Time limit: 50 minutes.
Instructions: For this test, you work in teams of eight to solve 15 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute the remainder when 2^{30} is divided by 1000.

2. Consider all right triangles with integer side lengths that form an arithmetic sequence. Compute the 2014th smallest perimeter of all such right triangles.

3. A segment of length 1 is drawn such that its endpoints lie on a unit circle, dividing the circle into two parts. Compute the area of the larger region.

4. A frog is hopping from $(0,0)$ to $(8,8)$. The frog can hop from (x,y) to either $(x+1,y)$ or $(x,y+1)$. The frog is only allowed to hop to point (x,y) if $|y-x| \leq 1$. Compute the number of distinct valid paths the frog can take.

5. Given a triangle ABC with integer side lengths, where BD is an angle bisector of $\angle ABC$, $AD = 4$, $DC = 6$, and D is on AC, compute the minimum possible perimeter of $\triangle ABC$.

6. Compute the largest integer N such that one can select N different positive integers, none of which is larger than 17, and no two of which share a common divisor greater than 1.

7. Eddy draws 6 cards from a standard 52-card deck. What is the probability that four of the cards that he draws have the same value?

8. Equilateral triangle DEF is inscribed inside equilateral triangle ABC such that DE is perpendicular to BC. Let x be the area of triangle ABC and y be the area of triangle DEF. Compute $\frac{x}{y}$.

9. Find the sum of all real numbers x such that $x^4 - 2x^3 + 3x^2 - 2x - 2014 = 0$.

10. Three real numbers x, y, and z are chosen independently and uniformly at random from the interval $[0,1]$. Compute the probability that x, y, and z can be the side lengths of a triangle.

11. In the following system of equations

$$|x+y| + |y| = |x-1| + |y-1| = 2,$$

find the sum of all possible x.

12. Find the last two digits of $\binom{200}{100}$. Express the answer as an integer between 0 and 99. (e.g. if the last two digits are 05, just write 5.)

13. Let α, β, γ be the three real roots of polynomial $x^3 - x^2 - 2x + 1 = 0$. Find all possible values of $\frac{\alpha}{\beta} + \frac{\beta}{\gamma} + \frac{\gamma}{\alpha}$.

14. Consider a round table on which 2014 people are seated. Suppose that the person at the head of the table receives a giant plate containing all the food for supper. He then serves himself and passes the plate either right or left with equal probability. Each person, upon receiving the plate, will serve himself if necessary and similarly pass the plate either left or right with equal probability. Compute the probability that you are served last if you are seated 2 seats away from the person at the head of the table.

15. A point is "bouncing" inside a unit equilateral triangle with vertices $(0,0)$, $(1,0)$, and $(1/2, \sqrt{3}/2)$. The point moves in straight lines inside the triangle and bounces elastically off an edge at an angle equal to the angle of incidence. Suppose that the point starts at the origin and begins motion in the direction of $(1,1)$. After the ball has traveled a cumulative distance of $30\sqrt{2}$, compute its distance from the origin.