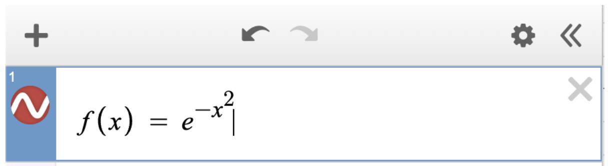
Name: ______________________________ **Algebra II**

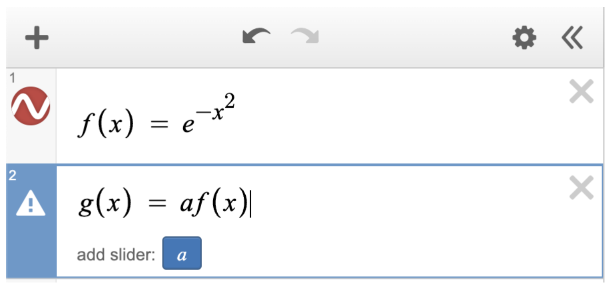
Date: **Homework 31**

For this assignment you will need to go to https://www.desmos.com/calculator. For each problem below, you will be given a different function $f(x)$, but your setup will be the same for each problem. We will use the function from Problem 1 to describe the setup and instructions.

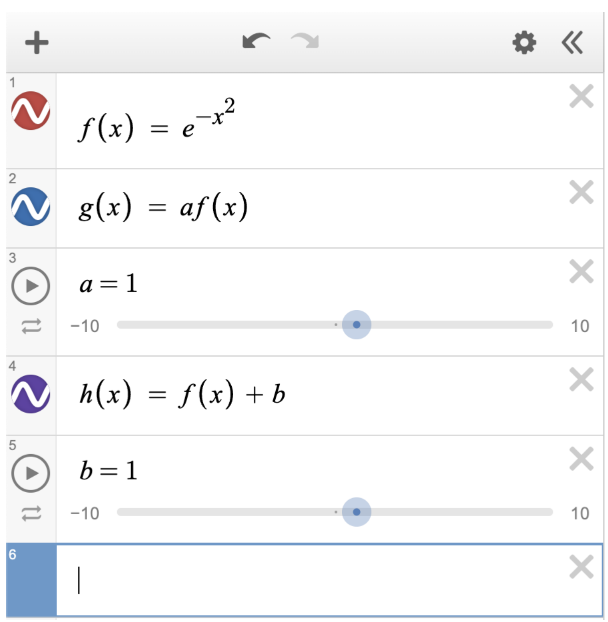
Given $f(x) = e^{-x^2}$ write this exactly in the input box. Note that to get an exponent, you'll need to use the ˆ symbol (hold Shift and press 6). Your screen should look like this:



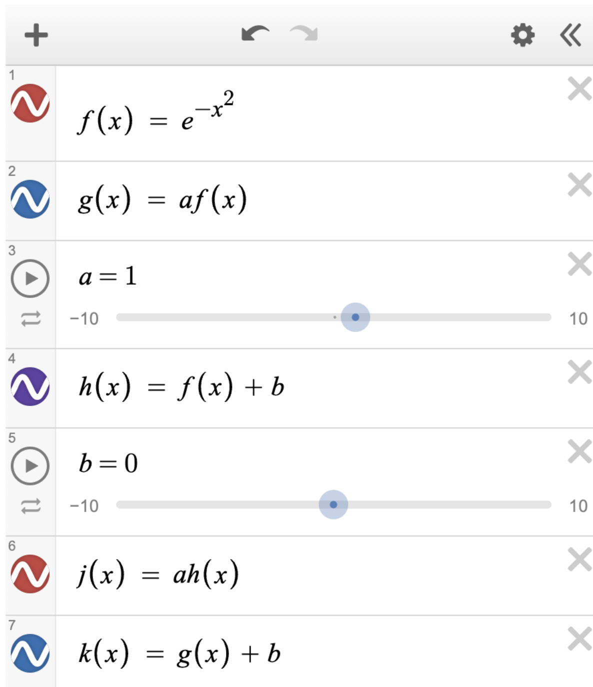
When you're done, hit return and a new box will show up under the one you just filled in. Type $g(x) = af(x)$. When you do you'll get a message to "add slider a":



You can either click on the a or simply hit return to add the slider. A new box will appear beneath the slider. Type $h(x) = f(x) + b$. You will again be prompted to "add slider b." Hitting return is the easiest way to continue. At this point your screen should look like this:

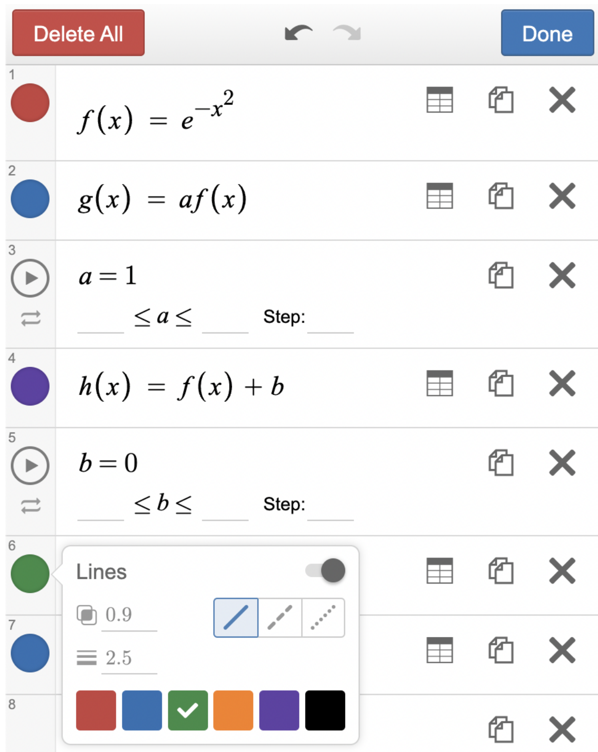


Type $j(x) = ah(x)$ into the box and hit return. Then type $k(x) = g(x) + b$ into the next box. Make sure that the a slider is set to 1 and that the b slider is set to 0. Your screen should look like this:

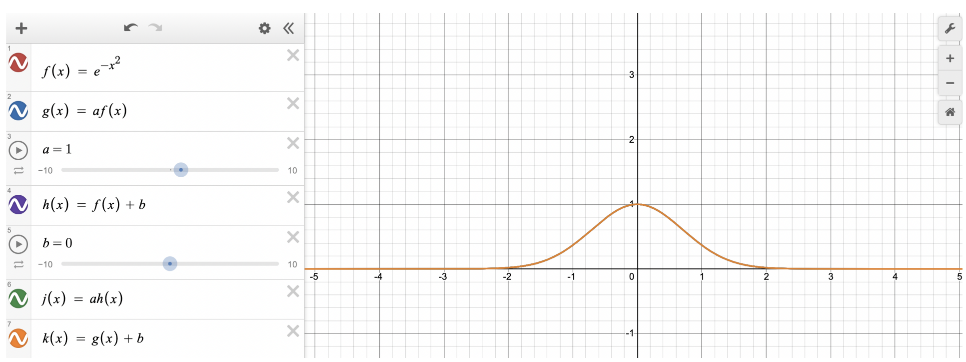


Unfortunately the default colors will give you repeats (in the above f and j are both red, g and k are both blue) or colors you don't want (like black). To deal with this, let's change the colors by clicking on the gear icon in the top right.

This will let you edit this list we've just made. You can change a color by clicking on the circle on the left and selecting a different color from the drop down box that appears. Below you can see the result of clicking on the gear, then clicking on the (previously) red circle for $j(x)$ in box 6, and selecting green from the dropdown.



Once you have the colors you want and you've ensured that all 5 function have unique colors, hit the blue "Done" button on the top right to finish editing. Your screen should look like this:



You'll notice that we're only seeing the orange function, $k(x)$. This is to be expected since we have $a = 1$ and $b = 0$. Then

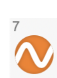
$$g(x) = af(x) = (1)f(x) = f(x)$$

$$h(x) = f(x) + b = f(x) + 0 = f(x)$$

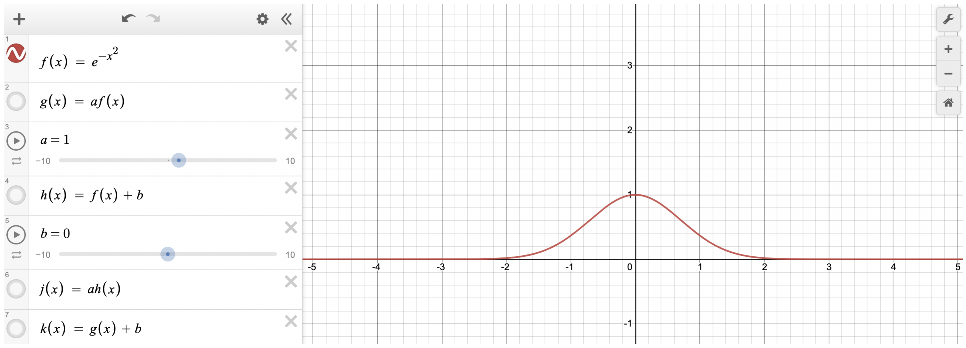
$$j(x) = ah(x) = a(f(x) + b) = 1(f(x) + 0) = f(x)$$

$$k(x) = g(x) + b = af(x) + b = (1)f(x) + 0 = f(x)$$

You can confirm that they're all being plotted by going through and clicking each of the circles on the left:



This toggles the plotting of that function on and off. Turn all of them off except for $f(x)$. You'll be ready to begin the assignment when your screen looks like this:



For the rest of this assignment, you will not need to edit g, h, j, and k. Each problem will begin with "Let $f(x) =$" some formula. Simply edit $f(x)$ in box 1 to match the problem and continue. You're all set to begin with Problem 1. I'll be walking you through an investigation of the behavior of various functions after being vertically shifted and/or stretched. Unless I explicitly write that I want you to sketch something, don't worry about writing anything down. Also, when I say to sketch something, don't worry about making it look nice or if it's "right or wrong." I'll be "grading" you on whether or not you attempted the problem, not if your guesses were accurate. In the end, just submit to me the "thumbnail sketches" I ask you to draw as we move along.

Problem 1. Let $f(x) = e^{-x^2}$.

$f(x)$ is related to what we call a probability density function of the normal distribution, but simplified. If you're interested check out https://en.wikipedia.org/wiki/Normal_distribution to learn more.

Plot $g(x)$ by toggling it on. Anticipate what's going to happen to $g(x)$ before you move the a slider to $a = 3$, then move the slider. Did it do what you expected? If not, take careful note of what did happen. Did every single point move up? This is easy to see near $x = 0$, but what about further out? What about at $x = 3$? Zoom in and check. (You can zoom in by putting your cursor on the point you want to zoom in on and scrolling up to move in, and scrolling down to move out.) Does it look like $g(x)$ is about 3 times higher than $f(x)$? Zoom back out so that you can see the bigger picture.

Imagine what would happen to $g(x)$ if you dragged the a slider to $a = -1$, and then move the slider. Does this make sense? Recall that $g(x) = af(x)$, so you're taking all the heights of f and multiplying them by $a = -1$. Is this an exact reflection about the x-axis? Now just play around with the a slider a little and try to imagine what will happen before you actually move the slider. Confirm what your mind's eye tells you you should see when you do move the slider. When you have a pretty good feel for what happens when you multiply $f(x)$ by a number, return the slider to $a = 1$.

Toggle $g(x)$ off and turn on $h(x)$. Imagine what you should see if you dragged the b slider up to $b = 1$, then do it. Did you think that the point at $x = 0$, $f(0) = 1$ would be shifted up one unit and end up at $y = 2$? If not, notice that it did indeed happen. You see how $f(x)$ gets closer and closer to $y = 0$ (i.e. the x-axis) as we take large positive and negative values of x? That is, the graph gets closer and closer to the x axis the farther left or right that we go. Now that we've shifted $h(x)$ up one unit, what y value should the graph approach? (Spoiler alert: it should be approaching $y = 1$ and the graph should confirm this.)

On a sheet of scratch paper, draw a quick sketch of what you should see if you dragged the slider to $b = -1$ **before** you actually move the slider. If the graph passes through one (or more) of the axes, indicate what value it goes through (and continue this for the rest of Problem 1). Now drag the slider to $b = -1$. If your picture differs from the graph of $h(x)$, take a minute to reconcile these differences and convince yourself that you should indeed have the graph that desmos has plotted for $h(x)$.

Now that $h(x)$ has been shifted down below the x-axis, imagine what a reflection about the x-axis would look like. Sketch it on your scratch paper. How can you do this in desmos? Keep in mind the differences between $j(x)$ and $k(x)$: $j(x) = ah(x)$ so we have to do h first, that is, we shift up or down by b units, and *then* we compress/stretch/flip by a units. $k(x) = g(x) + b$ so we have to do g first, that is, we compress/stretch/flip by a units, and *then* we shift up or down by b units. Put differently:

$$j(x) = ah(x) = a(f(x) + b) \qquad \text{shift up/down, then stretch/reflect}$$

$$k(x) = g(x) + b = af(x) + b \qquad \text{stretch/reflect, then shift up/down}$$

So we want to take our shifted $h(x)$ and reflect it about the x-axis. Since $j(x)$ shifts first, then stretches/reflects, what value can we set for a to give us a reflection about the x-axis? Set a to that value and toggle $j(x)$ on.

You should have set $a = -1$ and you should be plotting f, h, and j. The b slider should still be at $b = -1$ and $j(x)$ should match your sketch. Convince yourself that $j(x)$ as drawn is exactly what you'd get if you took the original function $f(x)$, shifted it down one unit, and then reflected it about the x-axis.

In a minute I'll ask you to toggle $k(x)$ on but before you do, sketch what it should look like and indicate where it goes through any axes it intersects (if it does). Note the order of k. We first multiply by a and *then* we shift by b. Sketch it now. Okay, you should have figured that multiplying by $a = -1$ reflects $f(x)$ about the x-axis, and adding $b = -1$ drags it down another one unit. Toggle $k(x)$ on. Is this what you sketched? Convince yourself that what desmos is plotting is accurate and understand why. Notice that $j(x)$ and $k(x)$ are *not* the same graph. As we said in class, stretching and *then* shifting is not necessarily the same as shifting and *then* stretching. Order matters.

Feel free to play with these guys as little or as much as you'd like before moving on to the next problem. When you're ready, toggle all the functions off except for $f(x)$, set $a = 1$ and $b = 0$.

Problem 2. Let $f(x) = \cos(x)$.

This is a trigonometric function. Notice how it oscillates between 1 and -1 forever. Go ahead and zoom out. See? It just goes on forever and ever. Now zoom back in so that the vertical axis does show the 1 and -1. If you're zoomed out too much, you won't see it. $f(x)$ is periodic with period 2π. That means that every time you go $2\pi \approx 6.283$ units over, you end up at the same output value. For instance, when you're at $x = 0$, then $f(0) = 1$, so if you go over to 2π you end up back where you started at $1 = f(2\pi)$.

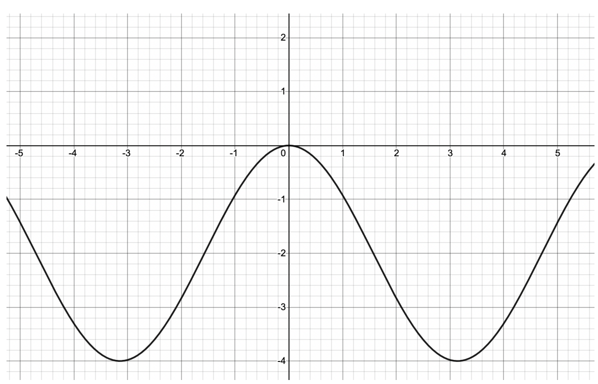
One of the hardest things to do in trig involves stretching and shifting these functions. Let's start by toggling $g(x)$ on and playing with the a slider. Note that the zeros (the x-intercepts) do not change as we vary the a values. By letting $a = 4$ you can change the amplitude ($4\cos(x)$ now oscillates between 4 and -4) but you haven't changed the speed at which it does so (the zeros have remained fixed and you're still back where you started when $x = 2\pi$). What would you need to set the slider to in order to make the wave oscillate between 2 and -2 but have it start at -2 when $x = 0$? That is, how do we stretch it by a factor of 2 and flip it upside down? (Answer: $b = -2$. Confirm this on desmos.)

Toggle $g(x)$ off and turn on $h(x)$. Shift h up and down by playing with the b slider. Set $b = 0$ and imagine what your graph will look like if you set $b = 1$. For now, we'll completely ignore the x values. (We'll come back to that when we multiply inputs. Now we're just multiplying outputs.) Sketch what you expect to see if we shifted $f(x)$ up by one unit (i.e. what $h(x)$ will look like when you set $b = 1$.) Now set $b = 1$ and see if you were right? Were you?

Now that you've shifted the original graph up by one unit, sketch what it would look like if you reflected it about the x-axis. Be sure to note where it goes through the y-axis. Confirm this in desmos. Think of how you can check a vertical shift by one unit and *then* a reflection before reading my spoiler. (Spoiler alert: you can confirm this by setting $a = -1$, $b = 1$, and toggling $j(x)$ on, since j shifts by b first, *then* stretches/reflects by a.) With $a = -1$ and $b = 1$ sketch what $k(x)$ should look like. Then check it by toggling $k(x)$ on.

Toggle all the functions off except for $f(x)$. Set $a = 2$ and $b = 1$. Sketch what $j(x)$ and $k(x)$ should be be sure to label where the functions pass through the y-axis, and then check it.

Finally, describe 2 ways that we can obtain the following transformation. One way by a shift and *then* a stretch/reflection. The other by a strecth/reflection and *then* a shift. Write your answers in terms of $\cos(x)$.



For instance, if I (wrongly) thought that the function above could be obtained by stretching by a factor of three and then shifting down by two units, I'd write $3\cos(x) - 2$. And if I (wrongly) thought that it could be obtained by shifting up by 4 units and then stretching/reflecting by a factor of -2, I'd write $-2(\cos(x) + 4)$. Do check your answers in desmos and try to find the proper transformations!