Name: ______________________ **Algebra II**
Date: **Quiz 4**

Problem 1. Simplify and factor completely.

$$\sqrt{x^5 - x^4} - \sqrt{16x - 16}$$
$$= \sqrt{x^4(x-1)} - \sqrt{16(x-1)}$$
$$= x^2\sqrt{x-1} - 4\sqrt{x-1}$$
$$= (x^2-4)\sqrt{x-1}$$
$$= (x+2)(x-2)\sqrt{x-1}$$

Problem 2. Simplify.

$$\sqrt[3]{y^4}\,\sqrt[3]{16y^5}$$
$$= \sqrt[3]{y^4 16y^5}$$
$$= \sqrt[3]{16y^4y^5}$$
$$= \sqrt[3]{16y^9}$$
$$= \sqrt[3]{2^4(y^3)^3}$$
$$= \sqrt[3]{2 * 2^3(y^3)^3}$$
$$= 2y^3\sqrt[3]{2}$$

Problem 3. First write as one single radical. Then simplify if possible.

$$(x^3)^{\frac{1}{2}}(xy^2)^{\frac{1}{3}}(x^2y^3)^{\frac{1}{6}}$$
$$= (x^3)^{\frac{1}{2}*\frac{3}{3}}(xy^2)^{\frac{1}{3}*\frac{2}{2}}(x^2y^3)^{\frac{1}{6}}$$
$$= (x^3)^{\frac{3}{6}}(xy^2)^{\frac{2}{6}}(x^2y^3)^{\frac{1}{6}}$$
$$= \sqrt[6]{(x^3)^3}\,\sqrt[6]{(xy^2)^2}\,\sqrt[6]{(x^2y^3)^1}$$
$$= \sqrt[6]{x^9}\,\sqrt[6]{x^2y^4}\,\sqrt[6]{x^2y^3}$$
$$= \sqrt[6]{x^9x^2y^4x^2y^3}$$
$$= \sqrt[6]{x^{13}y^7} \text{ (this is written as one radical)}$$
$$= \sqrt[6]{x(x^2)^6yy^6}$$
$$= x^2y\sqrt[6]{xy}$$

Note that I did not say what flavor of radical to use. You could also have chosen to use, say, a 12th root, giving you

$$= (x^3)^{\frac{1}{2}*\frac{6}{6}}(xy^2)^{\frac{1}{3}*\frac{4}{4}}(x^2y^3)^{\frac{1}{6}*\frac{2}{2}}$$
$$= (x^3)^{\frac{6}{12}}(xy^2)^{\frac{4}{12}}(x^2y^3)^{\frac{2}{12}}$$
$$= \sqrt[12]{(x^3)^6}\,\sqrt[12]{(xy^2)^4}\,\sqrt[12]{(x^2y^3)^2}$$

$$
\begin{aligned}
&= \sqrt[12]{x^{18}}\ \sqrt[12]{x^4y^8}\ \sqrt[12]{x^4y^6} \\
&= \sqrt[12]{x^{18}x^4y^8x^4y^6} \\
&= \sqrt[12]{x^{26}y^{14}} \\
&= \sqrt[12]{x^{12}x^{12}x^2y^{12}y^2} \\
&= x^2y\ \sqrt[12]{x^2y^2}
\end{aligned}
$$

which is an acceptable answer. But notice that we can further reduce this since

$$x^2y\ \sqrt[12]{x^2y^2} = x^2y(x^2y^2)^{\frac{1}{12}} = x^2y((xy)^2)^{\frac{1}{12}} = x^2y(xy)^{\frac{2}{12}} = x^2y(xy)^{\frac{1}{6}} = x^2y\sqrt[6]{xy}$$

which is the first answer above.

Problem 4. Compute. For full credit write your answer in reduced form.

$$\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^2$$

$$
\begin{aligned}
&= \left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) \\
&= \frac{1}{4}+i\frac{\sqrt{3}}{4}+i\frac{\sqrt{3}}{4}+i^2\frac{3}{4} \\
&= \frac{1}{4}+i\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right)-\frac{3}{4} \\
&= \frac{1}{4}-\frac{3}{4}+i\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right) \\
&= -\frac{2}{4}+i\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right) \\
&= -\frac{1}{2}+i\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right) \\
&= -\frac{1}{2}+i\left(\frac{\sqrt{3}+\sqrt{3}}{4}\right) \\
&= -\frac{1}{2}+i\left(\frac{2\sqrt{3}}{4}\right) \\
&= -\frac{1}{2}+i\frac{\sqrt{3}}{2}
\end{aligned}
$$

Problem 5. Write in the form $a+ib$. Be sure to show your work!

$$\frac{1}{2-3i}$$

$$= \frac{1}{(2-3i)} \frac{(2+3i)}{(2+3i)}$$

$$= \frac{(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{(2+3i)}{4+6i-6i-9i^2}$$

$$= \frac{(2+3i)}{4-9(-1)}$$

$$= \frac{(2+3i)}{4+9}$$

$$= \frac{(2+3i)}{13}$$

$$= \frac{2}{13} + \frac{3}{13}i$$

Problem 6. Write in the form $a+ib$. Be sure to show your work!

$$\frac{\sqrt{2}-3i}{2-i\sqrt{3}}$$

$$= \frac{(\sqrt{2}-3i)}{(2-i\sqrt{3})} \frac{(2+i\sqrt{3})}{(2+i\sqrt{3})}$$

$$= \frac{(\sqrt{2}-3i)(2+i\sqrt{3})}{4+2i\sqrt{3}-2i\sqrt{3}-3i^2}$$

$$= \frac{(\sqrt{2}-3i)(2+i\sqrt{3})}{4-3(-1)}$$

$$= \frac{(\sqrt{2}-3i)(2+i\sqrt{3})}{4+3}$$

$$= \frac{(\sqrt{2}-3i)(2+i\sqrt{3})}{7}$$

$$= \frac{2\sqrt{2}+i\sqrt{2}\sqrt{3}-6i-3i^2\sqrt{3}}{7}$$

$$= \frac{2\sqrt{2}+i\sqrt{6}-6i-3(-1)\sqrt{3}}{7}$$

$$= \frac{2\sqrt{2}+i\sqrt{6}-6i+3\sqrt{3}}{7}$$

$$= \frac{2\sqrt{2}+3\sqrt{3}+i\sqrt{6}-6i}{7}$$

$$= \frac{(2\sqrt{2}+3\sqrt{3})+i(\sqrt{6}-6)}{7}$$

$$= \left(\frac{2\sqrt{2}+3\sqrt{3}}{7}\right) + i\left(\frac{\sqrt{6}-6}{7}\right)$$

Problem 7. Write in the form $a+ib$. Be sure to show your work!

$$\begin{aligned}
&\frac{x+iy}{u+iv}\\
&=\frac{(x+iy)}{(u+iv)}\frac{(u-iv)}{(u-iv)}\\
&=\frac{xu-ixv+iyu-i^2yv}{u^2-iuv+iuv-i^2v^2}\\
&=\frac{xu-ixv+iyu-(-1)yv}{u^2-(-1)v^2}\\
&=\frac{xu-ixv+iyu+yv}{u^2+v^2}\\
&=\frac{xu+yv+iyu-ixv}{u^2+v^2}\\
&=\frac{(xu+yv)+i(yu-xv)}{u^2+v^2}\\
&=\left(\frac{xu+yv}{u^2+v^2}\right)+i\left(\frac{yu-xv}{u^2+v^2}\right)
\end{aligned}$$

Extra Credit. Compute

$$\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$$

Let $x=\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ and notice that we must have $0<x$ since at every level, this is the square root of a positive number and therefore positive. Then

$$x=\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$$

$$\implies x^2=2+\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}.$$

But we can substitute $x=\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ to give us

$$x^2=2+\left(\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}\right)$$

$$\begin{aligned}
x^2&=2+x\\
\iff x^2-x-2&=0\\
\iff (x+1)(x-2)&=0
\end{aligned}$$

which is true whenever $x=-1$ or when $x=2$. But since we must have $0<x$, it must be the case that $x=2$. We'll check our work by letting $x=2$ in $x=\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ so that

$$2=\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$$

$$\begin{aligned}
&\implies 2^2 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} \\
&\iff 4 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} \\
&\iff 4 - 2 = 2 - 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} \\
&\iff 2 = 0 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} \\
&\iff 2 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}
\end{aligned}$$

which is what we want. Therefore,

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} = 2.$$