

Name: _____

Date: _____

Algebra II
Homework 27**Problem 1.** Fill in the missing information in each equation.

For example, given $(x \quad)^2 = x^2 - 4x + (\quad)^2$, you'll need to complete it with the -2 on the left hand side, and with a 2 on the right giving you $(x - 2)^2 = x^2 - 4x + (2)^2$. (Notice that it's not necessary to write $(-2)^2$ since it's gonna be positive anyway. I mean, $(-2)^2 = (2)^2 = 4$, so why waste your time writing an unnecessary symbol?)

(a) $(x \quad)^2 = x^2 + 2x + (\quad)^2$

(b) $(x \quad)^2 = x^2 - 6x + (\quad)^2$

(c) $(x \quad)^2 = x^2 + 3x + (\quad)^2$

(d) $(x \quad)^2 = x^2 - 11x + (\quad)^2$

(e) $(x \quad)^2 = x^2 + \frac{1}{2}x + (\quad)^2$

(f) $(x \quad)^2 = x^2 - \frac{2}{5}x + (\quad)^2$

(g) $(x \quad)^2 = x^2 + \frac{7}{3}x + (\quad)^2$

(h) $(x \quad)^2 = x^2 + \frac{b}{a}x + (\quad)^2$

Problem 2. Solve for x . Write your answers in a solution set. Check your answers. You'll notice that keeping your answers in the form $\frac{a}{b} \pm \frac{c}{b}$ will make it easier to check than if you'd written it like $\frac{a \pm c}{b}$. That is, since you're going to plug your answers back in to the original equations, then writing your answers with a common denominator will just make things more difficult. Here's an example of what I'm looking for:

$$5 \left(x + \frac{1}{2} \right)^2 + 10 = 0$$

$$5 \left(x + \frac{1}{2} \right)^2 = -10$$

$$\left(x + \frac{1}{2} \right)^2 = -2$$

$$\sqrt{\left(x + \frac{1}{2} \right)^2} = \pm \sqrt{-2}$$

$$x + \frac{1}{2} = \pm i\sqrt{2}$$

$$x = -\frac{1}{2} \pm i\sqrt{2}$$

$$x \in \left\{ -\frac{1}{2} + i\sqrt{2}, -\frac{1}{2} - i\sqrt{2} \right\}$$

Now we'll check both answers simultaneously...

$$5 \left(x + \frac{1}{2} \right)^2 + 10 = 5 \left(-\frac{1}{2} \pm i\sqrt{2} + \frac{1}{2} \right)^2 + 10$$

$$\begin{aligned}
&= 5 \left(\pm i\sqrt{2} \right)^2 + 10 \\
&= 5i^2(2) + 10 \\
&= -5(2) + 10 \\
&= -10 + 10 = 0
\end{aligned}$$

Note that the only reason that we *can* check both answers simultaneously is that both $(a)^2 = a^2$ and $(-a)^2 = a^2$. And as we just saw, this is even true for complex numbers since

$$(i\sqrt{2})^2 = i\sqrt{2} * i\sqrt{2} = i^2\sqrt{2}\sqrt{2} = (-1)2 = -2$$

and

$$(-i\sqrt{2})^2 = -i\sqrt{2} * -i\sqrt{2} = (-1)i\sqrt{2} * (-1)i\sqrt{2} = (-1)^2 i^2 \sqrt{2}\sqrt{2} = (1)(-1)2 = -2$$

So as long as we're on the same page and understand that when we write $(\pm a)^2 = a^2$ we mean that both $(a)^2 = a^2$ and that $(-a)^2 = a^2$, then we can simplify our work by killing two birds with one stone.

Finally, notice how much harder it would have been to check if I'd written my answers with a common denominator. I would have had

$$\begin{aligned}
x &= -\frac{1}{2} \pm i\sqrt{2} \\
x &= -\frac{1}{2} \pm \frac{2i\sqrt{2}}{2} \\
x &= \frac{-1 \pm 2i\sqrt{2}}{2}
\end{aligned}$$

which we would have plugged in to get a giant headache:

$$5 \left(\frac{-1 \pm 2i\sqrt{2}}{2} + \frac{1}{2} \right)^2 + 10$$

It's much easier to keep these guys split up instead of putting them all over a common denominator. (It's *usually* easier to not rationalize denominators too!)

(a) $(x - 2)^2 - 9 = 0$

(b) $(x + 3)^2 - 3 = 0$

(c) $(x - 3)^2 + 3 = 0$

(d) $2(x + 1)^2 - 8 = 0$

(e) $2 \left(x - \frac{1}{4} \right)^2 - \frac{3}{8} = 0$

(f) $3 \left(x + \frac{2}{7} \right)^2 + \frac{5}{6} = 0$

(g) $a(x + h)^2 - k = 0$ (assume $0 < k, 0 < a$)

(h) $a(x - h)^2 + k = 0$ (assume $0 < k, 0 < a$)