MFE 405 Project 5
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1. Pricing American Put Options with Least-Square Monte Carlo

Using antithetic variates for variance reduction in simulating stock paths, and implementing the Least-Square Monte Carlo method, we can compute the price for put options of different strike prices and maturities, as shown in the table below. Weighted polynomials used are Laguerre polynomials, Hermite polynomials, and simple monomials, each with $k = 2, 3, 4$. The results are as below (also outputted by code as *"put_prices.csv"*):

$k = 2$:

	Strike = $36			Strike = $40			Strike = $44		
	T=0.5	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**
Hermite	$4.06	$4.26	$4.47	$1.77	$2.25	$2.73	$0.63	$1.09	$1.62
Laguerre	$4.00	$4.00	$4.00	$1.41	$1.49	$1.60	$0.51	$0.64	$0.77
Monomials	$4.06	$4.26	$4.47	$1.77	$2.25	$2.73	$0.63	$1.09	$1.62

$k = 3$:

	Strike = $36			Strike = $40			Strike = $44		
	T=0.5	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**
Hermite	$4.07	$4.25	$4.42	$1.77	$2.24	$2.70	$0.63	$1.09	$1.62
Laguerre	$4.04	$4.06	$4.08	$1.71	$1.86	$1.96	$0.62	$0.89	$1.04
Monomials	$4.07	$4.25	$4.42	$1.77	$2.24	$2.70	$0.63	$1.09	$1.62

$k = 4$:

	Strike = $36			Strike = $40			Strike = $44		
	T=0.5	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**	**T=0.5**	**T=1**	**T=2**
Hermite	$4.06	$4.25	$4.42	$1.77	$2.24	$2.70	$0.63	$1.09	$1.61
Laguerre	$3.81	$4.00	$3.77	$1.68	$2.07	$2.36	$0.60	$1.02	$1.43
Monomials	$4.06	$4.25	$4.42	$1.77	$2.24	$2.70	$0.63	$1.09	$1.61

Looking at the tables, it seems that the Hermite polynomials and simple monomials are quite precise for $k = 2, 3, 4$, yet the Laguerre polynomials are most precise at around $k = 3$. All methods find very similar results.

2. Pricing Forward-Start Options

(a) Forward-Start European Put-Option

We can estimate the price of a forward-start European put option by using simple Monte Carlo simulation. After simulating the stock paths, we define $S_{0.2}$ as the strike price, and using the payoff function, we can find the estimated expected payoff, which is the estimated price. After simulating 10,000 paths with 100 periods each, we find the price to be around **$3.14**.

(b) Forward-Start American Put-Option

We can estimate the price of a forward-start American put option by using the Least-Square Monte Carlo method. However, in this case, for each simulated path we would use a different strike price, determined by $S_{0.2}$. Also, since exercise can only start after time $t = 0.2$, we only go through the LSMC process and change the index matrix starting from the end and all the way to the beginning until the point $t = 0.2$. Using this method, simulating 10,000 paths with 100 periods each, and using simple monomials of $k = 3$, we compute the price to be around **$3.34**, which makes sense as it is higher than the corresponding European put option price.