EXPONENT AND PARTIAL

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These are just two personal notation conventions that people might find useful.

Knuth's up arrow notation

It can be handy to denote $x \uparrow y = x^y$, as in:

$$5 + 7 = 12, \qquad 5 \cdot 7 = 35, \qquad 5 \uparrow 7 = 78125.$$

This notation is normally used for generating really large numbers like $8 \uparrow\uparrow\uparrow 9$, but just one arrow can make things more readable, actually. Some people have trouble with exponents in middle school because it's hard to keep track of the slightly smaller numbers. They can be easy to typeset but also float around when computing things by hand, leading to errors (especially in general relativity where superscript is used to denote tensor index). Some more examples:

- $e \uparrow i\theta = \cos\theta + i \sin\theta$.
- $\sqrt[3]{8x^2} = (8x \uparrow 2) \uparrow (3 \uparrow (-1))$.
- $a \uparrow (b + c) = a \uparrow b \cdot a \uparrow c$.
- $\log_a b = c \iff b = a \uparrow c$.

The resulting notation is a good tool for being more careful, for example when solving problems in general chemistry where the exponent often carries crucial information. Note that $X \uparrow Y$ with capital letters is something different (it denotes the NAND of the logical expressions X and Y, which is commutative unlike what we have here).

Partial derivatives

This next one mainly just saves space. I denote by $\partial_2(f(u, v))$ the first partial derivative of $f(u, v)$ with respect to v:

$$\partial_1\partial_2(4x^5y + 671 \sin y\sqrt{z}) = \partial_1(4x^5 + 671\sqrt{z}\cos y) = 20x^4$$

In the single variable case, I eliminate the subscript entirely:

$$\partial^3(\sin x) = -\cos x,$$

since taking a partial derivative for a function of a single variable reduces to just working out the ordinary derivative of that function. Note that when there are no mixed terms in an expression, we have $\partial_x\partial_y = 0$. I denote by $\partial_v(f)$ the directional derivative $\nabla f \cdot v$. This way I avoid seeing d with $\frac{d}{dx}f(x)$ (which is a silly fraction to write anyway) and can use it exclusively when integrating.