MFE 405 Project 7
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1. Solving Log-Transformation of Black-Scholes PDE for European Put Prices

Using the $X = ln(S)$ transformation of the Black-Scholes PDE, we can discretize the PDE and find the price for European put options. Three methods used are the Explicit Finite Difference, Implicit Finite Difference, and Crank-Nicolson Finite Difference methods. Since there is a close-formed solution for a European put option, as described by the Black-Scholes formula, we can use that exact solution to compare with our numerical results and observe the errors.

All computed data are shown in the table below:

Black-Scholes Price: 0.464696

	Price			**Error for $\sigma\sqrt{\Delta_t}$**
ΔX	$\sigma\sqrt{\Delta_t}$	$\sigma\sqrt{3\Delta_t}$	$\sigma\sqrt{4\Delta_t}$	
EFD	0.464126	0.464153	0.463878	-0.00057
IFD	0.464142	0.463592	0.463317	-0.000554
CNFD	0.464421	0.463873	0.463598	-0.000275

We can see from this table that when $\Delta X = \sigma\sqrt{\Delta_t}$, the estimations are most accurate. We also see that the Crank-Nicolson Finite Difference method converges most with the Black-Scholes result.

2. Solving Black-Scholes PDE for American Call & Put Prices

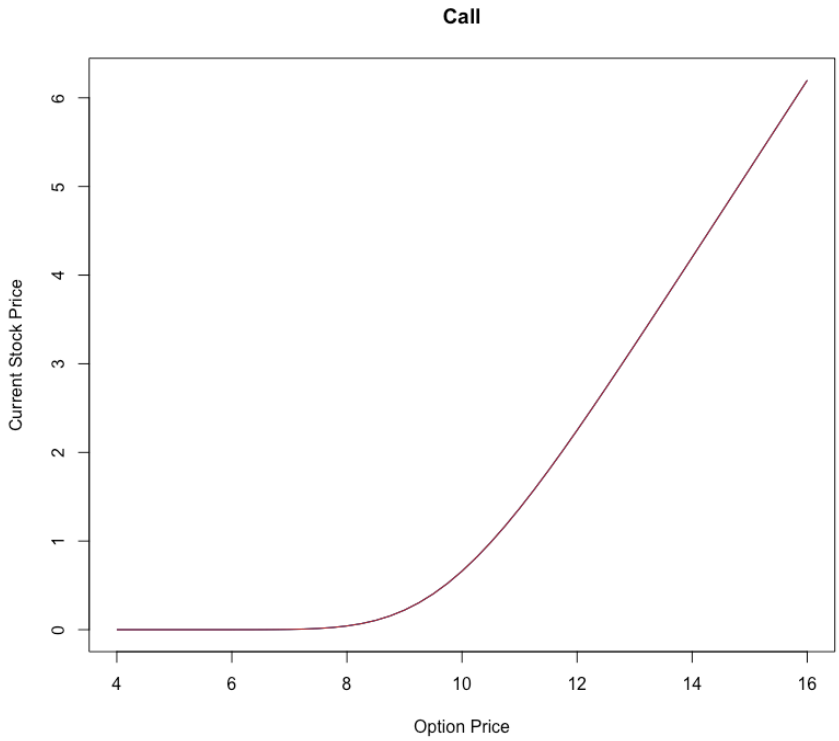
Again, using the three different PDE estimation methods, we can computer call and put prices for American call and put options. The minor modification is that at each time *j*, we need to compare the computed continuation value with the exercise value and choose the higher value.

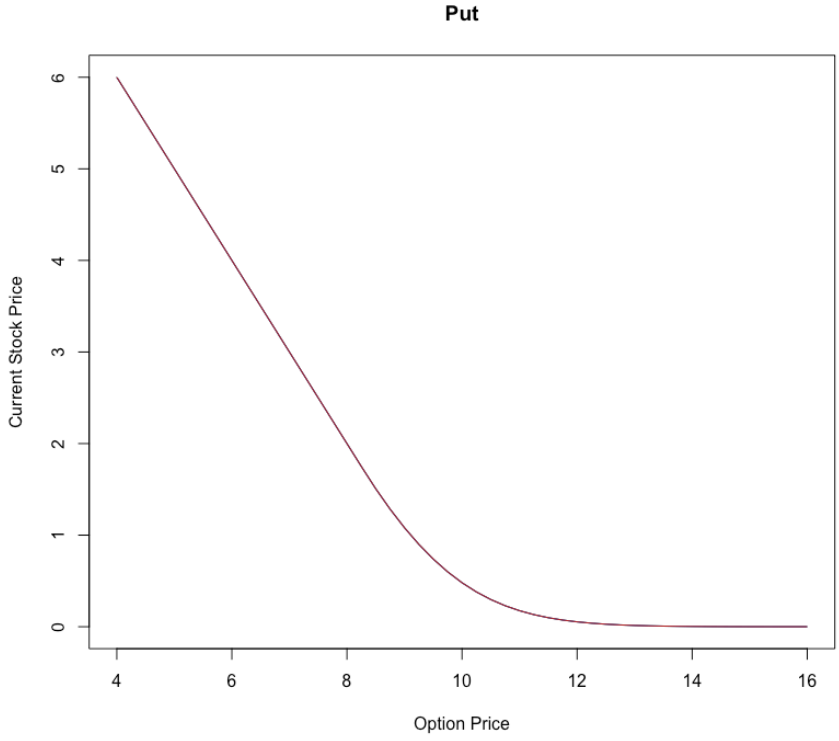
All computed data are shown in the tables below:

	Put Price		
ΔS	**0.25**	**1**	**1.25**
EFD	0.480664	0.439979	0.350726
IFD	0.47973	0.439081	0.35007
CNFD	0.480192	0.43953	0.350398

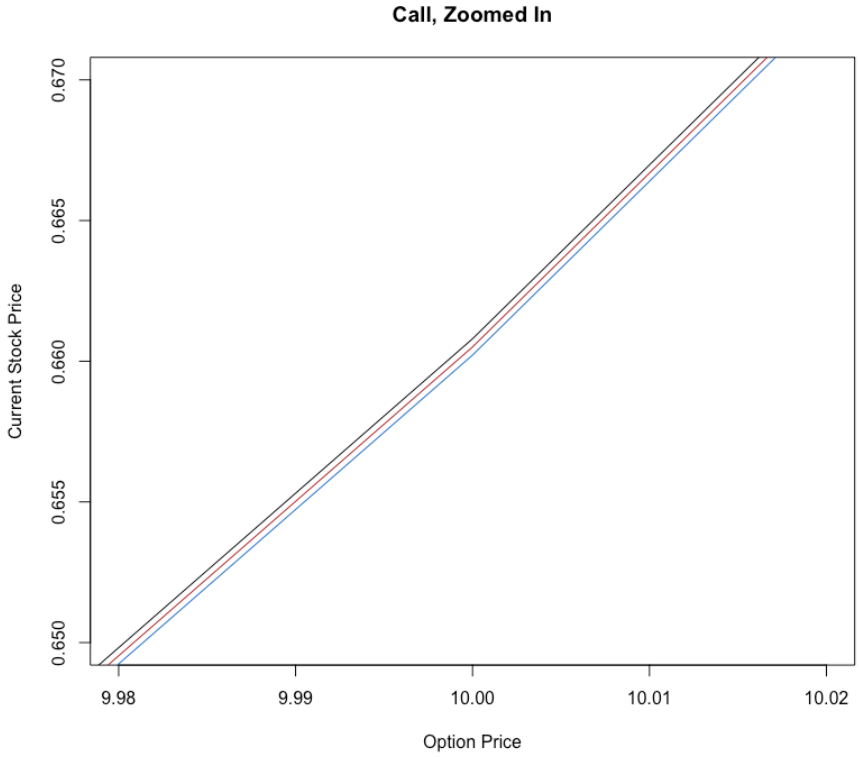
	Call Price		
ΔS	**0.25**	**1**	**1.25**
EFD	0.660798	0.623048	0.793297
IFD	0.660217	0.622291	0.792655
CNFD	0.660508	0.62267	0.792976

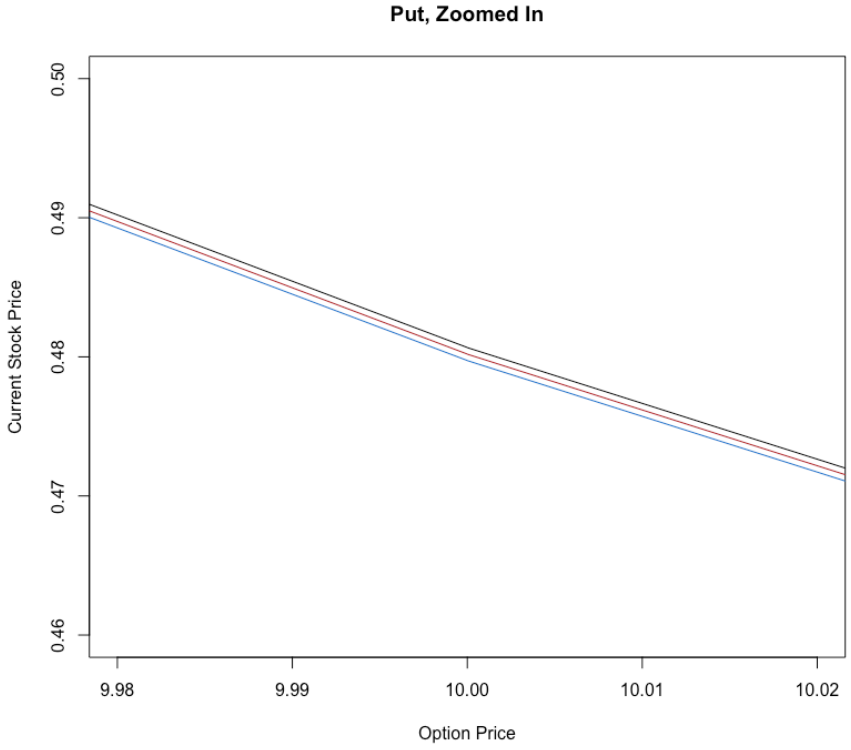
Looking at the tables, we see that when $\Delta S = 0.25$, the results are more accurate. Now, using this $\Delta S = 0.25$, and using a range of current stock prices from \$4 to \$16, we can compute different corresponding option prices. The graph of such option prices against the stock price, using each of the three methods, are as below:





In each plot there are actually three lines, but the results are too close to see. Therefore, if we zoom in on parts of the graph, we will see something like this:





In all of these graphs, the black line is the Explicit Finite Difference, the blue line is the Implicit Finite Difference, and the red line is the Crank-Nicolson Finite Difference. We again see that the Crank-Nicolson method is most accurate.