NYU-Tandon School of Engineering

MA1024/MA1324

Review Problems for Exam 3

(1) Find the rate of change of the distance between the origin and a point moving on the graph of $y = \log_2(x)$ if $\dfrac{dx}{dt} = 2$ and $x = 10$. Round your answer to 2 decimal places.

(2) A cube has edges that are expanding at a constant rate of 4.5 cm/sec. At the moment when the edge is 7 cm, find the rate of change of

(a) the cube's volume (b) the cube's surface area.

(3) A balloon is rising vertically over a point A on the ground at the rate of 20 ft/sec. Another point B on the ground is 30 ft from point A. When the balloon is 40 ft above point A, at what rate is its distance from point B changing?

(4) Water is leaking out of an inverted conical tank (vertex pointed down) at a rate of $10,000$ cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

(5) A train is heading due west from St. Louis. At noon, a plane flying horizontally due north at a fixed altitude of 4 miles passes directly over the train. When the train has traveled another mile, it is going 80 mph, and the plane has traveled another 5 miles and is going 500 mph. At that moment, how fast is the distance between the train and the plane increasing?

(6) A 7-meter ladder is leaning against a house so that the top of the ladder is 3 meters above the ground. If a worker pushes the base of the ladder in towards the house at a rate of 2 m/sec, how fast is the top of the ladder rising when the top is 4 meters above the ground?

(7) A cone with a circular base and a vertex pointed downwards has a height of 14 feet and a base diameter of 8 feet. If sand is being pumped into the cone at a rate of 2 cubic feet per second, how fast is the sand level rising when the sand in the cone is 3 feet high?

(8) A function f satisfies the following conditions:

$$f(5) = 0, \quad f'(5) = 3, \quad \text{and } f''(x) > 0 \text{ for } x \geq 5.$$

Which of the following are possible values for $f(7)$? Circle all that apply. Explain your choice.

5 6 7

YOUR SIGNATURE:

(9) Let $f(x) = \dfrac{1}{1-x} - \dfrac{1}{x}$, where $x \in [-2, 3]$. Find

(a) all critical points and the intervals on which f is increasing or decreasing
(b) the local maxima and minima
(c) all points of inflection and the intervals on which the graph of f is concave up or concave down
(d) all x- or y- intercepts if any is possible

(10) For the function $f(x) = xe^{-x}$, classify the critical points of the function as local maxima or local minima or neither.

(11) For the function $f(x) = x + \dfrac{1}{x}$, classify the critical points of the function as local maxima or local minima or neither.

(12) For the function $f(x) = x^2 e^{-2x/5}$, classify the critical points of the function as local maxima or local minima or neither.

(13) (a) If a is a nonzero constant, find all critical points of $f(x) = \dfrac{a}{x^2} + x$.
(b) Use the second derivative test to show that if a is positive then the graph has a local minimum, and if a is negative, then the graph has a local maximum.

(14) Let $f(x) = \dfrac{x^2}{x-1}$.
(a) Find all critical points of $f(x)$.
(b) Find all local max and local min of $f(x)$.

(15) Let $f(x) = x(x^2 - 1)^{1/3}$.
(a) Find all critical points of $f(x)$.
(b) Find all local max and local min of $f(x)$.

(16) Describe the concavity of the graph of $f(x) = (1-x)^2(1+x)^2$ and find the points of inflection (if any).

(17) Describe the concavity of the graph of $f(x) = 2\cos^2 x - x^2$, $x \in [0, \pi]$ and find the points of inflection (if any).

(18) Let $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. Under what conditions on a, b, and c will f have:
(a) two local extrema?
(b) only one local extremum?
(c) no local extrema?

(19) Let $f(x) = 2x^3 + x^2 - 4x + 3$. Find
(a) all critical points and the intervals on which f is increasing or decreasing;
(b) the local maxima and minima;

(c) all points of inflection and the intervals on which the graph of f is concave up or concave down.

(20) Let $f(x) = 3x - \cos(6x)$, where $-1 \le x \le 1$. Find
(a) all critical points and the intervals on which f is increasing or decreasing
(b) the local maxima and minima
(c) all points of inflection and the intervals on which the graph of f is concave up or concave down
(d) all x- or y- intercepts if any is possible

(21) Determine if each of the following statements is TRUE or FALSE for a function f whose domain is all real numbers. If it is true, explain how you know. If it is false, give a counterexample.
(a) If $f'(x) \ge 0$ for all x, then $f(a) \le f((b)$ whenever $a \le b$.
(b) If $f'(x) \le g'(x)$ for all x, then $f(x) \le g(x)$ for all x.
(c) If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.
(d) If $f'(x) \le 1$ for all x and $f(0) = 0$, then $f(x) \le x$ for all x.

(22) Suppose that f'' and g'' exist and that f and g are concave up for all x. Determine if each of the following statements is TRUE or FALSE for all such f and g. If it is true, explain how you know. If it is false, give a counterexample.
(a) $f(x) + g(x)$ is concave up for all x.
(b) $f(x)g(x)$ is concave up for all x.
(c) $f(x) - g(x)$ can not be concave up for all x.
(d) $f(g(x))$ is concave up for all x.

(23) Let $p(x) = x^5 + 8x^4 - 30x^3 + 30x^2 - 31x + 22$. What is the relationship between $p(x)$ and $f(x) = 5x^4 + 32x^3 - 90x^2 + 60x - 31$? What do the values of $p(1)$ and $p(2)$ tell you about the values of $f(x)$?

(24) Let $p(x)$ be a 8th degree polynomial with 8 distinct zeros. How many zeros does $p'(x)$ have? Why?

(25) Use the Racetrack Principle to show that $\ln(x) \le x - 1$ for any $x > 0$.

(26) Prove that if $f'(x) \le 1$ for all x and $f(0) = 0$, then $f(x) \le x$ for all $x \ge 0$.

(27) Find an equation for the line through the point $(3, 4)$ that cuts from the first quadrant a triangle of minimum area.

(28) Find the global maximum and global minimum of f on respective intervals.
(a) $f(x) = x + \dfrac{1}{x^2}, \quad 1 \le x \le \sqrt{2}$.

(b) $f(x) = \sin^2(x) - \sqrt{3}\cos(x), \quad 0 \le x \le \pi.$

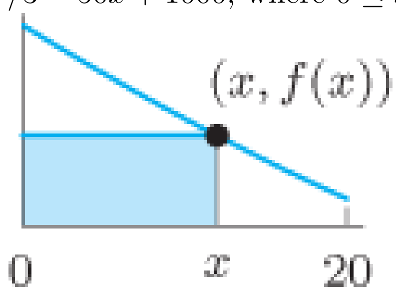
(29) Find the global maximum and global minimum of

$$f(x) = \begin{cases} 2 - 2x - x^2, & -2 \le x \le 0 \\ |x-2|, & 0 < x < 3 \\ \frac{1}{3}(x-2)^3, & 3 \le x \le 4 \end{cases}$$

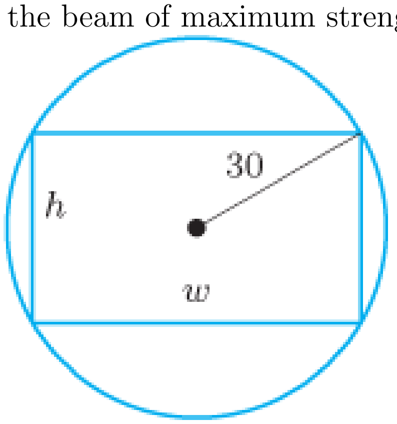
(30) Find a formula for the function of the form $y = ax^b \ln(x)$, where a and b are nonzero constants, which has a local maximum at the point $(e^2, 6e^{-1})$.

(31) Find a formula for the function of the form $y = A\sin(Bx) + C$ with a maximum at $(5, 2)$ and a minimum at $(15, 1.5)$, and no critical points between these two points.

(32) Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = x^2/3 - 50x + 1000$, where $0 \le x \le 20$.



(33) A rectangular beam is cut from a cylindrical log of radius 30 cm. The strength of a beam of width w and height h is proportional to wh^2. Find the width and height of the beam of maximum strength.



(34) Which point on the curve $y = \sqrt{1-x}$ is closest to the origin?

(35) A hemisphere of radius 1 sits on a horizontal plane. A cylinder stands with its axis vertical, the center of its base at the center of the sphere, and its top circular rim touching the hemisphere. Find the radius and height of the cylinder of maximum volume.

(36) Find the minimum and maximum values of the expression $2x + y$ where x and y are lengths in the figure below and $0 \leq x \leq 10$.