Title: Application of hollow wooden cylinders to investigate the relationship between moment of inertia and translational acceleration.

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Abstract:

This study investigates how the moment of inertia of hollow cylinders affects their translational acceleration under constant torque. The experiment used hollow wooden cylinders with varying inner radii (0.2 cm to 2.7 cm) and a constant outer radius of 0.35 cm to quantify the effect of wall thickness on rolling motion. A Vernier Go Direct motion detector, sampling at 100 Hz, measured the cylinders' acceleration down a 6-degree inclined plane. The moment of inertia was calculated by assuming uniform mass distribution and integrating over volume, and Newton's second law was used to derive an acceleration equation in terms of internal radius. The results show an inverse relationship between internal radius and acceleration. The cylinder with a 0.2 cm internal radius had the highest average acceleration (0.70 ms^{-2}), while the cylinder with a 2.7 cm internal radius had the lowest (0.57 ms^{-2}). The relationship was linearized by plotting 1/acceleration against the square of the internal radius, and the least squares method was used to generate a line of best fit. The discrepancy between the experimental slope and the expected value was 6.48%, while the y-intercept discrepancy was 2.03%. Instrumental uncertainty and experimental uncertainties was measured and introduced back to the regression equation of least squares. Results show instrumental uncertainty accounted for 71% of the total uncertainty in slope, indicating high precision. This experiment is accurate and easily replicable with common equipment, making it an effective demonstration for pedagogy.

Introduction

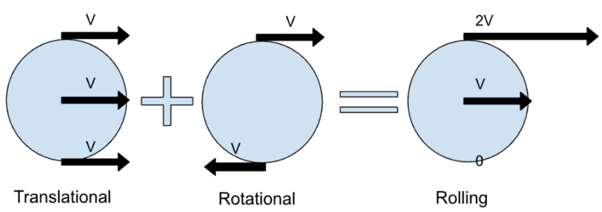
The rotation of a solid body is a classical model in rigid body mechanics. The body's angular acceleration, and therefore translational acceleration, is influenced by moment of inertia, a measure of how resistant a body is to changes in its rotational motion. This topic can range from simple 2D shapes to complicated 3D shapes that require extensive physics knowledge.

This topic is significant because of various real world implications. In mechanical engineering, the moment of inertia is calculated for different components to determine how a design reacts under torque. This can be crucial in the safety of vehicles, or stability of buildings. Personally, I'm an avid cyclist, and the moment of inertia of a bicycle wheel can have a great impact on cycling performance. I would choose wheels with this in mind to maximize my acceleration.

I chose the cylinder for investigation because it is radially symmetrical and therefore simplifies the process of calculation. By changing the thickness of the cylinder, I can modify its moment of inertia, and therefore create observable effects on its acceleration. This will be measured through a motion detector.

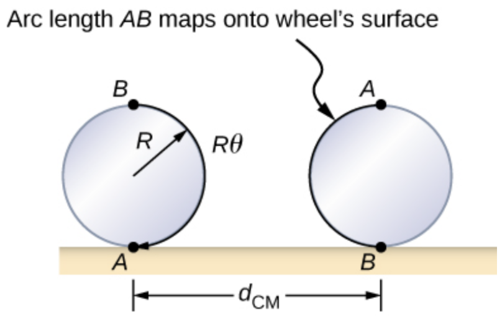
Background

First, we shall define rolling motion.
Rolling motion is a combination of translational and rotational motion.
Consider an object with radius (r) rolling with velocity (v) without slipping.



The torque is applied on the object's contact point, opposite to the direction of velocity. This results in a velocity of zero on the object's contact point, and a tangential velocity of 2v on the opposite end. The centre is not affected by rotation, therefore maintains the velocity v.

During this process, the displacement of the centre(s) is equivalent to displacement in arc length(l). We can picture this by straightening out the arc and seeing it as the path travelled. Since we know the arc length equation, displacement can be written as



$$s = l = \theta \cdot r$$

By the same notion, translational velocity is

$v = \omega \cdot r$

And acceleration is

$a = \alpha \cdot r$

Second, we shall define the moment of inertia(I).
Moment of inertia(I), is defined as angular momentum(L) over angular velocity(ω)

$$I = \frac{L}{\omega}$$

Substituting the equations for angular momentum and angular velocity we have

$$I = \frac{mvr}{\frac{v}{r}} = mr^2$$

The equations L and ω are based on the assumption that m is a point mass, always a distance r from the centre of rotation. However, it is not for solid bodies, where mass is distributed throughout the volume. Each point of mass has a different distance r to the centre, and therefore a different I. To find the moment of inertia of a solid body, a summation should be found.

$$\sum_i = m_i r_i^2$$

If the mass is evenly distributed in the solid and divisible into infinitely small pieces(dm), then we can solve for this equation using integration

$$I_{total} = \int r^2 dm$$

To integrate this function, the differential of mass(dm) should be written as the differential of radius (dr). Since we know

$$m = \rho \cdot V$$

Then we can write (dm) as

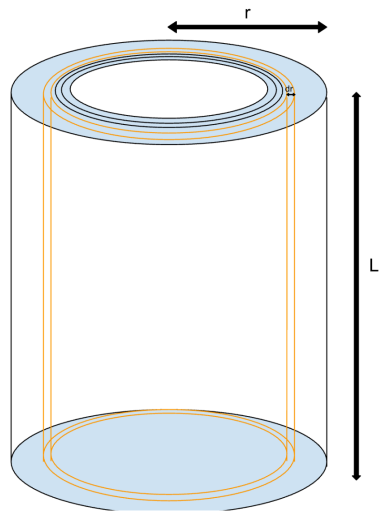
$$dm = \rho \cdot dV$$

We can interpret dV as one of the infinitely thin shells that form the hollow cylinder. The volume of this thin shell would be its length(l) multiplied by its circumference(2πr) multiplied by its thickness(dr), leaving us with

$$dv = 2\pi rL \cdot dr$$

And dm would be

$$dm = \rho 2\pi rL \cdot dr$$



Substituting this back to our equation for I_{total} yields:

$$I_{total} = \int \rho 2\pi L r^3 dr$$

The lower boundary for integration is the cylinders inner radius (r1) while the outer boundary outer radius (r). This is because there is only mass between the two radii. The centre is hollow and therefore has no mass.T

$$I_{total} = \int_{r_1}^{r_2} \rho 2\pi L r^3 dr$$

$$= 2\pi\rho L \left[\frac{1}{4}r^4\right]_{r_1}^{r}$$

$$= 2\pi\rho L(\frac{1}{4}r^4 - \frac{1}{4}r_1^4)$$

$$= \frac{1}{2}\pi\rho L(r^4 - r_1^4)$$

We also know that the volume of a hollow cylinder is $\pi L(r^2 - r_1^2)$, so we can write the density as

$$\rho = \frac{M}{\pi L(r^2 - r_1^2)}$$

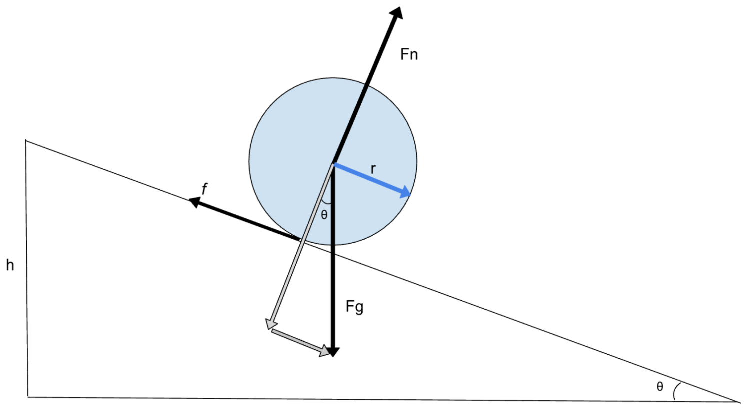
Substituting this back to the original equation gives us

$$I_{total} = \frac{1}{2}\pi L \frac{M}{\pi L(r^2 - r_1^2)}(r^4 - r_1^4)$$

$$= \frac{1}{2}\pi L \frac{M}{\pi L(r^2 - r_1^2)}(r^2 - r_1^2)(r^2 + r_1^2)$$

$$= \frac{1}{2}M(r^2 + r_1^2)$$

Third, we shall discover the relationship between rolling and moment of inertia. Consider a cylinder rolling down an inclined plane as indicated below.



Notice that the source of torque is the friction on the plane. Therefor, we can write

$\tau_{net} = F \cdot R = f \cdot r$

We also have the equation

$$\tau_{net} = I\alpha$$

Equating the two yields

$$f \cdot r = I\alpha$$

Since $a = \alpha \cdot r$, we can rewrite α as

$$f \cdot r = I \cdot \frac{a}{r}$$

Solving for f gives us

$$f = \frac{I \cdot a}{r^2}$$

We can list a second equation in translational motion, where the tangential force from gravity subtracted by frictional force is equivalent to the object's tangential force.

$$M \cdot g \cdot sin\theta - f = ma$$

Substituting the first equation for f into the second equation gives us

$$M \cdot g \cdot sin\theta - \frac{I \cdot a}{r^2} = ma$$

Now solving for acceleration a gives us

$$r^2 \cdot M \cdot g \cdot sin\theta - I \cdot a = ma \cdot r^2$$

$$r^2 \cdot M \cdot g \cdot sin\theta = ma \cdot r^2 + I \cdot a$$

$$r^2 \cdot M \cdot g \cdot sin\theta = a(mr^2 + I)$$

$$a = \frac{r^2 \cdot M \cdot g \cdot sin\theta}{mr^2 + I}$$

Finally, let us put all of our calculations together to find the final equation. Substituting the equation for I into the equation for a gives

$$a = \frac{r^2 \cdot M \cdot g \cdot sin\theta}{Mr^2 + \frac{1}{2}M(r^2 + r_1^2)}$$

$$= \frac{r^2 \cdot M \cdot g \cdot sin\theta}{\frac{3}{2}Mr^2 + \frac{1}{2}Mr_1^2}$$

$$= \frac{2r^2 \cdot g \cdot sin\theta}{3r^2 + r_1^2}$$

For our experiment, the outer radius r2, gravitational acceleration g, angle θ are all constants at 0.035m, 9.81ms^{-2}, and 6° respectively. We can substitute the values for the constants to simplify our equation.

$$a = \frac{2 \cdot (0.035)^2 \cdot 9.81 \cdot sin(6)}{3 \cdot (0.035)^2 + r_1^2}$$

$$= \frac{0.0025}{0.0037 + r_1^2}$$

To linearize a graph, we should manipulate the equation in the form of y=mx+b. We will do this by inverting the two sides first.

$$\frac{1}{a} = \frac{0.0037 + r_1^2}{0.0025}$$

Then we perform devision on the numerators to isolate the x value

$$\frac{1}{a} = \frac{0.0037}{0.0025} + \frac{r_1^2}{0.0025}\cdot$$

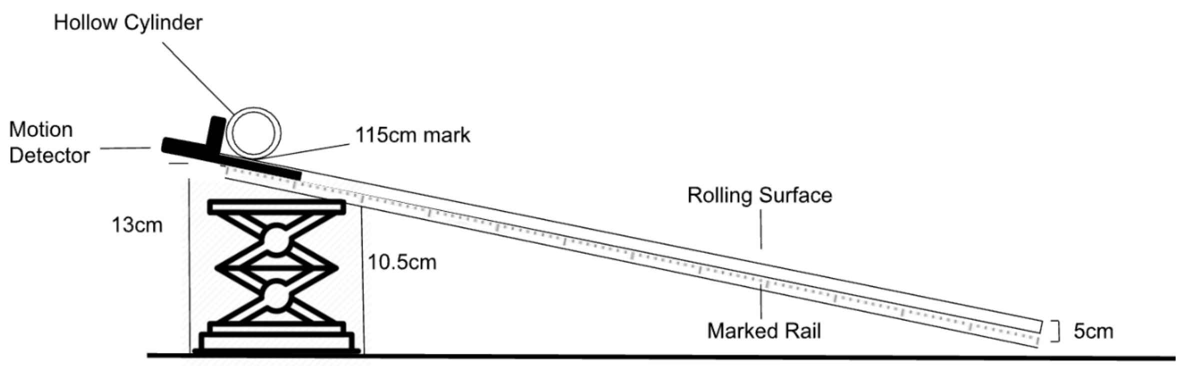
$$= 1.48 + 400r_1^2$$

Our linearization is complete. $\frac{1}{a}$ will be set as the y value, and r_1^2 as the x value. From this equation, the expected slope is 400, and the y intercept is 1.48

Apparatus

Equipment	Quantity	Uncertainty	Purpose
Vernier Motion Detector	1	±0.001 m/s	To track the change in velocity
1 m ruler	1	0.5mm	To measure the start point of rolling.
Iphone protractor	1	0.1°	To measure the incline of plane.
Lifting Jack	1	NA	To adjust the degree of incline of plane.
1 m Padded Plane	2	NA	Frictional platform that allows rolling without slipping
Hollow wood cylinder (7cm OD, 2.7CM ID)	1	NA	Test object
Hollow wood cylinder (7cm OD, 2.2CM ID)	1	NA	Test object
Hollow wood cylinder (7cm OD, 1.7CM ID)	1	NA	Test object
Hollow wood cylinder (7cm OD, 1.2CM ID)	1	NA	Test object
Hollow wood cylinder (7cm OD, 0.7CM ID)	1	NA	Test object
Hollow wood cylinder (7cm OD, 0.2CM ID)	1	NA	Test object

Diagram



Variables

	Range/quantity	Units	Method for changing/ measuring
IV	Six variables with the internal diameter of 0.005, 0.015, 0.025, 0.035, 0.045, 0.055	Meters(m)	Switching cylinders with different internal diameters
DV	Measurement for five values with five trails each. Total of 25 measurements	Acceleration(ms^{-2})	Recording velocity using motion detector and taking the tangent line.

Controls

Control Variable	Control Method	Control Reason
Incline of the rolling surface	The friction plane will be placed on a grooved rail, which is heavy and sturdy, preventing it from shifting back and forth.	This will affect the horizontal component of gravitational acceleration and therefor changes the constant coefficient of the equation.
Outer diameter of the cylinder	All cylinders will be cut from the same large cylinder. Only the hollow part would be cut separately.	The outer diameter of the cylinder affects its moment of inertia and therefor changes its acceleration.
Starting point for rolling	All cylinders will start from 115 cm of the marked rail.	This gives more reliability to motion detector readings because there is more compatibility between data.
Rolling path of the cylinder	A secondary ruler will be placed perpendicular the the rail edge. This is placed against the curved side of the cylinder to ensure a perfectly straight alignment.	The velocity measured by the motion-detector is only in one dimension. If cylinder rolls diagonally, the recorded velocity from the detectors perspective will be slower than actual velocity, therefor affecting the acceleration.

Method

1. Set the lifting jack to a height of 10.5cm

2.Place the 120cm guided rail on top of the lifting jack while using a protractor to adjust its incline to 6 degrees. Uncertainty is 0.1 degrees.

3. Insert the motion detector into the slot of the lifting jack and push it until a mechanical stop.

4.Cover the guided rail with the rolling surface, ensuring that the edge of the surface lies on the 115cm mark. Uncertainty is 0.1 degrees.

5.Place a cushion on the end of the rolling surface to prevent the cylinders falling out.

6.Connect the motion detector to computer.

7.Gently hold the cylinder with 5.5cm internal diameter against the end of the rolling surface. Hold it against a ruler to make sure the curved surface is aligned with the edge.

8.Start recording on the motion detector with ±0.001 m/s

9.Release the cylinder by lifting finger perpendicularly upward.

10. After the cylinder hits the cushion, end the motion detector.

11. Generate a linear fit for the velocity graph between v_0 and v_{max} recording the slope and y-intercept.

12. Repeat steps 7-11 four more times.

13. Repeat steps 7-12 for the remaining cylinders with an internal diameter of 4.5mm, 3.5mm, 2.5mm, 1.5mm, and 0.5mm, with five trails each.

14. Calculate the average slope and y intercept for each independent variable using their five corresponding trails.

Safety

Equipment Used	Risks Associated	Methods for Risk Reduction
Marked Rail	This equipment is hard, heavy, and has sharp edges. People could accidentally knock the rail of the lifting jack, leading to a risk of cutting and smashing fingers.	The rail will be placed away from the edges of the table to prevent passerby from knocking it over. When adjusting the rails direction, fingers or feet shouldn't be placed under the rail.
Hollow wooden cylinder	The wood cylinder can roll off the table and hurt someones feet. The cylinder lying on the floor also poses a risk of slipping.	A cushion will be placed at the end of the rolling surface to stop the cylinder.

Ethical and Environmental Considerations

While there is no ethical considerations of this experiment, it is important to ensure that other people aren't disturbed if a cylinder falls to the ground. This is done by leaving 2m of space between other experimental apparatus and using a cushion.

This experiment requires specialized cylinders with measured diameters. For sustainability, I chose available cylinders, with the limitation being that there were only five variables. Also, the cylinders are made from wood which are renewable and recyclable.

Raw Data

Inner Radiusr_1 (m) $\pm 5\times10^{-4}$m	Acceleration (ms^{-2}) ± 0.01 ms^{-2}						Uncertainty (ms^{-2})	
	Trail 1	Trail 2	Trail 3	Trail 4	Trail 5	Average	$\Delta_{average}$	$\Delta_{instrument}$
2×10^{-3}	0.71	0.69	0.70	0.69	0.71	0.70	±0.01	±0.01
7×10^{-3}	0.69	0.67	0.67	0.68	0.67	0.68	±0.01	±0.01
1.2×10^{-2}	0.65	0.65	0.65	0.64	0.68	0.65	±0.02	±0.01
1.7×10^{-2}	0.64	0.66	0.64	0.64	0.63	0.64	±0.02	±0.01
2.2×10^{-2}						0.60	±0.02	±0.01
2.7×10^{-2}	0.57	0.57	0.57	0.58	0.58	0.57	±0.01	±0.01

Uncertainty for the inner diameter of the cylinders is $\pm 5\times10^{-4}$. This comes the uncertainty rule for analogue instruments, which is ± half the smallest increment(mm). All values are rounded to three units behind decimal (note that 2×10^{-3} and 2.7×10^{-2} have the same precision) because it cannot exceed the smallest increment of uncertainty.

Uncertainty for acceleration on trail 1-5 is ±0.01. This comes from the uncertainty rule for digital instruments, which is ± the smallest increment. The smallest increment is 0.01 ms^{-2} because the sample rate on the motion detector limits time measurements to two decimal places. All values are rounded to two decimal places because it cannot exceed the precision of uncertainty.

Sample Calculation

The acceleration for row 1(2×10^{-3}) was calculated using the formula

$$a_{average} = \frac{a_1+a_2+a_3+a_4+a_5}{5} = \frac{0.71+0.69+0.70+0.69+0.71}{5} = 0.70$$

The $\Delta_{average}$ for row 1 was calculated using the formula

$$\Delta_{average} = \frac{a_{max}-a_{min}}{2} = \frac{0.71-0.69}{2} = 0.01$$

The $\Delta_{instrument}$ is the uncertainty coming purely from instrumental imprecisions, where all acceleration measurements have an uncertainty of 0.01

$$\Delta_{average} = \frac{\Delta_1+\Delta_2+\Delta_3+\Delta_4+\Delta_5}{\#trails} = \frac{0.01+0.01+0.01+0.01+0.01}{5} = 0.01$$

Processed Data

Inner Radius squared r_1^2 (m^2)	$\Delta\ r_1^2$ (m^2)	1/acceleration ($m^{-1}s^2$)						Uncertainty ($m^{-1}s^2$)	
		Trail 1	Trail 2	Trail 3	Trail 4	Trail 5	1/ average	$\Delta_{1/average}$	$\Delta_{instrument}$
4×10^{-6}	2×10^{-6}	1.41	1.45	1.43	1.45	1.41	1.43	0.02	0.01
4.9×10^{-5}	7×10^{-6}	1.45	1.49	1.49	1.47	1.49	1.47	0.04	0.02
1.4×10^{-4}	1.2×10^{-5}	1.54	1.54	1.54	1.56	1.47	1.54	0.04	0.02
2.9×10^{-4}	1.7×10^{-5}	1.56	1.52	1.56	1.56	1.59	1.56	0.02	0.01
4.8×10^{-4}	2.2×10^{-5}	1.71	1.63	1.67	1.63	1.71	1.67	0.02	0.01
7.3×10^{-4}	2.7×10^{-5}	1.75	1.75	1.72	1.72	1.72	1.75	0.02	0.01

Sample Calculation
The inner diameter squared value for row 1(4×10^{-6}) was calculated as below:

$$r^2 = (2\times10^{-3})^2 = 2\times10^{-6}$$

The uncertainty $\Delta\ r_1^2$ for row 1 was calculated using the formula:
All values of r_1^2 are rounded to the six digit behind decimal because it cannot exceed the smallest increment of uncertainty.

$$\Delta_{percent} r_1^2 = \Delta_{percent} r_1 \times 2$$
$$\Delta r_1^2 = \Delta_{percent} r_1 \times 2 \times r_1^2$$
$$= \frac{\Delta r_1}{r_1} \times 2 \times r_1^2$$
$$= \frac{5\times10^{-4}}{2\times10^{-3}} \times 2 \times (4\times10^{-6})$$
$$= 2\times10^{-6}$$

The 1/average value for row 1 was calculated as below:

$$\frac{1}{avg} = \frac{1}{0.7} = 1.43$$

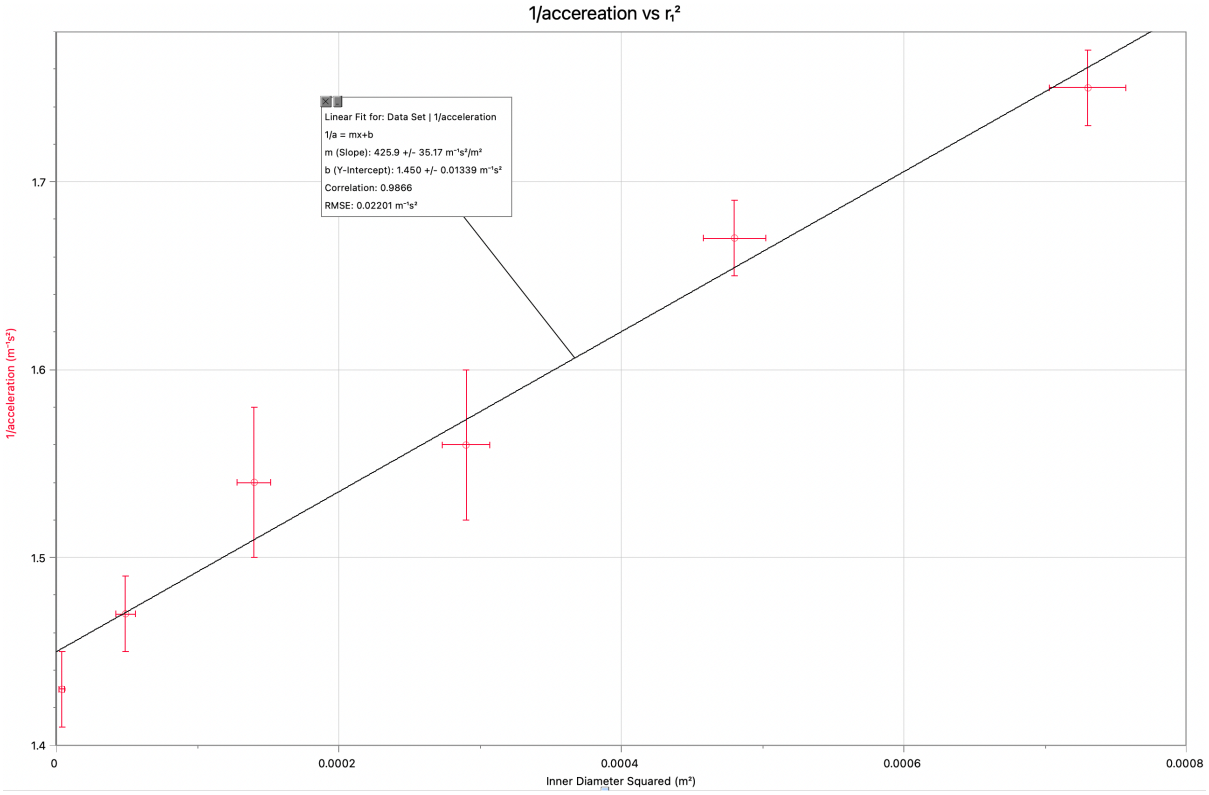
The uncertainty $\Delta_{1/average}$ for row 1 was calculated using the formula:
Both the uncertainty and values for 1/acceleration are rounded to two digits behind decimal place because it has to align with the precision of raw data.

$$\Delta_{percent} \frac{1}{Average} = \Delta_{percent} Average$$
$$\Delta \frac{1}{Average} = \Delta_{percent} Average \times \frac{1}{Average}$$
$$= \frac{\Delta Average}{Average} \times \frac{1}{Average}$$
$$= \frac{0.01}{0.70} \times \frac{1}{0.70}$$
$$= 0.02$$

The uncertainty $\Delta_{instrument}$ for row 3(1.4×10^{-4}) is calculated using the formula:

$$\Delta_{device} \frac{1}{Average} = \frac{\Delta Device}{Average} \times \frac{1}{Average}$$
$$= \frac{0.01}{0.65} \times 1.54$$
$$= 0.02$$

Graph



Graph Analysis

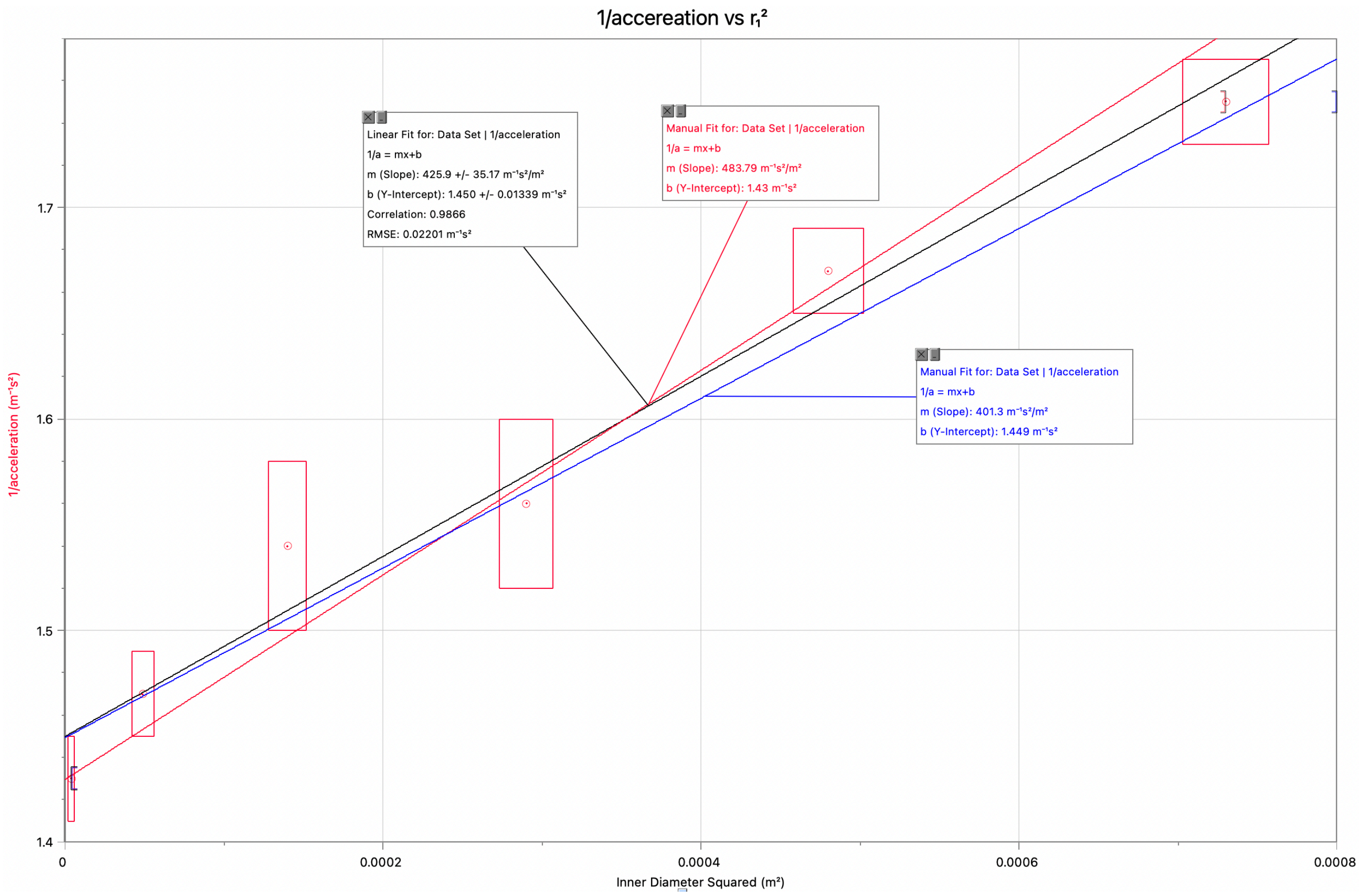
The graph shows a positive, linear relationship between x and y. Correlation is relatively high visually, translating to a coefficient of 0.987. The line of best fit (black) was generated using a linear regression y=mx+b in logger pro. Logger pro uses the least squares method, which aims to minimize the MSE (square of average difference between the predicted and actual value). This can be expressed mathematically as:

$$m = \frac{n\sum(x_i y_i) - \sum x_i \sum y_i}{n\sum(x_i^2) - (\sum x_i)^2} \qquad b = \bar{y} - m\bar{x}$$

The calculated equation is:

$$y = 425x + 1.45$$

The RMSE value, which is dependent on MSE, is minimized at 0.022.
The uncertainties calculated by logger pro (±35.17 for slope and ±0.13 for y intercept) is unsuitable for our application. This is because logger pro uses a standard curve fitting algorithm, and it doesn't account for horizontal and vertical error bars.
Instead, uncertainties in the gradient and y intercept will be calculated by finding the maximum and minimum slope. These equations will be manually fitted within the vertical error bar(Δ1/average) and the horizontal error bars(Δ r_1^2) . Note that the uncertainty Δ1/average consists of random error, while the uncertainty Δ r_1^2 only comes from the rulers precision limits. This is important because instrumental uncertainties should be separated from experimental uncertainties. The difference can tell us what kind of uncertainty is acceptable and what kind is caused by errors.



This is a graph including experimental uncertainties. We can manually fit the maximum and minimum equations and ensure that the lines fit within each uncertainty box.

Max Gradient: 483.79

Minimum Gradient: 401.30

Uncertainty Gradient $\frac{Maximum\ Gradient - Minimum\ Gradient}{2} = \frac{483.79-401.30}{2} = 40.75$

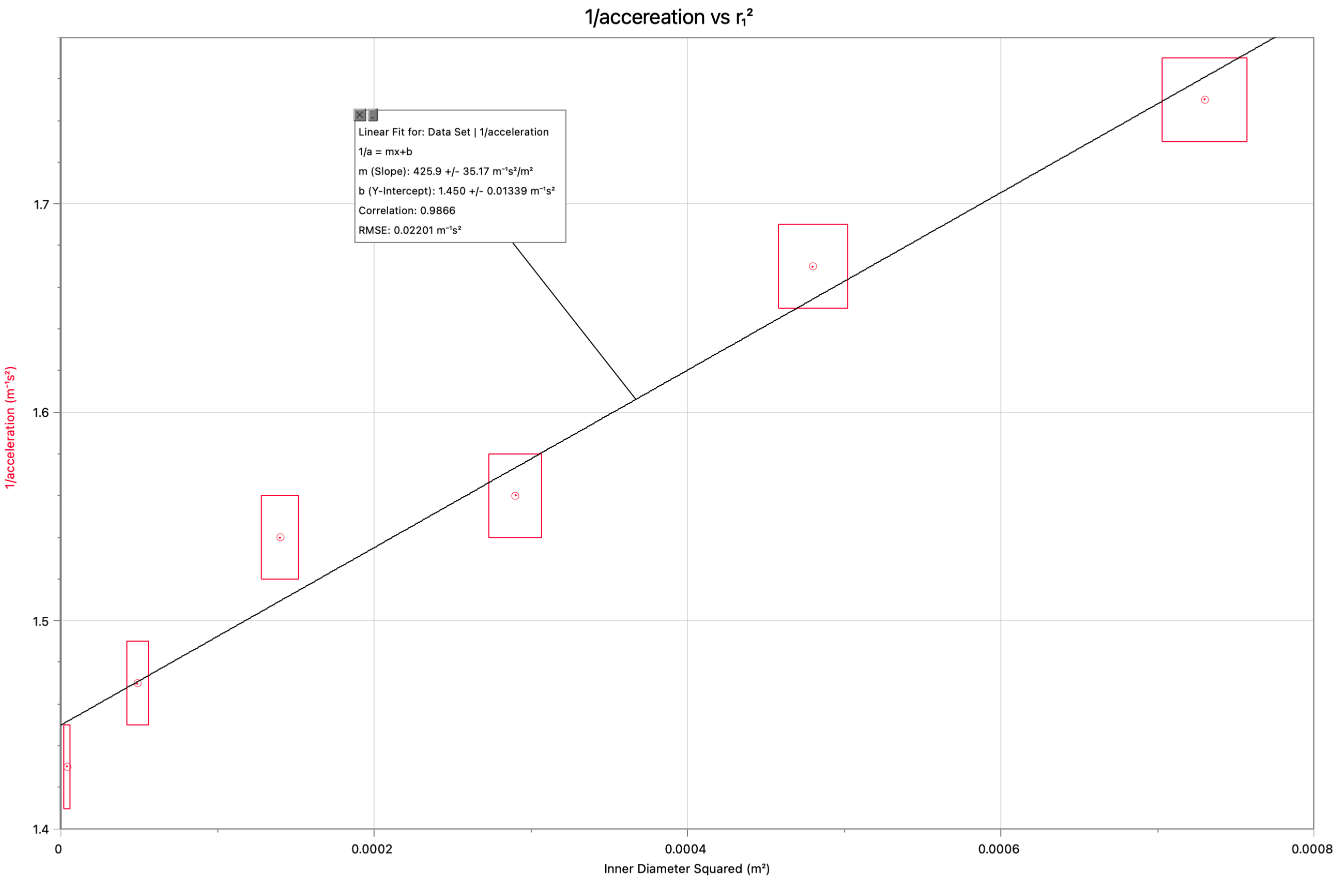
Alternatively, we can calculated uncertainty more accurately using mathematics. We know the formula for linear regression. We also know that there is an uncertainty corresponding to each x value and each y value. Therefor we can modify the linear regression formula to calculate uncertainties.

$$m = \frac{n\sum(x_i y_i) - \sum x_i \sum y_i}{n\sum(x_i^2) - (\sum x_i)^2}$$

When we introduce uncertainties, this equation becomes:

$$\Delta m = \sqrt{\frac{1}{n-2} \cdot \left(\frac{\sum(\Delta y)^2}{\sum(r_1^2 - \bar{r}_1^2)^2} + \frac{\sum(\Delta r_1^2)^2}{\sum(r_1^2 - \bar{r}_1^2)^2} \right)}$$

Working this out we get a uncertainty in slope of 55.35.



This is a graph including only instrumental uncertainties. We cannot use the max - min gradient method as before, because it is impossible to fit a line within all error bars.

Mathematically, we can try and calculate the uncertainty.

$$\Delta m = \sqrt{\frac{1}{n-2} \cdot \left(\frac{\sum(\Delta y)^2}{\sum(r_1^2 - \bar{r_1^2})^2} + \frac{\sum(\Delta r_1^2)^2}{\sum(r_1^2 - \bar{r_1^2})^2} \right)}$$

Working this out we get a uncertainties in slope of 39.14.

To summarize, instrumental uncertainty is 39.14, while the actual uncertainty is 55.5. The difference between the two is caused by random errors. We can see random errors in the data tables before, where the half range is larger than the uncertainty of the motion detector. An example of this is the third value (1.4×10^{-4}), where the experimental uncertainty is 0.04, but the expected instrumental uncertainty is 0.02. Because differences on these data points, the final calculation for experimental uncertainty has a higher value.

Anomalies like the third value. can have a large impact on the linear regression calculation, especially when we only have six values. In this case, it has raised the average slope to higher than our expectation.

Conclusion

Generally, the graph successfully showed the mathematical relationship between acceleration and inner diameter of a hollow cylinder. The data is more accurate than precise.

The discrepancy will be calculated with the line of best fit: $y = 425x + 1.45$

$$Slope\ Discrepancy = \left|\frac{expected\ value - experimental\ value}{expected\ value}\right| \times 100\%$$
$$= \left|\frac{400 - 425.9}{400}\right| \times 100\%$$
$$= 6.48\%$$

$$y - int\ Discrepancy = \left|\frac{expected\ value - experimental\ value}{expected\ value}\right| \times 100\%$$
$$= \left|\frac{1.48 - 1.45}{1.48}\right| \times 100\%$$
$$= 2.03\%$$

The discrepancy in slope represents instrumental uncertainty and random error. Most of the imprecision is resulted from instrumental uncertainty, as it accounts for 71% percent of the total uncertainty. The uncertainty in the y values (1/acceleration) can be reduced by increasing the sampling rate of the motion detector. The uncertainty in the x values (r_1^2) can be reduced by using a vernier caliper instead of ruler to measure the cylinders radius.

The remaining 31% of the uncertainty is a result of random error. This may be attributed to the fact that the inner radius of some cylinders, such as the anomaly 3, are not perfectly round. Also, I noticed that the rolling surface flexed slightly, so under difference human forces and masses of the cylinder the slope could change. This subsequently affects the gravitational acceleration acted on the cylindrical body. Furthermore, from raw data in vernier graphical analysis, I noticed that the initial acceleration was always higher. This might have caused inaccuracies.

The discrepancy in the y intercept represent systematic errors. The experimental value for y intercept is 2.03% lower than the expected value for y intercept. This means that on average every data point is lower than expected value by 2.03%. The main reasons for are air resistance which we ignored, and small undulations on the rolling surface which might result in more friction and slow the rolling of the cylinder

From my data, I can conclude that a hollow cylinder with a thicker wall has a smaller moment of inertia, making it faster to accelerate down an inclined plane. The specific relationship between the hollow cylinder's internal radius and its acceleration down an inclined plane with degree θ can be modeled below:

$$a = \frac{2r^2 \cdot g \cdot \sin(\theta)}{3r^2 + r1^2}$$

I have reached this conclusion with confidence. This is because the discrepancy value of 6.46% and 2.03 show accuracy in my data.
Other researchers have reached similar conclusions on the negative correlation between a slide moment of inertia and its acceleration down an inclined plane, some by using different methods.

(Mulhayatiah et al., 2018) conducted a similar experiment by rolling 4 cylinders of different radii down an incline with the height of h and the angle of θ. The movement of the object is recorded and its acceleration is analyzed through tracker software. Instead of deriving the acceleration equation in terms of I, he derived the equation for I in terms of acceleration. Then, by comparing the calculated experimental results with the anticipated result, he verified that I is 1/2m(r12+r22) for hollow cylinders with an discrepancy of 5.32%. This is a similar discrepancy from our experiment, which makes our conclusion valid. His also verified that larger the radius, larger the moment of inertia, and smaller the acceleration. Refer back to equation $a = \dfrac{2r^2 \cdot g \cdot \sin(\theta)}{3r^2 + r1^2}$

(Eadkhong et al., 2012) has taken into count the linear coefficient of rolling friction, and reduced the discrepancy value for expected I value to 0.4% and 3.0% percent.

Researchers have also measured the correlation between I and α for more realistic applications. Uys et al., 2006 has used rigid body oscillations to measure the pitch, roll, and yaw for the moment of inertia of an off-road vehicle.

Evaluation

Process	Strengths	Weaknesses	Improvements
Derivation of Expected Equation	Intuitive Derivation Cancels the effect of mass	Slope and the y-intercept has no scientific meaning	Consider expressing the linearized acceleration in terms of I, so it could be easily identifiable as the slope. This connects the theory and experiment better.
Selection of Apparatus	Easily assessable apparatus in lab.	Radius of the cylinder is too small, demanding high precision from measuring devices. Some cylinders have a slightly offset internal drill	Use cylinders with larger radius which decreases the percent of error as the precision of cutting devices remain the same.
Experiment Setup	Easy to setup under 5 minutes. Cylinder rolls in a straight trajectory.	Rolling surface is not perfectly flat and flexes under cylinders of different masses, causing a varying slope.	Use a stiffer and thicker material as the rolling surface.
Data Collection Method	Start and end of data collection period is easily identifiable, omitting human error.	Sampling rate at 100Hz limits the precision of acceleration to 0.01 ms^{-2}	Set the motion detector to a higher sampling rate.
Processing of Collect Data	All data points are rounded to the correct precision. There is minimal calculation for linearization, reducing the uncertainty as much as possible. Uncertainty derived from half range and instrumental uncertainty are calculated separately. This allows me to identify the portion of human error.	Data values are already very small and the calculation with scientific notations increases complexity.	Use cylinders with larger radius

Graph Analysis	The mathematical method of calculating uncertainties is reliable and accurate for both types of uncertainty. The two graphs featuring separate uncertainties visually demonstrate their implications.	The graph featuring instrumental uncertainties is too imprecise, making it impossible to determine max & min slope. Mathematical method is not fully expanded and may hinder the readers understanding.	Expand the mathematical way to calculate uncertainty and explain using degrees of freedom why 1/(n-2) is added to the equation. Consider adding more trails to increase the correlation of the graph.

Limitations and Applicability of Conclusion Made

The experiment is conducted by rolling a radially symmetric object down an inclined plane. The main driver of torque is friction, therefor a difference conclusion might be reached if the torque is created by other sources. Furthermore, although the cylinder rolls without slipping, it still experiences a small degree of static friction that is not accounted in the calculation of data. Similar to friction, air resistance is also ignored. This has minimal effect because the cylinder is relatively smooth and has a relatively small size. However, when the experiment is scaled up, different results may occur.
Another aspect worth considering is that since a cylinders cross section remains constant and is radially symmetrical along its rolling axis. However, shapes like spheres, ellipses, rectangles does not meet these requirements. Therefor, the correlation between radius and translational acceleration could not be concluded for all shapes.