

Comment: This is version 0.2, ready for earliest testsolving. (Moor only?)

- LJ15** 1. Patricia has a rectangular painting that she wishes to frame. The frame must also be rectangular and will extend 3 cm outward from each of the four sides of the painting. The area of the frame alone (without the painting) is 108 cm^2 . What is the perimeter of the painting (without the frame)?

Answer: 24 cm

Solution: Let the painting have width w and height h . The frame's area is $(w + 6)(h + 6) - wh = 108$. The perimeter we want to find is $2(w + h)$. We expand $(w + 6)(h + 6) - wh = wh + 6w + 6h + 36 - wh = 3(2(w + h)) + 36 = 108$, so $2(w + h) = \frac{108 - 36}{3} = \boxed{24 \text{ cm}}$.

- LJ17** 2. Antoine, Benoît, Claude, Didier, Étienne, and Françoise go to the cinéma together to see a movie. The six of them want to sit in a single row of six seats. But Antoine, Benoît, and Claude are mortal enemies and refuse to sit next to either of the other two. How many different arrangements are possible?

Answer: 144

Solution: In order not to sit next to each other, A, B, and C must take seats $\{1, 3, 5\}$, $\{1, 3, 6\}$, $\{1, 4, 6\}$, or $\{2, 4, 6\}$ in any order ($3!$). D, E, and F take the other three seats in any order (also $3!$). So there are $4 \times 3! \times 3! = \boxed{144}$ permutations.

- LJ16** 3. $ABCD$ is a parallelogram, and circle S (with radius 2) is inscribed inside $ABCD$ such that S is tangent to all four line segments AB , BC , CD , and DA . One of the internal angles of the parallelogram is 60° . What is the maximum possible area of $ABCD$?

Answer: $\frac{32}{\sqrt{3}}$

Solution: All tangential quadrilaterals that are parallelograms must also be rhombuses, so $ABCD$ is a rhombus, with two internal angles of 60° and two of 120° . This fully constrains the rhombus (so no maximum needs to be found, just the only solution). The rhombus can be split into four 30-60-90 triangles with altitude 2, which will have legs of 4 and $\frac{4}{\sqrt{3}}$. So the total area

$$\text{is } 4\left(\frac{1}{2} \times 4 \times \frac{4}{\sqrt{3}}\right) = \boxed{\frac{32}{\sqrt{3}}}.$$