CHAPTER 5 SUMMARY. **Energy**

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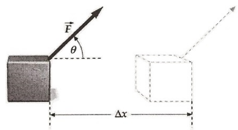
Work

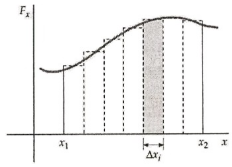


The **work** done by a constant force $\vec{F}$ that moves an object a displacement Δx is defined as

$$W \equiv F \Delta x \cos\theta.$$

So $W = \vec{F} \cdot \Delta\vec{x}$.

Work is a scalar. The SI unit of work is the *joule* (J), 1 J = 1 N·m = 1 kg·m^2/s^2.



A **particle** is any object where all of its parts undergo equal Δx over any Δt. The total work done on a particle is the same as the work done by the net force on the particle, so the work done is the area under the F_x-versus-x curve:

$$W = \sum \vec{F}_i \cdot \Delta\vec{x} = \vec{F}_{\text{net}} \cdot \Delta\vec{x}.$$

Kinetic Energy

Under a constant *net* force F_{net} acting along a straight line on a particle of mass m, which is displaced by Δx along the straight line, the work done on the particle is

$$W = F_{\text{net}} \Delta x.$$

Applying Netwon's second law $F_{\text{net}} = ma$ and the kinematic relation $v^2 - v_0^2 = 2a\Delta x$, we have

$$W = F_{\text{net}}\Delta x = ma\Delta x = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

The quantity $\frac{1}{2}mv^2$ is defined as the **kinetic energy** of the particle

$$K \equiv \frac{1}{2}mv^2.$$

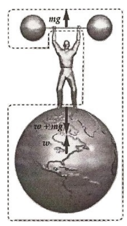
Kinetic energy is a scalar. The SI unit of kinetic energy is the same as work: kg · m^2/s^2 or J.

Kinetic energy depends on the mass and speed of the particle but not the direction of motion.

$W = \Delta K$. This is true even when the force is varying. This is known as the **work-energy theorem**.

Potential Energy

The **potential energy** of a system is the energy associated with the configuration of the system. Often the work done by external forces on a system may result in an increase in the potential energy of the system.



Gravitational Potential Energy The gravitational force between an object of mass m and the Earth is $\vec{F} = -mg\,\hat{j}$, where h, $h_0 \ll r_E$, so the work done by gravity is

$$W_g = \vec{F} \cdot \Delta x = -mg\,\hat{j} \cdot \Delta\vec{x} = -mg\Delta h = -mg\,(h - h_0)\,.$$

When the object is near the surface of the Earth, the gravitational potential energy

$$U_g \equiv mgh.$$

Thus, the work done by gravity is at the expense of the gravitational potential energy:

$$W_g \equiv -\Delta U_g.$$

Potential Energy of a Spring The work done by the spring force, $F = -kx$, is given as

$$W_s = -\frac{1}{2}(kx_1 + kx_2)(x_2 - x_1) = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right).$$

When the spring potential energy is zero at $x = 0$, the spring potential energy can be defined as

$$U_s \equiv \frac{1}{2}kx^2.$$

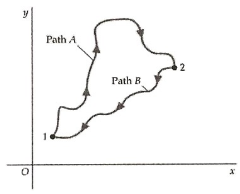
The work done by the spring force is then at the expense of the spring potential energy

$$W_s = -\Delta U_s.$$

Conservative Force and Potential-Energy Function

A force is conservative if on a particle $W_{\text{net}} = 0$ around *any* closed path.
We can use this property to define a **potential-energy function** U such that the force is the negative of the slope of the potential-energy U-versus-x curve:

$$W = \sum_i \vec{F} \cdot \Delta\vec{x}_i = -\Delta U.$$



Non-conservative forces are forces that are not conservative.

Conservation of Mechanical Energy

A **system** is a collection of particles. All forces are either **external** or **internal**. The change in E_{net} of a system is done through work and heat. Since $K = \sum K_i$, we obtain by the work-energy theorem

$$W_{\text{net}} = \sum \Delta K_i = \Delta K = W_{\text{ext}} + W_{\text{nc}} + W_{\text{c}}.$$

The work done by all internal conservative forces can be recast as the change in the total potential energy of the system:

$$W_{\text{c}} = -\Delta U.$$

The sum $E_{\text{mech}} = K + U$ is known as the total mechanical energy of the system,

$$W_{\text{ext}} + W_{\text{nc}} = \Delta K + \Delta U = \Delta(K + U) = \Delta E_{\text{mech}}.$$

When $W_{\text{ext}} = 0$ and $W_{\text{nc}} = 0$, we get the **conservation of mechanical energy**:

$$K_f + U_f = K_i + U_i.$$

Conservation of Energy

For an isolated system, we have $W_{\text{ext}} = 0$ and we may account of W_{nc} by changes in forms of energy other than mechanical energy. **The law of energy conservation:**

$$E = E_{\text{mech}} + E_{\text{therm}} + E_{\text{chem}} + E_{\text{other}}.$$

Work and heat are the ways to transfer energy in or out of a system. When $\Delta Q = 0$, we have:

$$W_{\text{ext}} = \Delta E = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} + \Delta E_{\text{chem}} + \Delta E_{\text{other}}.$$

Power

Power is the rate at which energy is transferred. The average **power** supplied by a force $\vec{F}$ is the rate at which the force does work:

$$\bar{P} = \frac{\Delta W}{\Delta t} = \vec{F} \cdot \frac{\Delta\vec{x}}{\Delta t} = \vec{F} \cdot \vec{v}_{av},$$

$$P = \lim_{\Delta t \to 0} \vec{F} \cdot \vec{v}.$$

The SI unit of power is J/s, also called the **watt**. 1 W = 1 J/s = 1 kg $\cdot$ m^2/s^3.