Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. A college math class has N teaching assistants. It takes the teaching assistants 5 hours to grade homework assignments. One day, another teaching assistant joins them in grading and all homework assignments take only 4 hours to grade. Assuming everyone did the same amount of work, compute the number of hours it would take for 1 teaching assistant to grade all the homework assignments.

2. Let a and b be positive integers such that $a > b$ and the difference between $a^2 + b$ and $a + b^2$ is prime. Compute all possible pairs (a, b).

3. Compute all prime numbers p such that $8p + 1$ is a perfect square.

4. Let $f(x) = \sum_{i=1}^{2014} |x - i|$. Compute the length of the longest interval $[a, b]$ such that $f(x)$ is constant on that interval.

5. A positive integer k is 2014-ambiguous if the quadratics $x^2 + kx + 2014$ and $x^2 + kx - 2014$ both have two integer roots. Compute the number of integers which are 2014-ambiguous.

6. Compute $\cos\left(\frac{\pi}{9}\right) - \cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{3\pi}{9}\right) - \cos\left(\frac{4\pi}{9}\right)$.

7. $f(x)$ is a quartic polynomial with a leading coefficient of 1 where $f(2) = 4$, $f(3) = 9$, $f(4) = 16$, and $f(5) = 25$. Compute $f(8)$.

8. Consider the recurrence relation
$$a_{n+3} = \frac{a_{n+2}a_{n+1} - 2}{a_n}$$
with initial condition $(a_0, a_1, a_2) = (1, 2, 5)$. Let $b_n = a_{2n}$ for nonnegative integral n. It turns out that $b_{n+2} + xb_{n+1} + yb_n = 0$ for some pair of real numbers (x, y). Compute (x, y).

9. A sequence $\{a_n\}_{n\geq 0}$ obeys the recurrence $a_n = 1 + a_{n-1} + \alpha a_{n-2}$ for all $n \geq 2$ and for some $\alpha > 0$. Given that $a_0 = 1$ and $a_1 = 2$, compute the value of α for which
$$\sum_{n=0}^{\infty} \frac{a_n}{2^n} = 10$$

10. Let $p(x) = c_1 + c_2 \cdot 2^x + c_3 \cdot 3^x + c_4 \cdot 5^x + c_5 \cdot 8^x$. Given that $p(k) = k$ for $k = 1, 2, 3, 4, 5$, compute $p(6)$.