Name: ______________________________ **Algebra II**
Date: **Quiz 2**

Problem 1. Write $(x-1)^2(x+1)^2$ in standard form. That is, as a polynomial in descending powers of x. (Your answer should have the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$. *Hint*: There's an easy way to do this, and a hard way.)

$$\begin{aligned} &(x-1)^2(x+1)^2 \\ &= ((x-1)(x+1))^2 \\ &= (x^2-1)^2 \\ &= (x^2-1)(x^2-1) \\ &= x^4 - 2x^2 + 1 \end{aligned}$$

Problem 2. Factor $25x^2 - 36y^2$ completely.

$$\begin{aligned} &25x^2 - 36y^2 \\ &= (5x)^2 - (6y)^2 \\ &= (5x+6y)(5x-6y) \end{aligned}$$

Problem 3. Factor $16z^4 - 81w^4$ completely.

$$\begin{aligned} &16z^4 - 81w^4 \\ &= (4z^2)^2 - (9w^2)^2 \\ &= (4z^2+9w^2)(4z^2-9w^2) \\ &= (4z^2+9w^2)((2z)^2-(3w)^2) \\ &= (4z^2+9w^2)(2z+3w)(2z-3w) \end{aligned}$$

Problem 4. Factor $y^3 + y^2 - 4y - 4$ completely.

$$\begin{aligned} &y^3 + y^2 - 4y - 4 \\ &= y^3 + y^2 - 4(y+1) \\ &= y^2(y+1) - 4(y+1) \\ &= (y^2-4)(y+1) \\ &= (y+2)(y-2)(y+1) \end{aligned}$$

Problem 5. Factor $10x^2 + 11x - 6$ completely.

$$10x^2 + 11x - 6$$

$$= (5x - 2)(2x + 3)$$

Problem 6. Factor $5x^3 + 40y^3$ completely.

$$5x^3 + 40y^3$$

$$= 5(x^3 + 8y^3)$$
$$= 5(x^3 + (2y)^3)$$
$$= 5(x + 2y)(x^2 - (x)(2y) + (2y)^2)$$
$$= 5(x + 2y)(x^2 - 2xy + 4y^2)$$

Problem 7. Let $f(x) = \frac{1}{x^2-2x-15}$. Write the domain of f in *interval notation*.

We need to find the allowable inputs to f. Clearly the only problem we'll have is if the denominator is equal to 0. We'll be able to take all real numbers as inputs except for the solutions to the equation

$$x^2 - 2x - 15 = 0.$$

And since

$$x^2 - 2x - 15 = (x + 3)(x - 5),$$

we have

$$(x + 3)(x - 5) = 0$$

precisely when $x = -3$ or $x = 5$. Therefore all real numbers except -3 and 5 are allowable inputs to f. To put this in interval notation, we think of the real numbers as an interval, $(-\infty, \infty)$, and pull out the numbers -3 and 5, leaving us

$$(-\infty, -3) \cup (-3, 5) \cup (5, \infty).$$

Thus,

$$\text{Dom}(f) = (-\infty, -3) \cup (-3, 5) \cup (5, \infty)$$