Name: ______________________________ **Algebra II**
Date: **Homework 28**

Problem 1. Write the following polynomials in the form $a(x \pm h)^2 \pm k$ using the method we covered in class (by completing the square). Show all your work. (The $\pm$ just means that the signs don't have to be a certain way, just do whatever's natural.)

(a) $x^2 + 2x + 3$

$$\begin{aligned} &= (x^2 + 2x + 1^2 - 1^2) + 3 \\ &= ((x+1)^2 - 1^2) + 3 \\ &= (x+1)^2 - 1 + 3 \\ &= (x+1)^2 + 2 \end{aligned}$$

(b) $2x^2 - 4x + 1$

$$\begin{aligned} &= 2(x^2 - 2x) + 1 \\ &= 2(x^2 - 2x + 1^2 - 1^2) + 1 \\ &= 2((x-1)^2 - 1^2) + 1 \\ &= 2(x-1)^2 - 2 + 1 \\ &= 2(x-1)^2 - 1 \end{aligned}$$

(c) $2x^2 + 3x - 2$

$$\begin{aligned} &= 2\left(x^2 + \frac{3}{2}x\right) - 2 \\ &= 2\left(x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) - 2 \\ &= 2\left(\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) - 2 \\ &= 2\left(x + \frac{3}{4}\right)^2 - 2\left(\frac{3}{4}\right)^2 - 2 \\ &= 2\left(x + \frac{3}{4}\right)^2 - 2 * \frac{9}{16} - 2 \\ &= 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8} - \frac{16}{8} \\ &= 2\left(x + \frac{3}{4}\right)^2 - \frac{25}{8} \end{aligned}$$

(d) $-3x^2 + 2x + 1$

$$= -3\left(x^2 - \frac{2}{3}x\right) + 1$$

$$= -3\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + 1$$

$$= -3\left(\left(x - \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + 1$$

$$= -3\left(x - \frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2 + 1$$

$$= -3\left(x - \frac{1}{3}\right)^2 + 3 * \frac{1}{9} + 1$$

$$= -3\left(x - \frac{1}{3}\right)^2 + \frac{1}{3} + \frac{3}{3}$$

$$= -3\left(x - \frac{1}{3}\right)^2 + \frac{4}{3}$$

(e) $\frac{1}{4}x^2 + x - 1$

$$= \frac{1}{4}(x^2 + 4x) - 1$$

$$= \frac{1}{4}\left(x^2 + 4x + 2^2 - 2^2\right) - 1$$

$$= \frac{1}{4}\left((x+2)^2 - 2^2\right) - 1$$

$$= \frac{1}{4}(x+2)^2 - \frac{1}{4} * 2^2 - 1$$

$$= \frac{1}{4}(x+2)^2 - \frac{4}{4} - 1$$

$$= \frac{1}{4}(x+2)^2 - 1 - 1$$

$$= \frac{1}{4}(x+2)^2 - 2$$

(f) $3x^2 - \frac{1}{2}x + 3$

$$= 3\left(x^2 - \frac{1}{3} * \frac{1}{2}x\right) + 3$$

$$= 3\left(x^2 - \frac{1}{6}x\right) + 3$$

$$= 3\left(x^2 - \frac{1}{6}x + \left(\frac{1}{12}\right)^2 - \left(\frac{1}{12}\right)^2\right) + 3$$

$$= 3\left(\left(x - \frac{1}{12}\right)^2 - \left(\frac{1}{12}\right)^2\right) + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - 3\left(\frac{1}{12}\right)^2 + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - 3 * \frac{1}{12^2} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - 3 * \frac{1}{(4 * 3)^2} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - 3 * \frac{1}{4 * 4 * 3 * 3} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - \frac{1}{4 * 4 * 3} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - \frac{1}{16 * 3} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - \frac{1}{48} + 3$$

$$= 3\left(x - \frac{1}{12}\right)^2 - \frac{1}{48} + \frac{3 * 48}{48}$$

$$= 3\left(x - \frac{1}{12}\right)^2 - \frac{1}{48} + \frac{144}{48}$$

$$= 3\left(x - \frac{1}{12}\right)^2 + \frac{143}{48}$$

(g) $5x^2 + 7x - 2$

$$= 5\left(x^2 + \frac{7}{5}\right) - 2$$

$$= 5\left(x^2 + \frac{7}{5} + \left(\frac{7}{10}\right)^2 - \left(\frac{7}{10}\right)^2\right) - 2$$

$$= 5\left(\left(x + \frac{7}{10}\right)^2 - \frac{7^2}{10^2}\right) - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - 5 * \frac{49}{(2 * 5)^2} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - 5 * \frac{49}{2^2 * 5^2} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{2^2 * 5} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{2^2 * 5} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{4 * 5} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{20} - 2$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{20} - \frac{2 * 20}{20}$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{20} - \frac{40}{20}$$

$$= 5\left(x + \frac{7}{10}\right)^2 - \frac{89}{20}$$

(h) $-5x^2 - 3x + 7$

$$= -5\left(x^2 + \frac{3}{5}x\right) + 7$$

$$= -5\left(x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) + 7$$

$$= -5\left(\left(x + \frac{3}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) + 7$$

$$= -5\left(x + \frac{3}{10}\right)^2 - (-5)\left(\frac{3}{10}\right)^2 + 7$$

$$= -5\left(x + \frac{3}{10}\right)^2 + 5 * \frac{9}{100} + 7$$

$$= -5\left(x + \frac{3}{10}\right)^2 + \frac{9}{20} + 7$$

$$= -5\left(x + \frac{3}{10}\right)^2 + \frac{9}{20} + \frac{7 * 20}{20}$$

$$= -5\left(x + \frac{3}{10}\right)^2 + \frac{9}{20} + \frac{140}{20}$$

$$= -5\left(x + \frac{3}{10}\right)^2 + \frac{149}{20}$$

(i) $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{1}{5}$

$$= \frac{1}{2}\left(x^2 + \frac{2}{1} * \frac{1}{3}x\right) + \frac{1}{5}$$

$$= \frac{1}{2}\left(x^2 + \frac{2}{3}x\right) + \frac{1}{5}$$

$$= \frac{1}{2}\left(x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + \frac{1}{5}$$

$$= \frac{1}{2}\left(\left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + \frac{1}{5}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 - \frac{1}{2}\left(\frac{1}{3}\right)^2 + \frac{1}{5}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 - \frac{1}{2} * \frac{1}{9} + \frac{1}{5}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 - \frac{1}{18} + \frac{1}{5}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 - \frac{1 * 5}{18 * 5} + \frac{1 * 18}{5 * 18}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 - \frac{5}{90} + \frac{18}{90}$$

$$= \frac{1}{2}\left(x + \frac{1}{3}\right)^2 + \frac{13}{90}$$

(j) $2x^2 - \frac{1}{3}x - 1$

$$= 2\left(x^2 - \frac{1}{2 * 3}x\right) - 1$$

$$= 2\left(x^2 - \frac{1}{6}x\right) - 1$$

$$= 2\left(x^2 - \frac{1}{6}x + \left(\frac{1}{12}\right)^2 - \left(\frac{1}{12}\right)^2\right) - 1$$

$$= 2\left(\left(x - \frac{1}{12}\right)^2 - \left(\frac{1}{12}\right)^2\right) - 1$$

$$= 2\left(x - \frac{1}{12}\right)^2 - 2\left(\frac{1}{12}\right)^2 - 1$$

$$= 2\left(x - \frac{1}{12}\right)^2 - 2 * \frac{1}{144} - 1$$

$$= 2\left(x - \frac{1}{12}\right)^2 - \frac{1}{72} - 1$$

$$= 2\left(x - \frac{1}{12}\right)^2 - \frac{1}{72} - \frac{72}{72}$$

$$= 2\left(x - \frac{1}{12}\right)^2 - \frac{73}{72}$$

(k) $ax^2 + bx + c$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

$$= a\left(x+\frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

$$= a\left(x+\frac{b}{2a}\right)^2 - a * \frac{b^2}{4a^2} + c$$

$$= a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \qquad \text{(technically we're done)}$$

$$= a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4a * c}{4a}$$

$$= a\left(x+\frac{b}{2a}\right)^2 + \frac{-b^2+4ac}{4a}$$

$$= a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$$

Problem 2. Check your answers from Problem 1 by converting them back into the form $ax^2 \pm bx \pm c$.

(a) $(x+1)^2+2$

$$= (x+1)(x+1)+2$$
$$= x^2+2x+1+2$$
$$= x^2+2x+3$$

(b) $2(x-1)^2-1$

$$= 2\left((x-1)(x-1)\right)-1$$
$$= 2\left(x^2-2x+1\right)-1$$
$$= \left(2x^2-4x+2\right)-1$$
$$= 2x^2-4x+2-1$$
$$= 2x^2-4x+1$$

(c) $2\left(x+\frac{3}{4}\right)^2-\frac{25}{8}$

$$= 2\left(\left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right)\right)-\frac{25}{8}$$
$$= 2\left(x^2+\frac{3}{2}x+\frac{9}{16}\right)-\frac{25}{8}$$
$$= 2x^2+3x+\frac{9}{8}-\frac{25}{8}$$
$$= 2x^2+3x-\frac{16}{8}$$
$$= 2x^2+3x-2$$

(d) $-3\left(x-\frac{1}{3}\right)^2+\frac{4}{3}$

$$\begin{aligned}
&= -3\left(\left(x-\frac{1}{3}\right)\left(x-\frac{1}{3}\right)\right)+\frac{4}{3}\\
&= -3\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+\frac{4}{3}\\
&= -3x^2+2x-\frac{1}{3}+\frac{4}{3}\\
&= -3x^2+2x+\frac{3}{3}\\
&= -3x^2+2x+1
\end{aligned}$$

(e) $\frac{1}{4}(x+2)^2-2$

$$\begin{aligned}
&= \frac{1}{4}\left((x+2)(x+2)\right)-2\\
&= \frac{1}{4}\left(x^2+4x+4\right)-2\\
&= \frac{1}{4}x^2+x+1-2\\
&= \frac{1}{4}x^2+x-1
\end{aligned}$$

(f) $3\left(x-\frac{1}{12}\right)^2+\frac{143}{48}$

$$\begin{aligned}
&= 3\left(\left(x-\frac{1}{12}\right)\left(x-\frac{1}{12}\right)\right)+\frac{143}{48}\\
&= 3\left(x^2-\frac{1}{6}x+\frac{1}{144}\right)+\frac{143}{48}\\
&= 3x^2-\frac{1}{2}x+\frac{3}{144}+\frac{143}{48}\\
&= 3x^2-\frac{1}{2}x+\frac{1}{48}+\frac{143}{48}\\
&= 3x^2-\frac{1}{2}x+\frac{144}{48}\\
&= 3x^2-\frac{1}{2}x+3
\end{aligned}$$

(g) $5\left(x+\frac{7}{10}\right)^2-\frac{89}{20}$

$$\begin{aligned}
&= 5\left(\left(x+\frac{7}{10}\right)\left(x+\frac{7}{10}\right)\right)-\frac{89}{20}\\
&= 5\left(x^2+\frac{7}{5}x+\frac{49}{100}\right)-\frac{89}{20}\\
&= 5x^2+7x+\frac{5*49}{100}-\frac{89}{20}\\
&= 5x^2+7x+\frac{49}{20}-\frac{89}{20}\\
&= 5x^2+7x-\frac{40}{20}\\
&= 5x^2+7x-2
\end{aligned}$$

(h) $-5\left(x+\frac{3}{10}\right)^2+\frac{149}{20}$

$$= -5\left(\left(x+\frac{3}{10}\right)\left(x+\frac{3}{10}\right)\right)+\frac{149}{20}$$
$$= -5\left(x^2+\frac{3}{5}x+\frac{9}{100}\right)+\frac{149}{20}$$
$$= -5x^2-3x-\frac{5*9}{100}+\frac{149}{20}$$
$$= -5x^2-3x-\frac{9}{20}+\frac{149}{20}$$
$$= -5x^2-3x+\frac{140}{20}$$
$$= -5x^2-3x+7$$

(i) $\frac{1}{2}\left(x+\frac{1}{3}\right)^2+\frac{13}{90}$

$$= \frac{1}{2}\left(\left(x+\frac{1}{3}\right)\left(x+\frac{1}{3}\right)\right)+\frac{13}{90}$$
$$= \frac{1}{2}\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)+\frac{13}{90}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{1}{18}+\frac{13}{90}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{5*1}{5*18}+\frac{13}{90}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{5}{90}+\frac{13}{90}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{18}{90}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{18}{5*18}$$
$$= \frac{1}{2}x^2+\frac{1}{3}x+\frac{1}{5}$$

(j) $2\left(x-\frac{1}{12}\right)^2-\frac{73}{72}$

$$= 2\left(\left(x-\frac{1}{12}\right)\left(x-\frac{1}{12}\right)\right)-\frac{73}{72}$$
$$= 2\left(x^2-\frac{1}{6}x+\frac{1}{144}\right)-\frac{73}{72}$$
$$= 2x^2-\frac{1}{3}x+\frac{2}{144}-\frac{73}{72}$$
$$= 2x^2-\frac{1}{3}x+\frac{1}{72}-\frac{73}{72}$$
$$= 2x^2-\frac{1}{3}x-\frac{72}{72}$$
$$= 2x^2-\frac{1}{3}x-1$$

(k) $a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$

$$\begin{aligned}
&= a\left(\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)\right)+\frac{4ac-b^2}{4a}\\
&= a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+\frac{4ac-b^2}{4a}\\
&= ax^2+bx+\frac{ab^2}{4a^2}+\frac{4ac-b^2}{4a}\\
&= ax^2+bx+\frac{b^2}{4a}+\frac{4ac}{4a}-\frac{b^2}{4a}\\
&= ax^2+bx+\frac{b^2}{4a}-\frac{b^2}{4a}+\frac{4ac}{4a}\\
&= ax^2+bx+0+\frac{4ac}{4a}\\
&= ax^2+bx+\frac{4ac}{4a}\\
&= ax^2+bx+c
\end{aligned}$$

Problem 3. Show that both $\frac{-b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}$ and $\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}$ solve the equation $ax^2+bx+c=0$ by separately plugging each in for x. It may (or may not) be easier to write these as $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$.

Let $x=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and substitute into ax^2+bc+c to obtain

$$\begin{aligned}
&ax^2+bx+c\\
&= a\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)^2+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= a\left(\frac{(-b+\sqrt{b^2-4ac})^2}{(2a)^2}\right)+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= a\left(\frac{(-b+\sqrt{b^2-4ac})(-b+\sqrt{b^2-4ac})}{4a^2}\right)+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= a\left(\frac{b^2-2b\sqrt{b^2-4ac}+b^2-4ac}{4a^2}\right)+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= \frac{b^2-2b\sqrt{b^2-4ac}+b^2-4ac}{4a}+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= \frac{b^2}{4a}-\frac{2b\sqrt{b^2-4ac}}{4a}+\frac{b^2}{4a}-\frac{4ac}{4a}+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c\\
&= \frac{b^2}{4a}+\frac{b^2}{4a}-\frac{2b\sqrt{b^2-4ac}}{4a}-\frac{4ac}{4a}+b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+c
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b^2}{4a} - \frac{2b\sqrt{b^2-4ac}}{4a} - \frac{4ac}{4a} + b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) + c \\
&= \frac{2b^2}{4a} - \frac{2b\sqrt{b^2-4ac}}{4a} - \frac{4ac}{4a} + \frac{-b^2+b\sqrt{b^2-4ac}}{2a} + c \\
&= \frac{2b^2}{4a} - \frac{2b\sqrt{b^2-4ac}}{4a} - \frac{4ac}{4a} + \frac{-b^2}{2a} + \frac{b\sqrt{b^2-4ac}}{2a} + c \\
&= \frac{b^2}{2a} - \frac{b\sqrt{b^2-4ac}}{2a} - \frac{4ac}{4a} + \frac{-b^2}{2a} + \frac{b\sqrt{b^2-4ac}}{2a} + c \\
&= \frac{b^2}{2a} + \frac{-b^2}{2a} - \frac{b\sqrt{b^2-4ac}}{2a} - \frac{4ac}{4a} + \frac{b\sqrt{b^2-4ac}}{2a} + c \\
&= \frac{b^2}{2a} + \frac{-b^2}{2a} - \frac{b\sqrt{b^2-4ac}}{2a} + \frac{b\sqrt{b^2-4ac}}{2a} + c - \frac{4ac}{4a} \\
&= \left(\frac{b^2}{2a} + \frac{-b^2}{2a}\right) + \left(\frac{-b\sqrt{b^2-4ac}}{2a} + \frac{b\sqrt{b^2-4ac}}{2a}\right) + c - \frac{4ac}{4a} \\
&= (0) + (0) + c - \frac{4ac}{4a} \\
&= c - \frac{4ac}{4a} \\
&= c - c \\
&= 0
\end{aligned}
$$

Therefore, $ax^2 + bx + c = 0$ when $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$. A similar calculation will also work for $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$.