Name: ______________________________ **Algebra II**
Date: **Homework 30**

Problem 1. Write the following functions in the form $f(x) = a(x \pm h)^2 \pm k$ by completing the square. Describe how x^2 is shifted to obtain $f(x)$. Graph $f(x)$, label the vertex, label all axis intersections. An example of what I expect is given below.

(a) $f(x) = -2x^2 + 8x + 10$

(b) $f(x) = 3x^2 - 12x + 2$

(c) $f(x) = -2x^2 + 3x + 2$

(d) $f(x) = \frac{1}{3}x^2 + x + \frac{1}{4}$

(e) $f(x) = \frac{3}{5}x^2 - 3x - \frac{1}{4}$

Example. $f(x) = -2x^2 - 5x + 1$

$$\begin{aligned}
f(x) &= -2x^2 - 5x + 1 \\
&= -2\left(x^2 + \frac{5}{2}x\right) + 1 \\
&= -2\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) + 1 \\
&= -2\left(\left(x + \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) + 1 \\
&= -2\left(x + \frac{5}{4}\right)^2 - (-2)\left(\frac{5}{4}\right)^2 + 1 \\
&= -2\left(x + \frac{5}{4}\right)^2 + \frac{2 * 25}{16} + 1 \\
&= -2\left(x + \frac{5}{4}\right)^2 + \frac{25}{8} + \frac{8}{8} \\
&= -2\left(x + \frac{5}{4}\right)^2 + \frac{33}{8}
\end{aligned}$$

Then $f(x) = -2\left(x + \frac{5}{4}\right)^2 + \frac{33}{8}$ is the function x^2 shifted left $\frac{5}{4}$ units, stretched vertically by a factor of 2, reflected about the x-axis, and shifted up $\frac{33}{8}$ units. To find x-intercepts, we set $f(x) = 0$ and solve for x:

$$\begin{aligned}
-2\left(x + \frac{5}{4}\right)^2 + \frac{33}{8} &= 0 \\
-2\left(x + \frac{5}{4}\right)^2 &= -\frac{33}{8} \\
\left(x + \frac{5}{4}\right)^2 &= -\frac{33}{(-2)8}
\end{aligned}$$

$$\left(x+\frac{5}{4}\right)^2=\frac{33}{16}$$

$$\sqrt{\left(x+\frac{5}{4}\right)^2}=\pm\sqrt{\frac{33}{16}}$$

$$x+\frac{5}{4}=\pm\frac{\sqrt{33}}{\sqrt{16}}$$

$$x+\frac{5}{4}=\pm\frac{\sqrt{33}}{4}$$

$$x=-\frac{5}{4}\pm\frac{\sqrt{33}}{4}$$

Note that $-\frac{5}{4}+\frac{\sqrt{33}}{4}$ is positive and $-\frac{5}{4}-\frac{\sqrt{33}}{4}$ is negative. To find the y-intercept, we set $x=0$ and find $f(0)=-2(0)^2-5(0)+1=1$. Thus our y-intercept is at $y=1$. Noting that our vertex is above the x-axis, on the left of the y-axis, and that the parabola is flipped so that it opens down, it makes sense that one of our x-intercepts is positive and the other is negate. Be sure that all intercepts are labeled and that the vertex is indicated as in the graph below.