COMP90057 Advanced Theoretical Computer Science
Assignment 3
Second (Spring) Semester 2015

As at September 28, 2015
Posted on LMS: Monday, 28 September 2015
Due: Monday, 19 October 2015 [10.00am]

Important: Your submissions for this assignment must be your own individual work. There is a further notice about this below. This document has *two* (2) pages.

Questions

1. (1 mark) Let $S = (0, 1)$, i.e.,
   $$S = \{x : x \text{ is a real number and } 0 < x < 1\}$$
   and let $T = (0, \infty)$, i.e.,
   $$T = \{x : x \text{ is a real number and } x > 0\}.$$
   Prove that S and T have the same cardinality.
   (Hint: consider $f(x) = (1/x) - 1$.)

2. (1 mark) If $A \leq_m B$ and B is a regular language, does that imply A is a regular language? Justify your answer. *(Hint: consider $A = \{0^n 1^n : n \geq 0\}$, $B = \{1\}$)*

3. (2 marks)

   (i) Consider a set S of natural numbers with the property that there is a program P that prints out all the elements of S in increasing order. Show that $L = \{x : x \in S\}$ is decidable.

   (ii) Show that if a set T of natural numbers is infinite and recursively enumerable, it has an infinite decidable subset.

4. (3 marks) Consider the following language: $L = \{\langle M \rangle : M \text{ accepts } 0010\}$. In this question you are asked for two (different) proofs that L is undecidable.

   (i) By reduction from A_{TM}, prove that L is undecidable.

   (ii) Using Rice's Theorem, prove that L is undecidable.

5. (3 marks) Let $L = \{w : \text{ either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither L nor $\overline{L}$ is recursively enumerable.

6. (3 marks) Show that, if $NP \subseteq BPP$ then $NP = RP$. *(Hint: first show that if $SAT \subseteq BPP$ then $SAT \subseteq RP$, and in doing so, search for a satisfying assignment.)*

7. (2 marks) Let BPL be the collection of languages that are decided by probabilistic log-space Turing machines with error probability $1/3$. Prove that $BPL \subseteq P$.

   (Hint: Consider how to estimate in polynomial time the probability that the "BPL" machine would accept the given input. You may also need some notation for the running time of this "BPL" machine.)

Submissions

Submit your answers to these assignment questions on the LMS. The file must be in `.pdf` format. You are expected to *test* for yourself that the document you submit can be printed! If necessary, carry out a trial run with a preliminary version of your submission. *The University offers scanning facilities for students.* A "raw" photograph of your handwritten solution is not sufficient: the submission should look like a scanned document.

   You may submit *multiple* versions of your solutions. Your final *on-time* submission will be the one that is assessed for the purpose of determining a mark. However, for the purpose of ensuring academic honesty, any material that is submitted, regardless of when it was submitted, or whether further or previous submissions were made, may be inspected and will be regarded as an attempt to submit that material for assessment.

Administrative issues

When is late? What do I do if I am late? The due date and time are printed on the front of this document. The lateness policy for this assignment, subject to CIS Department policies, is that ten percent of the available marks for this assignment are lost for each day (or part thereof) that the submission is late.

Should you decide to make a late submission, you can still submit via the LMS. But you must also send an email directly to the lecturers as soon as you are aware you are making a late submission.

Should you experience circumstances affecting your study, as soon as possible: both consult the University's Special Consideration Procedure (MPF1297) and Extensions Procedure (MPF1029), *and* let the lecturers know that you are experiencing such circumstances.

The University Extensions Procedure requires appropriate supporting documentation (from an independent agent), except for instances covered by §3.4.

What are the marks? Recall that this assignment is worth 15% of your final score. There is also a hurdle requirement: to pass this subject, you must earn at least 15 marks out of a subtotal of 30 for the assignments and in-class quiz.

We will reward succinctly written solutions, and reserve the right to penalize prolix solutions.

Individual work You are reminded that your submission for this assignment is to be your own individual work. Where there is suspicion of plagiarism or collusion, the University policy and procedures for responding to academic misconduct will apply. The LMS submission process requires you to make a statement regarding academic honesty.

Finally *We are here to help!* Frequently asked questions about the assignment will be answered in the LMS discussion group. For confidential questions, please contact the lecturers directly.

LizS & AIW
SEPTEMBER 28, 2015