**Name:** ______________________________ **Algebra II**
**Date:** **Homework 32**

**Problem 1.** Given a vertex and another point on a parabola, write in the form $f(x) = a(x-h)^2+k$. Find the zeros of this function by setting $f(x) = 0$ and solving for $x$. Rewrite each function in the form $f(x) = ax^2 + bx + c$. Use the quadratic formula to find the zeros and the vertex of the parabola. Graphing the parabola may help you to keep all of this straight. It's not necessary, but I highly recommend doing it so that you're not just "plugging numbers" into the right places, you're actually seeing why you're plugging in inputs and getting outputs. An example is given below.

(a) Vertex: $(-2, 3)$, Point: $(-1, 2)$

$$f(x) = a(x+2)^2 + 3$$
$$f(-1) = 2$$
$$a(-1+2)^2 + 3 = 2$$
$$a(1) = 2 - 3$$
$$a = -1$$
$$f(x) = -(x+2)^2 + 3$$
$$f(x) = 0 = -(x+2)^2 + 3$$
$$(x+2)^2 = 3$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 \pm \sqrt{3}$$
$$x \in \left\{-2-\sqrt{3}, -2+\sqrt{3}\right\}$$
$$f(x) = -(x^2 + 4x + 4) + 3$$
$$f(x) = -x^2 - 4x - 4 + 3$$
$$f(x) = -x^2 - 4x - 1$$
$$a = -1, b = -4, c = -1$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = -\frac{(-4)}{2(-1)} \pm \frac{\sqrt{(-4)^2 - 4(-1)(-1)}}{2(-1)}$$
$$= -2 \pm \frac{\sqrt{16-4}}{2} = -2 \pm \frac{\sqrt{12}}{2} = -2 \pm \frac{\sqrt{4*3}}{2} = -2 \pm \frac{2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$
$$x \in \left\{-2-\sqrt{3}, -2+\sqrt{3}\right\}$$

Clearly the midpoint is $x = -2$. Alternately, the midpoint is at $x = -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = -2$. Also

$$f(-2) = -(-2)^2 - 4(-2) - 1 = -(4) + 8 - 1 = 4 - 1 = 3$$

Then the vertex is at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (-2, f(-2)) = (-2, 3)$$

(b) Vertex: $(3, -1)$, Point: $(0, 1)$

$$f(x) = a(x-3)^2 - 1$$

$$f(0) = 1$$

$$a(0-3)^2 - 1 = 1$$

$$a(9) = 2$$

$$a = \frac{2}{9}$$

$$f(x) = \frac{2}{9}(x-3)^2 - 1$$

$$f(x) = \frac{2}{9}(x-3)^2 - 1 = 0$$

$$\frac{2}{9}(x-3)^2 = 1$$

$$(x-3)^2 = (1)\frac{9}{2}$$

$$x - 3 = \pm\sqrt{\frac{9}{2}}$$

$$x = 3 \pm \frac{\sqrt{9}}{\sqrt{2}}$$

$$x = 3 \pm \frac{3}{\sqrt{2}}$$

$$x \in \left\{3 - \frac{3}{\sqrt{2}}, 3 + \frac{3}{\sqrt{2}}\right\}$$

$$f(x) = \frac{2}{9}(x-3)^2 - 1$$

$$f(x) = \frac{2}{9}(x^2 - 6x + 9) - 1$$

$$f(x) = \frac{2}{9}x^2 - \frac{2}{9}(6)x + \frac{2}{9}(9) - 1$$

$$f(x) = \frac{2}{9}x^2 - \frac{2}{3}(2)x + 2 - 1$$

$$f(x) = \frac{2}{9}x^2 - \frac{4}{3}x + 1$$

$$a = \frac{2}{9}, b = -\frac{4}{3}, c = 1$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = -\frac{(-\frac{4}{3})}{2(\frac{2}{9})} \pm \frac{\sqrt{(-\frac{4}{3})^2 - 4(\frac{2}{9})(1)}}{2(\frac{2}{9})}$$

$$= \frac{(\frac{4}{3})}{\frac{4}{9}} \pm \frac{\sqrt{\frac{16}{9} - \frac{8}{9}}}{\frac{4}{9}} = \left(\frac{4}{3}\right)\frac{9}{4} \pm \frac{\sqrt{\frac{8}{9}}}{\frac{4}{9}} = 3 \pm \frac{\frac{\sqrt{8}}{\sqrt{9}}}{\frac{4}{9}} = 3 \pm \frac{\frac{\sqrt{8}}{3}}{\frac{4}{9}}$$

$$= 3 \pm \left(\frac{\sqrt{8}}{3}\right)\frac{9}{4} = 3 \pm \frac{3\sqrt{8}}{4} = 3 \pm \frac{3\sqrt{8}}{\sqrt{16}} = 3 \pm 3\sqrt{\frac{8}{16}} = 3 \pm 3\sqrt{\frac{1}{2}}$$

$$x \in \left\{3 - \frac{3}{\sqrt{2}}, 3 + \frac{3}{\sqrt{2}}\right\}$$

Clearly the midpoint is $x = 3$. Alternately, the midpoint is at $x = -\frac{b}{2a} = -\frac{(-\frac{4}{3})}{2(\frac{2}{9})} = 3$. Also

$$f(3) = \frac{2}{9}(3)^2 - \frac{4}{3}(3) + 1 = \frac{2}{9}(9) - 4 + 1 = 2 - 4 + 1 = -1$$

Then the vertex is at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (3, f(3)) = (3, -1)$$

(c) Vertex: $(2, 1)$, Point: $(1, 2)$

$$f(x) = a(x-2)^2 + 1$$
$$f(1) = 2$$
$$a(1-2)^2 + 1 = 2$$
$$a(-1)^2 = 1$$
$$a = 1$$
$$f(x) = (x-2)^2 + 1$$
$$f(x) = (x-2)^2 + 1 = 0$$
$$(x-2)^2 + 1 = 0$$
$$(x-2)^2 = -1$$
$$x - 2 = \pm\sqrt{-1}$$
$$x = 2 \pm i$$
$$x \in \{2 - i, 2 + i\}$$
$$f(x) = (x-2)^2 + 1$$
$$f(x) = x^2 - 4x + 4 + 1$$
$$f(x) = x^2 - 4x + 5$$
$$a = 1, b = -4, c = 5$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = -\frac{(-4)}{2(1)} \pm \frac{\sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = -(-2) \pm \frac{\sqrt{16 - 20}}{2}$$

$$= 2 \pm \frac{\sqrt{-4}}{2} = 2 \pm \frac{i\sqrt{4}}{2} = 2 \pm \frac{2i}{2} = 2 \pm i$$

$$x \in \{2 - i, 2 + i\}$$

Clearly the midpoint is $x = 2$. Alternately, the midpoint is at $x = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$. Also

$$f(2) = (2)^2 - 4(2) + 5 = 4 - 8 + 5 = -4 + 5 = 1$$

Then the vertex is at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (2, f(2)) = (2, 1)$$

(d) Vertex: $(-5,-2)$, Point: $(-1,6)$

$$f(x) = a(x+5)^2 - 2$$
$$f(-1) = 6$$
$$a(-1+5)^2 - 2 = 6$$
$$a(4)^2 = 8$$
$$16a = 8$$
$$a = \frac{8}{16}$$
$$a = \frac{1}{2}$$
$$f(x) = \frac{1}{2}(x+5)^2 - 2$$
$$f(x) = \frac{1}{2}(x+5)^2 - 2 = 0$$
$$\frac{1}{2}(x+5)^2 = 2$$
$$(x+5)^2 = 2(2)$$
$$x+5 = \pm\sqrt{4}$$
$$x+5 = \pm 2$$
$$x = -5 \pm 2$$
$$x \in \{-5-2, -5+2\}$$
$$f(x) = \frac{1}{2}(x+5)^2 - 2$$
$$f(x) = \frac{1}{2}(x^2+10x+25) - 2$$
$$f(x) = \frac{1}{2}x^2 + \frac{1}{2}(10)x + \frac{1}{2}(25) - 2$$
$$f(x) = \frac{1}{2}x^2 + 5x + \frac{25}{2} - \frac{4}{2}$$
$$f(x) = \frac{1}{2}x^2 + 5x + \frac{21}{2}$$
$$a = \frac{1}{2}, b = 5, c = \frac{21}{2}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = -\frac{(5)}{2(\frac{1}{2})} \pm \frac{\sqrt{(5)^2 - 4(\frac{1}{2})(\frac{21}{2})}}{2(\frac{1}{2})} = -\frac{(5)}{1} \pm \frac{\sqrt{(5)^2 - 2(\frac{21}{2})}}{1}$$
$$= -5 \pm \sqrt{25-21} = -5 \pm \sqrt{4} = -5 \pm 2$$
$$x \in \{-5-2, -5+2\}$$

Clearly the midpoint is $x = -5$. Alternately, the midpoint is at $x = -\frac{b}{2a} = -\frac{(5)}{2(\frac{1}{2})} = -5$. Also

$$f(-5) = \frac{1}{2}(-5)^2 + 5(-5) + \frac{21}{2} = \frac{25}{2} - 25 + \frac{21}{2} = \frac{46}{2} - 25 = 23 - 25 = -2$$

Then the vertex is at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (-5, f(-5)) = (-5, -2)$$

(e) Vertex: $\left(\frac{2}{3}, \frac{2}{3}\right)$, Point: $(1, 1)$

$$f(x) = a\left(x - \frac{2}{3}\right)^2 + \frac{2}{3}$$
$$f(1) = 1$$
$$a\left(1 - \frac{2}{3}\right)^2 + \frac{2}{3} = 1$$
$$a\left(1 - \frac{2}{3}\right)^2 = 1 - \frac{2}{3}$$
$$a\left(\frac{3}{3} - \frac{2}{3}\right)^2 = \frac{3}{3} - \frac{2}{3}$$
$$a\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$
$$a\left(\frac{1}{9}\right) = \frac{1}{3}$$
$$a = \frac{1}{3}(9)$$
$$a = 3$$
$$f(x) = 3\left(x - \frac{2}{3}\right)^2 + \frac{2}{3}$$
$$f(x) = 3\left(x - \frac{2}{3}\right)^2 + \frac{2}{3} = 0$$
$$3\left(x - \frac{2}{3}\right)^2 + \frac{2}{3} = 0$$
$$3\left(x - \frac{2}{3}\right)^2 = -\frac{2}{3}$$
$$\left(x - \frac{2}{3}\right)^2 = -\frac{2}{3}\left(\frac{1}{3}\right)$$
$$\left(x - \frac{2}{3}\right)^2 = -\frac{2}{9}$$
$$x - \frac{2}{3} = \pm\sqrt{-\frac{2}{9}}$$
$$x - \frac{2}{3} = \pm i\sqrt{\frac{2}{9}}$$
$$x - \frac{2}{3} = \pm i\frac{\sqrt{2}}{\sqrt{9}}$$

$$x-\frac{2}{3}=\pm i\frac{\sqrt{2}}{3}$$

$$x=\frac{2}{3}\pm i\frac{\sqrt{2}}{3}$$

$$x\in\left\{\frac{2}{3}-i\frac{\sqrt{2}}{3},\frac{2}{3}+i\frac{\sqrt{2}}{3}\right\}$$

$$f(x)=3\left(x-\frac{2}{3}\right)^2+\frac{2}{3}$$

$$f(x)=3\left(x^2-\frac{4}{3}x+\frac{4}{9}\right)+\frac{2}{3}$$

$$f(x)=3x^2-(3)\frac{4}{3}x+(3)\frac{4}{9}+\frac{2}{3}$$

$$f(x)=3x^2-4x+\frac{4}{3}+\frac{2}{3}$$

$$f(x)=3x^2-4x+\frac{6}{3}$$

$$f(x)=3x^2-4x+2$$

$$a=3,b=-4,c=2$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=-\frac{(-4)}{2(3)}\pm\frac{\sqrt{(-4)^2-4(3)(2)}}{2(3)}=\frac{2}{3}\pm\frac{\sqrt{16-24}}{2(3)}$$

$$=\frac{2}{3}\pm\frac{\sqrt{-8}}{2(3)}=\frac{2}{3}\pm\frac{i\sqrt{8}}{2(3)}=\frac{2}{3}\pm\frac{i\sqrt{4*2}}{2(3)}=\frac{2}{3}\pm\frac{2i\sqrt{2}}{2(3)}=\frac{2}{3}\pm i\frac{\sqrt{2}}{3}$$

$$x\in\left\{\frac{2}{3}-i\frac{\sqrt{2}}{3},\frac{2}{3}+i\frac{\sqrt{2}}{3}\right\}$$

Clearly the midpoint is $x=\frac{2}{3}$. Alternately, the midpoint is at $x=-\frac{b}{2a}=-\frac{(-4)}{2(3)}=\frac{2}{3}$. Also

$$f\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^2-4\left(\frac{2}{3}\right)+2=3\left(\frac{4}{9}\right)-\frac{8}{3}+2$$

$$=\frac{4}{3}-\frac{8}{3}+2=-\frac{4}{3}+2=-\frac{4}{3}+\frac{6}{3}=\frac{2}{3}$$

Then the vertex is at

$$\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)=\left(\frac{2}{3},f\left(\frac{2}{3}\right)\right)=\left(\frac{2}{3},\frac{2}{3}\right)$$