Rotation

Physics Club

November 13, 2014

Basic Definitions

Radian

Unit of angular measure. By definition, the angle in radians is equal to the length subtended.
One radian is approximately $180^\circ/\pi = 57.30^\circ$.

Angular Displacement $\Delta\theta$

$$\Delta s = r\Delta\theta$$

Angular Velocity ω

$$v = r\omega$$

Angular Acceleration α

$$a_t = r\alpha$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

Angular Quantities

	Translational	Rotational
Displacement	Δx	$\Delta\theta$
Velocity	v	ω
Acceleration	a	α
Equation #1	$\Delta x = \bar{v}t$	$\Delta\theta = \bar{\omega}t$
Equation #2	$v = v_0 + at$	$\omega = \bar{\omega}t$
Equation #3	$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\omega = \omega_0 t + \frac{1}{2}\alpha t^2$
Equation #4	$\Delta x = v_0 t - \frac{1}{2}at^2$	$\Delta\omega = \omega_0 t - \frac{1}{2}\alpha t^2$
Equation #5	$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

The Right-Hand Rule

Angular quantities are vectors. Angular displacement, angular velocity, and angular acceleration are all vectors.

Use the right-hand rule to find the direction of the angular velocity ω vector.

The angular acceleration α vector is the same direction if the angular velocity ω is increasing with time, opposite if decreasing.

Rotational Kinetic Energy & Moment of Inertia

Rotational Kinetic Energy K

$$K = \sum \left(\frac{1}{2} m_i v_i \right) = \frac{1}{2} \sum \left(m_i r_i^2 \omega^2 \right) = \frac{1}{2} I \omega^2$$

Moment of Inertia I

$$I = \sum m_i r_i^2$$

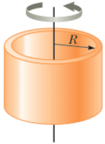
The Parallel-Axis Theorem

$$I = I_{\text{cm}} + Mh^2$$

Moments of Inertia

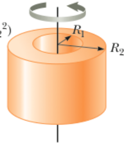
Hoop or thin cylindrical shell

$I_{CM} = MR^2$



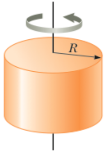
Hollow cylinder

$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$



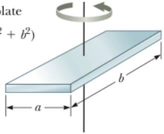
Solid cylinder or disk

$I_{CM} = \frac{1}{2}MR^2$



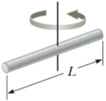
Rectangular plate

$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



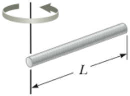
Long, thin rod with rotation axis through center

$I_{CM} = \frac{1}{12}ML^2$



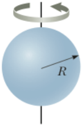
Long, thin rod with rotation axis through end

$I = \frac{1}{3}ML^2$



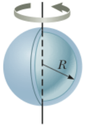
Solid sphere

$I_{CM} = \frac{2}{5}MR^2$



Thin spherical shell

$I_{CM} = \frac{2}{3}MR^2$



Newton's Second Law for Rotation

Torque τ

$$\tau = rF_t = rF \sin \phi = F\ell$$

Work W

$$\Delta W = F_t r \Delta\theta$$

Newton's Second Law for Rotation

$$\sum \tau_{\text{ext}} = I\alpha$$

Power

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau \Delta\theta}{\Delta t} = \tau\omega$$

Angular Momentum

Angular Momentum L

$$L = I\omega = r \times p$$

Conservation of Angular Momentum

L_{sys} is constant if the external torque on the system is 0.

Rolling

When rigid objects roll, there are two types of basic rolling: "with slipping" and "without slipping".

$$v_{\text{cm}} = R\omega$$

Everything Else

Gravitation, Static Equilibria, Fluids, Oscillations

Physics Club

December 4, 2014