CHAPTER 3 SUMMARY. **Vectors and Two-Dimensional Motion**

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General Properties of Vectors

A **vector quantity** has both a magnitude and a direction.
A **scalar quantity** has magnitude, but no direction.

Equality $\vec{A} = \vec{B}$ if $A = B$ and their directions are the same.

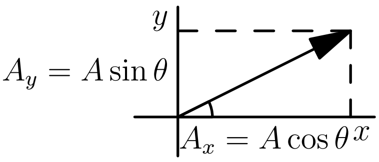
Addition Vectors can be added both geometrically and algebraically.
Geometrically *Head-to-tail* and *parallelogram* methods can be used.
Thus, the **resultant vector** $\vec{R} = \vec{A} + \vec{B}$ is the sum of two or more vectors.

Negative The negative of a vector has the same magintude but opposite direction.

Subtraction $\vec{A} + \vec{B} = \vec{A} + \left(-\vec{B}\right)$.

Multiplication by a Scalar $s\vec{A}$ has magnitude sA and has the same direction as $\vec{A}$ if s if positive and opposite direction if s is negative.

Components The component of $\vec{A}$ in the direction of a directed line S is $A_S = A\cos\theta$, where θ is the angle between S and $\vec{A}$.
A vector can be specified by its rectangular components along the x- and y-axes



$$A_x = A\cos\theta,$$
$$A_y = A\sin\theta.$$

It can also be specified by its magnitude and direction through the Pythagorean theorem and the definition of tangent

$$A = \sqrt{A_x^2 + A_y^2},$$
$$\tan\theta = \frac{A_y}{A_x}.$$

We can hence define algebraic addition of vectors such that

$$\vec{A} = \vec{A} + \vec{B} \iff C_x = A_x + B_x \text{ and } C_y = A_y + B_y.$$

Position, Velocity, and Acceleration

The **position vector** of a particle at a point (x, y) is a vector from the origin to the point

$$\vec{r} = x\hat{i} + y\hat{j}.$$

The **displacement** is the change in position

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

The **average velocity** in the time interval $\Delta t = t_2 - t_1$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}.$$

The **instantaneous velocity**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}.$$

The **average acceleration** in the time interval $\Delta t = t_2 - t_1$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}.$$

The **instantaneous acceleration**

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}.$$

Relative Velocity

Measurements of velocity depend on the reference frame of the observer. A reference frame is just a coordinate system.
Consider a particle of velocity $\vec{v}_{pA}$ relative to reference frame A, which has velocity $\vec{v}_{AB}$ relative to frame B, the velocity of the particle relative to frame B is then

$$\vec{v}_{pB} = \vec{v}_{pA} + \vec{v}_{AB}.$$

However, this relative-velocity realtion is only true when the velocities are small compared to the speed of light.

Projectile Motion

Consider an object projected with an initial velocity $\vec{v}_0$ at angle angle θ_0 with the horizontal surface. The components of the velocity are

$$v_{0x} = v_0 \cos\theta_0,$$

$$v_{0y} = v_0 \sin\theta_0.$$

In the absense of air resistance, the motion of the projectile is the superposition of a constant-velocity motion in the x-direction and a constant-acceleration in the y-direction.

$$a_x = 0,$$

$$a_y = -g.$$

Thus, the kinematics of one-dimensional motion can be applied

$$\Delta x = v_{0x} t,$$

$$v_y = v_{0y} - gt,$$

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2,$$

$$v_y^2 - v_{0y}^2 = -2g\Delta y.$$

We can also show that the path of the projectile is a parabola

$$y = x \tan\theta_0 - \frac{1}{2} \frac{g}{v_0^2 \cos 2\theta_0}.$$