2 Figure

3.3 Theorem

The most common definition of Murphy's Law is as follows.

Theorem (Murphy (1949))
Anything that can go wrong, will go wrong.

Proof. A special case of this theorem is proven in the textbook. □

Remark
This is a remark.

Algorithm
This is an algorithm.

4 Citations

An example of the `\cite` command to cite within the presentation:

This statement requires citation [1].

References

[1] J. M. Smith and A. B. Jones. *Book Title*. Publisher, 7th edition, 2012.

Questions?