

Smith Chart

Find $Z(l)$, $V(l)$ and $I(l)$ given Z_L & Z_0

- 1. Calculate $Z_L = Z_L/Z_0$
- 2. Mark Z_L on SC
- 3. Center of SC to Z_L is dist $|\Gamma_L|$
 $\angle \Gamma_L$ is measured CCW from Re axis
- 4. Find $Z(l)$: rotate CW (toward generator) by $\theta = \ell (360/0.5\lambda)$
- 5. Calculate $Z(l) = z(l)/Z_0$, $\Gamma(l) = |\Gamma_L| \angle \Gamma_L$
- 6. Use VDR: $V(l) = V_g [Z(l)/(Z_0 + Z(l))]^{1/2}$
- 7. Calc V^+ : $V(l) = V^+ (e^{-j\beta l} + \Gamma_L e^{j\beta l})$
- 8. Calculate $V(l)$ for any l by sub $V^+ \rightarrow (4)$
- 9. $I(l) = V(l)/Z(l)$ at any given l

Quarter Wave Transformer

- 1. Calculate $Z_L = Z_L/Z_0$
- 2. Mark Z_L on SC
- 3. Dist center SC $\rightarrow Z_L$ is $|\Gamma_L|$, $\angle \Gamma_L$ is measured CCW from real axis
- 4. Draw circle centered on SC w/ $r = |\Gamma_L|$
- 5. Z_L' point (intersection on real axis)
(impedance is purely real)
- 6. Rotate toward generator (CW)
 $Z_L \rightarrow Z_L'$. Angle is dist d , of quarter-wave transformer from load in λ units
- 7. Impedance at d is $Z(d_1) = Z_0 Z_L'$
- 8. Characteristic impedance of quarter-wave transformer $Z_{q0} = \sqrt{Z_0 Z(d_1)}$

Calculate Z_L given VSWR

- 1. Calculate d_{min} or d_{max} using fact that on TL, 2 minima or 2 maxima are separated by 0.5λ
- 2. On SC, mark $Z(d_{max}) = VSWR + j0$
 $Z(d_{min}) = (1/VSWR) - j0$, as A, B, resp.
- 3. Radius $\rho = VSWR$ circle on SC thru A, B
- 4. start at pt B, rotate CCW (toward load) by dist d_{min} (in λ units). \rightarrow pt C
 Z_L of C read it.
- 5. Dist from center of SC to C is $|\Gamma_L|$
and angle $\angle \Gamma_L = -180^\circ + d_{min} (360/0.5\lambda)$
- 6. solve for $|V^+|$ and $|V^-|$ using $|V_{max}| = |V^+| + |V^-|$, $|V_{min}| = |V^+| - |V^-|$
- 7. $VSWR = |V_{max}|/|V_{min}|$

length of stubs, locate $y=0$ rotate on outermost circle toward gen (CW) until $y = -jx$
11. Rotation dist in λ units is len of stub (ts)

line	$\vec{E} = \hat{x} \frac{1}{2\pi\epsilon_0 r}$	$\vec{\nabla} \times \vec{E} = 0 \Leftrightarrow$ curl-free
sheet	$\vec{E} = \hat{x} \frac{\rho_s}{2\epsilon_0} \sin(x)$	gradient $\vec{\nabla} V(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}) \Rightarrow \vec{E} = -\vec{\nabla} V$
slab	$\vec{E} = \hat{x} \frac{\rho_x}{\epsilon_0}$	$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$
	$\vec{\nabla} \cdot \vec{D} = \rho$ divergence	$\oint_C \vec{E} \cdot d\vec{l} = \int_S \vec{\nabla} \times \vec{E} \cdot d\vec{S}$
	$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$	stoke's thm $\oint_C \vec{E} \cdot d\vec{l} = 0 \Leftrightarrow \vec{\nabla} \times \vec{E} = 0$
	$\vec{\nabla} \times \vec{E} = 0$, $\vec{\nabla} \cdot \vec{D} = \rho$, $\vec{D} = \epsilon_0 \vec{E}$	
	$\vec{v} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$	$\vec{F} = Q\vec{E}$, $\vec{E} = -\nabla V$, parallel pl: $C = \epsilon_0 \frac{A}{T}$
	$\vec{\nabla} \times \vec{E} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \times (E_x, E_y, E_z)$	$\vec{D} = \hat{x} \frac{Q}{A}$, $Q = CV$
	Curl	$= \hat{x} (\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) - \hat{y} (\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) + \hat{z} (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y})$

TEM Wave Solns

$\vec{\nabla}^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

$\vec{E}(\vec{r}, t) = \vec{E}_x(x, z, t)$ $r = \frac{1}{\sqrt{\mu\epsilon}}$

$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$ $\eta = \sqrt{\frac{\mu}{\epsilon}}$ intrinsic impedance

$E_x = \cos(\omega(t \mp \frac{z}{v}))$ depend on z, t

$\vec{H} = \pm \hat{y} \sqrt{\frac{\epsilon}{\mu}} \cos(\omega(t \mp \frac{z}{v}))$ polarized in x

$\vec{E} \cdot \vec{H} = \frac{1}{2} \hat{x} \hat{y} f(t \mp \frac{z}{v})$ on $z=0$

$\vec{E} = \hat{y} \cos(\omega(t \mp \frac{z}{v}))$ polarized in y

$\vec{H} = \hat{x} \cos(\omega(t \mp \frac{z}{v}))$ dependent on z, t

$\mu_0 = 4\pi \times 10^{-7} H/m$
 $\epsilon_0 = (36\pi \times 10^{-9})^{-1} F/m$
 $c = 3 \times 10^8 m/s$ $\eta_0 = 120\pi \Omega$
 $\eta = \sqrt{\frac{\mu}{\epsilon}}$

$C = \epsilon A$
 $N/C = V/m$
 $B = V \cdot s/m^2 = Wb/m^2 = T$
 $D = C/m^2$
 $H = A/m$
 $\rho = C/m^3$
 $J = A/m^2$

\vec{E} antiparallel to \vec{J}
 \vec{B} parallel to \vec{J}

$\vec{E} = E_0 \cos(\omega(t \mp \frac{z}{v}))$
 $\vec{B} = B_0 \cos(\omega(t \mp \frac{z}{v}))$
 $\vec{H} = H_0 \cos(\omega(t \mp \frac{z}{v}))$
 $\vec{D} = D_0 \cos(\omega(t \mp \frac{z}{v}))$
 $\vec{J} = J_0 \cos(\omega(t \mp \frac{z}{v}))$

$\vec{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\epsilon}$

Laplacian

curl $\vec{\nabla} \times \vec{E} = \hat{x} (\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) - \hat{y} (\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) + \hat{z} (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y})$

divergence $\vec{\nabla} \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$

$\vec{D} = \epsilon \vec{E}$

$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

$\vec{B} = \frac{\mu_0}{4\pi} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

$\vec{H} = \frac{1}{4\pi} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

$\vec{J} = \frac{1}{4\pi} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

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$\vec{H} = \frac{1}{4\pi} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

$\vec{J} = \frac{1}{4\pi} \sum_{n=0}^{\infty} (\frac{r_n}{r})^n P_n(\cos\theta)$

- 1. Calculate $Z_L = Z_L/Z_0$
- 2. Calculate $Y_L = 1/Z_L$
- 3. Mark Y_L on SC
- 4. $|\Gamma_L|$ is dist to Y_L on SC
- 5. Draw circle of $r = |\Gamma_L|$
- 6. Rotate along circle from Y_L toward gen.
- 7. Rotate dist d is dist from load to put stub
- 8. Admittance of stubs $Y_S = -jX$
- 9. For short circuit stub: Length of stub, locate $y=0$. Rotate on outermost circle ($r=0$) toward gen (CW) until reach $y = -jX$
- 10. For open circuit stub: Length of stub, locate $y=0$. Rotate on outermost circle ($r=0$) toward gen (CW) until reach $y = jX$
- 11. Rotation dist in λ units is len of stub (ts)