Advanced Theoretical Computer Science - Project 3

Kai Hirsinger

6th October 2015

1

Define the sets S and R

$$S = \{x \mid x \in \mathbb{R} and 0 < x < 1\}$$
$$R = \{x \mid x \in \mathbb{R} and x > 0\}$$

Let f be a function $f : S \to R$. The sets S and R have the same cardinality if f is a mapping from S to R. For f to be a mapping from S and R, it must be the case that every $r \in R$ has exactly one $s \in S$ where $f(s) = r$.

Proof The function $f(x) = \left(\frac{1}{x}\right) - 1$ is a mapping from $S \to R$.

Suppose this function is not a mapping from S to R. If this were the case, there must exist some $s \in S$ where either:

- $f(s) \neq r$ for any $r \in R$
- There exists more than a single value $r \in R$ where $f(s) = r$

Observe that if there is some value $s \in S$ where $f(s) = 0$ the first item can be satisfied. To identify such a value, we set $f(s) = 0$ and solve for s.

$$0 = \left(\frac{1}{s}\right) - 1$$
$$s = 1 - 1$$
$$s = 0$$

Substituting back into $f(s)$ gives:

$$f(0) = \left(\frac{1}{0}\right) - 1$$

Which is clearly invalid, indicating that no such value s exists. TODO: Prove the second point.