**Name:** ____________________________________ **Algebra II**
**Date:** **Homework 1**

**Problem 1.** Find the equation of the line passing through the point $(1, 1)$ and perpendicular to the line $2x + 3y = 5$.

We need a line in the form $y = mx + b$. Since the given line can be written as $y = -\frac{2}{3}x + \frac{5}{3}$ and our line is to be perpendicular to it, then our slope must be

$$m = \frac{-1}{-\frac{2}{3}} = (-1)\frac{-3}{2} = \frac{3}{2}$$

Then our line can be written as $y = \frac{3}{2}x + b$. All that remains is to find $b$. But we know that when we plug in a $1$, we get out a $1$. That is, when $x = 1$, we must have $y = 1$. So plugging this in to our equation $y = \frac{3}{2}x + b$ gives us

$$\begin{aligned} 1 &= \frac{3}{2}(1) + b \\ 1 &= \frac{3}{2} + b \\ 1 - \frac{3}{2} &= b \\ \frac{2}{2} - \frac{3}{2} &= b \\ -\frac{1}{2} &= b \end{aligned}$$

Therefore, the equation we want is $y = \frac{3}{2}x - \frac{1}{2}$.

**Problem 2.** Find the equation of the line passing through the point $(2, 3)$ and perpendicular to the line $y = -2x + 3$.

Using the same reasoning as above, in our equation $y = mx + b$ we must have

$$m = \frac{-1}{-2} = \frac{1}{2}$$

Then $y = \frac{1}{2}x + b$ must pass through $(2, 3)$ giving us

$$\begin{aligned} 3 &= \frac{1}{2}(2) + b \\ 3 &= \frac{2}{2} + b \\ 3 &= 1 + b \\ 3 - 1 &= b \\ 2 &= b \end{aligned}$$

So the line we want is $y = \frac{1}{2}x + 2$.

**Problem 3.** Find the equation of the line passing through the point $(3, 4)$ and perpendicular to the line $7x - 2y = 1$.

The given line can be written as $y = \frac{7}{2}x - \frac{1}{2}$. Using the same reasoning as above, in our equation $y = mx + b$ we must have

$$m = \frac{-1}{\frac{7}{2}} = -\frac{2}{7}$$

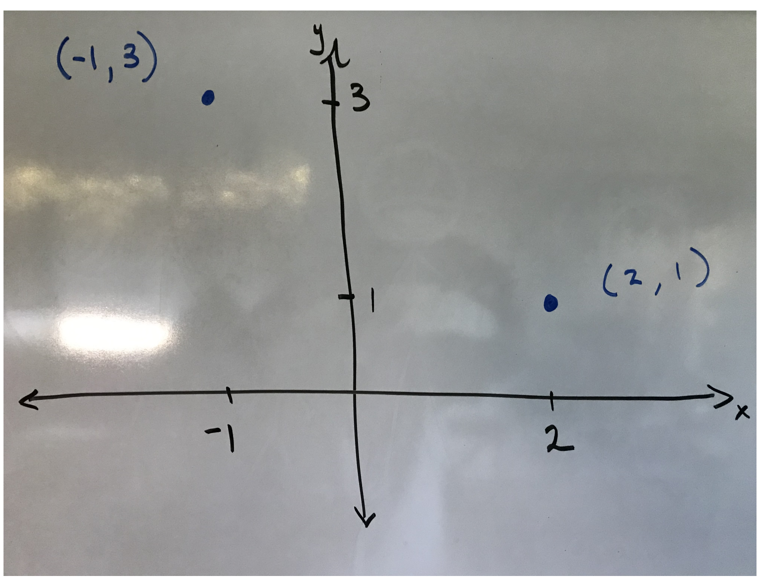
Then $y = -\frac{2}{7}x + b$ must pass through $(3, 4)$ giving us

$$\begin{aligned}
4 &= -\frac{2}{7}(3) + b \\
4 &= -\frac{6}{7} + b \\
4 + \frac{6}{7} &= b \\
(4)\frac{7}{7} + \frac{6}{7} &= b \\
\frac{4 * 7}{7} + \frac{6}{7} &= b \\
\frac{28}{7} + \frac{6}{7} &= b \\
\frac{34}{7} &= b
\end{aligned}$$

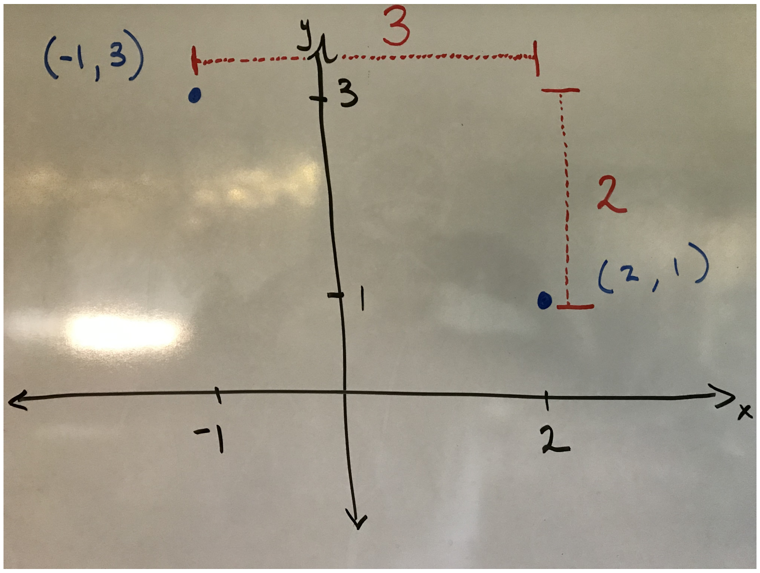
So the line we want is $y = -\frac{2}{7}x + \frac{34}{7}$.

**Problem 4.** Find the equation of the line passing through the points $(2, 1)$ and $(-1, 3)$.

We want an equation in the form $y = mx + b$. Let's find $m$. The first thing we should do is graph these two points. When you do this, you'll notice that one point is up and to the left, and the other is lower and to the right.



We should then find the horizontal and vertical distance between these points.




The visual makes it obvious that the run (or the horizontal distance) is 3. We can also easily see that the "rise" is actually a "fall," and so the 2 units of vertical distance must be negative. Thus we should have $m = \frac{-2}{3}$. I find this much easier (and more intuitive) than using the "slope formula" like so:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - 3}{2 - (-1)}$$

$$m = \frac{-2}{2 + 1}$$

$$m = \frac{-2}{3}$$

Either way we end up with $m = -\frac{2}{3}$. Returning to the equation we want, we have $y = -\frac{2}{3}x + b$. All that remains is to find $b$, which we can do by plugging in *either* given point and solving for $b$. This is true only because *both* of the given points are on the line that we're studying. You may encounter problems with multiple points, not all of which lie on the same line. Be careful. Since I hate using negative numbers, I'm going to choose the point $(2, 1)$. Then we must have $y = 1$ whenever $x = 2$ in our equation.

$$y = -\frac{2}{3}x + b$$

$$1 = -\frac{2}{3}(2) + b$$

$$1 = -\frac{4}{3} + b$$

$$1 + \frac{4}{3} = b$$

$$\frac{3}{3} + \frac{4}{3} = b$$

$$\frac{7}{3} = b$$

Just to emphasize that either point will work in this case because *both* points are on the *same* line, let's suppose that you chose instead to use the point $(-1, 3)$. Then you would have had

$$y = -\frac{2}{3}x + b$$

$$3 = -\frac{2}{3}(-1) + b$$

$$3 = \frac{2}{3} + b$$

$$3 - \frac{2}{3} = b$$

$$\frac{9}{3} - \frac{2}{3} = b$$

$$\frac{7}{3} = b$$

In any case, we find that our line is given by $y = -\frac{2}{3}x + \frac{7}{3}$.