1. Compute the number of three digit numbers such that all three digits are distinct and in descending order, and one of the digits is a 5.

   Answer: 36

   Solution: We split into three cases.

   First, suppose the first digit is a 5. Then the next two digits must be less than 5 and in descending order. There are $\binom{5}{2} = 10$ ways to choose them.

   Next, suppose the last digit is a 5. Then the first two digits must be greater than 5, so there are $\binom{4}{2} = 6$ choices.

   Finally, suppose the middle digit is 5. Then there are four choices for the first digit, and independently five choices for the last digit, giving 20 combinations.

   Hence, there are a total of $10 + 6 + 20 = \boxed{36}$ such numbers.

2. Compute the number of positive integers less than or equal to 10000 which are relatively prime to 2014.

   Answer: 4648

   Solution: We know $2014 = 2 \cdot 19 \cdot 53$. Hence, in general, the number of positive integers less than or equal to $N = 10000$ relatively prime to 2014 is, by inclusion-exclusion, $\lfloor \frac{N}{1} \rfloor - \lfloor \frac{N}{2} \rfloor - \lfloor \frac{N}{19} \rfloor - \lfloor \frac{N}{53} \rfloor + \lfloor \frac{N}{2\cdot 19} \rfloor + \lfloor \frac{N}{2\cdot 53} \rfloor + \lfloor \frac{N}{19\cdot 53} \rfloor - \lfloor \frac{N}{2\cdot 19\cdot 53} \rfloor = 10000 - 5000 - 526 - 188 + 263 + 94 + 9 - 4 = \boxed{4648}$.

3. A robot is standing on the bottom left vertex $(0, 0)$ of a 5×5 grid, and wants to go to $(5, 5)$, only moving to the right $(a, b) \mapsto (a + 1, b)$ or upward $(a, b) \mapsto (a, b + 1)$. However this robot is not programmed perfectly, and sometimes takes the upper-left diagonal path $(a, b) \mapsto (a-1, b+1)$. As the grid is surrounded by walls, the robot cannot go outside the region $0 \le a, b \le 5$. Supposing that the robot takes the diagonal path exactly once, compute the number of different routes the robot can take.

   Answer: 1650

   Solution: We compute the general answer for an $n \times n$ grid. Let $P = (x, y)$ be the point where the robot took the diagonal path. As $(x - 1, y + 1)$ should be also inside the grid, we have $1 \le x \le n$ and $0 \le y \le n - 1$. Let l_1 the route the robot took from $(0, 0)$ to (x, y), and let l_2 the route robot took from $(x - 1, y + 1)$ to (n, n).

   We shift the whole l_2 down by one square and to the right by one square so that it becomes the grid path from (x, y) to $(n + 1, n - 1)$. Then consider the concatenation l' of l_1 and l_2, which becomes the grid path from $(0, 0)$ to $(n + 1, n - 1)$. It is easy to see that the correspondence from the original path to the pair $(l', P \in l')$ is bijective, provided that $P = (x, y)$ satisfies the restriction $1 \le x \le n, 0 \le y \le n - 1$.

   Thus we need to count the number of pairs

   $$\begin{aligned} &\{(l', P) : l' \text{ is a grid path } (0,0) \to (n+1, n-1), \\ &\ P = (x, y) \text{ is a point on } l' \text{ satisfying } 1 \le x \le n, \le y \le n-1\}. \end{aligned}$$

   If we do not pose limits on the coordinate of P, there are a total of $\binom{2n}{n-1} \cdot (2n + 1)$ pairs, as there are $(2n+1)$ points on l'. We should subtract the number of pairs with P not in the region $1 \le x \le n, 0 \le y \le n - 1$. Those P are either $(0, k)$ or $(n + 1, k)$ for $0 \le k \le n - 1$. For $P = (0, k)$, there is a unique path from $(0, 0)$ to P and $\binom{2n-k}{n+1}$ paths from P to $(n + 1, n - 1)$.

For $P = (n+1, k)$, there are $\binom{n+1+k}{n+1}$ paths from $(0,0)$ to P, and a unique path from P to $(n+1, n-1)$. Thus the number of subtracted pairs is

$$\sum_{k=0}^{n-1}\left(\binom{2n-k}{n+1} + \binom{n+1+k}{n+1}\right) = 2\binom{2n+1}{n+2}.$$

Therefore the answer is

$$\begin{aligned}
&(2n+1)\binom{2n}{n-1} - 2\binom{2n+1}{n+2} \\
&= \frac{(n+2)(2n+1)!}{(n-1)!(n+2)!} - \frac{2(2n+1)!}{(n-1)!(n+2)!} \\
&= n\binom{2n+1}{n-1},
\end{aligned}$$

and for this problem, where $n = 5$, this value becomes $\boxed{1650}$.