COMPSCI 1JC3

Introduction to Computational Thinking

Fall 2017

Assignment 1

Dr. William M. Farmer

McMaster University

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The purpose of Assignment 1 is to write a program in Haskell that computes a solution of a cubic equation. The requirements for Assignment 1 and for an extra credit assignment, Assignment 1 Extra Credit, are given below. You are required to do Assignment 1, but Assignment 1 Extra Credit is optional. Please submit Assignment 1 as a single `.hs` file to the Assignment 1 folder on Avenue under Assessments/Assignments. If you choose to do Assignment 1 Extra Credit for extra marks, please submit it also as a single `.hs` file to the Assignment 1 Extra Credit folder on Avenue in the same place. Both Assignment 1 and Assignment 1 Extra Credit are due **September 29, 2017 before midnight.** Assignment 1 is worth 3% of your final grade, while Assignment 1 Extra Credit is worth 2 extra percentage points.

Although you are allowed to receive help from the instructional staff and other students, your submitted program must be your own work. Copying will be treated as academic dishonesty!

1 Background

Recall from high school mathematics that a quadratic equation

$$ax^2 + bx + c = 0,$$

where a, b, c are real numbers with $a \neq 0$, has two solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

If $b^2 - 4ac < 0$, the two solutions are non-real complex numbers; if $b^2 - 4ac = 0$, the two solutions are real numbers equal to each other; and if $b^2 - 4ac > 0$, the two solutions are distinct real numbers.

Less well known is that a cubic equation

$$ax^3 + bx^2 + cx + d = 0,$$

where a, b, c, d are real numbers with $a \neq 0$, has three solutions:

$$x_1 = S + T - \frac{b}{3a}.$$

$$x_2 = -\frac{S+T}{2} - \frac{b}{3a} + \frac{i\sqrt{3}}{2}(S - T).$$

$$x_3 = -\frac{S+T}{2} - \frac{b}{3a} - \frac{i\sqrt{3}}{2}(S - T).$$

where:

$$Q = \frac{3ac - b^2}{9a^2}.$$

$$R = \frac{9abc - 27a^2d - 2b^3}{54a^3}.$$

$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}}.$$

$$T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}.$$

The derivation of the solutions x_1, x_2, and x_3 is elegantly presented in the ProofWiki article "Cardano's Formula" found at

`https://proofwiki.org/wiki/Cardano's_Formula`.

x_1 is always a real number. If $Q^3 + R^2 < 0$, the three solutions are distinct real numbers; if $Q^3 + R^2 = 0$, the three solutions are real numbers and at least two are equal; and if $Q^3 + R^2 > 0$, the three solutions are one real number and two non-real complex numbers.

Historical note: The general solution for cubic equations was first devised by Niccolò Fontana Tartaglia (1499–1557) in 1530 and first published by Gerolamo Cardano (1501–1576) in 1545. The general solution for quartic equations (of degree four) was discovered by Lodovico Ferrari (1522–1565) in 1540. Niels Henrik Abel (1802–1829) gave the first complete proof in 1824 that general solutions do not exist for equations of degree five or greater.

2 Assignment 1

The purpose of this assignment is to compute an approximation of a real number solution of a cubic equation with floating point coefficients when $Q^3 + R^2 \geq 0$.

2.1 Requirements

1. The name of your Haskell file is `Assign_1_YourMacID.hs` where *YourMacID* is your actual MacID.

2. Your name, MacID, the date, and "Assignment 1" are given in comments at the top of your file.

3. The file includes a function named `cubicQ` of type `Float -> Float -> Float -> Float` that computes Q from a, b, and c. The file also includes a function named `cubicR` of type `Float -> Float -> Float -> Float -> Float` that R computes from a, b, c, and d.

4. The file includes a function named `cubicS` of type `Float -> Float -> Float` that computes S from q and r. The file also includes a function named `cubicT` of type `Float -> Float -> Float` that computes T from q and r. Using the `**` operator, define your own cubic root function of type `Float -> Float` that finds cubic roots of both negative and positive numbers. `cubicS` and `cubicT` should use only floating point operations and no complex number operations.

5. The file includes a function named `cubicRealSolution` of type `Float -> Float -> Float -> Float -> Float` that computes the solution

   $$x_1 = S + T - \frac{b}{3a}$$

   using `cubicQ`, `cubicR`, `cubicS`, and `cubicT`.

6. If the functions `cubicS`, `cubicT`, and `cubicRealSolution` are implemented properly, they should be undefined when $Q^3 + R^2 < 0$ since then the value of $\sqrt{Q^3 + R^2}$ is not a real number. That is, these functions should return `NaN` — which stands for "not a number"— when $Q^3 + R^2 < 0$.

   For example, if the cubic equation is

   $$x^3 - 3x = 0,$$

   then by factoring it is clear that $x_1 = 0$, $x_2 = \sqrt{3}$, and $x_3 = -\sqrt{3}$. However, $Q^3 + R^2 = -1$, so

   ```
   cubicRealSolution 1 0 (-3) 0
   ```

   should return `NaN` instead of 0.

7. Your file loads successfully into GHCi and all of your functions perform correctly when $Q^3 + R^2 \geq 0$.

2.2 Testing

You should test on your own all of the functions in the file you submit. You will be required to submit formal test plans for all subsequent assignments.

3 Assignment 1 Extra Credit

The purpose of this extra credit assignment is to compute an approximation of a real number solution of a cubic equation with floating point coefficients for any value of $Q^3 + R^2$. This is a very challenging assignment; do not be discouraged if you cannot complete it.

3.1 Requirements

1. The name of your Haskell file is `Assign_1_ExtraCredit_`*`YourMacID`*`.hs` where *YourMacID* is your actual MacID.

2. Your name, MacID, the date, and "Assignment 1 Extra Credit" are given in comments at the top of your file.

3. The file includes the same functions `cubicQ` and `cubicR` as in the file for Assignment 1.

4. The file includes two functions `cubicComplexS` and `cubicComplexT` of type `Float -> Float -> Complex` where `Complex` is some type of complex numbers whose real and imaginary parts are of type `Float`. You can either use a complex number type from a Haskell library or define your own. These two functions behave exactly like `cubicS` and `cubicT` when $Q^3 + R^2 \geq 0$, but they return a complex number when $Q^3 + R^2 < 0$.

5. The file includes `cubicRealSolution` of type `Float -> Float -> Float -> Float -> Float`. It produces real number solutions similarly to the function `cubicRealSolution` in the file for Assignment 1 when $Q^3 + R^2 \geq 0$, but it also produces real number solutions when $Q^3 + R^2 < 0$ using complex number arithmetic.

6. Your file successfully loads into GHCi and all of your functions perform correctly.

3.2 Testing

You you test on your own all of the functions in the file you submit. You will be required to submit formal test plans for all subsequent assignments.