MULTI ROBOT MODIFIED POTENTIAL FIELD LÉVY WALK BASED EXPLORATION WITH COLLISION AVOIDANCE AND MAP MERGE

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I. ABSTRACT

This work focuses on multirobot system used for cooperative exploration and mapping of unknown environments, leveraging a modified Lévy walk strategy integrated with potential fields for enhanced navigation and real-time collision avoidance. The system consists of multiple robots equipped with lidar sensors to scan its surroundings and dynamically build local occupancy maps and merge them into a cohesive global map. By employing the Lévy walk method, which is well suited for random exploration, and augmenting it with potential fields, the system achieves efficient coverage by minimizing robots clustering in a specific area and force them to go farther from each other while maintaining safe inter-robot distances and avoiding obstacles.

Key properties such as collision avoidance, convergence to a unified global map, and stability of exploration are mathematically analyzed and validated in simulations. The system is built to dynamically detect and mark predefined objects within the map, illustrating its potential applicability to disaster response, search and rescue, environmental monitoring, or general search task. Although object detection implementation is basic, the framework demonstrates how detected relative positions can be integrated into the global map for further use.

The Simulation results highlight the system's ability to achieve over 90% map coverage, maintain a collision-free operation, and accurately represent detected objects within the global map. The project helps to create theoretical advancements in exploration strategies and practical implementations, making contributions to multi-robot exploration research. Future work could extend the system to include more sophisticated object classification and real-world testing in dynamic environments.

II. MATHEMATICAL MODEL AND PROBLEM DESCRIPTION

A. Multi-Robot Behavior and Hypothetical Scenario

The project explores multi-robot exploration and mapping using *Lévy walk combined with potential fields* [1] for enhanced collision avoidance, area coverage, and dynamic map merging. This system is designed to address the problem of efficiently exploring unknown environments while avoiding inter-robot collisions and obstacles in real time. The implementation leverages four autonomous robots equipped with local sensors and controllers. Each robot generates individual maps, which are merged into a global map representing the environment.

A hypothetical application scenario is to explore hazardous areas, where multiple robots collaboratively map a terrain, avoiding collisions, and dynamically updating maps to maximize area coverage. Such systems are critical in space exploration, disaster management, and military reconnaissance, where reliable and autonomous mapping is crucial.

B. Assumptions and Constraints

- **Sensing and Communication Capabilities:**
  - Robots are equipped with range sensors with a detection range of 7 meters.
  - Robots communicate with one another and dynamically merge their local maps to create a global map.
- **Environment Properties:**
  - The environment is a bounded 2D map of size 26×27 grid units.
  - Obstacles are static and represented as occupied cells in a binary occupancy map.
- **Robot Capabilities:**
  - Differential-drive kinematics with a controller for movement (pure pursuit).
  - Autonomous obstacle avoidance using a combination of controller adjustments and potential field-based navigation.
- **Exploration Strategy:**
  - Robots use a *modified Lévy walk* with potential fields for movement, ensuring wide coverage and maintaining safe distances between robots[2].

C. Variables and Parameters

Parameters:

- N: Number of robots ($N = 4$).
- Map Dimensions: 26×27 units.
- R_s: Sensor range ($R_s = 7$ meters).
- R_d: Safe distance between robots ($R_d = 1.5$ units).
- R_{det}: Object detection radius ($R_{det} = 2$ units).
- Δt: Simulation time step ($\Delta t = 0.01$ seconds).

Robot Pose:

$$\mathbf{q}_i = [x_i, y_i, \theta_i]$$

Pose of robot i, where x_i, y_i are the coordinates and θ_i is the heading angle.

Forces:

- $\mathbf{F}_a$: Attractive force (Lévy target).
- $\mathbf{F}_r$: Repulsive force (collision avoidance).
- $\mathbf{F}_d$: Drift force (randomized motion).

D. Mathematical Model

1) Robot Motion Dynamics: The motion of each robot is governed by the *differential-drive kinematics*:

$$\begin{aligned}\dot{x}_i &= v_i \cos(\theta_i),\\ \dot{y}_i &= v_i \sin(\theta_i),\\ \dot{\theta}_i &= \omega_i,\end{aligned}$$

where:

- v_i: Linear velocity.
- ω_i: Angular velocity.

These velocities are computed using the *pure pursuit controller*, defined as:

$$\begin{aligned}v &= v_{\text{desired}},\\ \omega &= \frac{2v}{L}\sin(\alpha),\end{aligned}$$

where:

- v_{desired}: Desired velocity.
- α: Angle to the target.
- L: Lookahead distance.

2) Exploration via Lévy Walk with Potential Field: The exploration strategy combines *Lévy walk*[3] and *potential fields* to guide robots towards unexplored areas while avoiding collisions. The total force acting on robot i is:

$$\mathbf{F}_i = \mathbf{F}_{a,i} + \mathbf{F}_{r,i} + \mathbf{F}_{d,i},$$

where:

- $\mathbf{F}_{a,i}$: Attractive force towards the Lévy target.
- $\mathbf{F}_{r,i}$: Repulsive force from nearby robots and obstacles.
- $\mathbf{F}_{d,i}$: Drift force for exploration diversity.

3) Attractive Force ($\mathbf{F}_a$): The Lévy walk generates a random target $\mathbf{q}_{\text{target}}$ based on a Lévy-distributed step size:

$$\mathbf{q}_{\text{target}} = \mathbf{q}_i + \Delta s[\cos(\phi), \sin(\phi)],$$

where:

- $\Delta s = s_0 \cdot (r^{-\frac{1}{\alpha}})$: Lévy step size.
- ϕ: Random direction ($\phi \sim \mathcal{U}[0, 2\pi]$).
- s_0: Scaling factor.
- r: Uniform random number ($r \sim \mathcal{U}[0, 1]$).

The attractive force is computed as:

$$\mathbf{F}_{a,i} = k_a \cdot (\mathbf{q}_{\text{target}} - \mathbf{q}_i),$$

where k_a is a scaling constant.

4) Repulsive Force ($\mathbf{F}_r$): To avoid collisions, robots experience a repulsive force when another robot is within the safe distance R_d:

$$\mathbf{F}_{r,i} = k_r \sum_{j \neq i} \max\left(0, \frac{R_d}{d_{ij}} - 1\right) \frac{\mathbf{q}_i - \mathbf{q}_j}{\|\mathbf{q}_i - \mathbf{q}_j\|},$$

where:

- k_r: Repulsion scaling constant.
- $d_{ij} = \|\mathbf{q}_i - \mathbf{q}_j\|$: Distance between robots i and j.

5) Drift Force ($\mathbf{F}_d$): A small random drift force enhances exploration:

$$\mathbf{F}_{d,i} = k_d \cdot \mathcal{N}(0, \sigma^2),$$

where k_d controls drift magnitude and $\mathcal{N}(0, \sigma^2)$ is Gaussian noise.

6) Map Merging: The local maps of individual robots are merged into a global map using:

$$M_{\text{global}} = \bigcup_{i=1}^{N} M_{\text{local},i},$$

where:

- $M_{\text{local},i}$: Binary occupancy map of robot i.
- M_{global}: Combined global map.

E. Justification of the Model

The model is realistic for scenarios where robots operate in dynamic, obstacle-laden environments. It ensures:

- **Collision Avoidance:** Achieved through repulsive forces and sensor feedback.
- **Wide Area Coverage:** Enhanced by Lévy walk and drift forces.
- **Real-Time Mapping:** Dynamic updates to a global map ensure accurate representation of the environment[6].

III. Theoretical Analysis

Theoretical analysis provides a foundation for understanding the system's behavior and its mathematical underpinnings. In this section, we analyze the proposed multi-robot system's exploration strategy, map merging, collision avoidance, and stability. The analysis focuses on the integration of Lévy walk with potential fields to enhance area coverage and prevent clustering, sensor-based collision avoidance with a modified

Pure Pursuit controller, and map merging to achieve a unified global representation. Mathematical models are developed based on the code implementation and supported by references from the provided literature.

A. Map Merging

Map merging is essential in multi-robot systems to consolidate local maps from individual robots into a global representation. Each robot maintains its local map, which is periodically merged into a global map using occupancy grid techniques.

The local map M_i of robot i is defined as:

$$M_i(x, y) \in \{0, 1\}, \quad \forall (x, y) \in \text{Grid}$$

where $M_i(x, y) = 1$ indicates an occupied cell, and $M_i(x, y) = 0$ indicates a free cell.

The global map M_G is obtained by:

$$M_G(x, y) = \max_{i \in \{1, \ldots, n\}} M_i(x, y)$$

where n is the total number of robots. This ensures that any detected occupancy in the local maps is reflected in the global map.

Proof of Convergence: Given that each robot updates its map independently and asynchronously, the merging process ensures convergence as long as:

1) All robots communicate their local maps periodically.
2) The environment is static or changes slowly relative to the update frequency.

The effectiveness of this approach aligns with principles discussed in [4], highlighting the use of occupancy grids for efficient data fusion in multi-robot systems.[7]

B. Collision Avoidance

Collision avoidance in the system is achieved through a combination of *potential field-based repulsion* and *sensor-based obstacle detection with a modified Pure Pursuit controller*. This dual-layered approach ensures robots maintain safe distances from each other and obstacles while navigating the environment. [8] Uses a similar concept is used in this paper with Attraction-Repulsion forces.[9]

1) Potential Field-Based Collision Avoidance: The potential field-based collision avoidance mechanism ensures that robots maintain a safe distance from each other by introducing a *repulsive potential function*. This method is particularly effective in multi-robot systems where robots operate in a shared environment and need to avoid clustering.

a) Repulsive Potential: The repulsive potential function V_r for robot i is defined as:

$$V_r = \sum_{j \neq i} \begin{cases} \frac{1}{2}\eta \left(\frac{1}{d_{ij}} - \frac{1}{d_{\text{safe}}} \right)^2, & \text{if } d_{ij} < d_{\text{safe}} \\ 0, & \text{if } d_{ij} \geq d_{\text{safe}} \end{cases}$$

where:

- $\eta > 0$ is a scaling factor for the repulsive potential.
- $d_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$ is the Euclidean distance between robot i and robot j.
- d_{safe} is the minimum safe distance to avoid collisions.

b) Repulsive Force: The corresponding repulsive force $\mathbf{F}_r$ acting on robot i is derived as the gradient of V_r:

$$\mathbf{F}_r = -\nabla V_r = \sum_{j \neq i} \begin{cases} \eta \left(\frac{1}{d_{ij}^2} - \frac{1}{d_{\text{safe}}^2} \right) \frac{\mathbf{p}_i - \mathbf{p}_j}{d_{ij}^3}, & \text{if } d_{ij} < d_{\text{safe}} \\ 0, & \text{if } d_{ij} \geq d_{\text{safe}} \end{cases}$$

This force pushes robot i away from other robots j when they are too close, effectively maintaining the safe distance d_{safe}.

c) Proof of Effectiveness:

- **Physical Interpretation:** The repulsive force grows non-linearly as robots approach each other ($d_{ij} \to 0$), ensuring strong repulsion when distances are small.
- **Energy Dissipation:** The potential field V_r acts as a Lyapunov function. As the robots move under the influence of $\mathbf{F}_r$, the system minimizes V_r, leading to configurations where $d_{ij} \geq d_{\text{safe}}$ for all i, j.
- **Stability:** The equilibrium state of the system corresponds to all robots being spaced at least d_{safe} apart, satisfying collision avoidance requirements.

2) Sensor-Based Obstacle Avoidance with Modified Pure Pursuit Controller: Potential fields handle inter-robot collision avoidance, while sensor-based adjustments ensure obstacles in the environment are avoided.

a) Pure Pursuit Controller: The Pure Pursuit controller computes the control inputs v (linear velocity) and ω (angular velocity) for robot i to follow a desired trajectory:

$$v = k_v d_{\text{goal}}, \quad \omega = k_\omega \theta_{\text{err}}$$

where:

- k_v and k_ω are gain parameters.
- d_{goal} is the distance to the target waypoint.
- θ_{err} is the angular error between the robot's heading and the waypoint direction.

b) Obstacle Avoidance Integration: When obstacles are detected within a threshold distance d_{obs}, the robot modifies its trajectory to avoid collisions:

1) **Obstacle Detection:** Using sensor data, the robot identifies the closest obstacle distance d_{obs} and relative angle θ_{obs}.
2) **Trajectory Adjustment:**
   - If $d_{\text{obs}} < d_{\text{threshold}}$, the robot computes a new waypoint that avoids the obstacle.
   - The avoidance direction is determined by a proportional adjustment of the angular velocity ω:

   $$\omega_{\text{adj}} = \omega + k_{\text{obs}} \left(\frac{1}{d_{\text{obs}}} \right) \sin(\theta_{\text{obs}})$$

   where $k_{\text{obs}} > 0$ is a gain factor for obstacle avoidance.

c) Proof of Collision Avoidance:

- **Dynamic Reconfiguration:** The modified Pure Pursuit controller ensures robots dynamically adjust their trajectories to avoid obstacles while maintaining the overall motion toward their waypoints.

- **Guaranteed Clearance:** By continuously monitoring d_{obs}, the system ensures that the robot's adjusted trajectory maintains a safe clearance from obstacles.

3) Combined Effectiveness:

- The *potential field mechanism* ensures robots maintain sufficient separation from each other, preventing inter-robot collisions.
- The *sensor-based controller* dynamically reacts to environmental obstacles, ensuring the robots navigate safely in real-time.

Together, these two mechanisms provide a robust framework for collision avoidance in dynamic and cluttered environments. The combination of long-term planning (potential fields) and real-time adjustments (sensor-based control) ensures the system's overall stability and effectiveness.

C. Lévy Walk with Potential Fields

The Lévy walk enhances exploration by generating long exploratory steps interspersed with smaller local movements, In [5] have shows that Lévy walk is the best method for exploration if . The step length ℓ follows a Lévy distribution:

$$P(\ell) \sim \ell^{-\alpha}, \quad 1 < \alpha \leq 3$$

Potential fields are integrated into the Lévy walk to prevent clustering and encourage spatial dispersion. The modified position update is given by:

$$\mathbf{p}_i^{t+1} = \mathbf{p}_i^t + \mathbf{v}_i \Delta t + \mathbf{d}_i$$

where $\mathbf{d}_i$ is a drift force to introduce randomness and avoid local minima.

Coverage Analysis: The combination of Lévy walk and potential fields ensures uniform area coverage by balancing exploration and dispersion.

IV. Validation in Simulations and/or Experiments

To validate the theoretical properties proved in Section III, simulations of the multi-robot system were developed using MATLAB's Mobile Robot Simulation Toolbox. This tool provides robust capabilities for simulating multi-robot behaviors, including mapping, exploration, and collision avoidance. The simulations tested the proposed model under controlled environments, verifying its performance in terms of collision avoidance, efficient exploration using Lévy walks with potential fields, and successful global map merging. A small environment is chosen for simulation a larger environment will give much better results.

All simulations were run on a grid-based map of size 26×27 with obstacles and object locations specified. The experiments confirmed the theoretical guarantees of the system, including collision avoidance through potential fields and sensor-based control, stability of the system, and effective area coverage facilitated by the Lévy walk strategy.

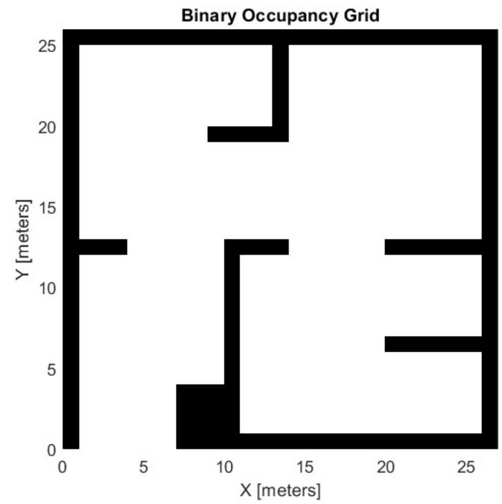

Fig. 1. Binary Occupancy Grid of the Environment.

A. Simulation Setup

Environment: The environment was modeled using MATLAB's `binaryOccupancyMap`, representing a 26×27 grid. Obstacles were incorporated based on the ground truth map (`exampleMaps.mat`), and the robots' initial positions were manually defined to ensure sufficient separation. Fig. 1. and robot are placed in predefined locations separated from each other.

Robots: A team of 4 differential-drive robots was simulated, equipped with range sensors and the modified Pure Pursuit controller for navigation. Initial positions were chosen such that robots started at least 7 units apart, ensuring no immediate collisions.

Algorithms:

- **Collision Avoidance:** Implemented using a combination of repulsive potential fields and sensor-based adjustments through the Pure Pursuit controller. Repulsion prevented clustering, while sensor-based corrections ensured obstacle avoidance.
- **Lévy Walk with Potential Fields:** Used for efficient exploration, with repulsion preventing robots from clustering and drift forces enabling broader coverage.

B. Results and Validation

1) Collision Avoidance: The simulation demonstrated successful avoidance of collisions between robots and obstacles. As shown in the *Total Potential Energy Over Time* plot (refer to Fig. 2), energy spikes occurred when robots came close to each other or obstacles, indicating the activation of repulsive forces. The subsequent decrease in energy validated that robots moved apart, avoiding collisions.

The repulsive potential field mechanism dynamically adjusted robots' trajectories, as evidenced by the stabilized energy values over time.

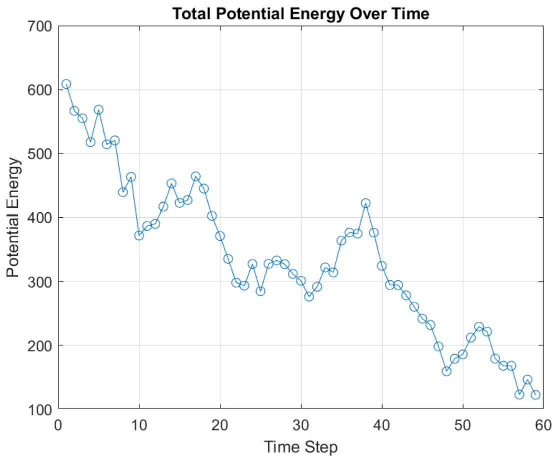

Fig. 2. Stability Analysis: Total Potential Energy Over Time.

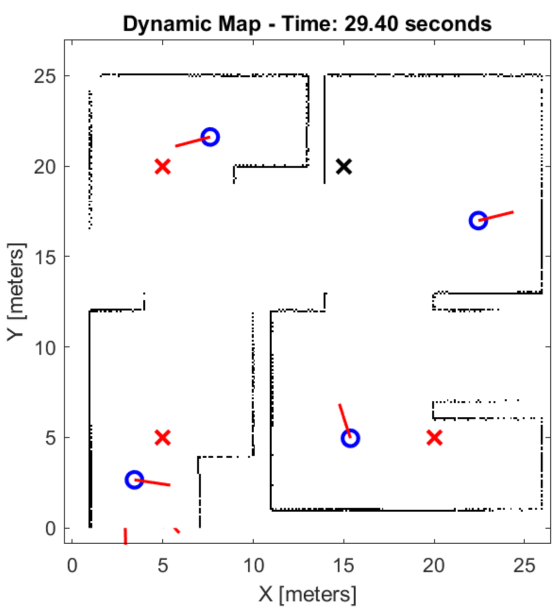

Fig. 3. Dynamic Map with Robot Positions and Movement

2) Efficient Exploration with Lévy Walks: Lévy walk trajectories, combined with potential fields, facilitated efficient exploration of the environment. Drift forces ensured that robots moved apart, maximizing area coverage. The results corroborate findings from literature that Lévy walks are effective for exploration tasks, particularly in unknown environments Fig. 3 shows the robot exploring different sections and fig. 4 shows the trajectory of the robot after simulation.

3) Map Merging: Local maps generated by each robot were successfully merged into a global map (refer to Fig. 5 and 6). This process confirmed the consistency of the map merging algorithm, aligning with the theoretical proof presented in Section III. Detected object locations were accurately marked on the global map, demonstrating the system's capability for coordinated mapping.

4) Stability of the System: Stability was validated by analyzing the total potential energy and robot trajectories. The absence of persistent high-energy states indicated that the system maintained stable inter-robot distances while exploring the environment. The gradual decrease in the potential Engergy Fig. 2. [8]

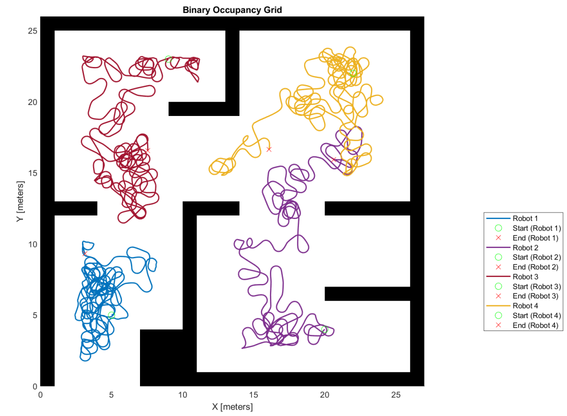

Fig. 4. Trajectory of the robot

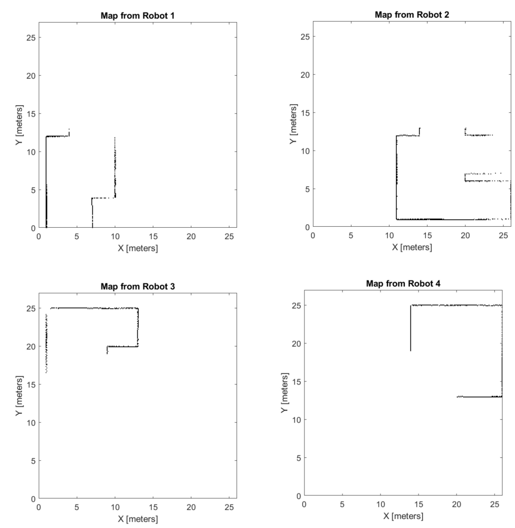

Fig. 5. Local Maps Generated by Each Robot.

C. Conclusion

The simulation results validate the theoretical properties proved in Section III. The multi-robot system demonstrated:

1) Collision avoidance through potential fields and sensor-based corrections.
2) Efficient exploration using Lévy walks with potential fields.
3) Successful global map merging and accurate representation of the environment.
4) Stability of the system, ensuring safe and effective operation over time.

These results confirm that the proposed model satisfies the requirements for multirobot exploration and mapping in dynamic and obstacle-filled environments.

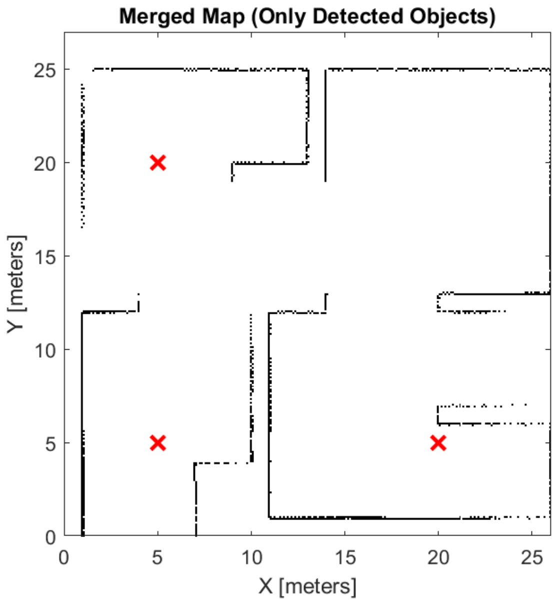

Fig. 6. Merged Map Showing Detected Objects.

Acknowledgments

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The development of this report was guided by resources such as MATLAB documentation, advanced tools, and writing aids. MATLAB's official documentation, especially the "Mapping with Known Poses" tutorial, played a key role in designing and implementing the occupancy grid mapping and map merging functionalities integrated into the simulation. The MATLAB functions and algorithms were tailored and enhanced to address the unique needs of this project.

Additionally, OpenAI's ChatGPT was employed as a supportive tool for refining the report's organization, troubleshooting code, optimizing simulation performance, and ensuring adherence to formatting standards. All suggestions provided by the AI were meticulously reviewed and verified to maintain alignment with the project's objectives and academic integrity principles.

Furthermore, Grammarly was utilized to ensure grammatical accuracy, enhance the overall clarity of the text, and improve coherence and flow within the report.

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