1 Physics Club Handout: Motion

For all these problems, assume acceleration due to gravity is 9.81 m/s^2.

Beginner problems:

1. An automobile traveling at 10 m/s accelerates to 20 m/s in 10 seconds. How far does the automobile travel during that time?

2. A roller-coaster car moves 200 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. It next travels 135 ft at an angle of 40.0° downward. What is its displacement from its starting point?

3. A projectile is fired such that the maximum height it reaches is three times the distance it travels before it lands. Find the angle of projection.

4. A bullet is fired from a gun on the surface of the earth (radius 6340 km) at velocity v. Find v such that the bullet will enter a stable orbit around the moon at its inital height.

Intermediate problems:

5. A projectile is launched at an angle θ to the horizontal. It rises to a height h and lands a distance d away. Find the ratio h/d in terms of θ.

6. For the vector $\vec{R} = 2\,\hat{i} + \hat{j} + 3\,\hat{k}$, find the magnitude of $\vec{R}$, and the angles between $\vec{R}$ and the x, y, and z axes.

7. A dive-bomber has a velocity of 280 m/s at an angle θ below the horizontal. It releases a bomb at an altitude of 2.15 km, which initially has the same velocity as the aircraft. The displacement from the point of release (not just horizontal displacement) is 4 km. Find θ.

Advanced problems:

8. The speed of a projectile at its greatest height is $\sqrt{\frac{6}{7}}$ of its speed when it is at half of its greatest height. Find the angle of projection.

9. The speed of a projectile at its greatest height h is m times its speed when it is at a height $n \cdot h$, where $n < 1$. Find the angle of projection in terms of m and n.

10. A ski jumper jumps off a ramp at angle θ to land on a hill that slopes downward and makes an angle ψ with the horizontal. Find θ such that the ski jumper's distance is maximized.

11. A ball is kicked off a hemispherical rock of radius R with horizontal velocity v. Assume no friction between the rock and the ball. Find the distance away from the center of the rock the ball lands in terms of R and v.

2 Physics Club Handout: Newton's Laws

For all these problems, assume acceleration due to gravity is 9.81 m/s^2.

Beginner problems:

1. A 50 kg human in freefall experiences a force from air resistance equal to $\vec{F}_{air} = -\vec{v}^2 * 0.3141 \frac{\text{kg}}{\text{m}}$, where $\vec{v}$ is the person's instantaneous velocity. Find the human's terminal velocity.

2. A hawk flies in a horizontal circle of radius 12 m at a constant velocity of 4 m/s. Find its centripital acceleration under these conditions. The hawk then increases its speed at a rate of 1.2 m/s^2; find its new acceleration (and the direction of its acceleration).

3. A 40.0-kg child swings in a swing supported by two chains, each 3.00 m long. The tension in each chain at the lowest point is 350 N. Find the child's speed at the lowest point and the maximum height to which the child rises.

Intermediate problems:

4. An object moving through a fluid (at sufficiently high velocity such that viscous forces are insignificant) experiences a retarding force of $\vec{F}_{drag} = -\frac{1}{2}\vec{v}^2 * C$, where $\vec{v}$ is the object's instantaneous velocity, and C is some constant based on the fluid's density and the object's shape.

   Given that an object of mass m is dropped in air at time $t = 0$, find the velocity of the object at any given time in terms of m, t, and C.

5. An amusement park ride is set up as a giant swing that starts at an angle of 80° to the vertical, and allows the swing to fall freely. For legal reasons, the maximum g-force a rider can experience is $5g$'s (where $g = 9.81$ m/s^2). Assuming no air resistance, what is the largest they can make the swing and still avoid litigation?

6. A plumb bob (a weight hanging from a string) usually does not hang perfectly vertically (i.e. along a line directed towards the center of the earth). By how much does a plumb bob deviate from vertical here in Palo Alto (latitude of 37.4° N), assuming the earth is spherical and has radius 6380 km?

Advanced problems:

7. An object moving througha a fluid experiences a force $\vec{F}_{drag} = -(ar\vec{v} + br^2\vec{v}^2)$ exerted on a sphere of radius r moving through a fluid at speed v, where a and b are constants based on the shape of the object and the surrounding atmosphere. For spherical objects in air at sea level, $a = 3.10 \times 10^{-4}$ Pa · s and $b = 0.870$ g/L. Find the velocity of a water droplet of 100 µm freefalling at time t, where t is the time elapsed since it was released from rest.

8. A 1 kg block is sitting on a table – the coefficient of static friction between the toaster and the table is 0.4. A string is attached to the block. You pull on the string to make the block move; at what angle should you pull the string to minimize the force necessary to move the block, and what is the force?

9. A time-dependant force $\vec{F} = (8.00\,\hat{i} - 4.00\,t\,\hat{j})$ N (where t is in seconds) is exerted on a 2 kg object initially at rest. At what time will it be moving at a speed of 15 m/s? What is the total displacement and distance of the object at this time?

3 Physics Club Handout: Energy

For all these problems, assume acceleration due to gravity is 9.81 m/s^2.

Beginner problems:

1. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the net work done on the block.

2. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Choose the origin to be at the location where the bullet begins to move. Then the force (in newtons) exerted by the expanding gas on the bullet is $15000 + 10000x - 25000x^2$, where x is in meters. Determine the work done by the gas on the bullet as the bullet travels the length of the barrel, and the speed of the bullet as it emerges from the barrel.

3. A ball is kicked off a 100 meter high cliff, at a speed of 42 m/s and an angle of 30° above the horizontal. Calculate the speed of the ball as it hits the ground below.

Intermediate problems:

4. The potential energy function for a particular system is given by $-x^3 + 2x^2 + 3x$. Find the force involved as a function of x, and where are stable and unstable equilibrium?

5. A toy car of mass m is sent through a circular loop-de-loop of radius r at velocity v. Find the normal force on the car at the bottom of the loop, and the top of the loop.

6. A block of mass M rests on a table. It is fastened to the lower end of a light, vertical spring with spring constant k. The upper end of the spring is fastened to a block of mass m. The spring is then compressed a distance d (relative to its unstretched state) by pushing down on the upper block. In this configuration, the upper block is released from rest. The spring lifts the lower block off the table. In terms of m, what is the greatest possible value for M?

Advanced problems:

7. A soccer ball is on an icy (i.e. frictionless) hill which is defined by the equation $z = 4\cos xy$, where x, y, z are in meters. If the ball is at position (x, y), which way will it roll, and how much will it accelerate? At what places could the ball settle; i.e. where are the places of stable and unstable equilibrium?

8. A ball of mass 300 g is connected by a strong string of length 80.0 cm to a pivot and held in place with the string vertical. A sudden gust of wind exerts constant force F to the right on the ball. The ball is released from rest. The wind makes it swing up to attain maximum height H above its starting point before it swings down again. Find H as a function of F.

9. Two stars of mass M are separated by a distance d. One star is moving at a velocity v relative to the other star, in a direction perpendicular to the line connecting the two stars. As time approaches infinity, how will the stars behave? (Will they enter a stable orbit with one another, will they collide, will they fly apart and never meet again?)

4 Physics Club Handout: Momentum & Impulse

For all these problems, assume acceleration due to gravity is 9.81 m/s^2.

Beginner problems:

1. A 60-kg man holding a 20-kg box rides on a skateboard at a speed of 7 m/s. He throws the box behind him, giving it a velocity of 5 m/s with respect to the ground. What is his velocity after throwing the object?

2. The kinetic energy of a ball is 100 J and its momentum is 40 kg $\cdot$ s. What is its mass?

3. If you jumped up so that you reached a height of 0.75 m, by how much would you change the earth's velocity?

Intermediate problems:

4. A force is applied to a 50 kg rock, initially at rest, for 4 seconds, such that the force $F = 10\,t^2$, where t is in seconds and F is in newtons. If this is the only force acting upon the rock, what is the final velocity of the rock?

5. A 5.0 kg block with a speed of 8.0 m/s travels 2.0 m along a horizontal surface where it makes a head-on, perfectly elastic collision with a 15.0 kg block which is at rest. The coeffcient of kinetic friction between both blocks and the surface is 0.35. How far does the 15.0 kg block travel before coming to rest?

6. Prove that if two identical objects undergo an elastic collision where one of them is at rest, the velocities of the masses afterwards will be perpendicular.

Advanced problems:

7. There is a 5 meter long chain of mass density 100 g/m. It is held above a scale such that the tip of the chain is just touching the scale, and then dropped. What is the reading on the scale as a function of time?

8. Two objects of mass m_1 and m_2 are traveling at velocities $\vec{v}_1$ and $\vec{v}_2$, respectively. They undergo a completely elastic collision. Find, in terms of these quantities, the final velocities of each mass.

9. A tennis ball of mass m_1 is on top of a basketball of mass m_2 and radius r. If they are dropped from a height h, to what height does the tennis ball bounce, assuming all collisions are elastic?

   Now consider a stack of N balls, where the bottom ball has mass m, radius r, and each subsequent ball has mass 1/27 that of the previous ball, and radius 1/3 that of the previous ball. Assuming all collisions are elastic and without air resistance (which is completely absurd), and the stack of balls is dropped from height h, what is the highest ball's velocity?

5 Physics Club Handout: Rotation

For all these problems, assume acceleration due to gravity is $9.81\ \frac{\text{m}}{\text{s}^2}$.

Beginner problems:

1. Derive the moment of inertia of a hollow cylinder about its axis: $I = MR^2$.
2. A solid cylinder 30 cm in diameter is released at the top of a 2 m high incline and rolls down. Assuming no loss of energy, what is the final angular velocity of the cylinder at the bottom?
3. A soccer ball, cheese wheel, and car tire are all rolled down a 45° hill. In what order do they reach the bottom, and why?

Intermediate problems:

4. Derive the moment of inertia of a rectangular plate about its axis: $I = \frac{1}{12}M(a^2 + b^2)$.
5. A billiard ball is struck by a cue. It starts slipping on the felt table, but eventually begins rolling. If it starts moving at velocity v_0, has radius r, has mass m, and the coefficient of friction between the ball and table is μ, how far does the ball travel before it stops slipping and rolls? What will be the speed of the ball at this point?
6. A yo-yo has outer radius R and inner radius r (the radius of the axle on which the string wraps around). (See figure, or remind Justin to draw it.) If the string is pulled with force F at an angle θ to the horizontal, what will be the acceleration of the yo-yo, and in what direction?
7. A breakdancer is in the middle of a headspin, with moment of inertia 4 kg/m^2 and angular velocity 5 rad/s. What is his current rotational kinetic energy? If he tucks his legs in so that his moment of inertia is now 1 kg/m^2 what is his new kinetic energy? What causes this discrepancy?

Advanced problems:

8. Derive the moment of inertia of a solid sphere about its axis: $I = \frac{2}{5}MR^2$.
9. A pendulum, when undergoing small displacements, follows simple harmonic motion. What are the equations of motion of the pendulum if the angle of displacement is not negligible (i.e. $\sin\theta \not\approx \theta$)? Can the position of the pendulum as a function of time be easily solved?
10. The Mississippi River runs north to south. Which bank should be most eroded?
11. For any object rotating about an axis, the radius of gyration is the distance the mass of that object may be concentrated to obtain the same moment of inertia. For example, a sphere of mass 2 kg and radius 1 m has moment of inertia $0.8\,\text{kg} \cdot \text{m}^2$ about the z-axis. A hoop of the same mass 2 kg and radius 0.63 m has the same moment of inertia about its axis 0.8 $\text{kg} \cdot \text{m}^2$. Therefore, a sphere of radius 1 m has a radius of gyration of 0.63 m about the z-axis.

    Having said that, consider a top with radii of gyration $r_z = 1.7$ cm and center of mass 3 cm above the tip. If the top is spun at 20 rpm and makes an angle of 25° with the vertical, at what frequency does the top precess?
12. A pencil is held on a table, point down, so that it is an angle θ away from being vertical. How long does it take to fall flat, assuming the point of the pencil does not move?