Name: ______________________________ **Algebra II**
Date: **Homework 32**

Problem 1. Given a vertex and another point on a parabola, write in the form $f(x) = a(x-h)^2+k$. Find the zeros of this function by setting $f(x) = 0$ and solving for x. Rewrite each function in the form $f(x) = ax^2 + bx + c$. Use the quadratic formula to find the zeros and the vertex of the parabola. Graphing the parabola may help you to keep all of this straight. It's not necessary, but I highly recommend doing it so that you're not just "plugging numbers" into the right places, you're actually seeing why you're plugging in inputs and getting outputs. An example is given below.

(a) Vertex: $(-2, 3)$, Point: $(-1, 2)$

(b) Vertex: $(3, -1)$, Point: $(0, 1)$

(c) Vertex: $(2, 1)$, Point: $(1, 2)$

(d) Vertex: $(-5, -2)$, Point: $(-1, 6)$

(e) Vertex: $\left(\frac{2}{3}, \frac{2}{3}\right)$, Point: $(1, 1)$

Example. Vertex: $(1, 2)$, Point: $(3, 1)$

Since the vertex is at $(1, 2)$, we can realize this function as the graph of x^2 shifted right one unit and up 2 units. Therefore

$$f(x) = a(x-1)^2 + 2$$

and we just need to find a. Since $(3, 1)$ is a point on the function, $y = 1$ when $x = 3$. That is, the output of f is 1 when the input is 3. Symbolically,

$$f(3) = 1$$

Using our $f(x)$ from above, we have

$$f(3) = a(3-1)^2 + 2 = 1$$

$$\iff a(2)^2 + 2 = 1$$

$$\iff a4 + 2 = 1$$

$$\iff 4a = -1$$

$$\iff a = -\frac{1}{4}$$

Plugging in our value for a into our original $f(x)$ gives us the answer to the first task:

$$f(x) = -\frac{1}{4}(x-1)^2 + 2$$

To find the zeros of this function we solve $f(x) = 0$ for x:

$$-\frac{1}{4}(x-1)^2 + 2 = 0$$

$$-\frac{1}{4}(x-1)^2 = -2$$

$$(x-1)^2 = -2(-4)$$
$$(x-1)^2 = 8$$
$$\sqrt{(x-1)^2} = \pm\sqrt{8}$$
$$x-1 = \pm\sqrt{2*2*2}$$
$$x-1 = \pm 2\sqrt{2}$$
$$x = 1 \pm 2\sqrt{2}$$
$$x \in \left\{1-2\sqrt{2}, 1+2\sqrt{2}\right\}$$

Rewriting $f(x) = -\frac{1}{4}(x-1)^2+2$ in the form $f(x) = ax^2+bx+c$ gives us

$$f(x) = -\frac{1}{4}(x-1)^2+2$$
$$= -\frac{1}{4}(x^2-2x+1)+2$$
$$= -\frac{1}{4}x^2+\frac{2}{4}x-\frac{1}{4}+2$$
$$= -\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{4}+\frac{8}{4}$$
$$= -\frac{1}{4}x^2+\frac{1}{2}x+\frac{7}{4}$$

So $f(x) = -\frac{1}{4}x^2+\frac{1}{2}x+\frac{7}{4}$ is in the desired form where $a = -\frac{1}{4}$, $b = \frac{1}{2}$, and $c = \frac{7}{4}$. Plugging this in to $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$
$$= -\frac{\frac{1}{2}}{2\left(-\frac{1}{4}\right)} \pm \frac{\sqrt{\left(\frac{1}{2}\right)^2-4\left(-\frac{1}{4}\right)\left(\frac{7}{4}\right)}}{2\left(-\frac{1}{4}\right)}$$
$$= -\frac{\frac{1}{2}}{\left(-\frac{1}{2}\right)} \pm \frac{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{4}{4}\right)\left(\frac{7}{4}\right)}}{\left(-\frac{1}{2}\right)}$$
$$= -(-1) \pm \frac{\sqrt{\left(\frac{1}{4}\right)+(1)\left(\frac{7}{4}\right)}}{\left(-\frac{1}{2}\right)}$$
$$= 1 \pm \frac{\sqrt{\frac{1}{4}+\frac{7}{4}}}{\left(-\frac{1}{2}\right)}$$
$$= 1 \pm \frac{\sqrt{\frac{8}{4}}}{\left(-\frac{1}{2}\right)}$$
$$= 1 \pm \frac{\sqrt{2}}{\left(-\frac{1}{2}\right)}$$

$$= 1 \pm \sqrt{2}\left(-\frac{2}{1}\right)$$
$$= 1 \mp 2\sqrt{2}$$

And therefore

$$x \in \left\{1 - 2\sqrt{2}, 1 + 2\sqrt{2}\right\}$$

Obviously the midpoint of these two zeros is 1 (which happens to be equal to $\frac{-b}{2a}$ ... imagine that!) and therefore the x-value of the vertex is $x = 1$. To find the vertex's y-value, we plug in $x = 1$ into our function $f(x) = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{7}{4}$ to get

$$f(1) = -\frac{1}{4}(1)^2 + \frac{1}{2}(1) + \frac{7}{4}$$
$$= -\frac{1}{4} + \frac{1}{2} + \frac{7}{4}$$
$$= -\frac{1}{4} + \frac{2}{4} + \frac{7}{4}$$
$$= -\frac{1}{4} + \frac{9}{4}$$
$$= \frac{8}{4}$$
$$= 2$$

Then the vertex is at $(1, 2)$, as desired.