Problem 1. Find the equation of the line passing through the point (1,1) and perpendicular to the line 2x + 3y = 5.

We need a line in the form y=mx+b. Since the given line can be written as $y=-\frac{2}{3}x+\frac{5}{3}$ and our line is to be perpendicular to it, then our slope must be

$$m = \frac{-1}{-\frac{2}{3}} = (-1)\frac{-3}{2} = \frac{3}{2}$$

Then our line can be written as $y = \frac{3}{2}x + b$. All that remains is to find b. But we know that when we plug in a 1, we get out a 1. That is, when x = 1, we must have y = 1. So plugging this in to our equation $y = \frac{3}{2}x + b$ gives us

$$1 = \frac{3}{2}(1) + b$$

$$1 = \frac{3}{2} + b$$

$$1 - \frac{3}{2} = b$$

$$\frac{2}{2} - \frac{3}{2} = b$$

$$-\frac{1}{2} = b$$

Therefore, the equation we want is $y = \frac{3}{2}x - \frac{1}{2}$.

Problem 2. Find the equation of the line passing through the point (2,3) and perpendicular to the line y = -2x + 3.

Using the same reasoning as above, in our equation y = mx + b we must have

$$m = \frac{-1}{-2} = \frac{1}{2}$$

Then $y = \frac{1}{2}x + b$ must pass through (2,3) giving us

$$3 = \frac{1}{2}(2) + b$$
$$3 = \frac{2}{2} + b$$
$$3 = 1 + b$$
$$3 - 1 = b$$
$$2 = b$$

So the line we want is $y = \frac{1}{2}x + 2$.

Problem 3. Find the equation of the line passing through the point (3,4) and perpendicular to the line 7x - 2y = 1.

The given line can be written as $y = \frac{7}{2}x - \frac{1}{2}$. Using the same reasoning as above, in our equation y = mx + b we must have

$$m = \frac{-1}{\frac{7}{2}} = -\frac{2}{7}$$

Then $y = -\frac{2}{7}x + b$ must pass through (3,4) giving us

$$4 = -\frac{2}{7}(3) + b$$

$$4 = -\frac{6}{7} + b$$

$$4 + \frac{6}{7} = b$$

$$(4)\frac{7}{7} + \frac{6}{7} = b$$

$$\frac{4 * 7}{7} + \frac{6}{7} = b$$

$$\frac{28}{7} + \frac{6}{7} = b$$

$$\frac{34}{7} = b$$

So the line we want is $y = -\frac{2}{7}x + \frac{34}{7}$.