Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute the smallest positive integer that is 3 more than a multiple of 5, and twice a multiple of 6.

2. Compute the number of integers between 1 and 100, inclusive, that have an odd number of factors. Note that 1 and 4 are the first two such numbers.

3. A mouse is playing a game of mouse hopscotch. In mouse hopscotch there is a straight line of 11 squares, and starting on the first square the mouse must reach the last square by jumping forward 1, 2, or 3 squares at a time (so in particular the mouse's first jump can be to the second, third, or fourth square). The mouse cannot jump past the last square. Compute the number of ways there are to complete mouse hopscotch.

4. Cynthia and Lynnelle are collaborating on a problem set. Over a 24-hour period, Cynthia and Lynnelle each independently pick a random, contiguous 6-hour interval to work on the problem set. Compute the probability that Cynthia and Lynnelle work on the problem set during completely disjoint intervals of time.

5. Compute the smallest 9-digit number containing all the digits 1 to 9 that is divisible by 99.

6. Consider 7 points on a circle. Compute the number of ways there are to draw chords between pairs of points such that two chords never intersect and one point can only belong to one chord. It is acceptable to draw no chords.

7. Two math students play a game with k sticks. Alternating turns, each one chooses a number from the set $\{1, 3, 4\}$ and removes exactly that number of sticks from the pile (so if the pile only has 2 sticks remaining the next player *must* take 1). The winner is the player who takes the last stick. For $1 \leq k \leq 100$, determine the number of cases in which the first player can guarantee that he will win.

8. Nick has a 3×3 grid and wants to color each square in the grid one of three colors such that no two squares that are adjacent horizontally or vertically are the same color. Compute the number of distinct grids that Nick can create.

9. Compute how many permutations of the numbers $1, 2, \ldots, 8$ have no adjacent numbers that sum to 9.

10. Find the remainder when $(1^2 + 1)(2^2 + 1)(3^2 + 1) \cdots (42^2 + 1)$ is divided by 43. Your answer should be an integer between 0 and 42.