CHAPTER 6 SUMMARY. **Momentum and Collisions**

Justin Yang

December 30, 2012

Momentum and Impulse

The product of mass and velocity of an object is defined as the **momentum** of the object

$$\vec{p} = m\vec{v}.$$

Momentum is a vector quantity. Its SI unit is kg · m/s. Larger momenta make objects harder to stop. Newton's second law was originally written as

$$\vec{F} = m\vec{a} = m \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}.$$

In terms of the average acceleration, we can define an average force

$$\vec{F}_{\text{av}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}.$$

Thus,

$$\vec{F}_{\text{av}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i.$$

Recall that displacement is given by the area under the velocity-versus-time curve. Analogously, the change in momentum is given by the area force-versus-time curve, defined as the **impulse** of the force. Thus, the impulse $\vec{I}$ and the average force are related by

$$\vec{I} = \vec{F}_{\text{av}}\Delta t = \Delta\vec{p}.$$

Impulse is a vector quantity. Its SI unit is N · s. Impulse produces a change in momentum.

Conservation of Momentum

The total momentum $\vec{P}$ of a system of particles is the sum of the momenta of the individual particles:

$$\vec{P} = \sum m_i\vec{v}_i = \sum \vec{p}_i = M\vec{v}_{\text{cm}}.$$

Thus,

$$\vec{F}_{\text{net,ext}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{P}}{\Delta t} = \frac{d\vec{P}}{dt},$$
$$\vec{I}_{\text{net,ext}} = \Delta\vec{P}.$$

The law of conservation of momentum When the net external force acting on a system of particles remains zero, the total momentum of the system remains constant:

$$\vec{P} = \sum m_i\vec{v}_i = M\vec{v}_{\text{cm}} = \text{constant if } \vec{F}_{\text{net,ext}} = 0.$$

Internal forces may change the mechanical energy of a system but they have no effect on the system's total momentum.

Collisions

In a **collision**, two objects interact strongly for a very short time.
During a collision, the only important forces acting on the two-object system are the interaction forces, which are equal and opposite, so the total momentum of the system remains unchanged.
Often the collision time is so short that during the collision any displacements of the colliding objects can be neglected.
Elastic collision: there is no change in kinetic energy before and after collision.
Inelastic collision: kinetic energies before and after collision are different, In a **perfectly inelastic collision**, the two objects stick together after collision. Thus, all of the kinetic energy relative to the center of mass is converted to thermal or internal energy of the system.

Collisions in one dimension

Consider a collision between two objects. Conservation of momentum gives

$$m1v_{1\mathrm{f}} + m_2 v_{2\mathrm{f}} = m_1 v_{1\mathrm{i}} + m_2 v_{2\mathrm{i}}.$$

The velocities are understood to be the components of the velocities so they can be positive or negative. To solve for the final velocities, we need one more relation, which depends on the type of collision.

Perfectly Inelastic Head-on Collisions The two objects stick together after collision so

$$v_{1\mathrm{f}} = v_{2\mathrm{f}} = v_{\mathrm{f}} = v_{\mathrm{cm}},$$

$$m_1 v_{1\mathrm{i}} + m_2 v_{2\mathrm{i}} = (m_1 + m_2)\, v_{\mathrm{f}}.$$

The kinetic energy of the system before and after collision can be written in terms the momentum of each particle as

$$K_i = \frac{p_{1\mathrm{i}}^2}{2m_1} + \frac{p_{2\mathrm{i}}^2}{2m_2},$$

$$K_f = \frac{P_{\mathrm{f}}^2}{2\,(m_1 + m_2)},$$

where the momentum of the system after collision $P_{\mathrm{f}} = p_{1\mathrm{i}} + p_{2\mathrm{i}}$. Show as an exercize that

$$\Delta K = K_f - K_i = -\frac{(m_1 p_{2\mathrm{i}} - m_2 p_{1\mathrm{i}})^2}{2\,(m_1 + m_2)\, m_1 m_2}.$$

Elastic head-on collisions Kinetic energy is conserved:

$$\frac{1}{2} m_1 v_{1\mathrm{i}}^2 + \frac{1}{2} m_2 v_{2\mathrm{i}}^2 = \frac{1}{2} m_1 v_{1\mathrm{f}}^2 + \frac{1}{2} m_2 v_{2\mathrm{f}}^2.$$

Using momentum conservation one can obtain a simpler relation:

$$v_{2\mathrm{f}} - v_{1\mathrm{f}} = v_{1\mathrm{i}} - v_{2\mathrm{i}}.$$

The speed of recession equals the speed of approach in an elastic collision.
The coefficient of restitution

$$e = \frac{v_{\mathrm{rec}}}{v_{\mathrm{app}}} = -\frac{v_{2\mathrm{f}} - v_{1\mathrm{f}}}{v_{2\mathrm{i}} - v_{1\mathrm{i}}}.$$

Special cases of head-on elastic collisions between two particles of the same mass

Collisions in Three Dimensions

Conservation of momentum is a vector equation in two or three dimensional collisions

$$m_1 \vec{v}_{1\mathrm{i}} + m_2 \vec{v}_{2\mathrm{i}} = m_1 \vec{v}_{1\mathrm{f}} + m_2 \vec{v}_{2\mathrm{f}}.$$

Conservation of momentum can be valid in only one of the three dimensions if only the corresponding component of the net external force is zero.

Perfectly Inelastic Collisions

$$m_1 \vec{v}_{1\mathrm{i}} + m_2 \vec{v}_{2\mathrm{i}} = (m_1 + m_2)\, \vec{v}_{\mathrm{f}}.$$

This means the three velocity vectors and thus the collision are in the same plane. Two equations with two unknowns so the final velocity (cm velocity) can be solved.

Elastic Collisions

Generally more complicated. A simplified case is when an object collides with another one that is initially at rest:

$$m_1 \vec{v}_{1\mathrm{i}} = m_1 \vec{v}_{1\mathrm{f}} + m_2 \vec{v}_{2\mathrm{f}}.$$

So the collision occurs in a plane, which we assume to have been determined experimentally and we take it as the xy plane. This collision is called a *glancing* collision.