Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Alex gets 8 points on an exam, while his friend gets 3 times as many points as Alex. What is the average of their scores?

2. Sally rolls an 8-sided die with faces numbered 1 through 8. Compute the probability that she gets a power of 2.

3. Boris is driving on a remote highway. His car's odometer reads 24942 km, which Boris notices is a palindromic number, meaning it is not changed when it is reversed. "Hm," he thinks, "it should be a long time before I see that again." But it takes only 1 hour for the odometer to once again show a palindromic number! How fast is Boris driving in km/h?

4. If Bobby's age is increased by 6, it's a number with an integral (positive) square root. If his age is decreased by 6, it's that square root. How old is Bobby?

5. Screws are sold in packs of 10 and 12. Harry and Sam independently go to the hardware store, and by coincidence each of them buys exactly k screws. However, the number of packs of screws Harry buys is different than the number of packs Sam buys. What is the smallest possible value of k?

6. In triangle ABC, we have that $AB = AC$, $BC = 16$, and that the area of $\triangle ABC$ is 120. Compute the length of AB.

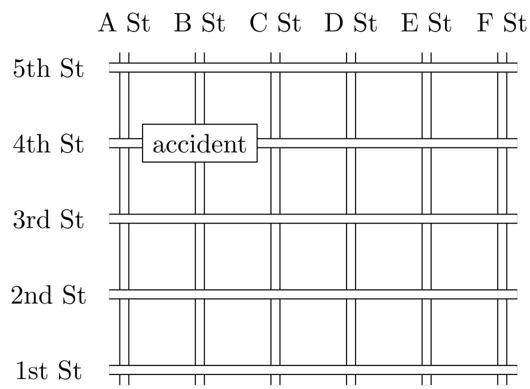
7. Ben works quickly on his homework, but tires quickly. The first problem takes him 1 minute to solve, and the second problem takes him 2 minutes to solve. It takes him N minutes to solve problem N on his homework. If he works for an hour on his homework, compute the maximum number of problems he can solve.

8. George and two of his friends go to a famous jiaozi restaurant, which serves only two kinds of jiaozi: pork jiaozi, and vegetable jiaozi. Each person orders exactly 15 jiaozi. How many different ways could the three of them order? Two ways of ordering are different if one person orders a different number of pork jiaozi in both orders.

9. The operation $\oplus$, called "reciprocal sum," is useful in many areas of physics. If we say $x = a \oplus b$, this means that x is the solution to
$$\frac{1}{x} = \frac{1}{a} + \frac{1}{b}\ .$$
Compute $4 \oplus 2 \oplus 4 \oplus 3 \oplus 4 \oplus 4 \oplus 2 \oplus 3 \oplus 2 \oplus 4 \oplus 4 \oplus 3$.

10. Find the area of the smallest possible square that contains the points $(2, -1)$ and $(4, 4)$.

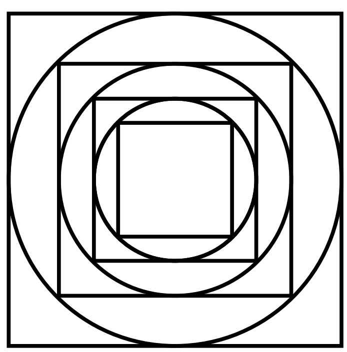
11. Mr. Ambulando is at the intersection of 5^{th} and A St, and needs to walk to the intersection of 1^{st} and F St. There's an accident at the intersection of 4^{th} and B St, which he'd like to avoid.



Given that Mr. Ambulando wants to walk the shortest distance possible, how many different routes through downtown can he take?

12. Consider a rectangular tiled room with dimensions $m \times n$, where the tiles are 1×1 in size. Compute all ordered pairs (m, n) with $m \leq n$ such that the number of tiles on the perimeter is equal to the number of tiles in the interior (i.e. not on the perimeter).

13. Square S_1 is inscribed inside circle C_1, which is inscribed inside square S_2, which is inscribed inside circle C_2, which is inscribed inside square S_3, which is inscribed inside circle C_3, which is inscribed inside square S_4.



Let a be the side length of S_4, and let b be the side length of S_1. What is $\frac{a}{b}$?

14. Patricia has a rectangular painting that she wishes to frame. The frame must also be rectangular and will extend 3 cm outward from each of the four sides of the painting. When the painting is framed, the area of the frame not covered by the painting is 108 cm^2. What is the perimeter of the painting alone (without the frame)?

15. Antoine, Benoît, Claude, Didier, Étienne, and Françoise go to the cinéma together to see a movie. The six of them want to sit in a single row of six seats. But Antoine, Benoît, and Claude are mortal enemies and refuse to sit next to either of the other two. How many different arrangements are possible?

16. Compute the number of geometric sequences of length 3 where each number is a positive integer no larger than 10.

17. Given that the line $y = mx + k$ intersects the parabola $y = ax^2 + bx + c$ at two points, compute the product of the two x-coordinates of these points in terms of a, b, c, k, and m.

18. A two-digit positive integer is *primeable* if one of its digits can be deleted to produce a prime number. A two-digit positive integer that is prime, yet not primeable, is *unripe*. Compute the total number of unripe integers.

19. Given that $f(x) + 2f(4 - x) = x + 8$, compute $f(16)$.

20. $ABCD$ is a parallelogram, and circle S (with radius 2) is inscribed inside $ABCD$ such that S is tangent to all four line segments AB, BC, CD, and DA. One of the internal angles of the parallelogram is 60°. What is the maximum possible area of $ABCD$?

21. A bitstring of length ℓ is a sequence of ℓ 0's or 1's in a row. How many bitstrings of length 2014 have at least 2012 consecutive 0's or 1's?

22. Apples cost 2 dollars. Bananas cost 3 dollars. Oranges cost 5 dollars. Compute the number of distinct baskets of fruit such that there are 100 pieces of fruit and the basket costs 300 dollars. Two baskets are distinct if and only if, for some type of fruit, the two baskets have differing amounts of that fruit.

23. Let triangle ABC have side lengths $AB = 11$, $BC = 7$, and $AC = 12$. Let D be a point on AC and E be a point on AB such that $\angle CDE = 90^\circ$ and the area of triangle CDE is maximized. Find the area of triangle CDE.

24. It's pouring down rain, and the amount of rain hitting point (x, y) is given by

$$f(x, y) = |x^3 + 2x^2y - 5xy^2 - 6y^3|,$$

If you start at the origin $(0, 0)$, find all the possibilities for m such that $y = mx$ is a straight line along which you could walk without any rain falling on you.

25. 300 couples (one man, one woman) are invited to a party. Everyone at the party either always tells the truth or always lies. Exactly $2/3$ of the men say their partner always tells the truth and the remaining $1/3$ say their partner always lies. Exactly $2/3$ of the women say their partner is the same type as themselves and the remaining $1/3$ say their partner is different. Find a, the maximum possible number of people who tell the truth, and b, the minimum possible number of people who tell the truth. Express your answer as (a, b).