Name: ____________________ **Algebra II**
Date: **Homework 30**

Problem 1. Write the following functions in the form $f(x) = a(x \pm h)^2 \pm k$ by completing the square. Describe how x^2 is shifted to obtain $f(x)$. Graph $f(x)$, label the vertex, label all axis intersections. An example of what I expect is given below.

(a) $f(x) = -2x^2 + 8x + 10$

$$\begin{aligned} f(x) &= -2x^2 + 8x + 10 \\ &= -2(x^2 - 4x) + 10 \\ &= -2(x^2 - 4x + 2^2 - 2^2) + 10 \\ &= -2((x-2)^2 - 4) + 10 \\ &= -2(x-2)^2 - (-2)4 + 10 \\ &= -2(x-2)^2 + 8 + 10 \\ &= -2(x-2)^2 + 18 \end{aligned}$$

So $f(x) = -2(x-2)^2 + 18$ is the function x^2 shifted right 2 units, stretched vertically by a factor of 2, reflected about the x-axis, and shifted 18 units up. This puts the vertex at $(2, 18)$. The y-intercept is at $f(0) = -2(0)^2 + 8(0) + 10 = 10$. We can find the x-intercepts by solving

$$-2(x-2)^2 + 18 = 0$$

$$-2(x-2)^2 = -18$$

$$(x-2)^2 = \frac{-18}{-2}$$

$$\sqrt{(x-2)^2} = \pm\sqrt{9}$$

$$x - 2 = \pm 3$$

$$x = 2 \pm 3$$

Then the x-intercepts are $\{-1, 5\}$.

(b) $f(x) = 3x^2 - 12x + 2$

$$\begin{aligned} f(x) &= 3x^2 - 12x + 2 \\ &= 3(x^2 - 4x) + 2 \\ &= 3(x^2 - 4x + 2^2 - 2^2) + 2 \\ &= 3((x-2)^2 - 4) + 2 \\ &= 3(x-2)^2 - 12 + 2 \\ &= 3(x-2)^2 - 10 \end{aligned}$$

So $f(x) = 3(x-2)^2 - 10$ is the function x^2 shifted right 2 units, stretched vertically by a factor of 3, and shifted 10 units down. This puts the vertex at $(2, -10)$.The y-intercept is at $f(0) = 3(0)^2 - 12(0) + 2 = 2$. We can find the x-intercepts by solving

$$3(x-2)^2 - 10 = 0$$

$$
\begin{aligned}
3(x-2)^2 &= 10 \\
(x-2)^2 &= \frac{10}{3} \\
\sqrt{(x-2)^2} &= \pm\sqrt{\frac{10}{3}} \\
x-2 &= \pm\sqrt{\frac{10}{3}} \\
x &= 2 \pm \sqrt{\frac{10}{3}}
\end{aligned}
$$

Then the x-intercepts are $\left\{2-\sqrt{\frac{10}{3}}, 2+\sqrt{\frac{10}{3}}\right\}$.

(c) $f(x) = -2x^2 + 3x + 2$

$$
\begin{aligned}
f(x) &= -2x^2 + 3x + 2 \\
&= -2\left(x^2 - \frac{3}{2}x\right) + 2 \\
&= -2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) + 2 \\
&= -2\left(\left(x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) + 2 \\
&= -2\left(x - \frac{3}{4}\right)^2 - (-2)\left(\frac{3}{4}\right)^2 + 2 \\
&= -2\left(x - \frac{3}{4}\right)^2 + 2\left(\frac{9}{16}\right) + 2 \\
&= -2\left(x - \frac{3}{4}\right)^2 + \frac{9}{8} + 2 \\
&= -2\left(x - \frac{3}{4}\right)^2 + \frac{9}{8} + \frac{16}{8} \\
&= -2\left(x - \frac{3}{4}\right)^2 + \frac{25}{8}
\end{aligned}
$$

So $f(x) = -2\left(x - \frac{3}{4}\right)^2 + \frac{25}{8}$ is the function x^2 shifted right $\frac{3}{4}$ units, stretched vertically by a factor of 2, reflected about the x-axis, and shifted $\frac{25}{8}$ units up. This puts the vertex at $\left(\frac{3}{4}, \frac{25}{8}\right)$. The y-intercept is at $f(0) = -2(0)^2 + 3(0) + 2 = 2$. We can find the x-intercepts by solving

$$
\begin{aligned}
-2\left(x - \frac{3}{4}\right)^2 + \frac{25}{8} &= 0 \\
-2\left(x - \frac{3}{4}\right)^2 &= -\frac{25}{8} \\
\left(x - \frac{3}{4}\right)^2 &= -\frac{25}{(-2)8}
\end{aligned}
$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{25}{16}$$

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \pm\sqrt{\frac{25}{16}}$$

$$x - \frac{3}{4} = \pm\frac{5}{4}$$

$$x = \frac{3}{4} \pm \frac{5}{4}$$

$$x \in \left\{-\frac{2}{4}, \frac{8}{4}\right\}$$

Then the x-intercepts are $\left\{-\frac{1}{2}, 2\right\}$.

(d) $f(x) = \frac{1}{3}x^2 + x + \frac{1}{4}$

$$\begin{aligned}
f(x) &= \frac{1}{3}x^2 + x + \frac{1}{4} \\
&= \frac{1}{3}\left(x^2 + 3x\right) + \frac{1}{4} \\
&= \frac{1}{3}\left(x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) + \frac{1}{4} \\
&= \frac{1}{3}\left(\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) + \frac{1}{4} \\
&= \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{1}{3}\left(\frac{3}{2}\right)^2 + \frac{1}{4} \\
&= \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{1}{3}\left(\frac{3^2}{4}\right) + \frac{1}{4} \\
&= \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{3}{4} + \frac{1}{4} \\
&= \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{2}{4} \\
&= \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{1}{2}
\end{aligned}$$

So $f(x) = \frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{1}{2}$ is the function x^2 shifted left $\frac{3}{2}$ units, compressed vertically by a factor of 3, and shifted $\frac{1}{2}$ units down. This puts the vertex at $\left(-\frac{3}{2}, -\frac{1}{2}\right)$. The y-intercept is at $f(0) = \frac{1}{3}(0)^2 + (0) + \frac{1}{4} = \frac{1}{4}$. We can find the x-intercepts by solving

$$\frac{1}{3}\left(x + \frac{3}{2}\right)^2 - \frac{1}{2} = 0$$

$$\frac{1}{3}\left(x + \frac{3}{2}\right)^2 = \frac{1}{2}$$

$$
\begin{aligned}
\left(x+\frac{3}{2}\right)^2 &= \frac{3}{2} \\
\sqrt{\left(x+\frac{3}{2}\right)^2} &= \pm\sqrt{\frac{3}{2}} \\
x+\frac{3}{2} &= \pm\sqrt{\frac{3}{2}} \\
x &= -\frac{3}{2} \pm \sqrt{\frac{3}{2}}
\end{aligned}
$$

Then the x-intercepts are $\left\{-\frac{3}{2}-\sqrt{\frac{3}{2}}, -\frac{3}{2}+\sqrt{\frac{3}{2}}\right\}$.

(e) $f(x) = \frac{3}{5}x^2 - 3x - \frac{1}{4}$

$$
\begin{aligned}
f(x) &= \frac{3}{5}x^2 - 3x - \frac{1}{4} \\
&= \frac{3}{5}\left(x^2 - \left(\frac{5}{3}\right)3x\right) - \frac{1}{4} \\
&= \frac{3}{5}\left(x^2 - 5x\right) - \frac{1}{4} \\
&= \frac{3}{5}\left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right) - \frac{1}{4} \\
&= \frac{3}{5}\left(\left(x-\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right) - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{3}{5}\left(\frac{5}{2}\right)^2 - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{3}{5}\left(\frac{5^2}{4}\right) - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{3*5^2}{5*4} - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{3*5}{4} - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{15}{4} - \frac{1}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - \frac{16}{4} \\
&= \frac{3}{5}\left(x-\frac{5}{2}\right)^2 - 4
\end{aligned}
$$

So $f(x) = \frac{3}{5}\left(x - \frac{5}{2}\right)^2 - 4$ is the function x^2 shifted right $\frac{5}{2}$ units, compressed vertically down to $\frac{3}{5}^{\text{ths}}$ of the original height, and shifted 4 units down. This puts the vertex at $\left(\frac{5}{2}, -4\right)$. The y-intercept is at $f(0) = \frac{3}{5}(0)^2 - 3(0) - \frac{1}{4} = -\frac{1}{4}$. We can find the x-intercepts by solving

$$\frac{3}{5}\left(x - \frac{5}{2}\right)^2 - 4 = 0$$

$$\frac{3}{5}\left(x - \frac{5}{2}\right)^2 = 4$$

$$\left(x - \frac{5}{2}\right)^2 = 4 * \frac{5}{3}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{4 * 5}{3}$$

$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm\sqrt{\frac{4 * 5}{3}}$$

$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm\frac{\sqrt{4 * 5}}{\sqrt{3}}$$

$$x - \frac{5}{2} = \pm\frac{2\sqrt{5}}{\sqrt{3}}$$

$$x - \frac{5}{2} = \pm 2\sqrt{\frac{5}{3}}$$

$$x = \frac{5}{2} \pm 2\sqrt{\frac{5}{3}}$$

Then the x-intercepts are $\left\{\frac{5}{2} - 2\sqrt{\frac{5}{3}}, \frac{5}{2} + 2\sqrt{\frac{5}{3}}\right\} = \left\{\frac{5}{2} - \sqrt{\frac{20}{3}}, \frac{5}{2} + \sqrt{\frac{20}{3}}\right\}$.