Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Consider a square of side length 1 and erect equilateral triangles of side length 1 on all four sides of the square such that one triangle lies inside the square and the remaining three lie outside. Going clockwise around the square, let A, B, C, D be the circumcenters of the four equilateral triangles. Compute the area of $ABCD$.

2. Let ABC be a triangle with sides $AB = 19$, $BC = 21$ and $AC = 20$. Let ω be the incircle of ABC with center I. Extend BI so that it intersects AC at E. If ω is tangent to AC at the point D, then compute the length of DE.

3. Compute the perimeter of the triangle that has area $3 - \sqrt{3}$ and angles $45°$, $60°$, and $75°$.

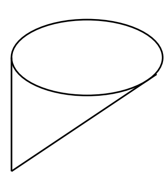
4. Consider a square $ABCD$ with side length 4 and label the midpoint of the side BC as M. Let X be the point along AM obtained by dropping a perpendicular from D onto AM. Compute the product of the lengths XC and MD.

5. Consider a triangle ABC with $AB = 4$, $BC = 3$, and $AC = 2$. Let D be the midpoint of line BC. Find the length of AD.

6. Consider a circle of radius 4 with center O_1, a circle of radius 2 with center O_2 that lies on the circumference of circle O_1, and a circle of radius 1 with center O_3 that lies on the circumference of circle O_2. The centers of the circle are collinear in the order O_1, O_2, O_3. Let A be a point of intersection of circles O_1 and O_2 and B be a point of intersection of circles O_2 and O_3 such that A and B lie on the same semicircle of O_2. Compute the length of AB.

7. Let $ABCD$ be a square piece of paper with side length 4. Let E be a point on AB such that $AE = 3$ and let F be a point on CD such that $DF = 1$. Now, fold $AEFD$ over the line EF. Compute the area of the resulting shape.

8. Moor made a lopsided ice cream cone. It turned out to be an oblique circular cone with the vertex directly above the perimeter of the base (see diagram below). The height and base radius are both of length 1. Compute the radius of the largest spherical scoop of ice cream that it can hold such that at least 50% of the scoop's volume lies inside the cone.



9. We have squares $ABCD$ and $EFGH$. Square $ABCD$ has points with coordinates $A = (1, 1, -1)$, $B = (1, -1, -1)$, $C = (-1, -1, -1)$ and $D = (-1, 1, -1)$. Square $EFGH$ has points with coordinates $E = (\sqrt{2}, 0, 1)$, $F = (0, -\sqrt{2}, 1)$, $G = (-\sqrt{2}, 0, 1)$, and $H = (0, \sqrt{2}, 1)$. Consider the solid formed by joining point A to H and E, point B to E and F, point C to F and G, and point D to G and H. Compute the volume of this solid.

10. In a convex quadrilateral $ABCD$ we are given that $\angle CAD = 10°$, $\angle DBC = 20°$, $\angle BAD = 40°$, $\angle ABC = 50°$. Compute angle BDC.