# PSO 5

Sorting

# Announcements

Project deadline extended to this Saturday

GL on 250 exam

**(Heap sort)** In the following questions, we consider Heap sort using **Heapify**.

(1) Show the array $\{3, 4, 1, 0, 9, 2\}$ as it goes through Heap sort (in the ascending order).

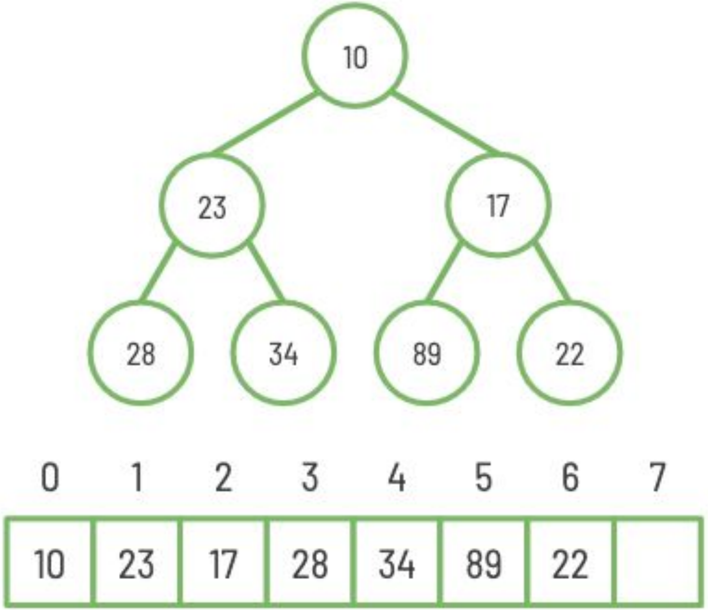
(2) Given $K$ number of sorted (ascending ordered) arrays each having $N/K$ elements in it, your task is to merge all these arrays to form a $N$-element final sorted array (also in the ascending order).

(2.1) Propose a simple solution to the problem which may run in $O(N \log(N))$ time.

(2.2) Can you propose a better algorithm to solve the problem? What is the time complexity of your proposed solution?

# Heap Insertion

1. Insert at next leaf
2. Sift up




Demonstration: insert(9)

# (Max) Heapify: Turning your arrays into Heaps

For each **non-leaf node** from the last to the first:

while it is less than its largest child:

swap it downward

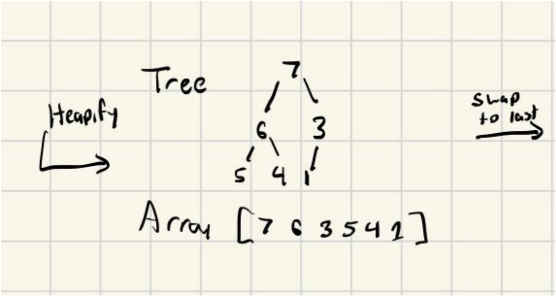
Demonstr. : Heapify [4 6 3 5 7 1]

# Heap Sort

Idea: In a max heap, the max element is always at the root, sort backwards, from largest to smallest

1. Heapify your array
2. Swap root with last leaf, excluding the elements you’ve already swapped
3. Fix heap by sifting down, excluding the elements you’ve already swapped
4. Repeat steps 2-4

[4 6 3 5 7 1]

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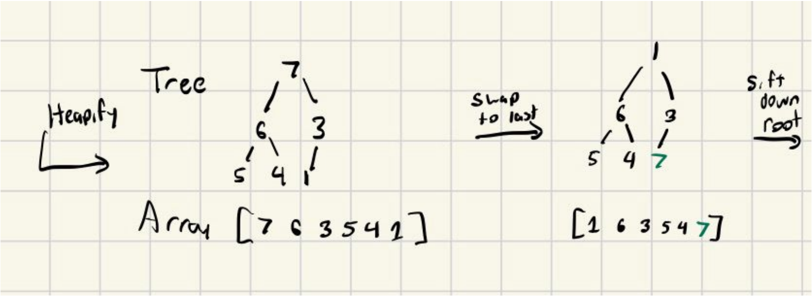


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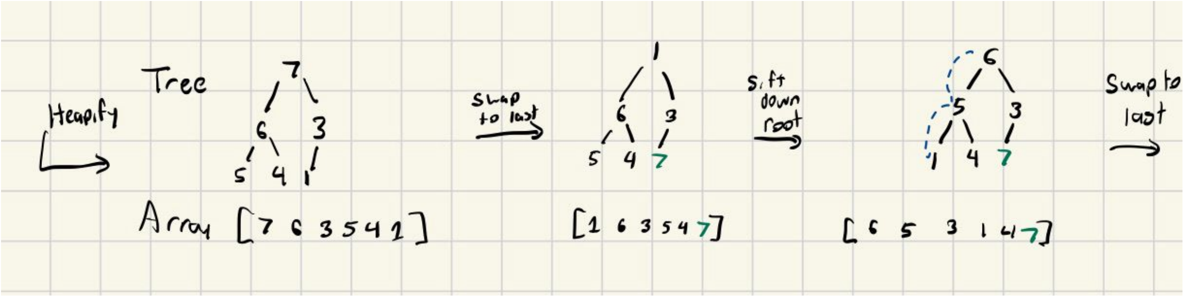


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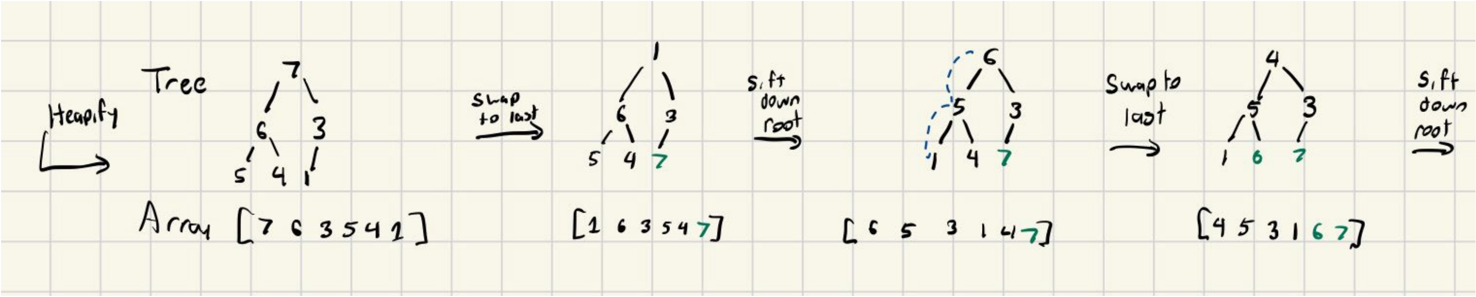


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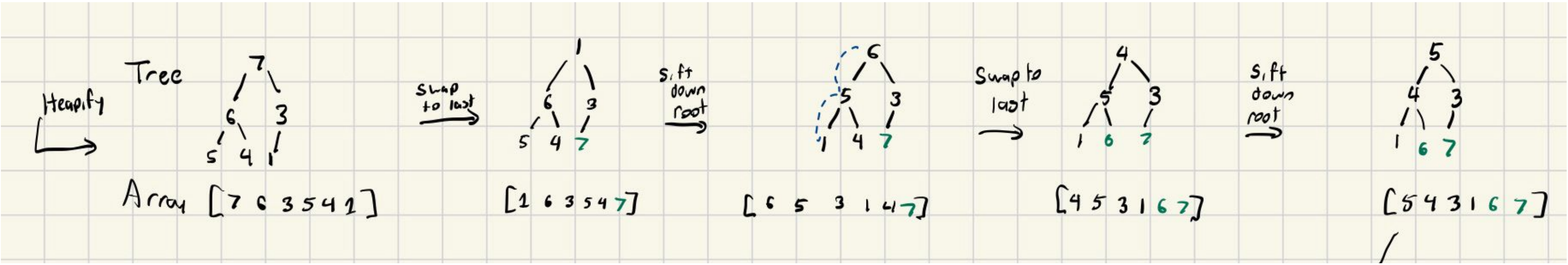


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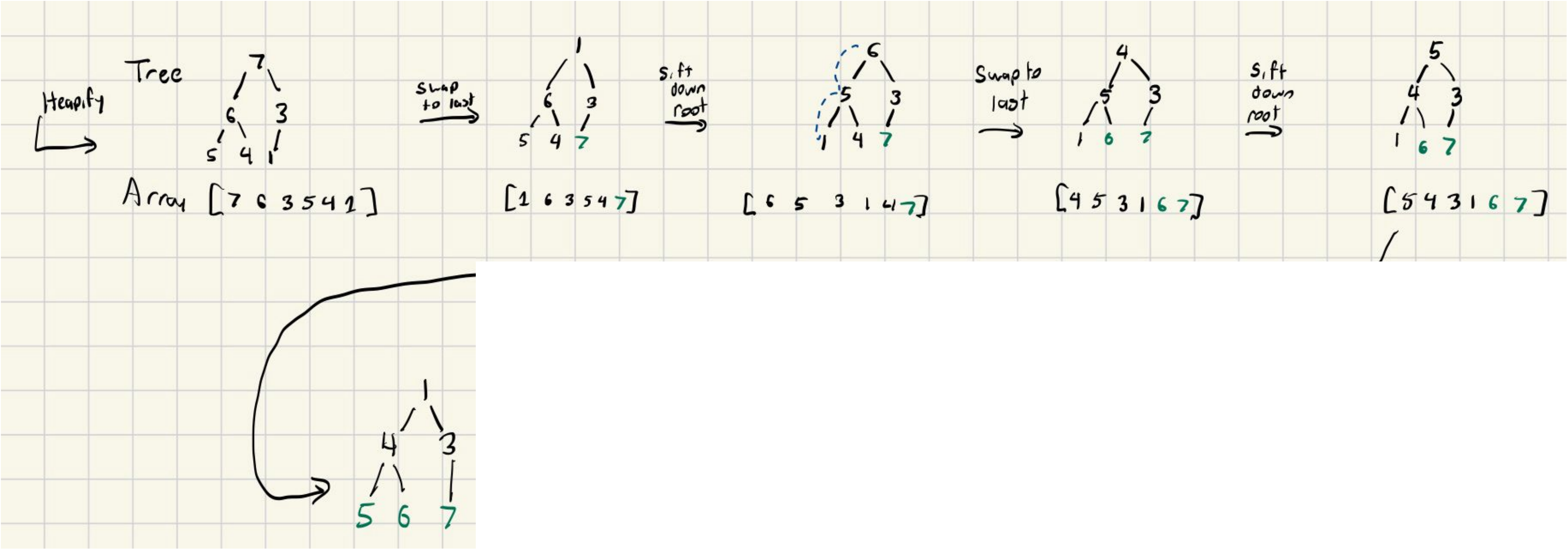


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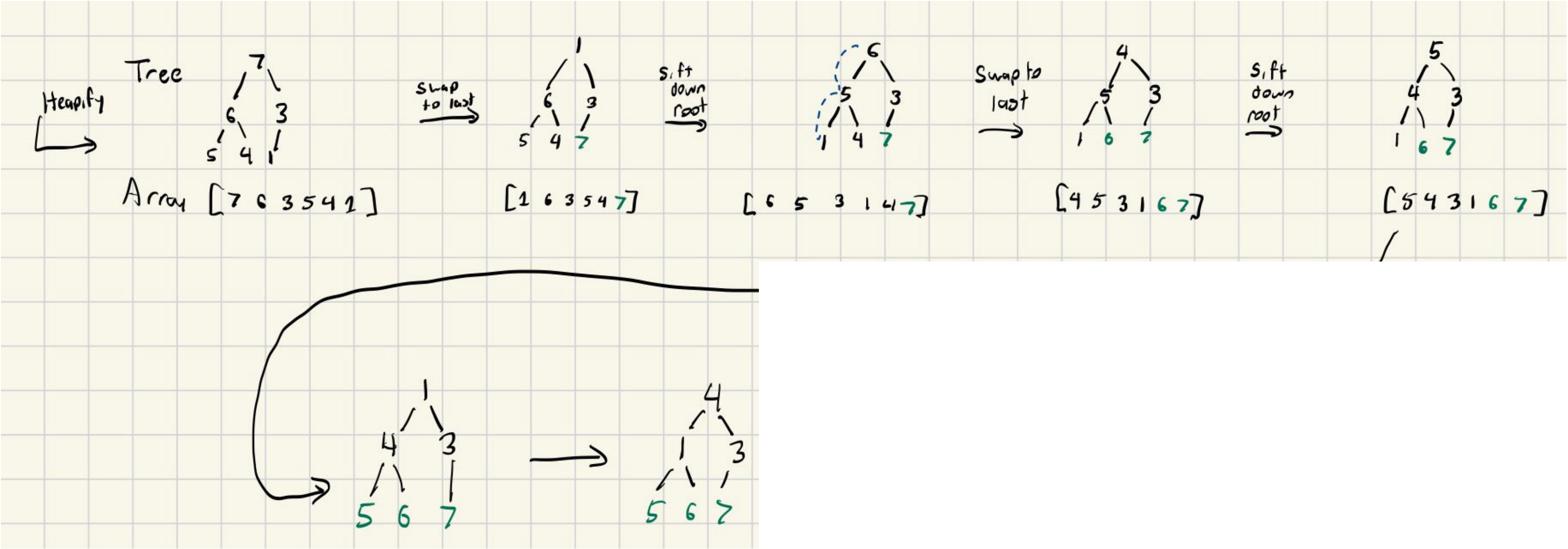


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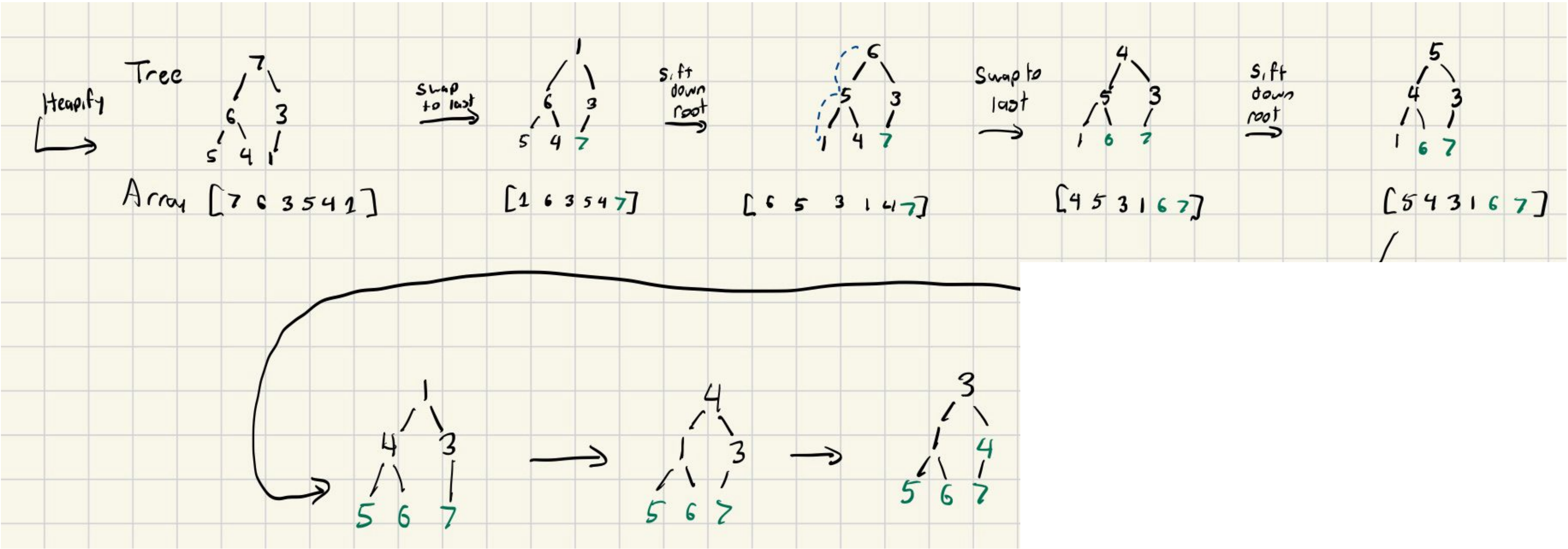


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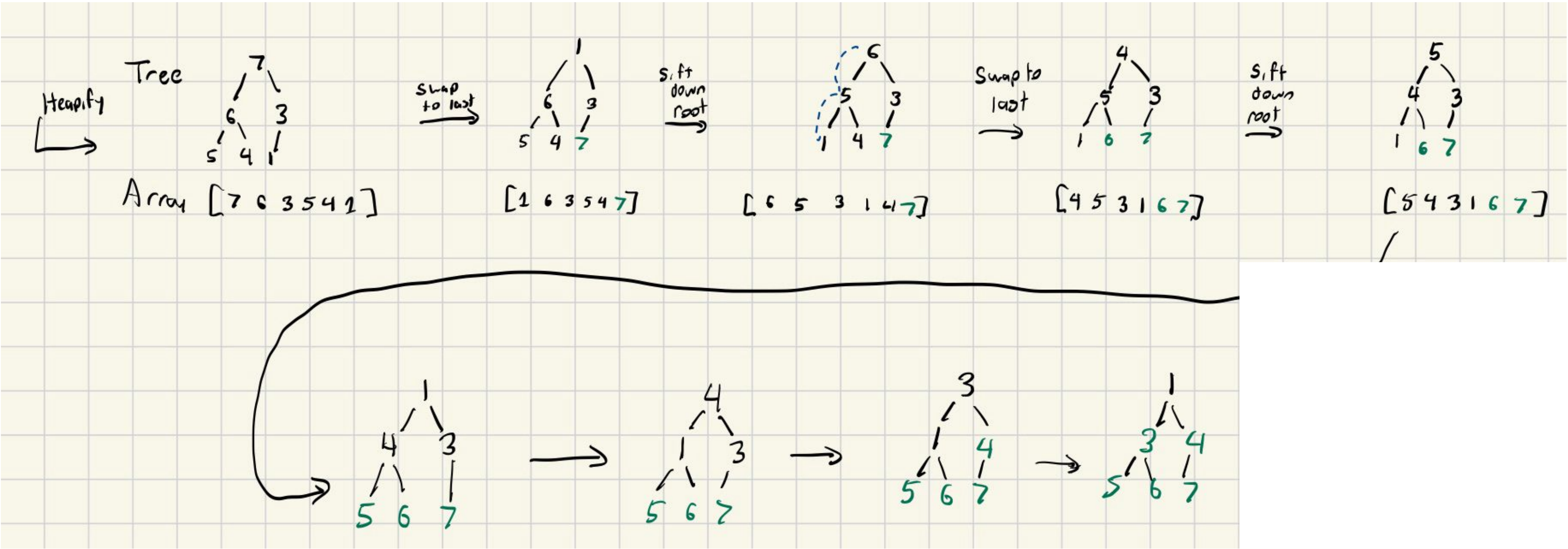


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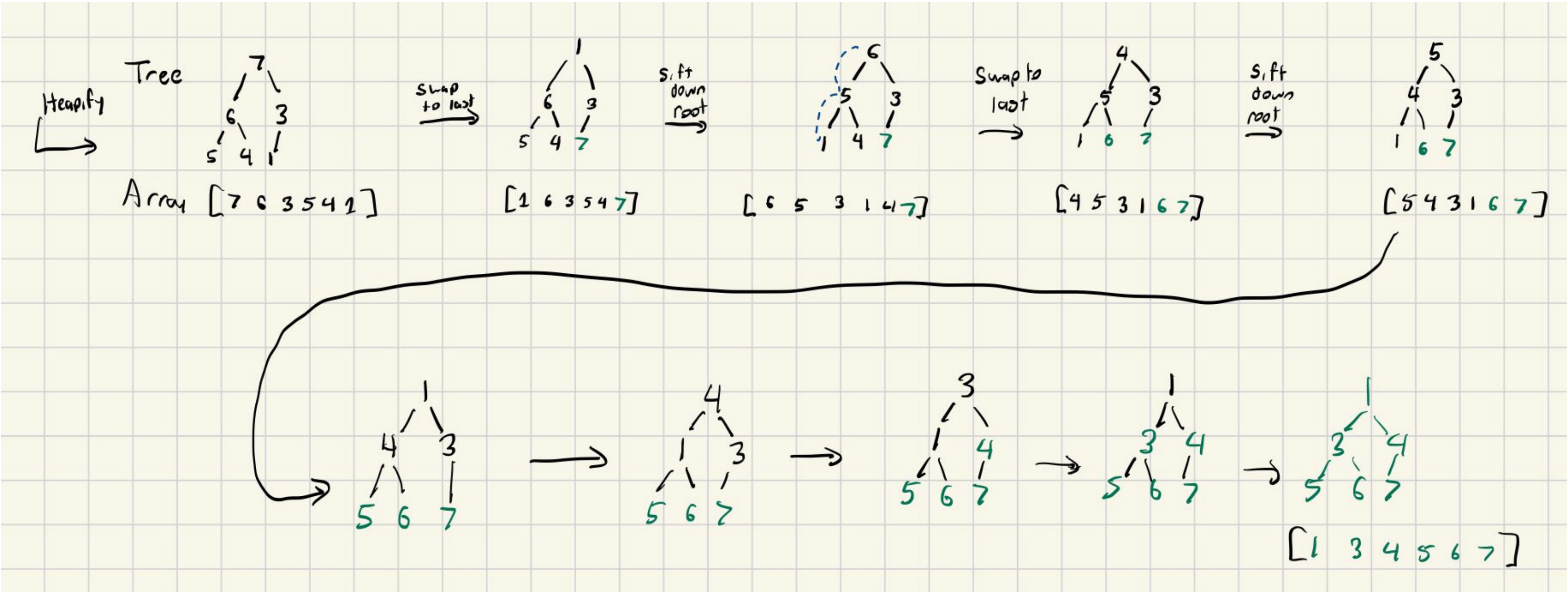


1. Heapify your array
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# Exercise: Equivalent Heap Sorts

## Working of Heap Sort

1. Since the tree satisfies Max-Heap property, then the largest item is stored at the root node.
2. **Swap:** Remove the root element and put at the end of the array (nth position) Put the last item of the tree (heap) at the vacant place.
3. **Remove:** Reduce the size of the heap by 1.
4. **Heapify:** Heapify the root element again so that we have the highest element at root.
5. The process is repeated until all the items of the list are sorted.

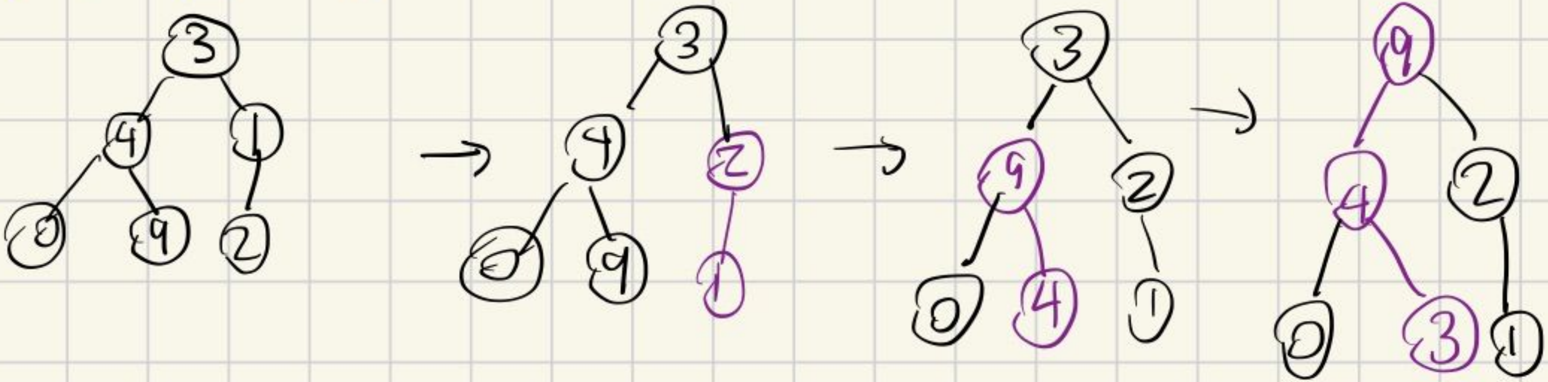
1. Heapify your array
2. Swap root with last leaf, excluding the elements you've already swapped
3. Fix heap by sifting down, excluding the elements you've already swapped
4. Repeat steps 2-4

**(Heap sort)** In the following questions, we consider Heap sort using **Heapify**.

(1) Show the array $\{3, 4, 1, 0, 9, 2\}$ as it goes through Heap sort (in the ascending order).

1) [3,4,1,0 9,2]

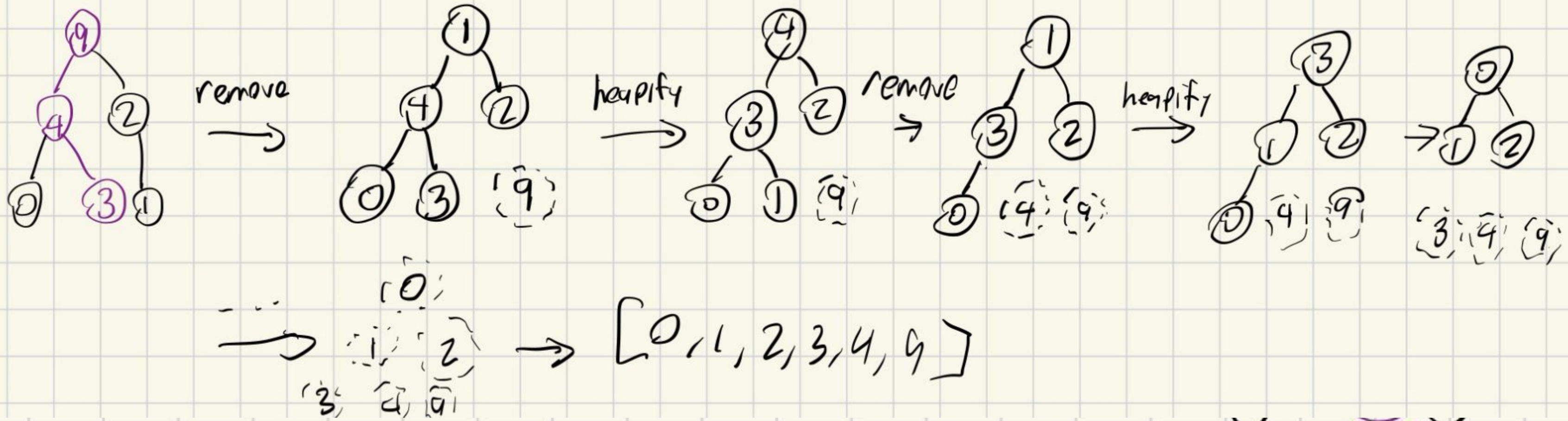
step1) array to heap



**(Heap sort)** In the following questions, we consider Heap sort using **Heapify**.

(1) Show the array $\{3, 4, 1, 0, 9, 2\}$ as it goes through Heap sort (in the ascending order).

step 2) The sort: remove root, replace by last leaf, reduce heapsize by 1, heapify, repeat.



# Heap Summary Costs

For a heap with 🎺 items,

Heapify: O(🎺)

Add/Pop: O(log 🎺)

Heap Sort: O(🎺log 🎺)

(2) Given $K$ number of sorted (ascending ordered) arrays each having $N/K$ elements in it, your task is to merge all these arrays to form a $N$-element final sorted array (also in the ascending order).

(2.1) Propose a simple solution to the problem which may run in $O(N\log(N))$ time.

(2) Given $K$ number of sorted (ascending ordered) arrays each having $N/K$ elements in it, your task is to merge all these arrays to form a $N$-element final sorted array (also in the ascending order).

(2.1) Propose a simple solution to the problem which may run in $O(N \log(N))$ time.

Just run merge sort on the combined array

(2) Given $K$ number of sorted (ascending ordered) arrays each having $N/K$ elements in it, your task is to merge all these arrays to form a $N$-element final sorted array (also in the ascending order).

(2.1) Propose a simple solution to the problem which may run in $O(N \log(N))$ time.

(2.2) Can you propose a better algorithm to solve the problem? What is the time complexity of your proposed solution?

Example (N = 12, K = 3):

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Can I use a heap somehow?

# Idea: index-wise heap sorting

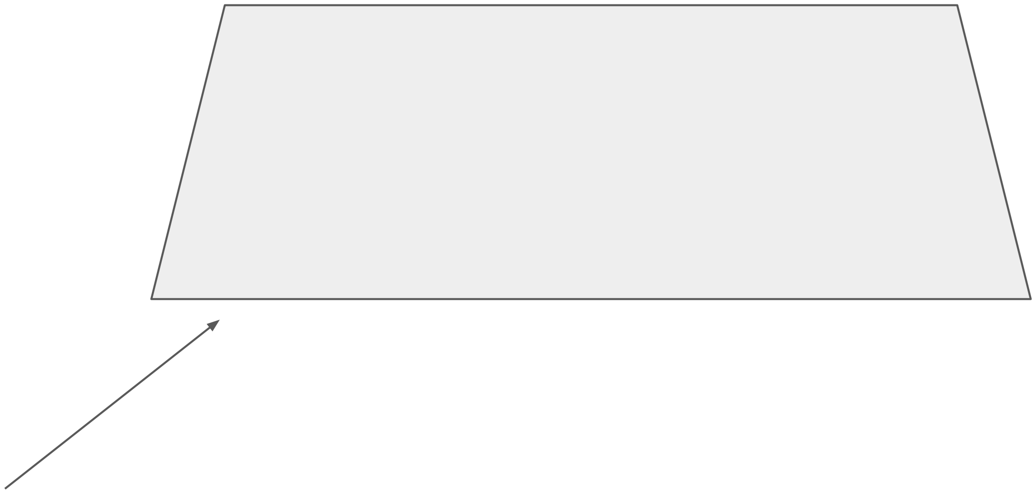
Example (N = 12, K = 3):

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap



This is a heap (as seen in the wild)

# Idea: index-wise heap sorting

Example (N = 12, K = 3):

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |
| 2 | 4 | 5 | 5 |
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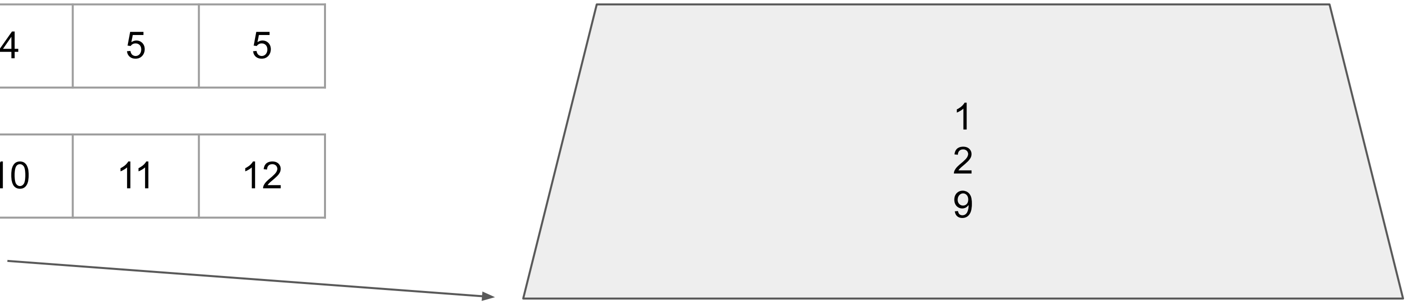
Iteratively sort and add items to the heap




1. Add first index elements to heap

# Idea: index-wise heap sorting

Example (N = 12, K = 3):z

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |

| | | | |
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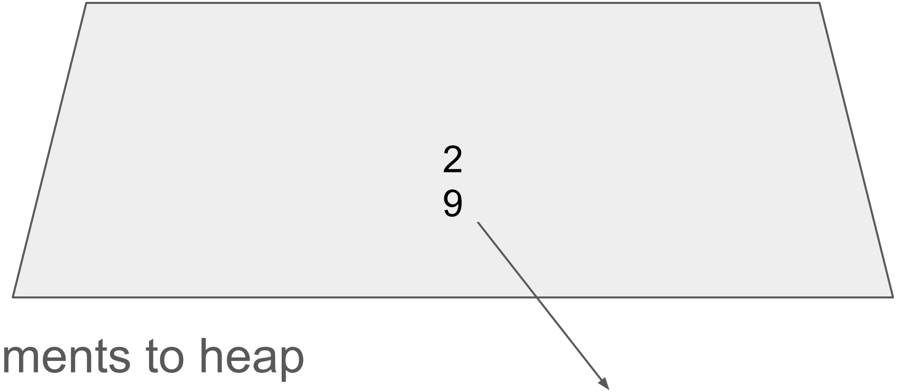
Iteratively sort and add items to the heap



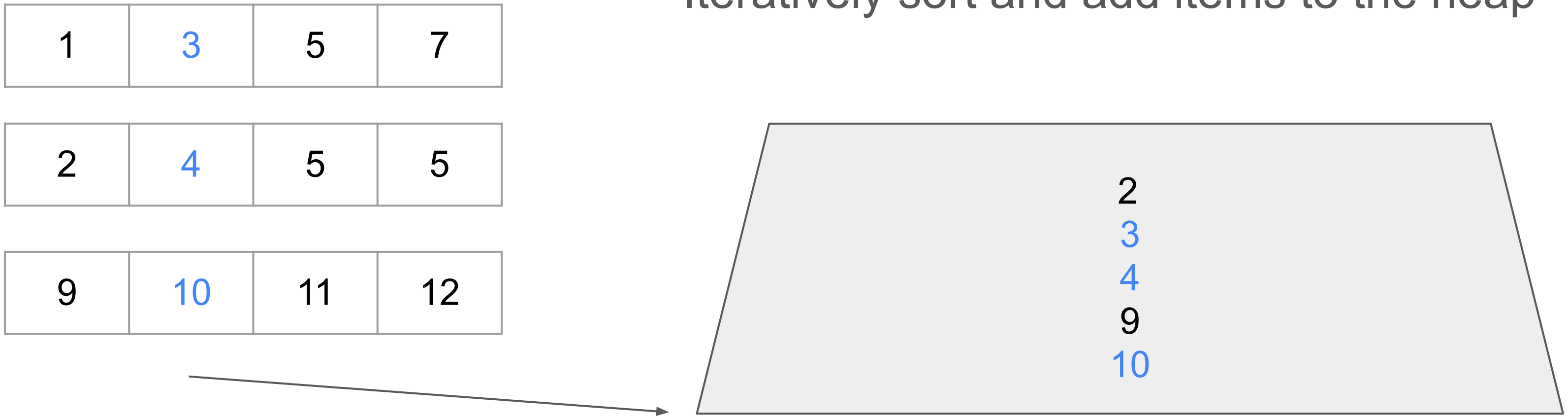

1. Add first index elements to heap
2. Pop heap and append to res array

res = 1

# Idea: index-wise heap sorting

Example (N = 12, K = 3):z

Iteratively sort and add items to the heap




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

res = 1

# Idea: index-wise heap sorting

**Problem**
Heap size at most N
Overall will be O(NlogN)

Example (N = 12, K = 3):z

It to the heap

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
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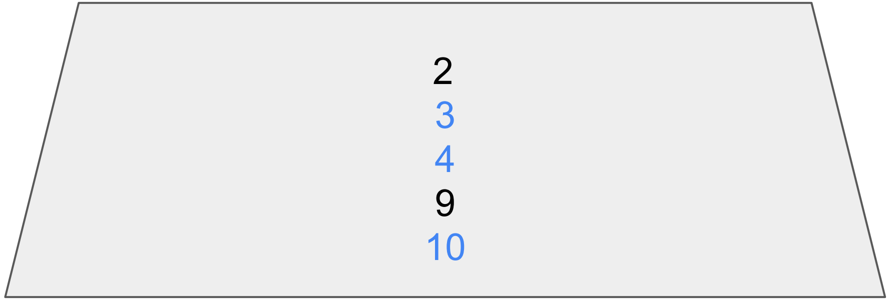


1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

res = 1

# Why will the heap has size O(n)?

**Problem**
Heap size at most
N
Overall will be
O(NlogN)

Example (N = 12, K = 3):z

Iterativ... ms to the heap

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
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| 9 | 10 | 11 | 12 |

3
4
5
5
9
10
11

1. Add first index elements to heap
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3. Repeat for each index?

res = 1

# Why will the heap has size O(n)?

**Problem**
Heap size at most
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Overall will be
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Example (N = 12, K = 3):z

Iterativ d ms to the heap

| | | | |
|---|---|---|---|
| 1 | 3 | 5 | 7 |
| 2 | 4 | 5 | 5 |
| 9 | 10 | 11 | 12 |

4
5
5
5
7
9
10
11
12

(12 - 3) = 9 elts

In general:
Will have <= (n-k) elts in heap

**Can we limit to just k?**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

res = 1

# Keeping our heap to size K

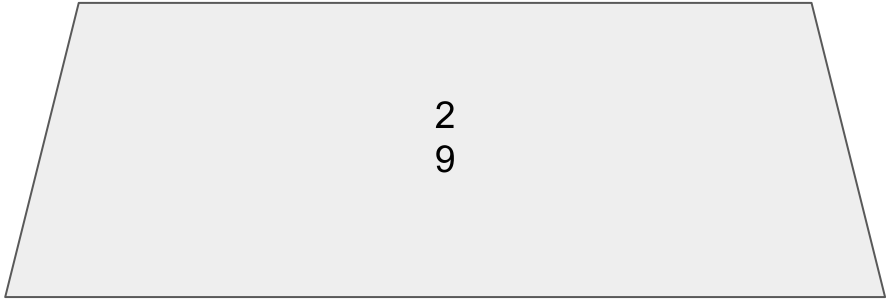
Example (N = 12, K = 3):z

| | | | |
|---|---|---|---|
| **1** | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

   **Only add next index element from popped array**

res = **1**

# Keeping our heap to size K

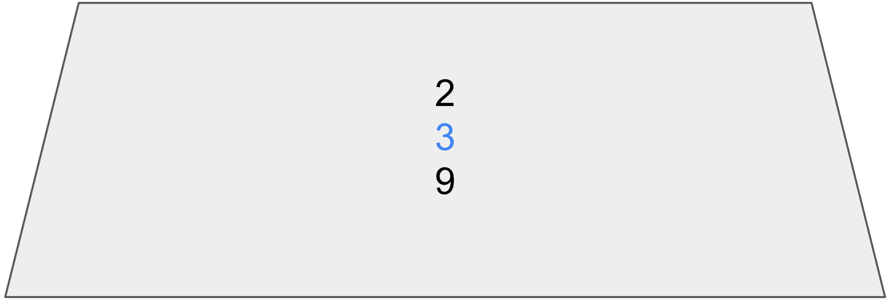
Example (N = 12, K = 3):z

| | | | |
|---|---|---|---|
| **1** | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| 2 | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

   **Only add next index element from popped array**

res = **1**

# Keeping our heap to size K

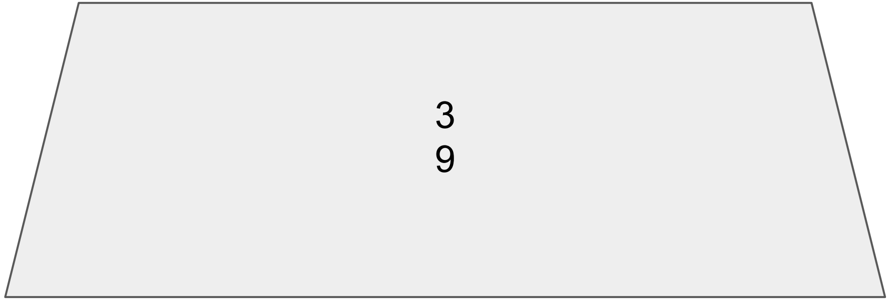
Example (N = 12, K = 3):z

| | | | |
|---|---|---|---|
| **1** | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| **2** | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap




res = **1 2**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

**Only add next index element from popped array**

# Keeping our heap to size K

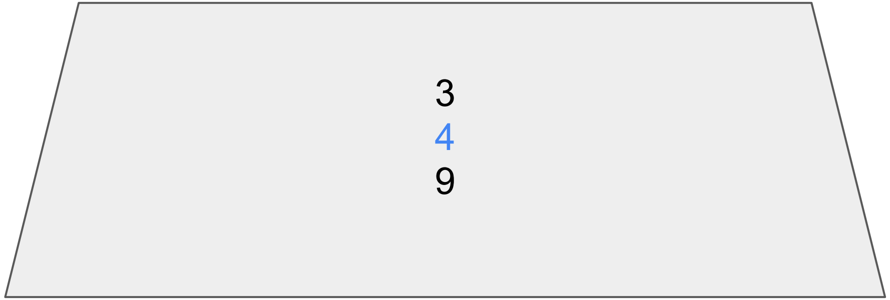
Example (N = 12, K = 3):z

| | | | |
|---|---|---|---|
| **1** | 3 | 5 | 7 |

| | | | |
|---|---|---|---|
| **2** | 4 | 5 | 5 |

| | | | |
|---|---|---|---|
| 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

   **Only add next index element from popped array**

res = **1 2**

# Keeping our heap to

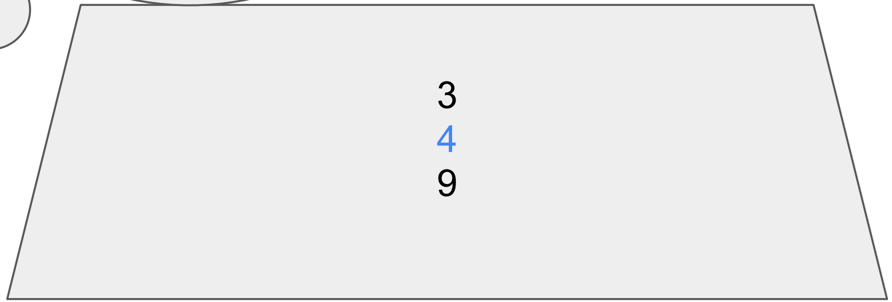
Okay.. but how do we know:

1. which array the popped element belongs to?
2. Its index in the array

Example (N = 12, K =

s to the heap

| | | | |
|---|---|---|---|
| **1** | 3 | 5 | 7 |
| **2** | 4 | 5 | 5 |
| 9 | 10 | 11 | 12 |




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?
   **Only add next index element from popped array**

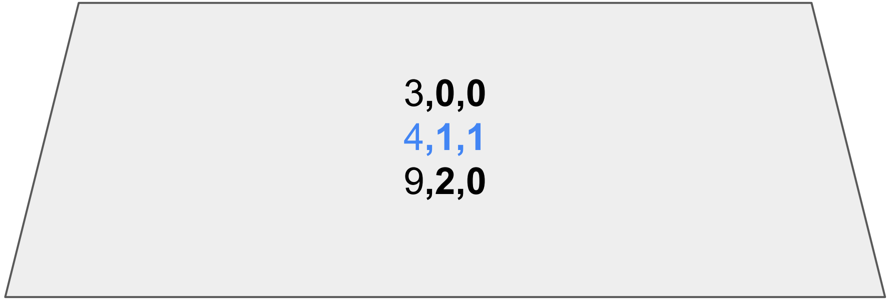
res = **1 2**

# Store the array number and the index!

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | 3 | 5 | 7 |
| **1** | **2** | 4 | 5 | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?
   **Only add next index element from popped array**

res = **1 2**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | 4 | 5 | 5 |
| **2** | 9 | 10 | 11 | 12 |

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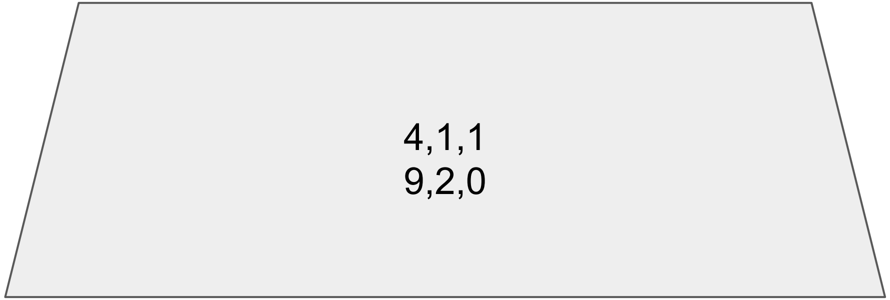



1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

**Only add next index element from popped array**

res = **1 2 3**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | 4 | 5 | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**

4,1,1
5,0,2
9,2,0

res = **1 2 3**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

**Only add next index element from popped array**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | 5 | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**

5,0,2
9,2,0

res = **1 2 3 4**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

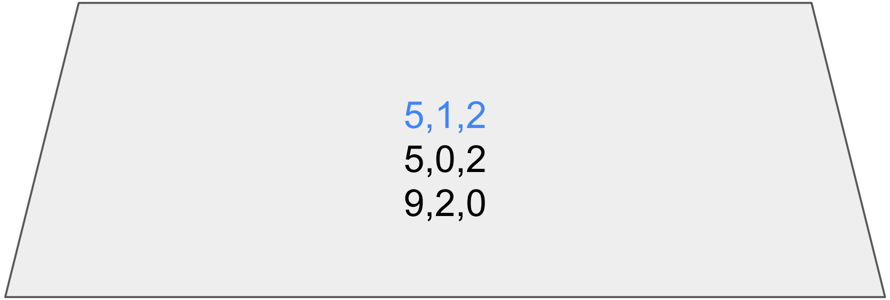
**Only add next index element from popped array**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | 5 | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

   **Only add next index element from popped array**

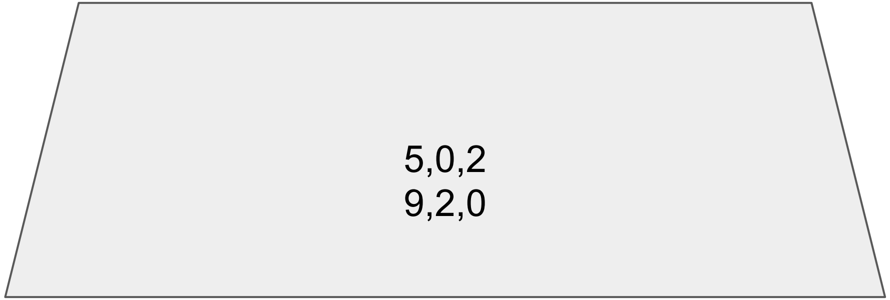
res = **1 2 3 4**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | **5** | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

**Only add next index element from popped array**

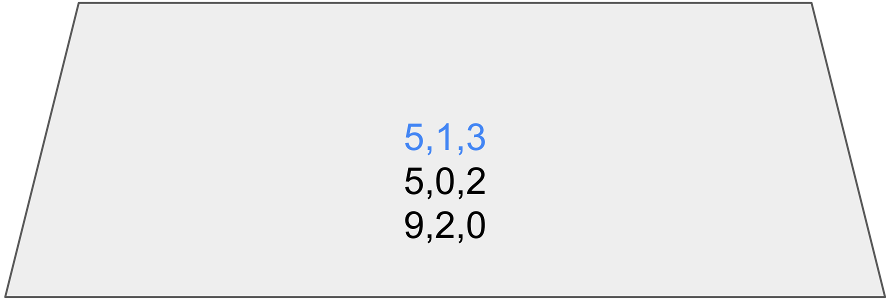
res = **1 2 3 4 5**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | **5** | 5 |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?
   **Only add next index element from popped array**

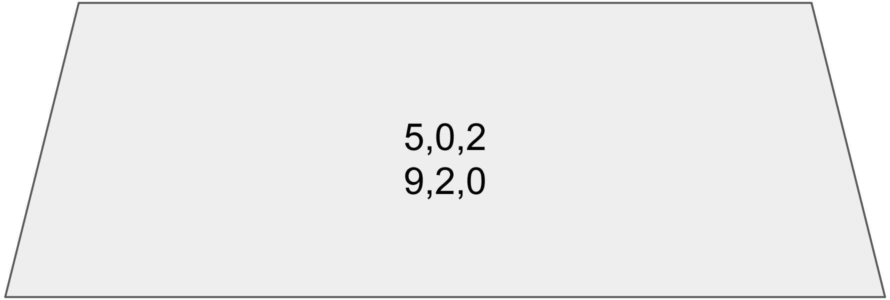
res = **1 2 3 4 5**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | **5** | **5** |
| **2** | 9 | 10 | 11 | 12 |

Keep sorting the remaining arrays




res = **1 2 3 4 5 5**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

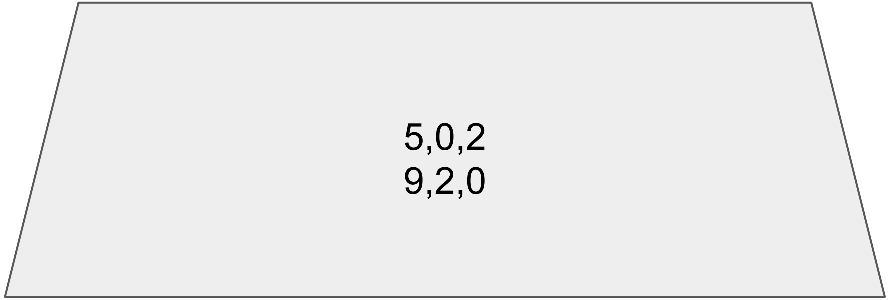
   **Only add next index element from popped array**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | 5 | 7 |
| **1** | **2** | **4** | **5** | **5** |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




res = **1 2 3 4 5 5**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

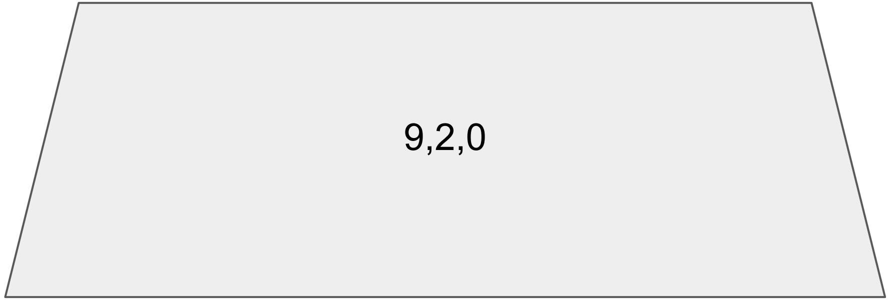
**Only add next index element from popped array**

# Store the index

Example (N = 12, K = 3):z

| | | | | |
|---|---|---|---|---|
| **0** | **1** | **3** | **5** | 7 |
| **1** | **2** | **4** | **5** | **5** |
| **2** | 9 | 10 | 11 | 12 |

Iteratively sort and add items to the heap **of the form x,array #,index**




res = **1 2 3 4 5 5 5**

1. Add first index elements to heap
2. Pop heap and append to res array
3. Repeat for each index?

**Only add next index element from popped array**

# Store the index

Example (N = 12, K = 3):z

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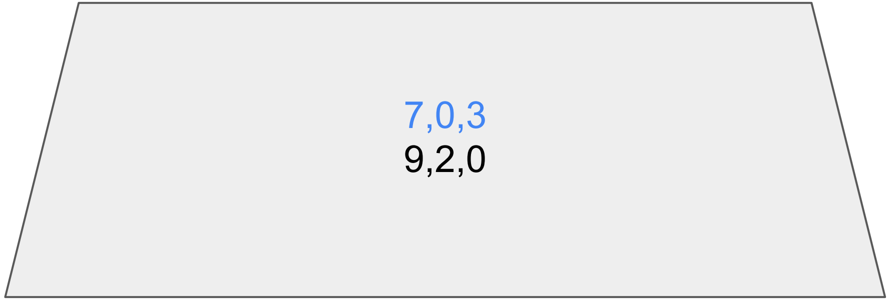


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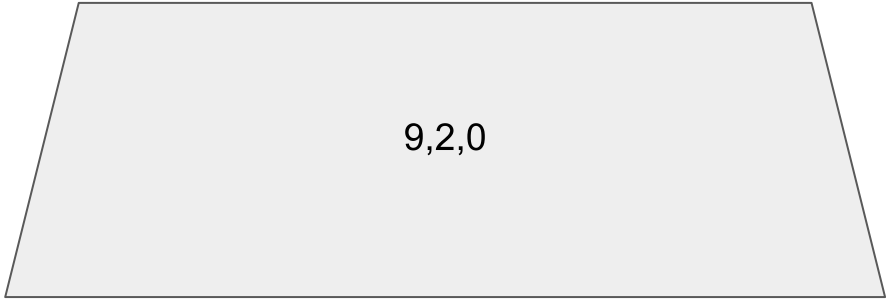


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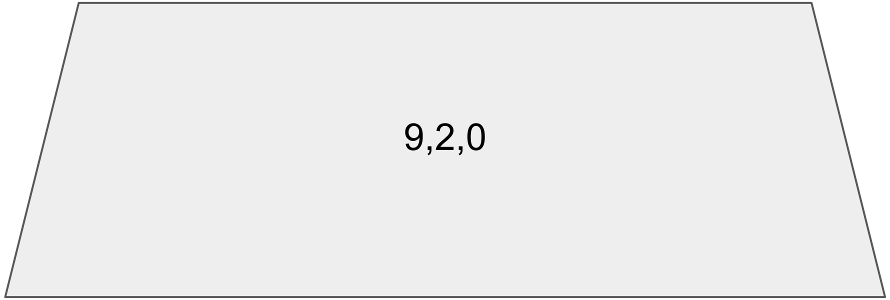
Add everything left from last array to res

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# Time complexity?

Example (N = 12, K = 3):z

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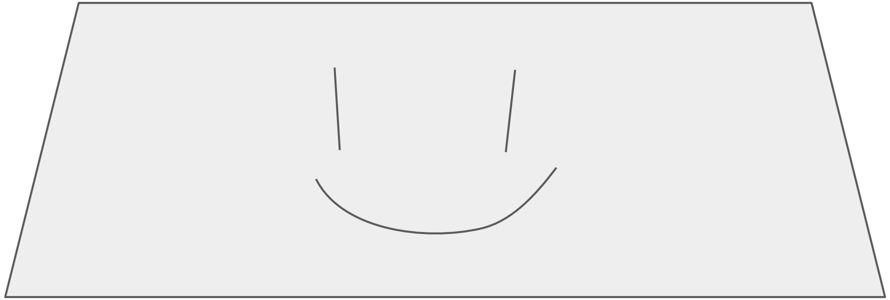

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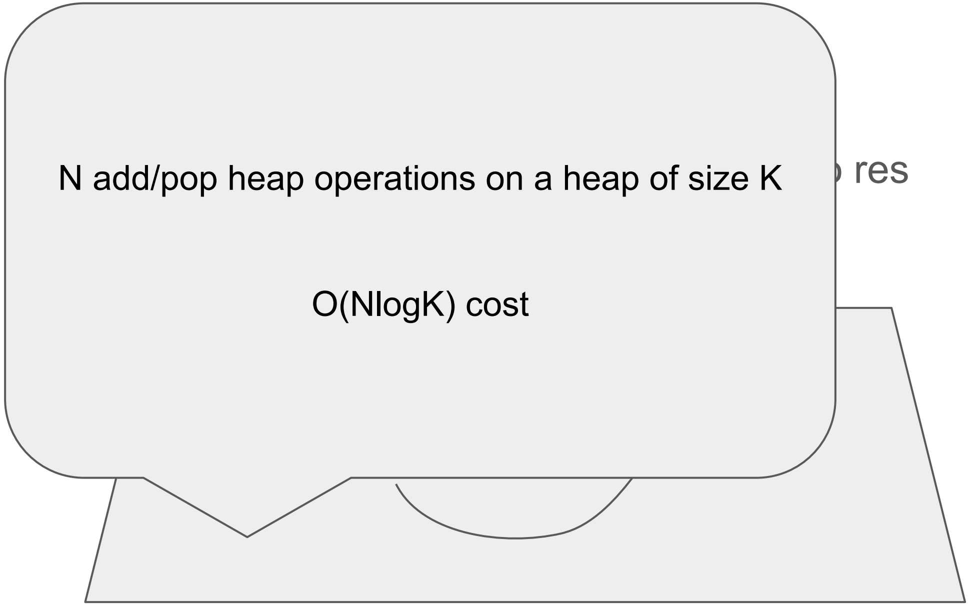


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**(Merge sort)** Merge sort is in its nature, a Divide-and-Conquer algorithm.

(1) Suppose that when doing a Mergesort you recursively break lists into 4 equal-sized sub-arrays instead of 2. Will you get a better runtime performance asymptotically?

(2) You are given two sorted arrays that are identical except that one of them is missing a single element. In other words, one array has length $n$ and the other has length $n-1$. The goal is to design an efficient algorithm with $O(\log n)$ runtime that finds the missing element.

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Whenever you see

1. Array is sorted
2. O(log n) time required

99%* of the time, you can use a **modified binary search**


* Source: It was revealed to me in a dream

# Example of Binary Search for x = 5

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

```
def binarySearch(A[1:n],x):
    l = 1, r = n
    while l <= r:
        m = ⌊(l + r) / 2⌋
        if A[m] == x:
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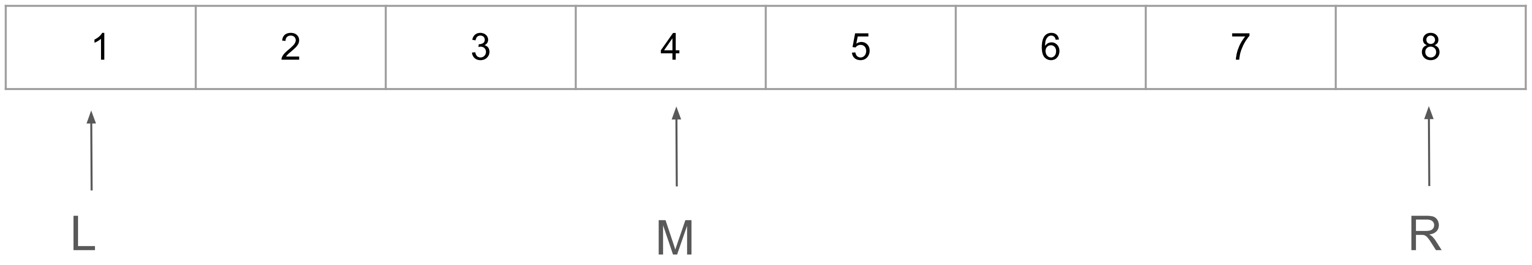


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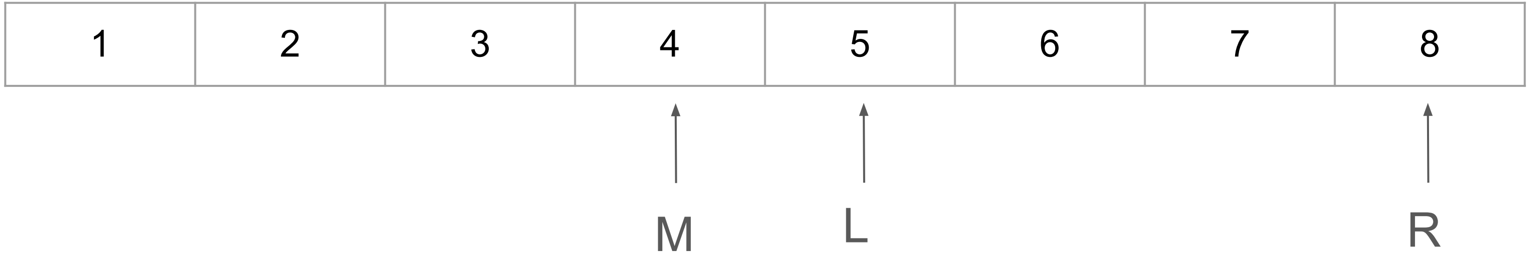


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            l = m + 1
        else:
            r = m - 1
    return -1
```

First iteration done.

# Example of Binary Search for x = 5

| | | | | | | | |
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L M R

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```

Since L = R, we found x!

# Important parts of Binary Search

```
def binarySearch(A[1:n],x):
    l = 1, r = n
    while l <= r:
        m = ⌊(l + r) / 2⌋
        if A[m] == x:
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    return -1
```

1. Search Range
2. Stop condition
3. Interval cutting

# Our case

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

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What should the search range be?

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def binarySearch(A[1:n],x):
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L R

What should the search range be?

L = 1, R = n, the missing index could be any index

```
def binarySearch(A[1:n],x):
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1. Search Range
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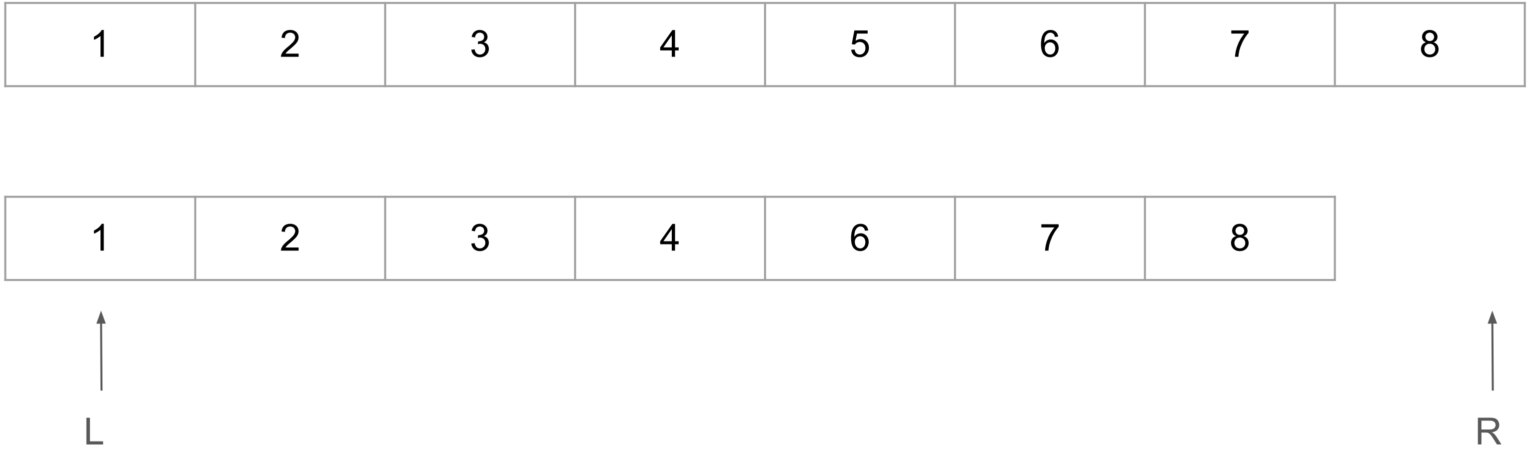


When should we stop?

This is often much trickier, run through the algorithm to figure this out.

```
def binarySearch(A[1:n],x):
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```

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L M R

When should we stop?

This is often much trickier to figure out next.

Instead, run through the binary search to figure out how to *interval cut*

```
def binarySearch(A[1:n],x):
    l = 1, r = n
    while l <= r:
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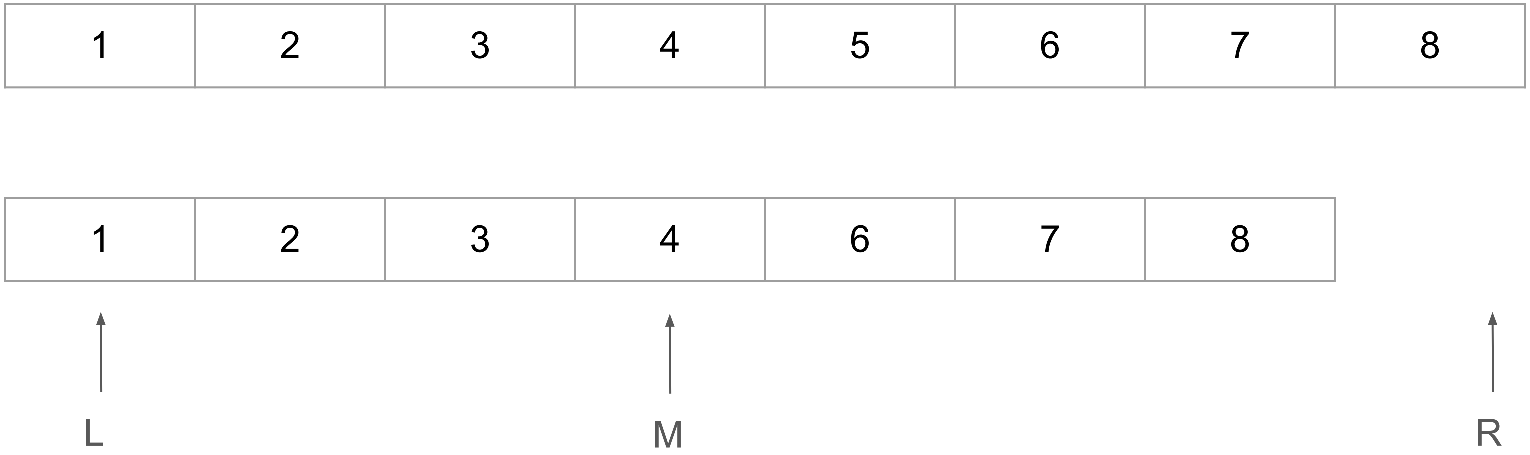


How should we cut the interval? What does A[m] and B[m] tell us?

```
def binarySearch(A[1:n],x):
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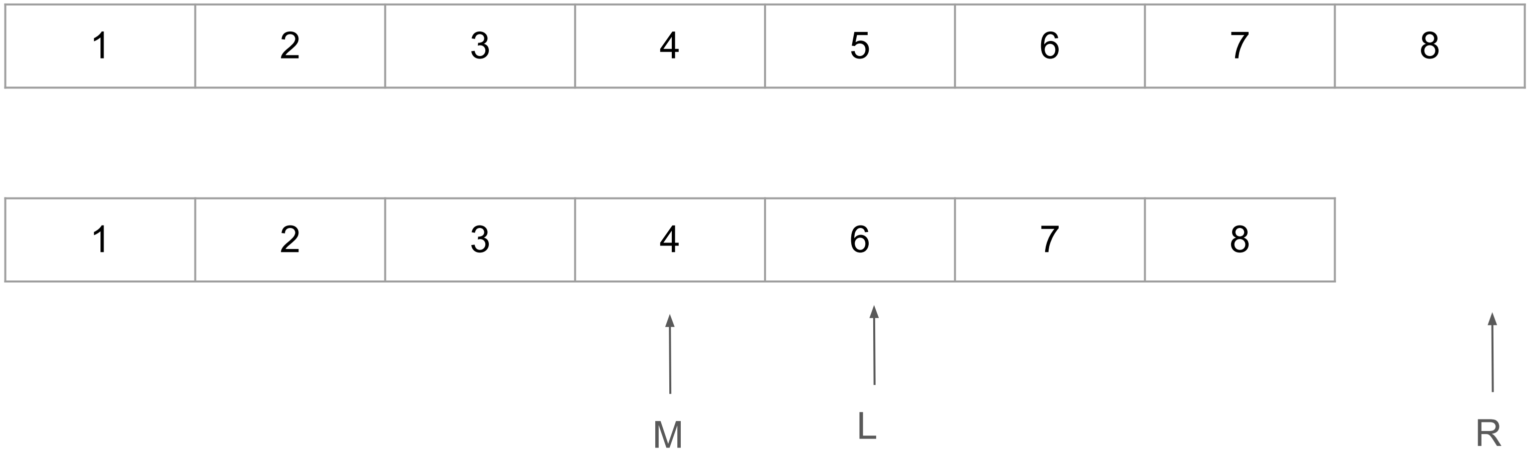


How should we cut the interval?

- **A[m] == B[m] → everything before is equal.**
  - **The missing element must be in the right half!**

```
def binarySearch(A[1:n],B[1:n-1]x):
    l = 1, r = n
    while l <= r:
        m = ⌊(l + r) / 2⌋
        if ???
            return m
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M L R

How should we cut the interval?

- A[m] == B[m] implies that everything before is equal. The missing element must be in the right half!

Continue running the algorithm to figure out when to stop

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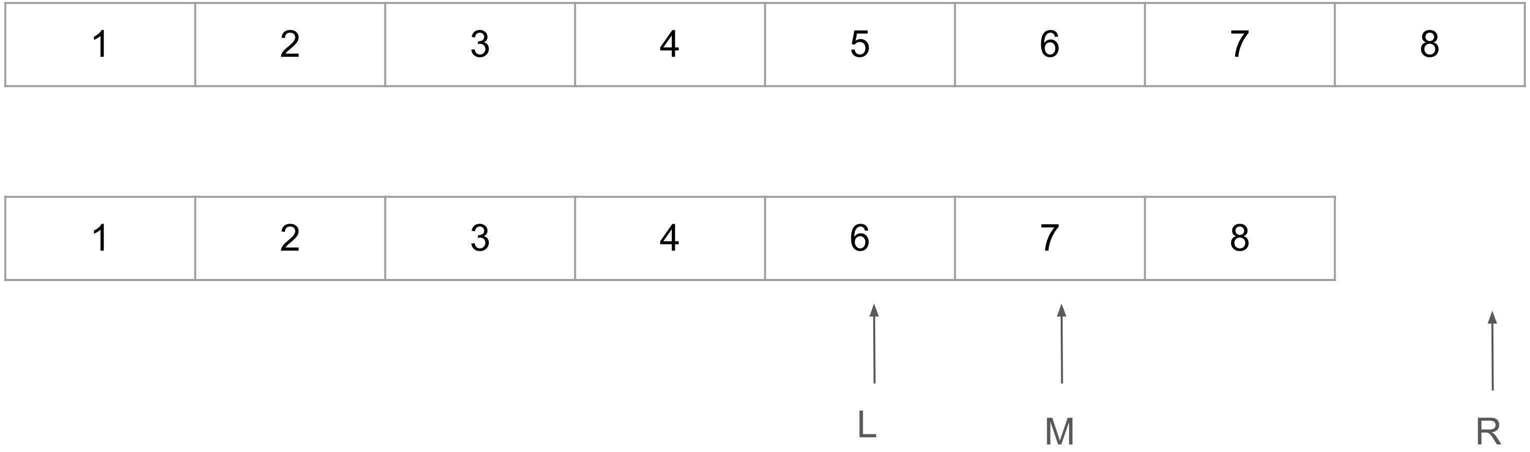


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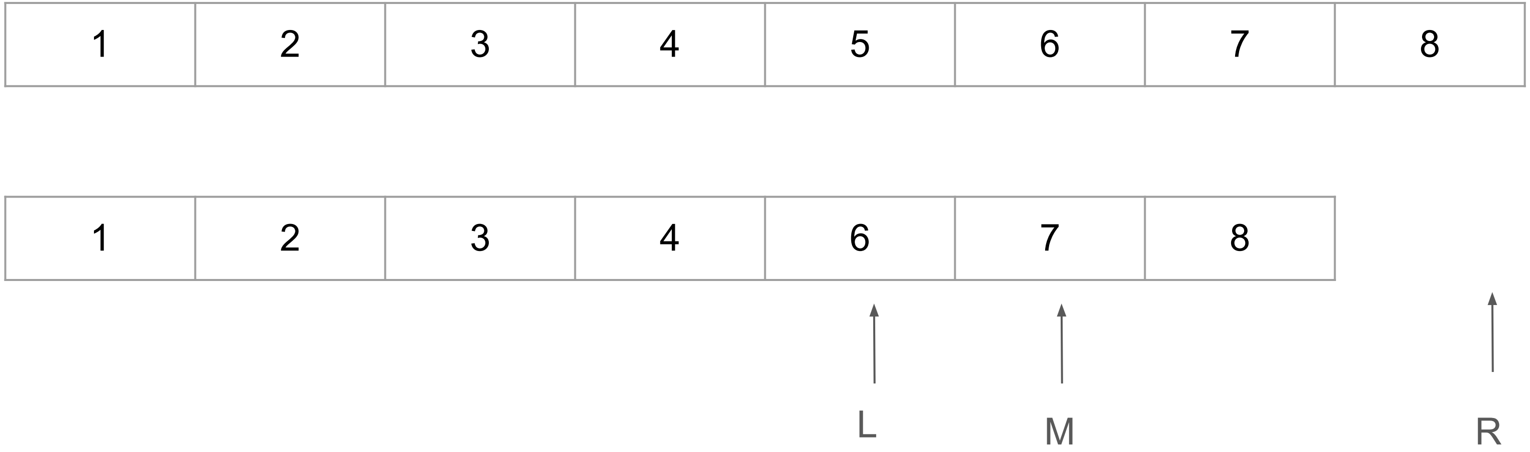


How should we cut the interval?

- A[m] == B[m] implies that everything before is equal. The missing element must be in the right half!
- **A[m] != B[m] → something in range [l,m] must be missing.**
  - **The missing element must be in the left half!**

Continue running the algorithm to figure out when to stop

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L R M

This seems like a good place to stop our algorithm. What's the stop condition?

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**When l == r**

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```

1. Search Range
2. Stop condition
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# Last book keeping

Written slightly cleaner since we are guaranteed to have a missing element

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1. Search Range
2. Stop condition
3. Interval cutting

```
def binarySearch(A[1:n],B[1:n-1]x):
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    while l < r:
        m = ⌊(l + r) / 2⌋
        if A[m] = B[m]:
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        else:
            r = m - 1
    return l
```

## Question 2

**(Quick sort)**

(1) Illustrate the operation of the **Partition** step in Quick sort on $A = [2, 8, 7, 1, 3, 5, 6, 4]$.

(2) Can we understand the average-case runtime of Quick sort? What is the best policy for selecting the pivot value in the quick sort?

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
   p ← A[r]
   i ← l - 1
   for j from l to r - 1 do
      if A[j] < p then
         i ← i + 1
         swap(A, i, j)
      end if
   end for
   i ← i + 1
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end algorithm
```

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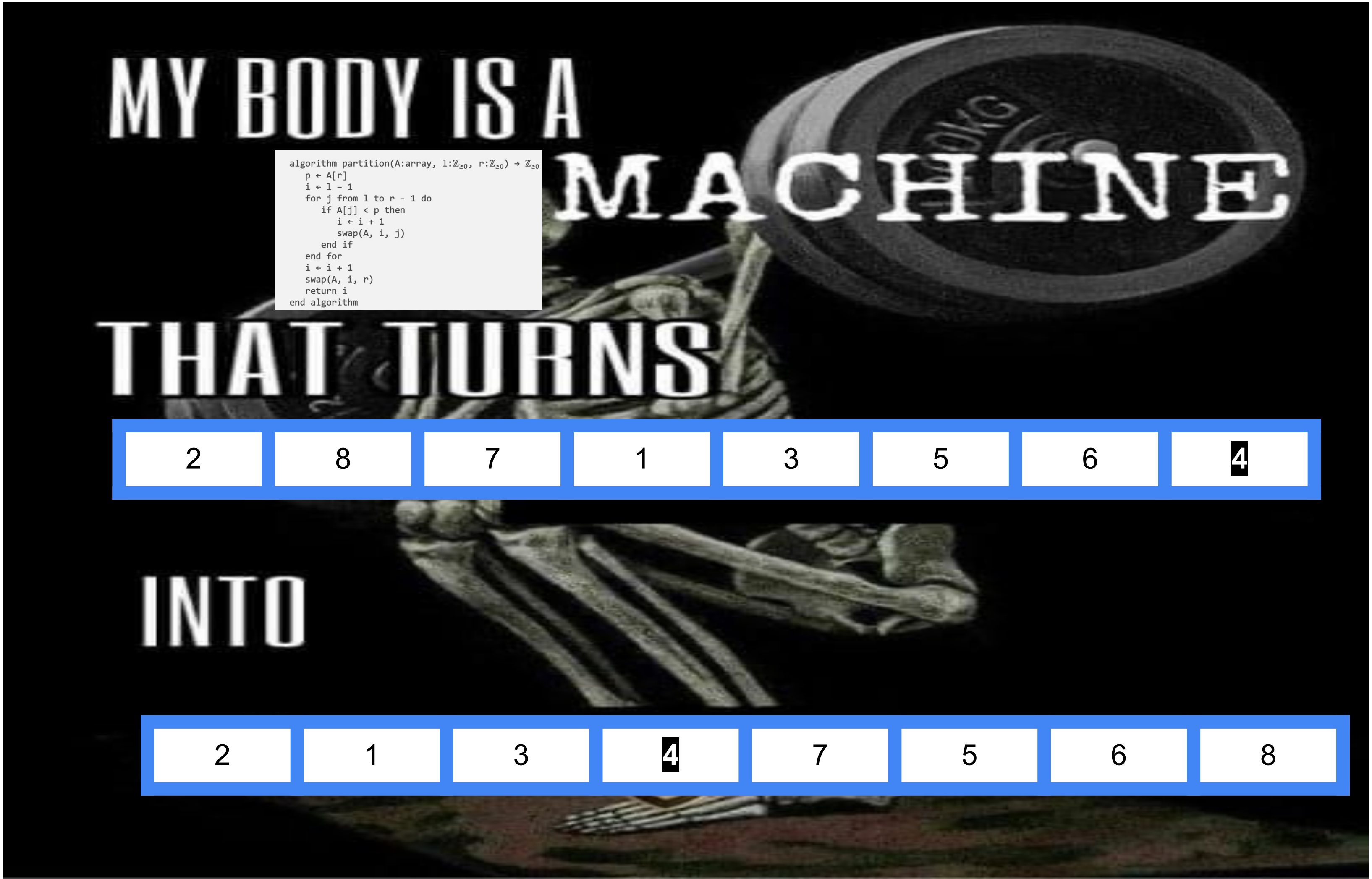
```

pivot p is set as last element

Ok… what does this do tho?

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```

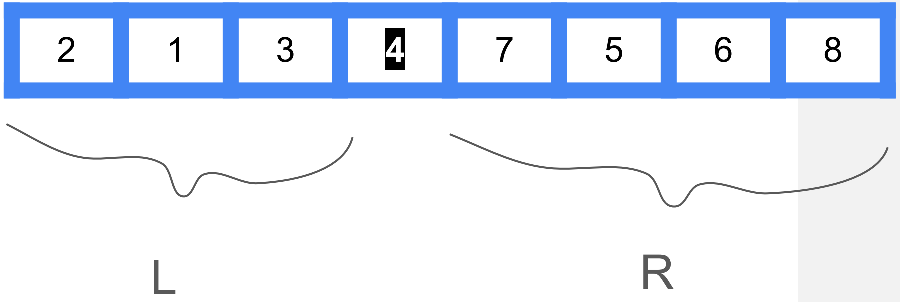
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 8 | 7 | 1 | 3 | 5 | 6 | 4 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 1 | 3 | 4 | 7 | 5 | 6 | 8 |

p: pivot

j: goes through entire array

i : growing index of L

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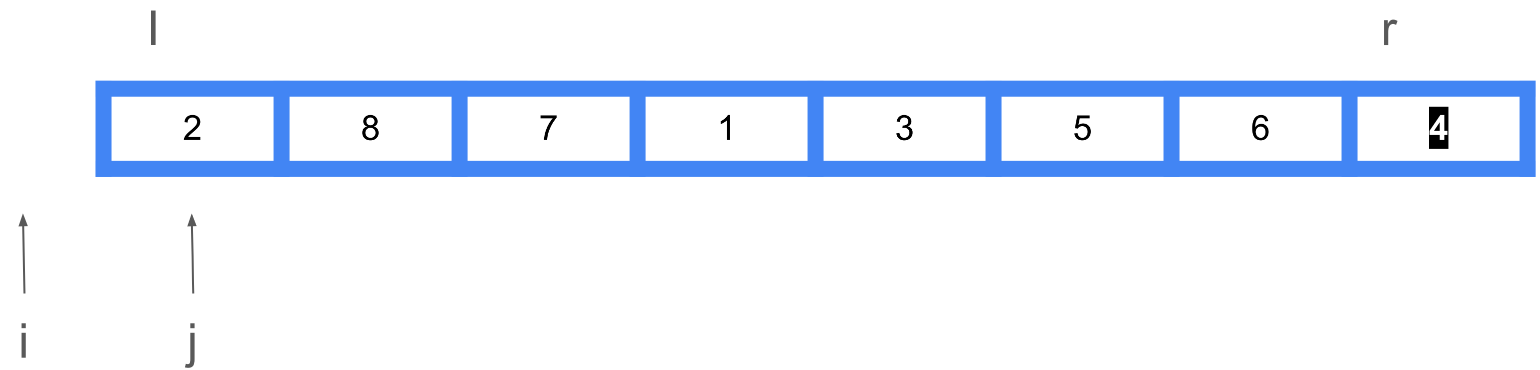


**Pivot = 4**

Start of the algorithm

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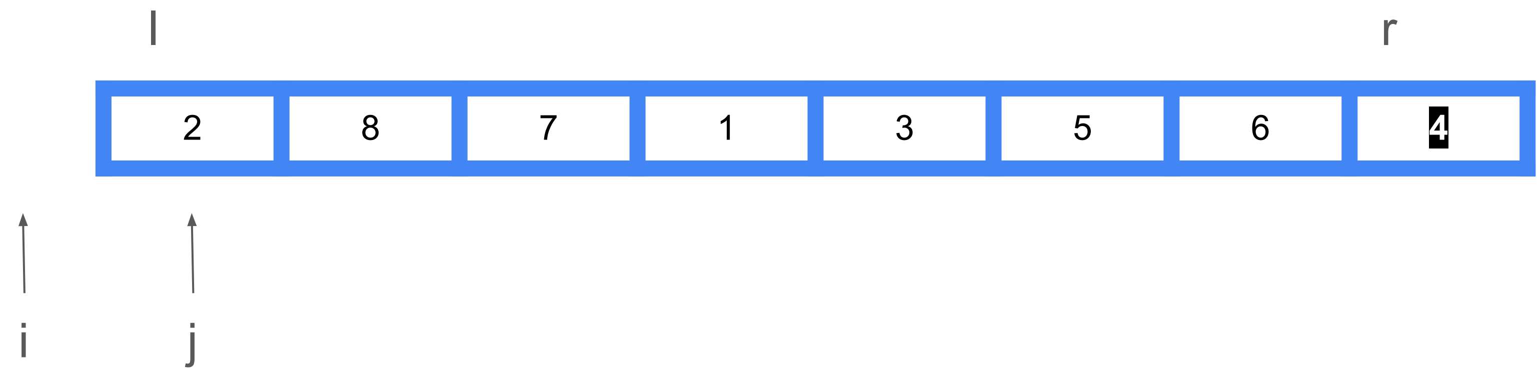


**Pivot = 4**

A[j] = 2 < 4

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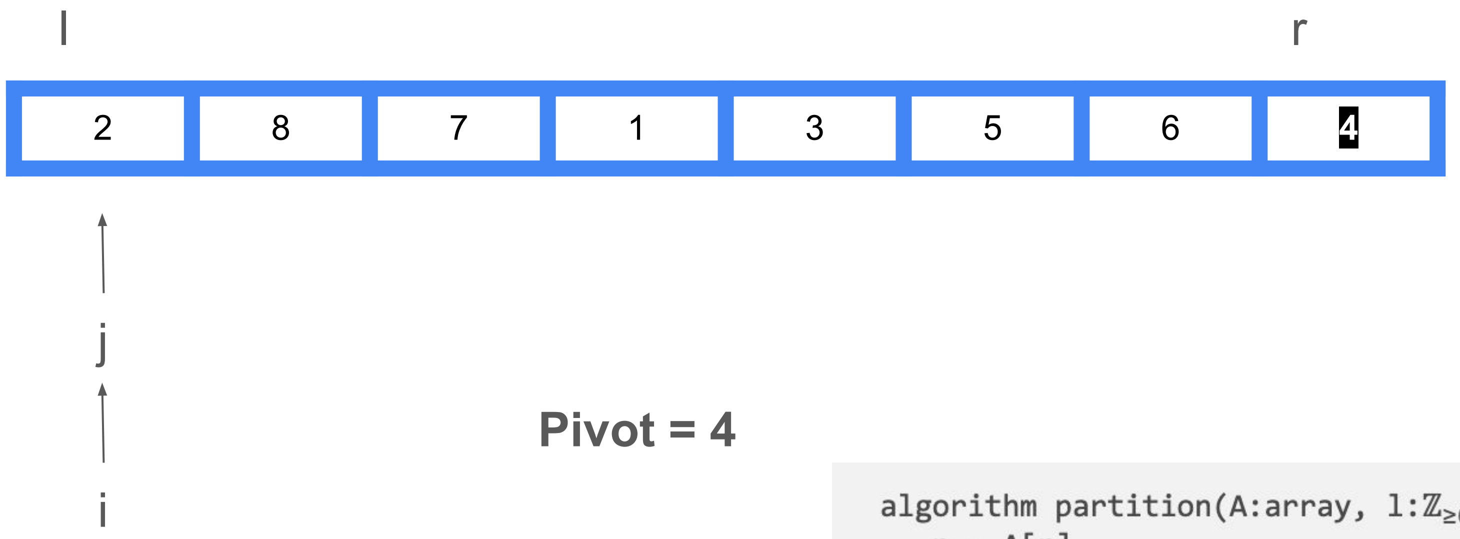


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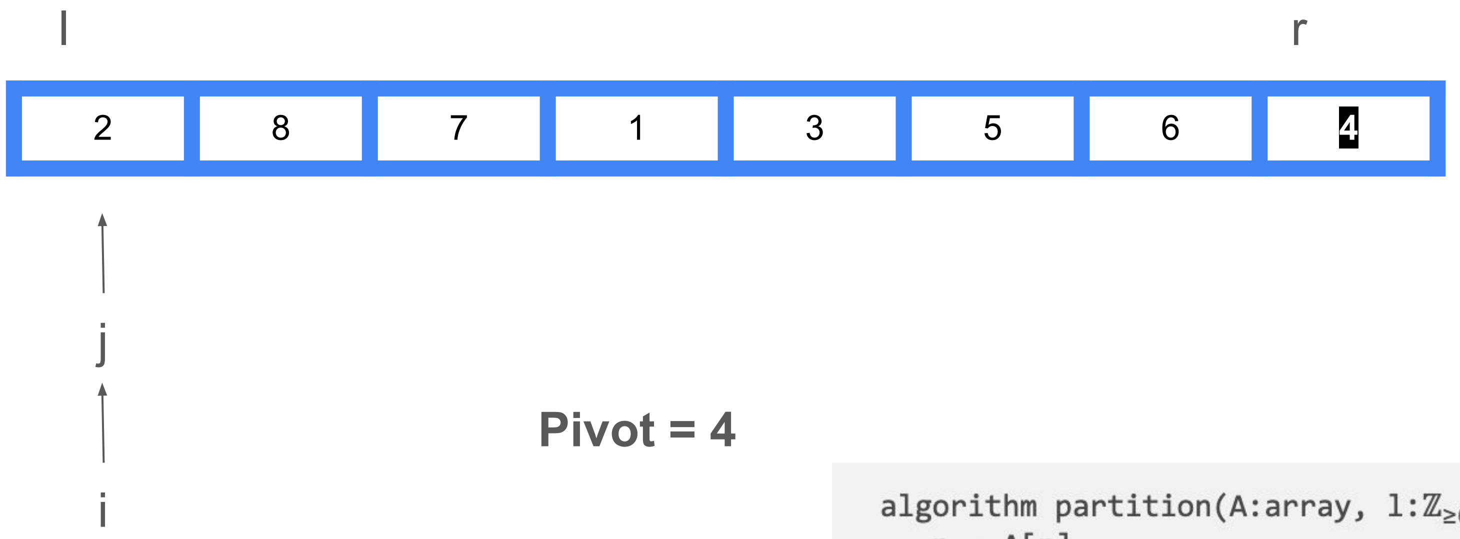
end algorithm
```




**Pivot = 4**

```
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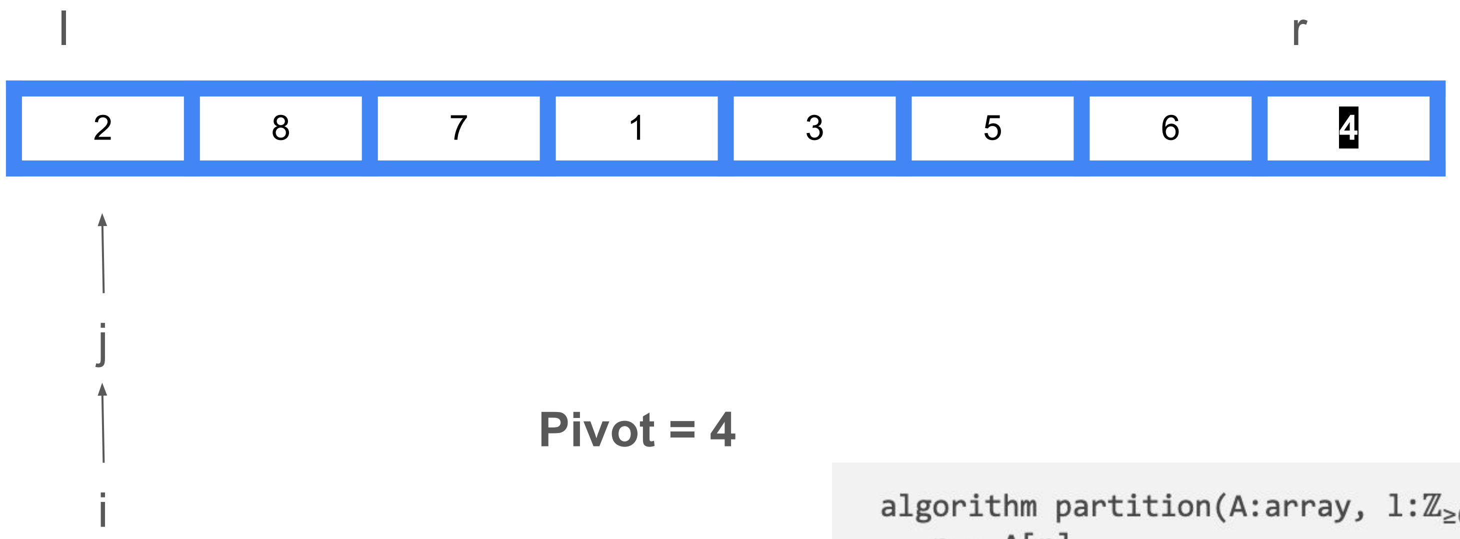


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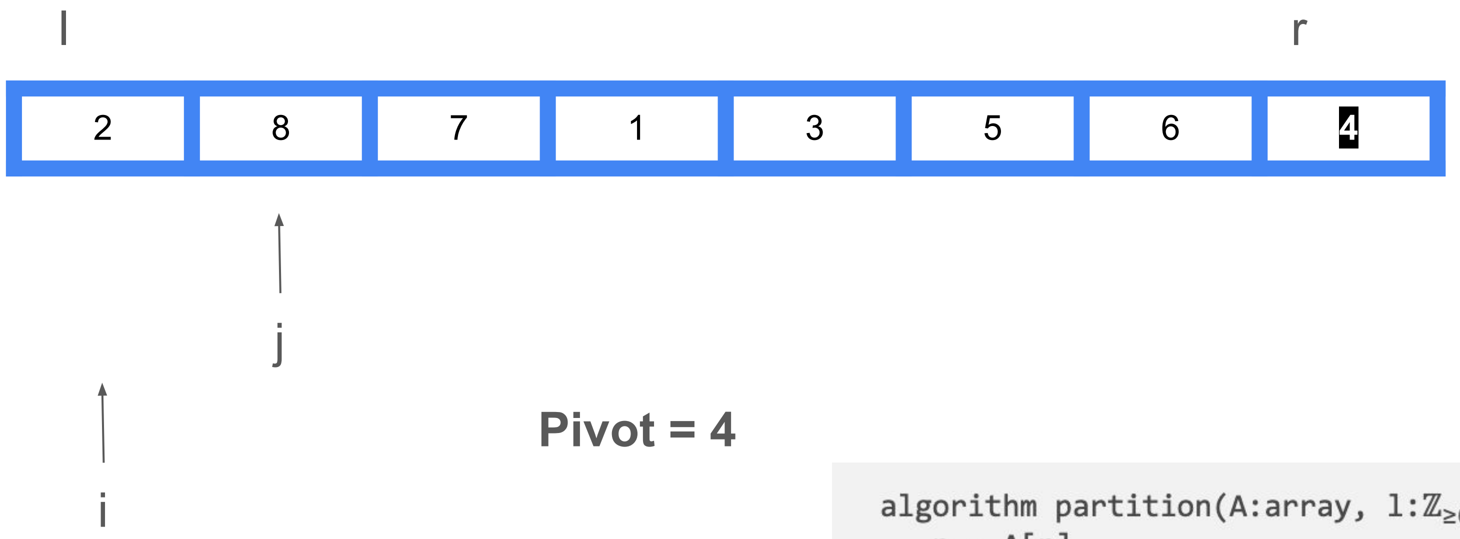


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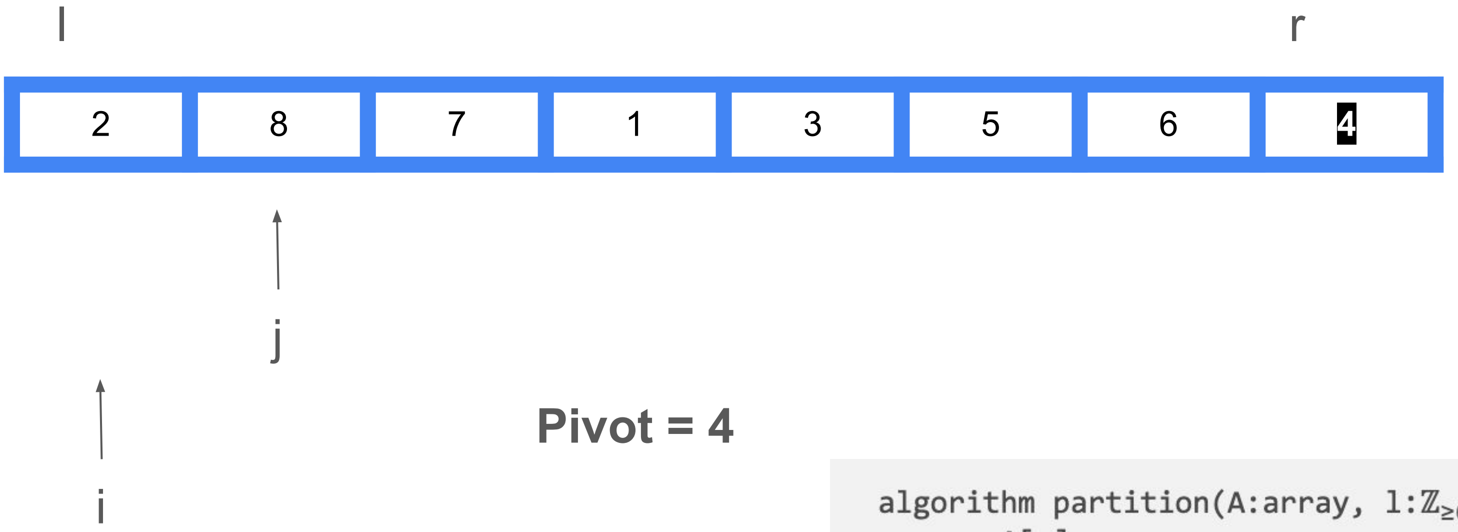


**Pivot = 4**

A[j] = 8 < 4? No

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
   p ← A[r]
   i ← l - 1
   for j from l to r - 1 do
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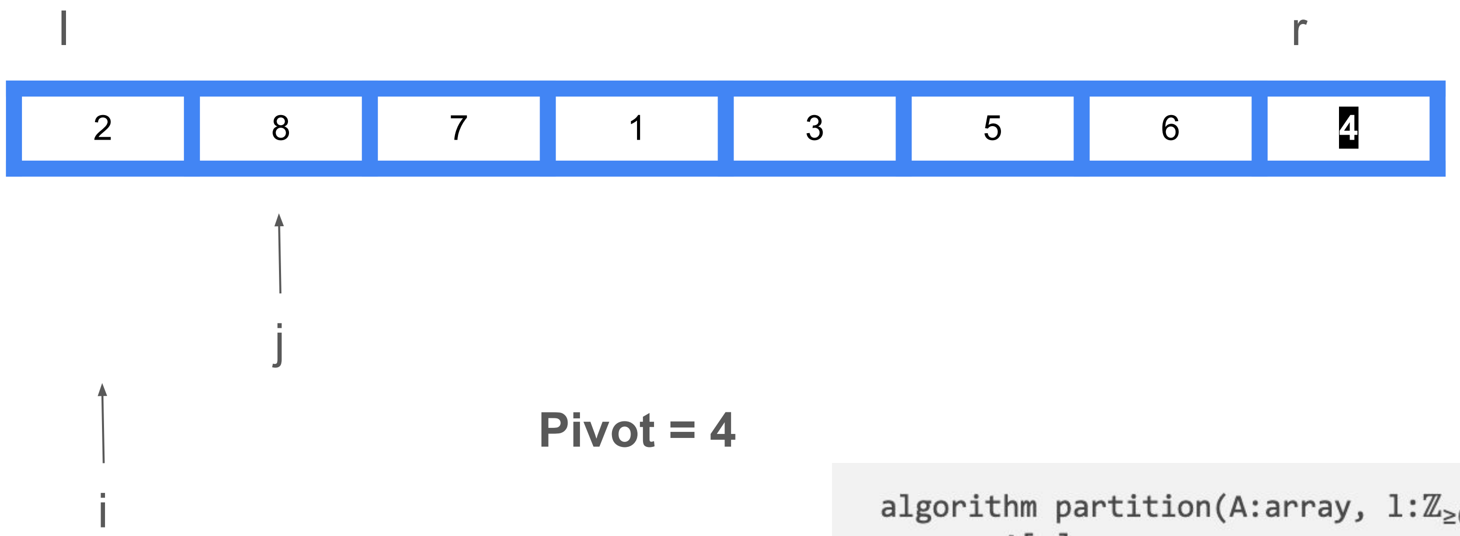


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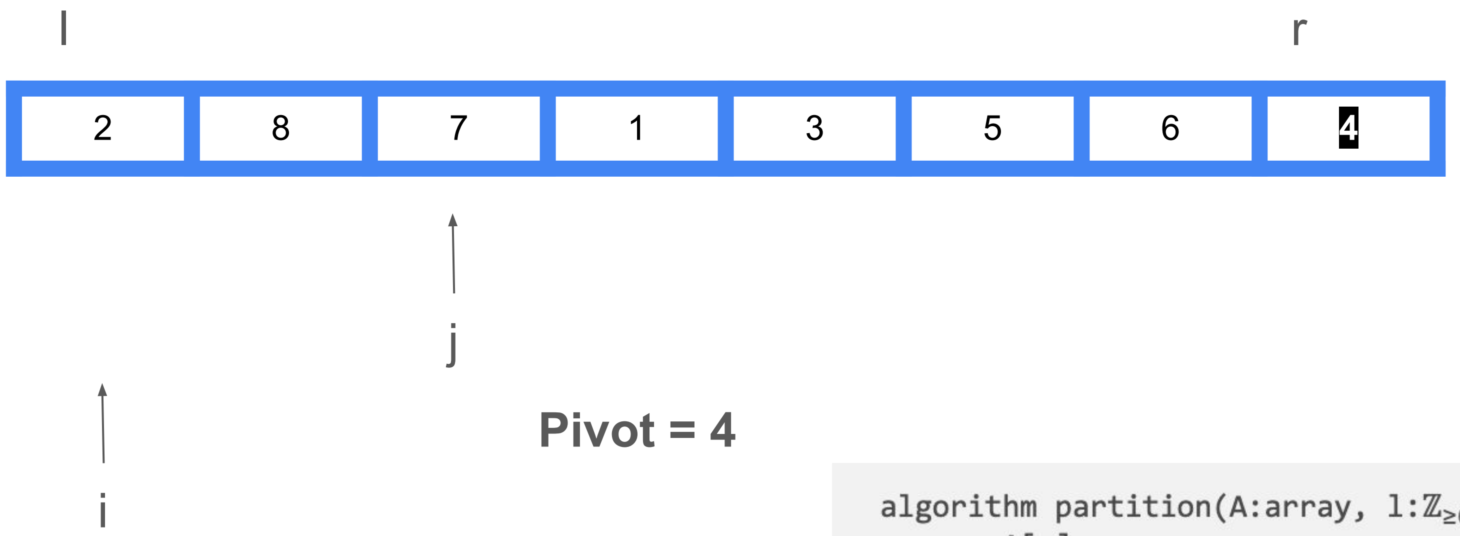


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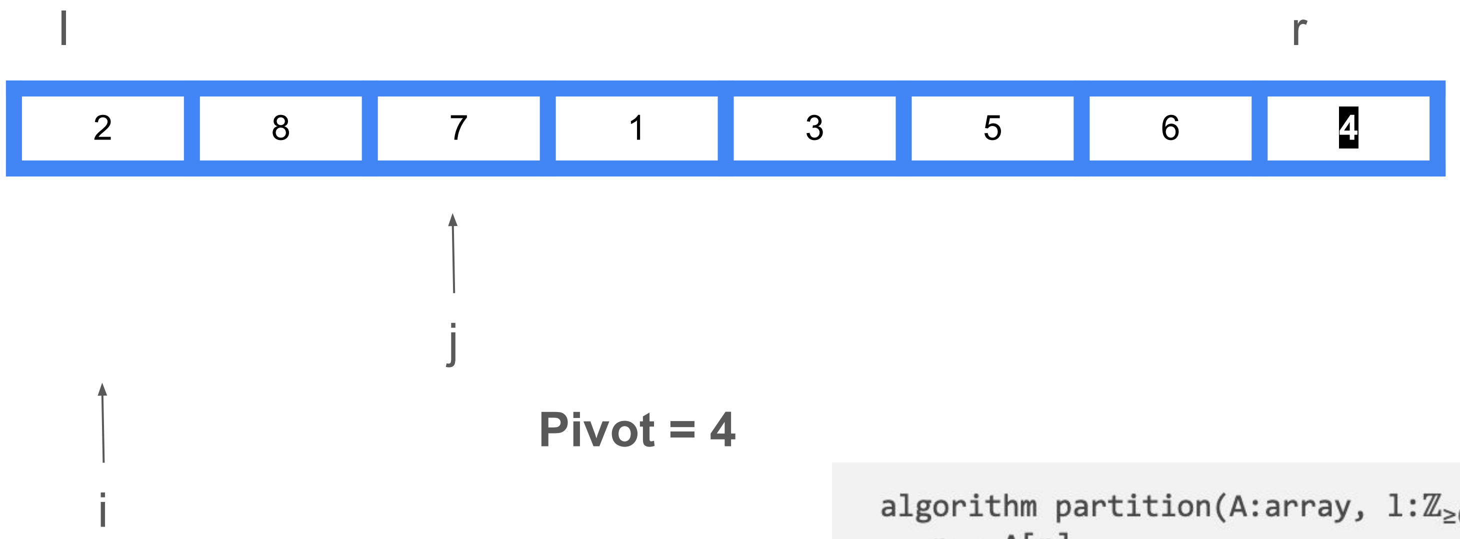


**Pivot = 4**

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   end for
   i ← i + 1
   swap(A, i, r)
   return i
end algorithm
```
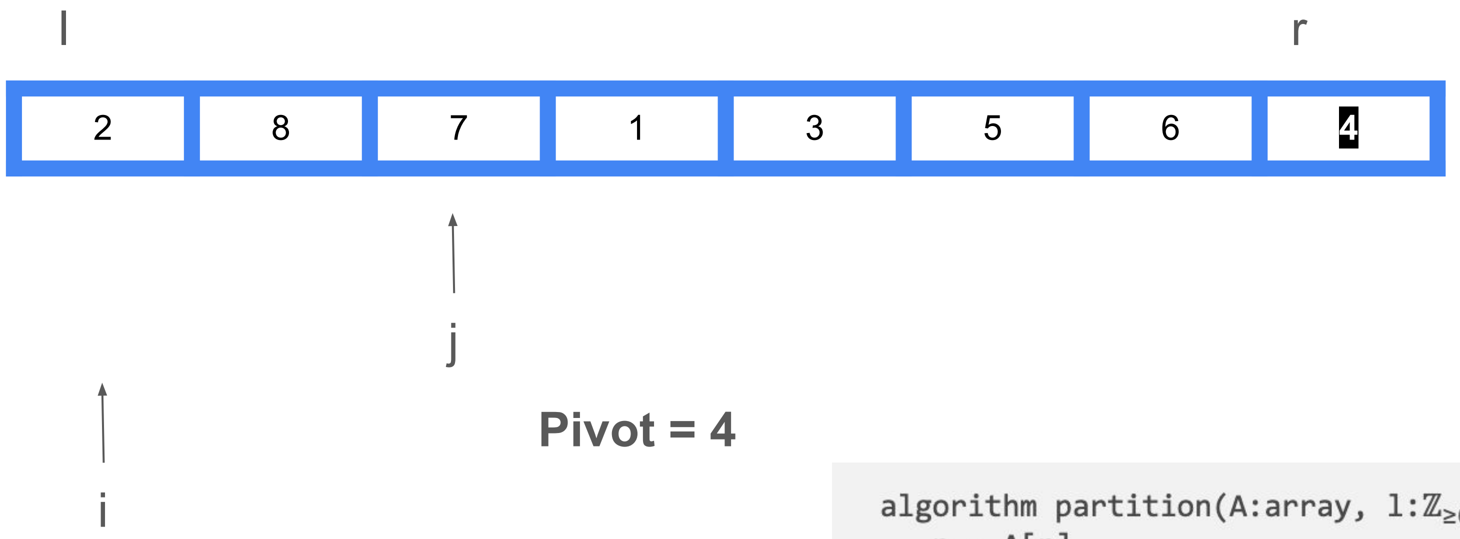

A[j] < 4? Nope

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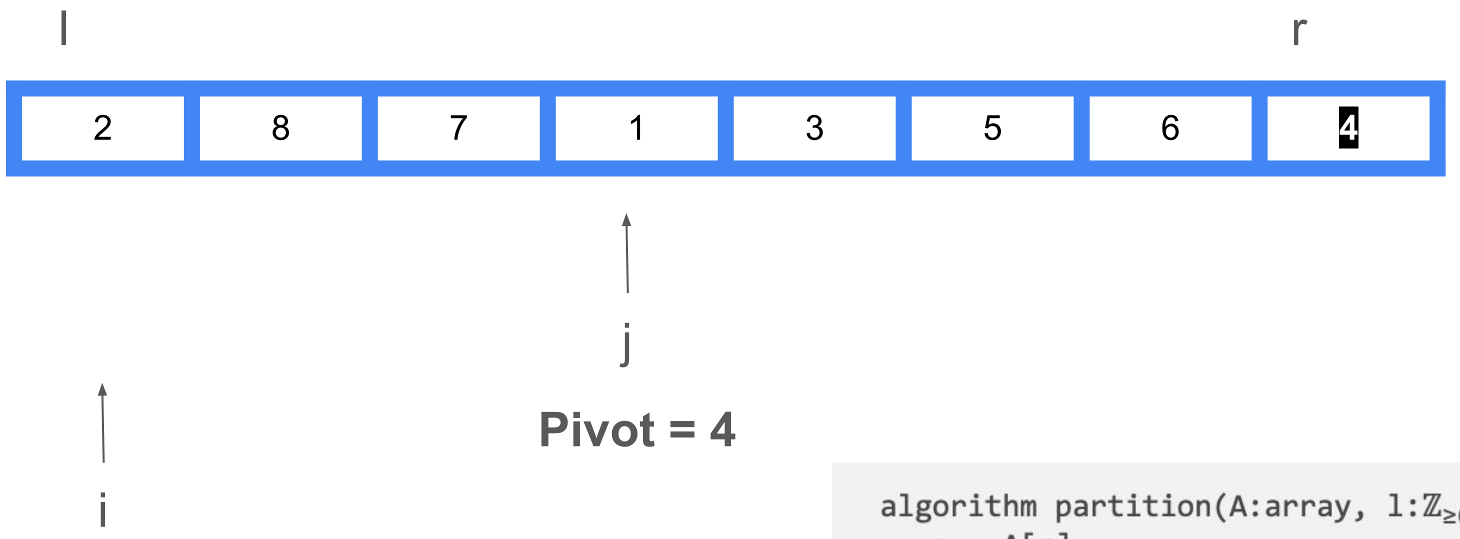


**Pivot = 4**

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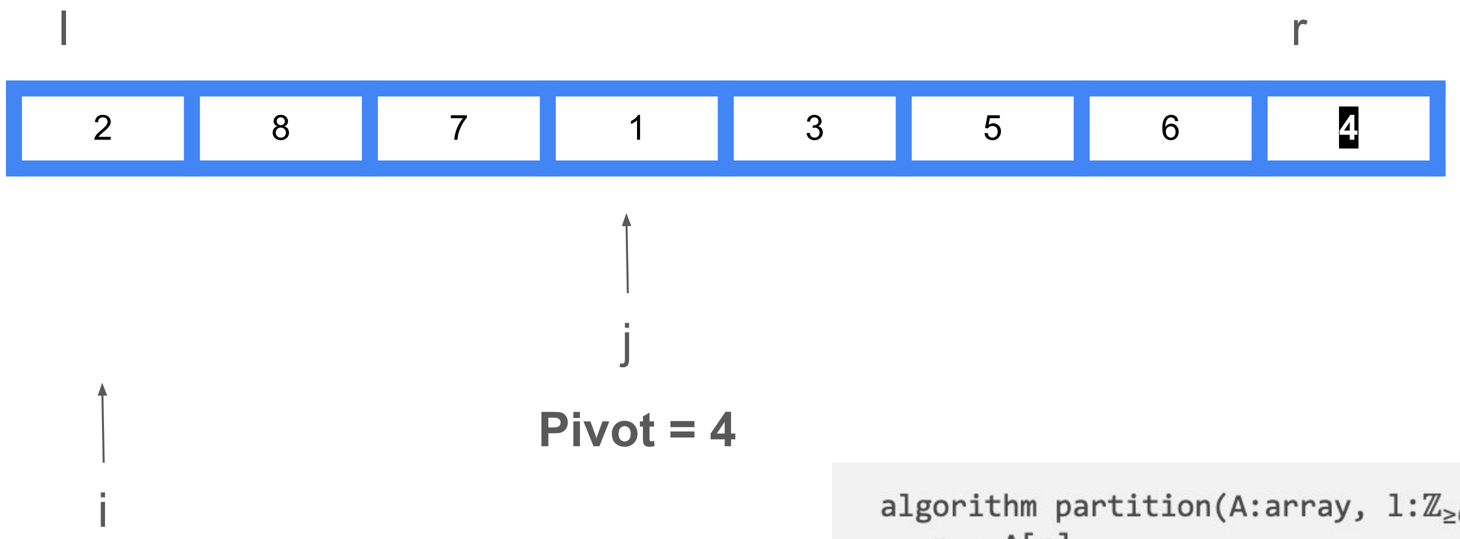


**Pivot = 4**

A[j] = 1 < 4

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
    p ← A[r]
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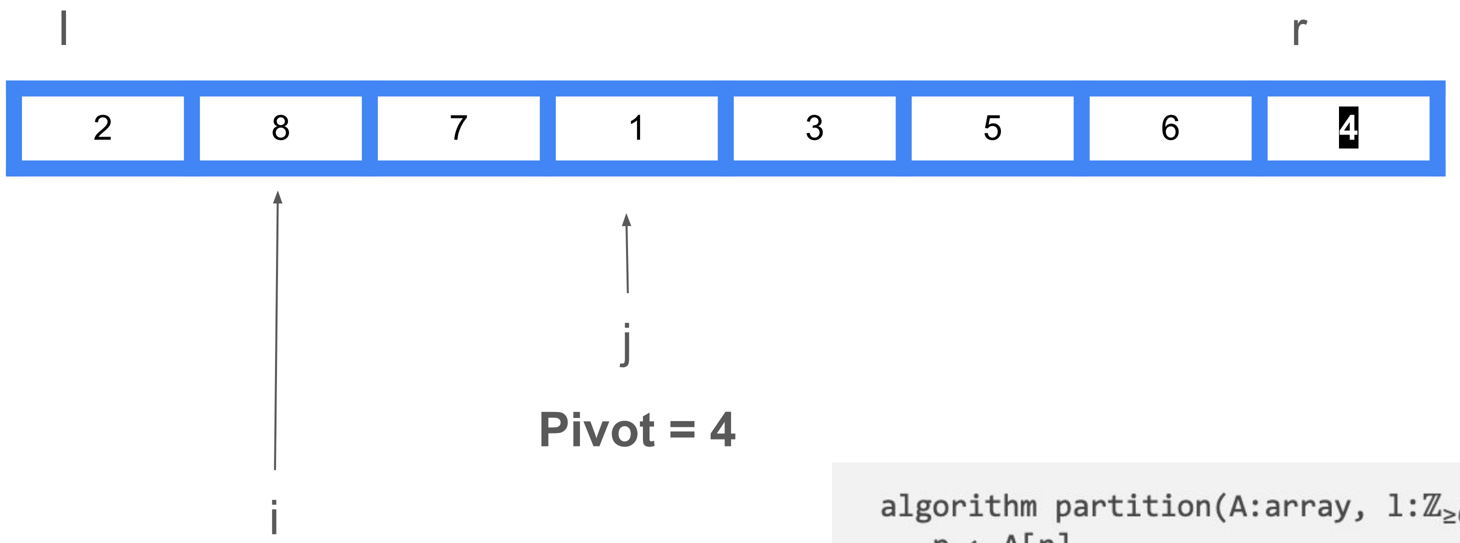


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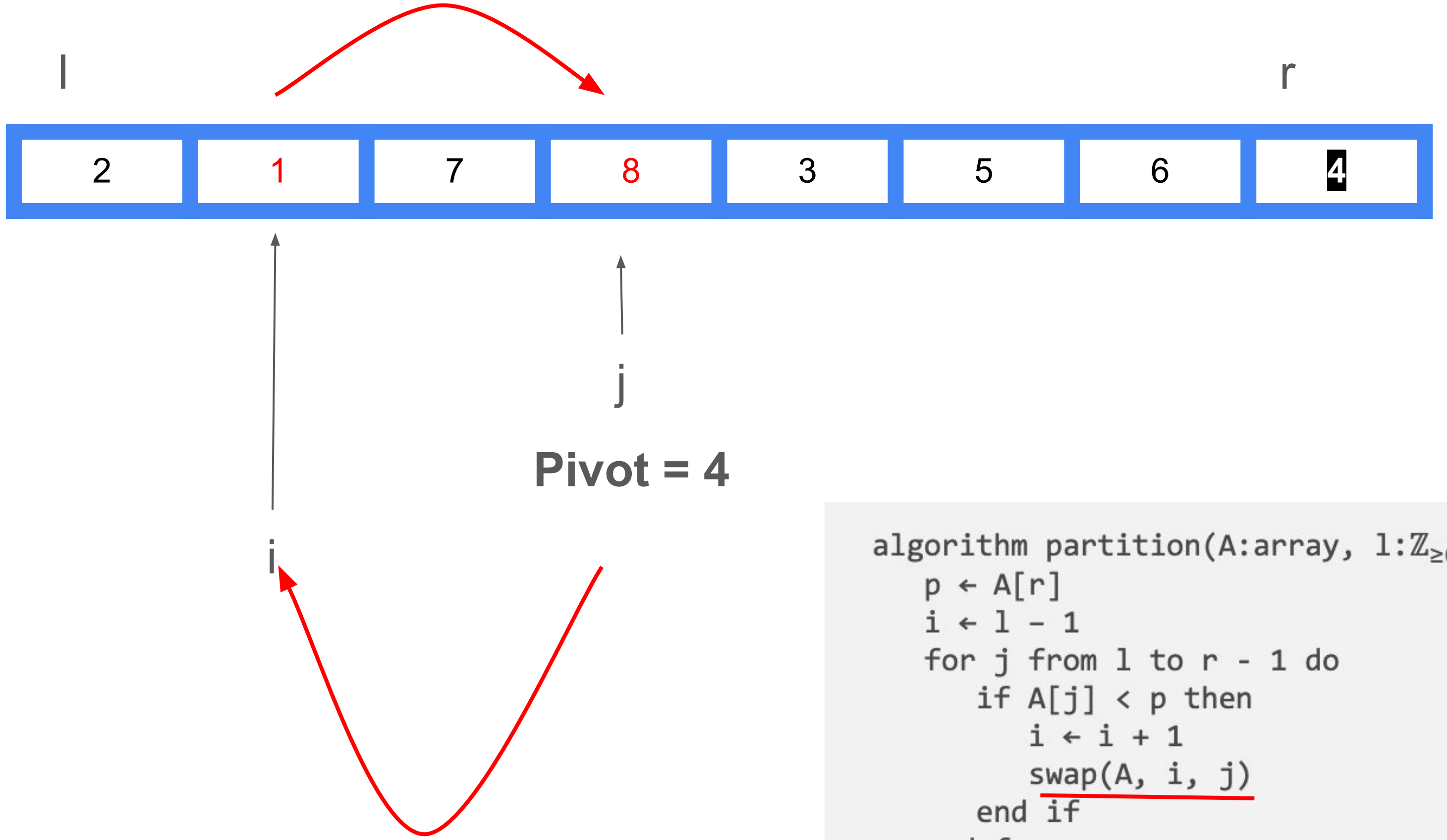


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```

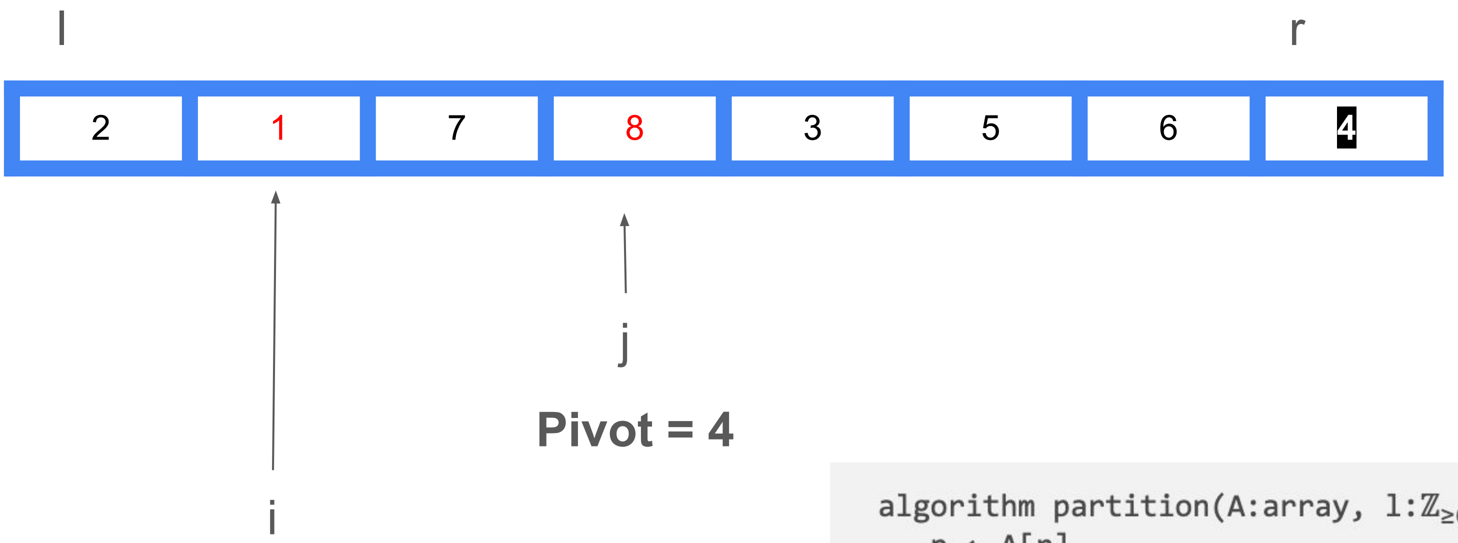


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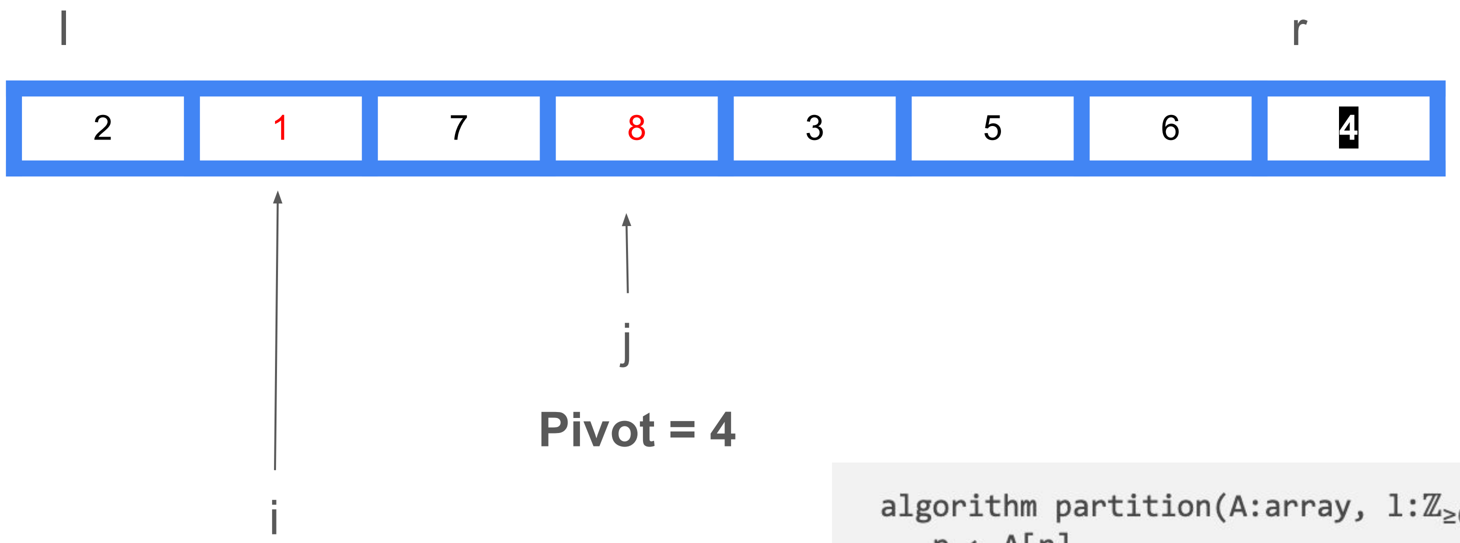
end algorithm
```

We’ve done two swaps now. Insight?




We’ve done two swaps now. Insight?

**Everything up to i is less than the pivot**

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
    p ← A[r]
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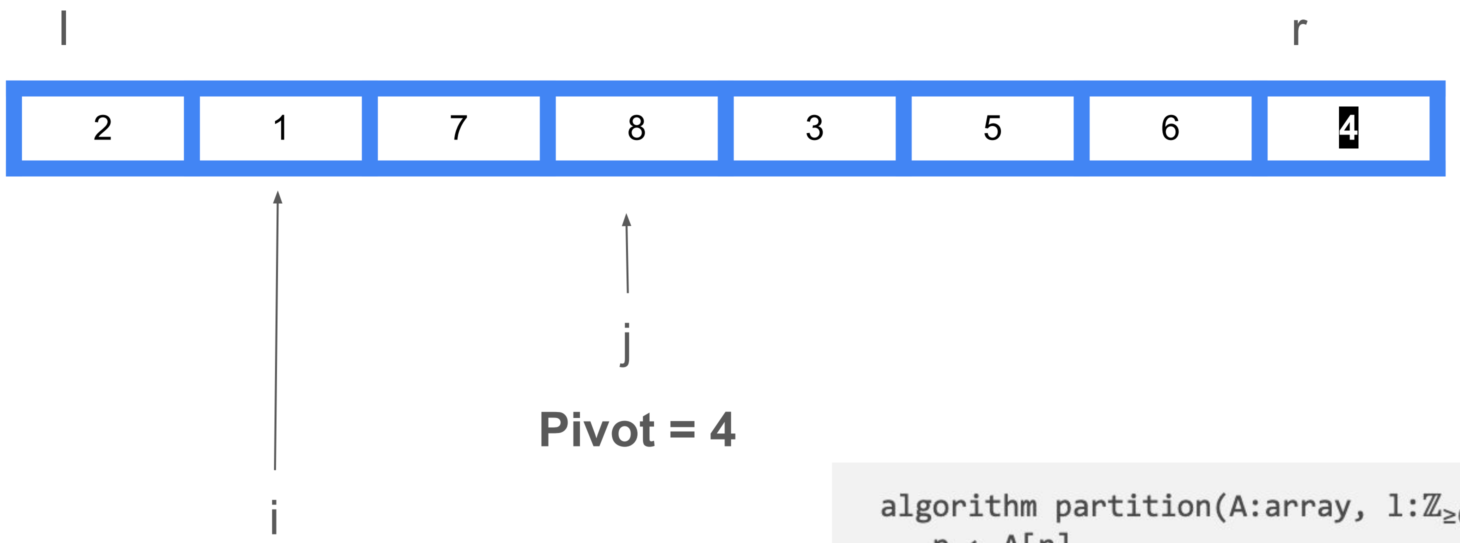


**Pivot = 4**

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
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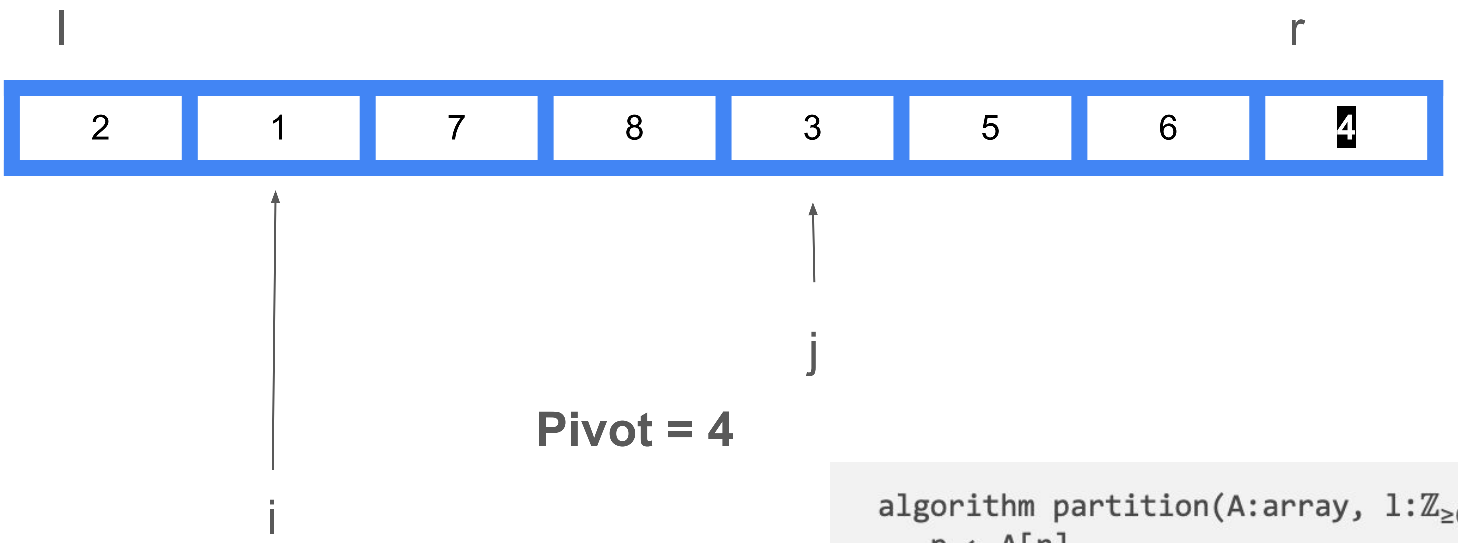


**Pivot = 4**

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end algorithm
```

l r

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 1 | 7 | 8 | 3 | 5 | 6 | 4 |

j

**Pivot = 4**

i

A[j] = 3 < 4, swap!

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
   p ← A[r]
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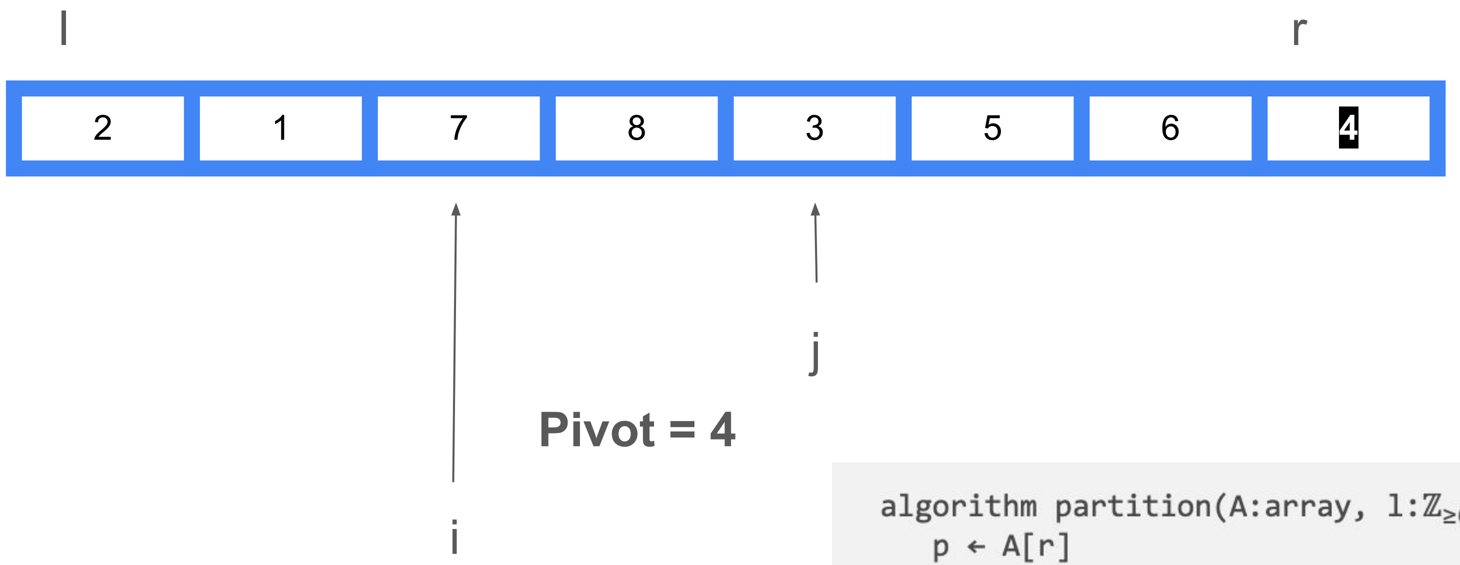


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```
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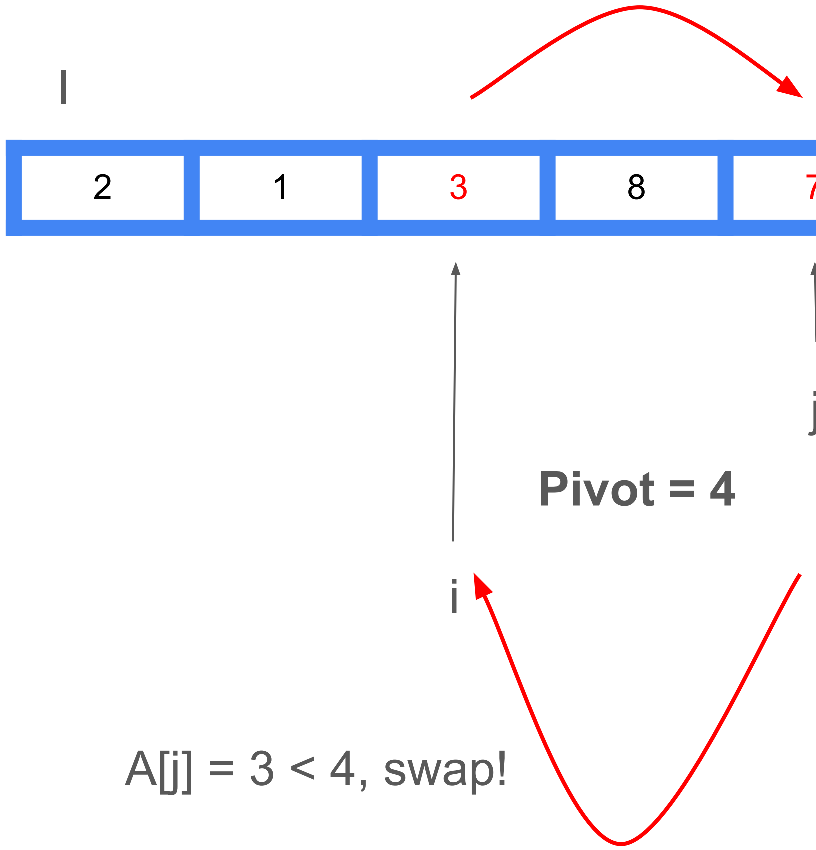


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l r

| | | | | | | | |
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j

**Pivot = 4**

i

**Everything up to i is less than the pivot**

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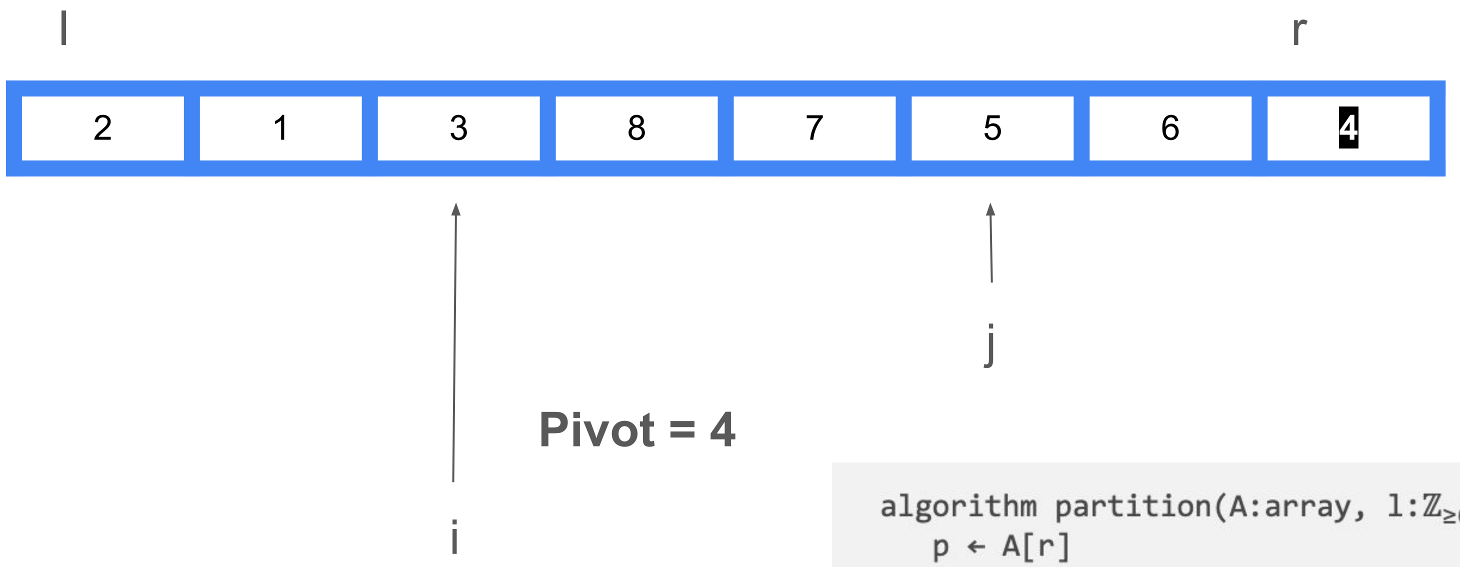


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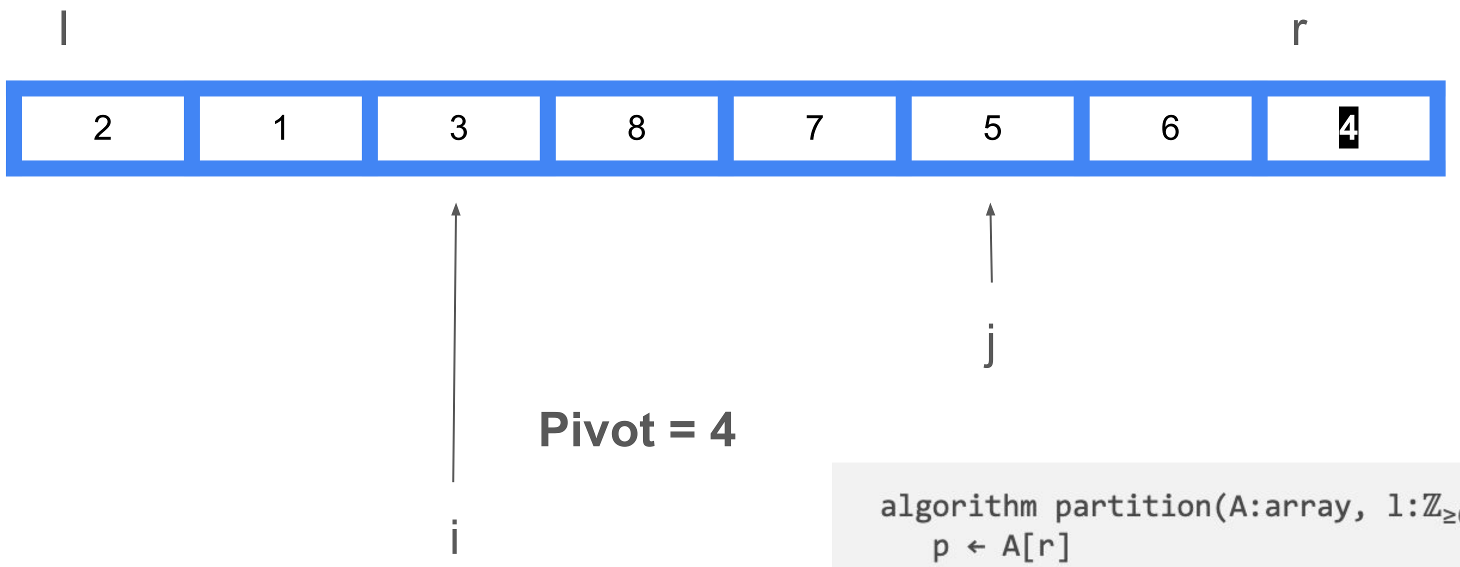


**Pivot = 4**

A[j] = 5 < 4? Nah

```
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   p ← A[r]
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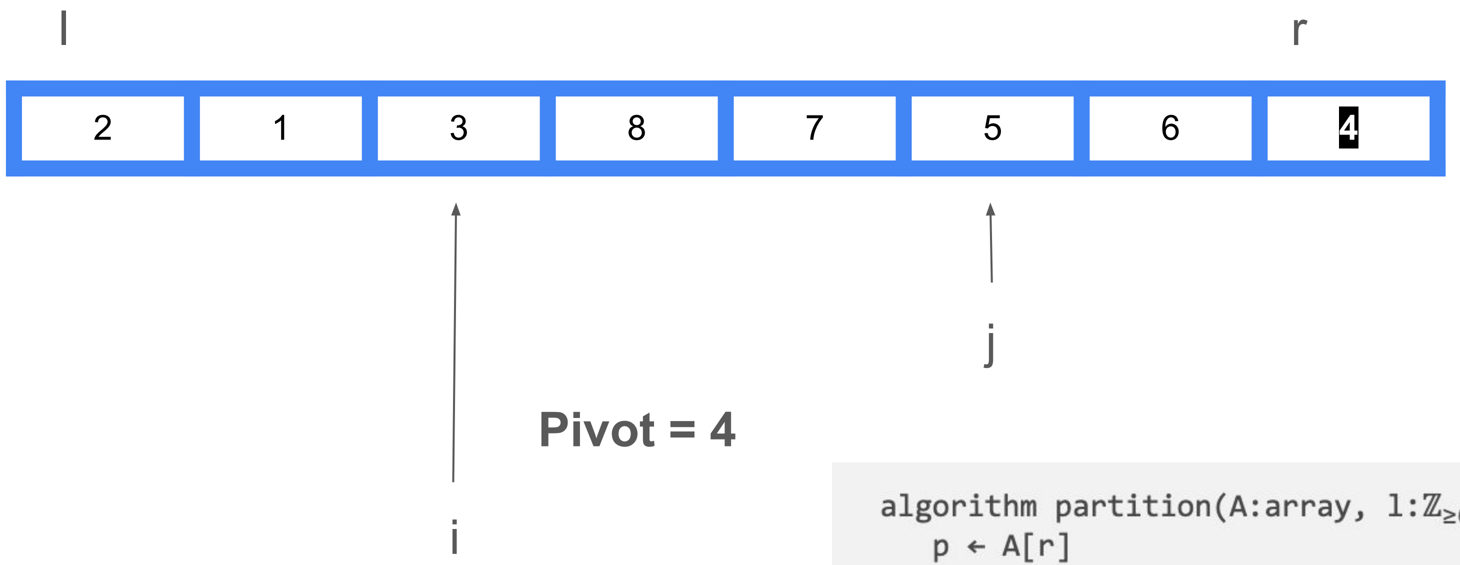


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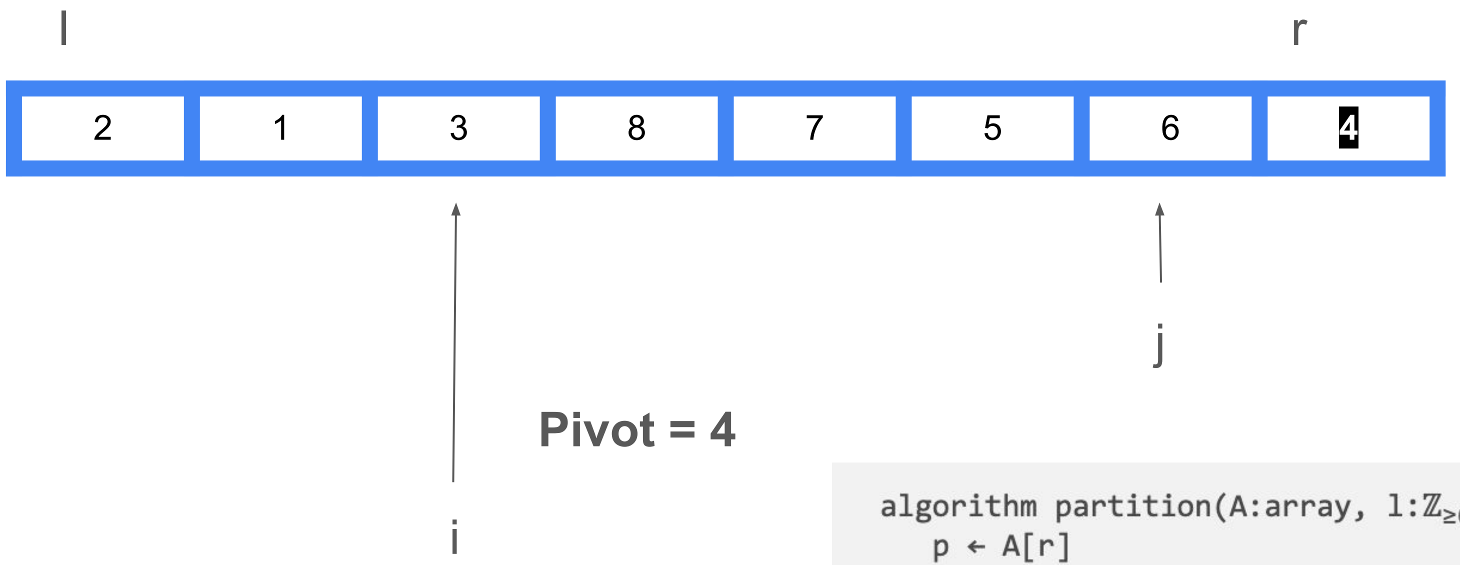


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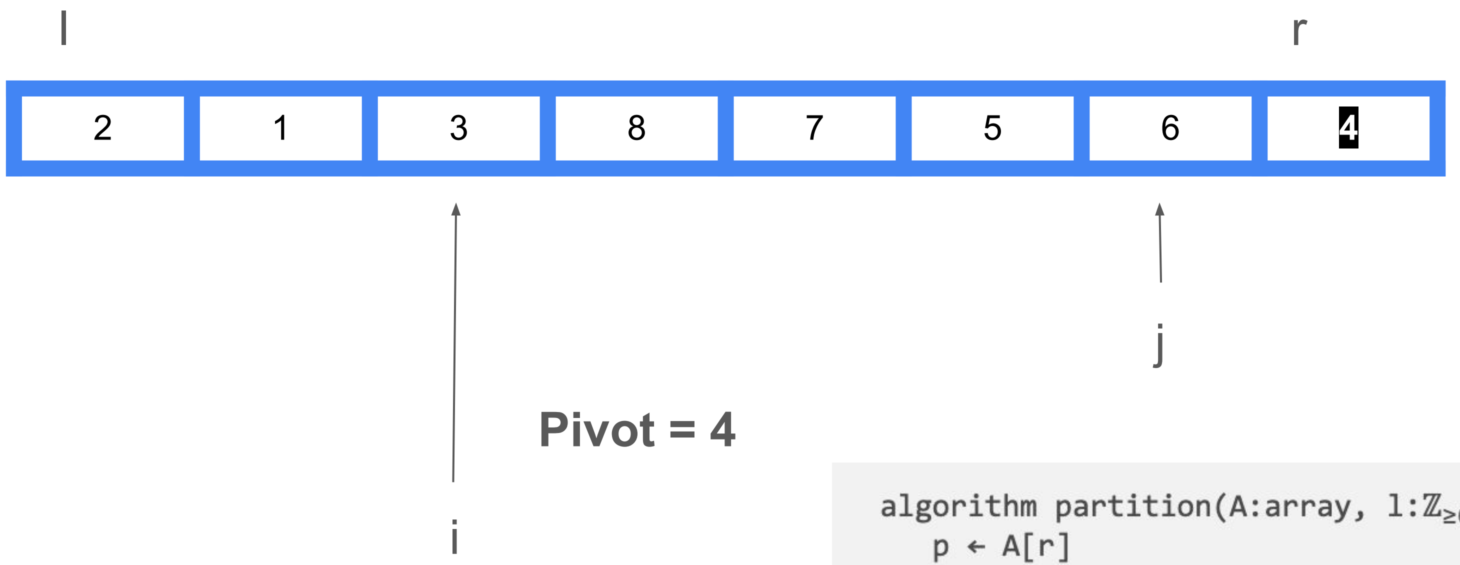


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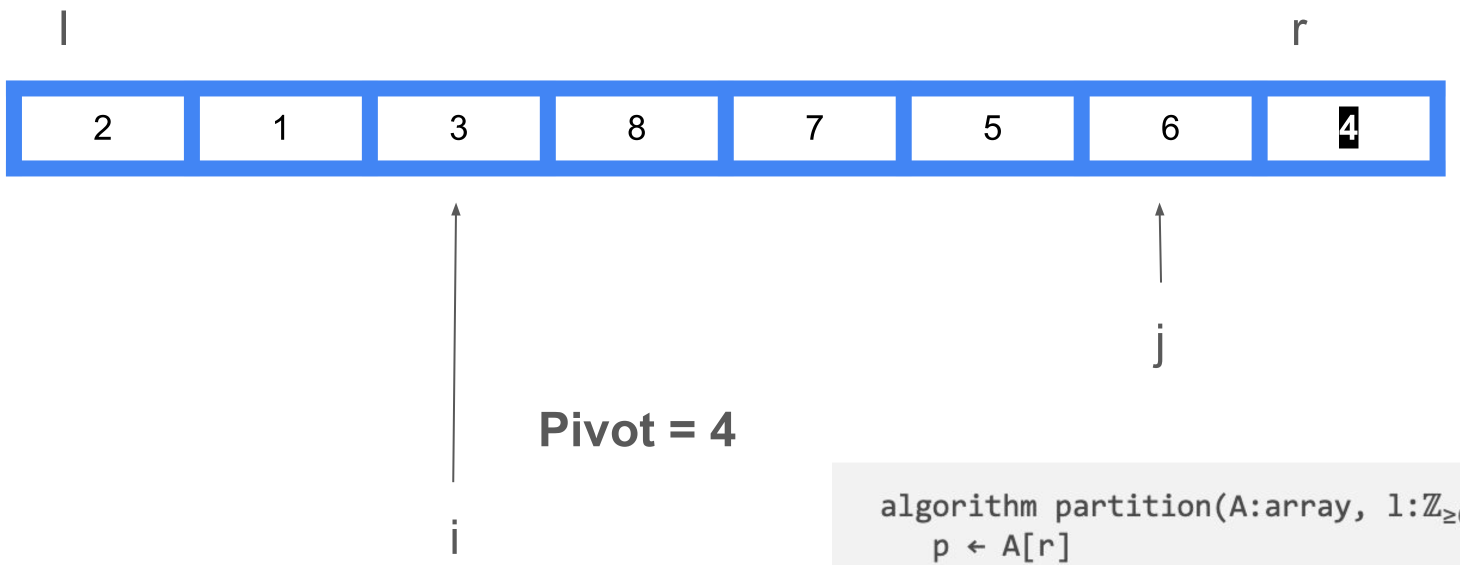


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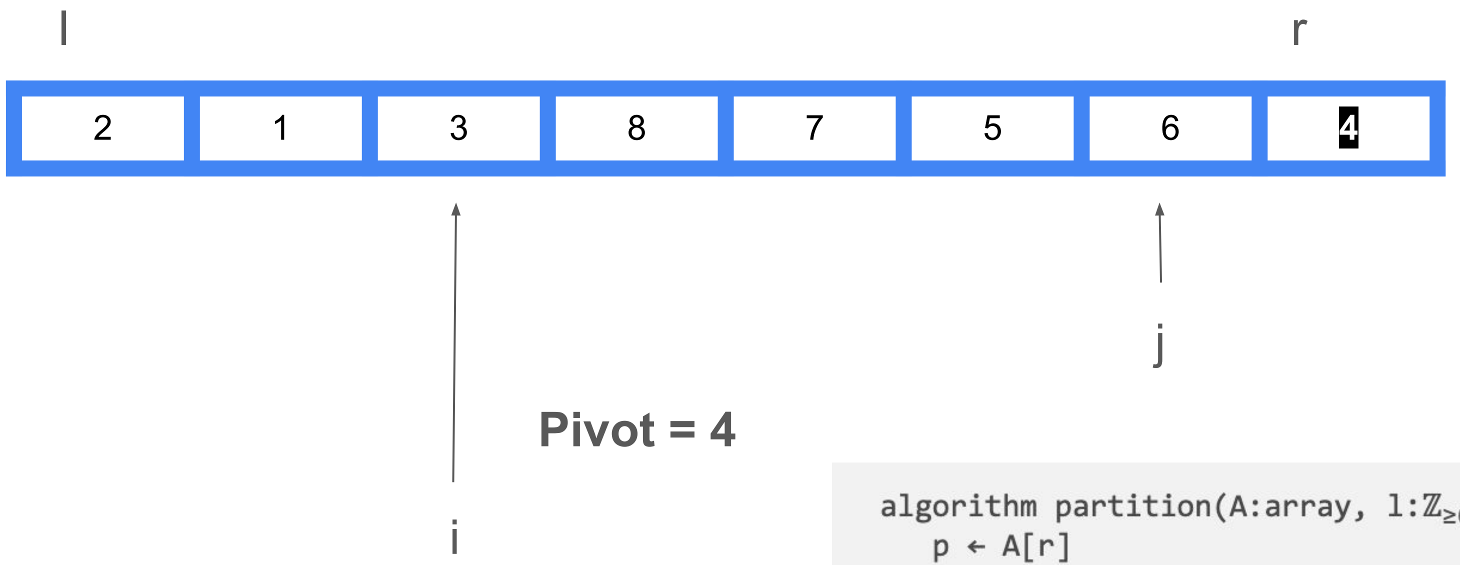


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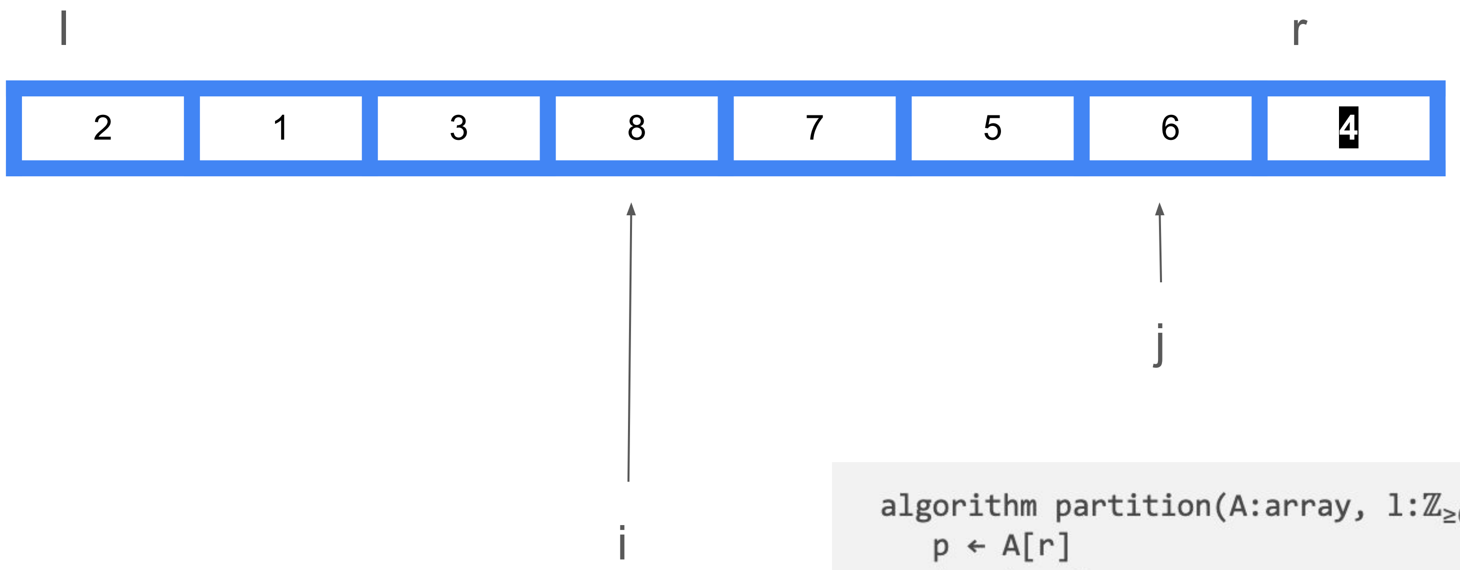
That was the last for loop iteration

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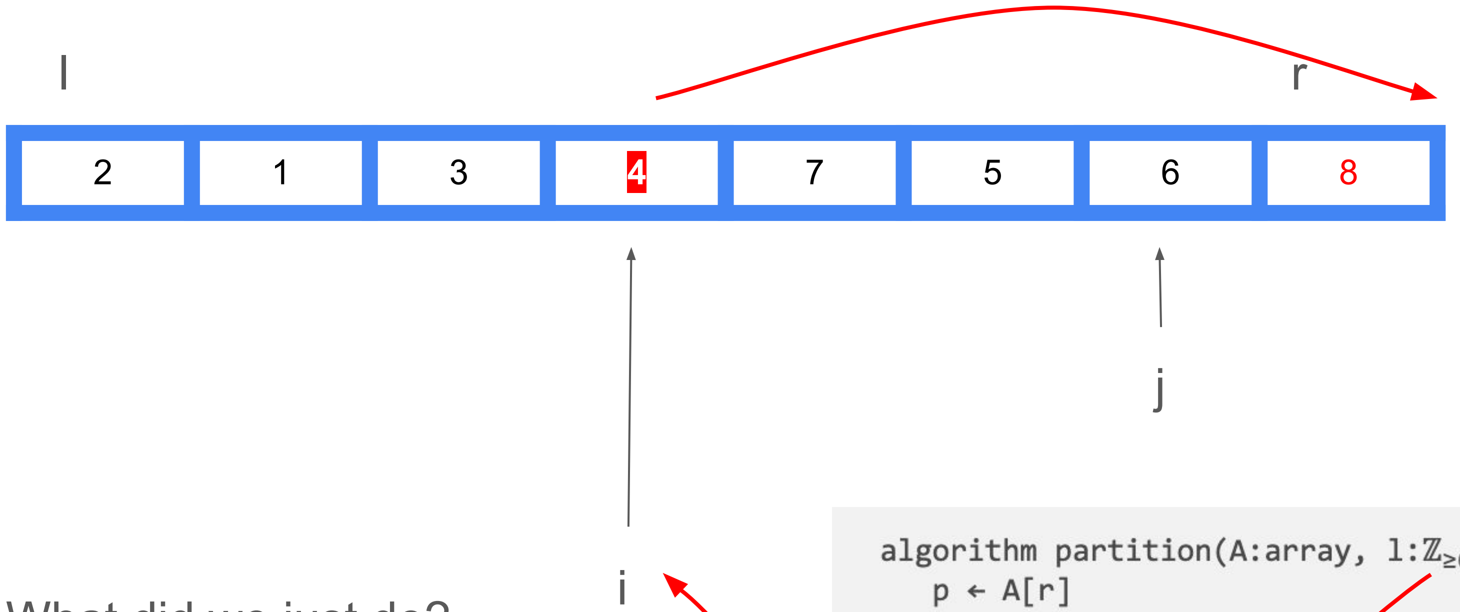


What did we just do?
Lets rewind a little

```
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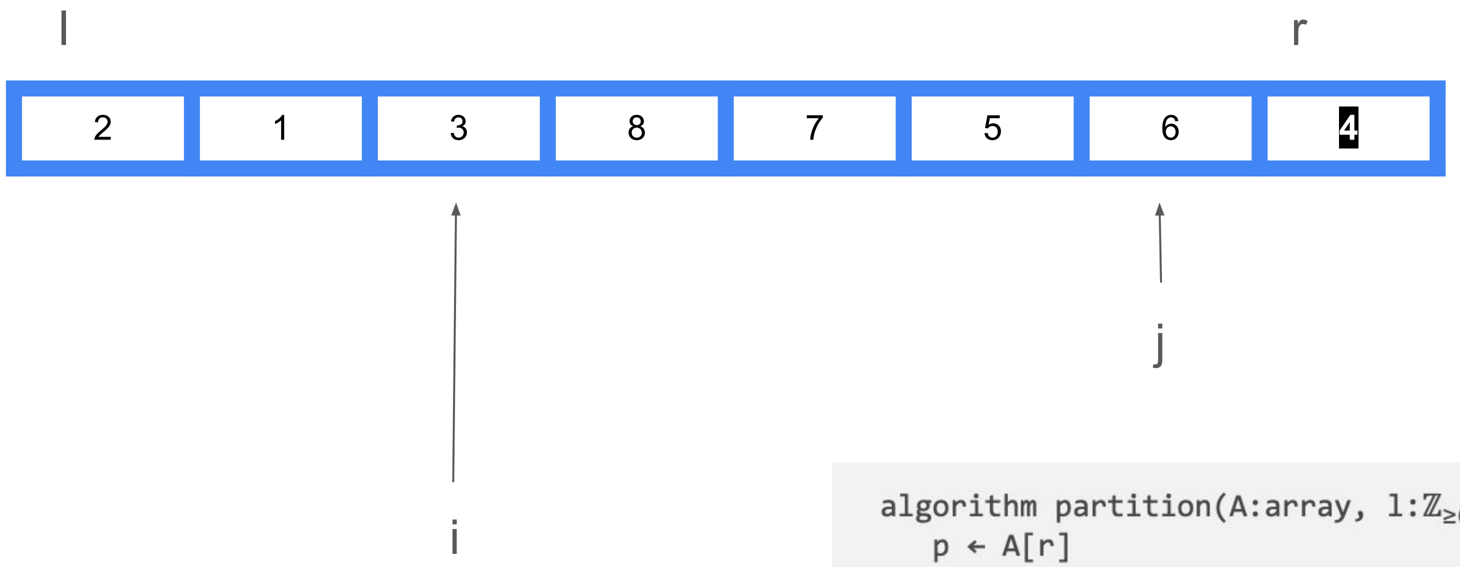


That was the last for loop iteration

**Everything up to i is less than the pivot**

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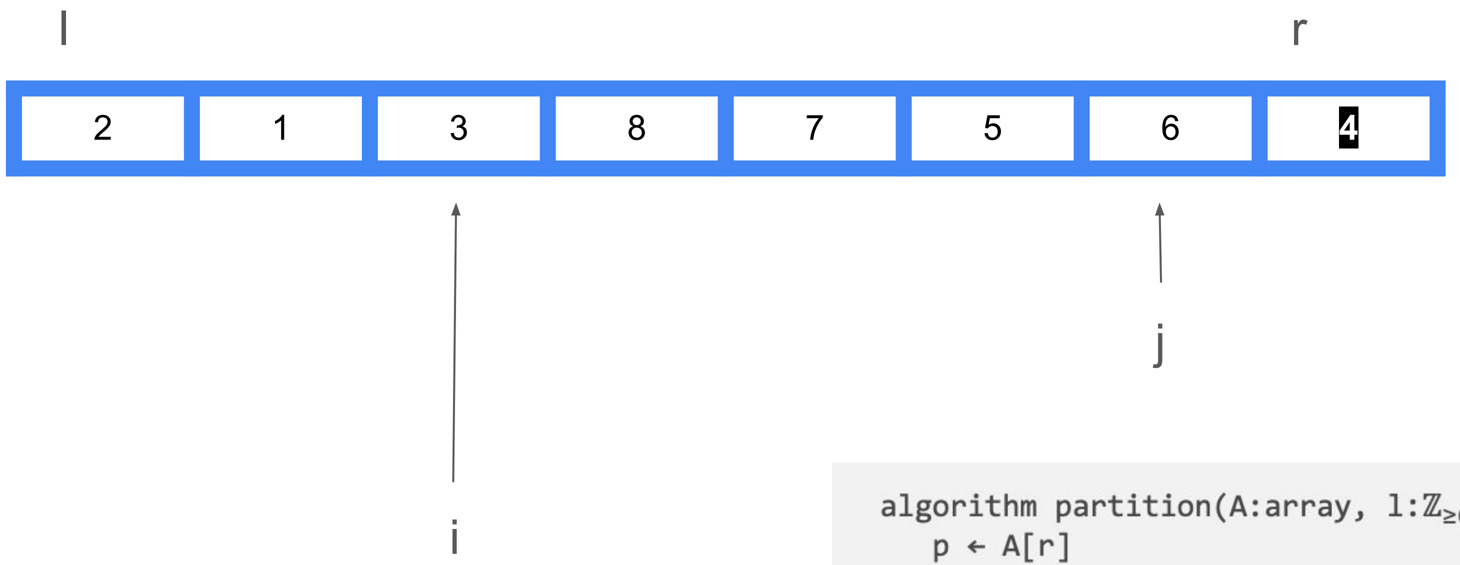


That was the last for loop iteration

Everything up to i is less than the pivot

**pivot index = i + 1**

```
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   p ← A[r]
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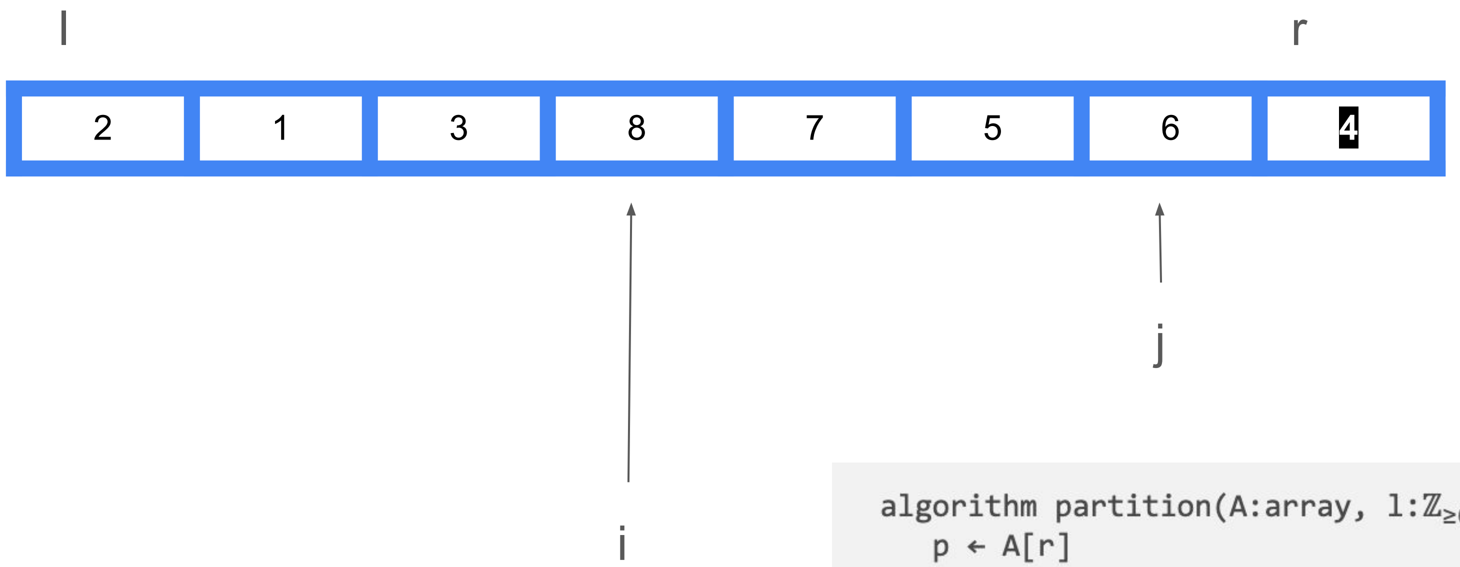


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```

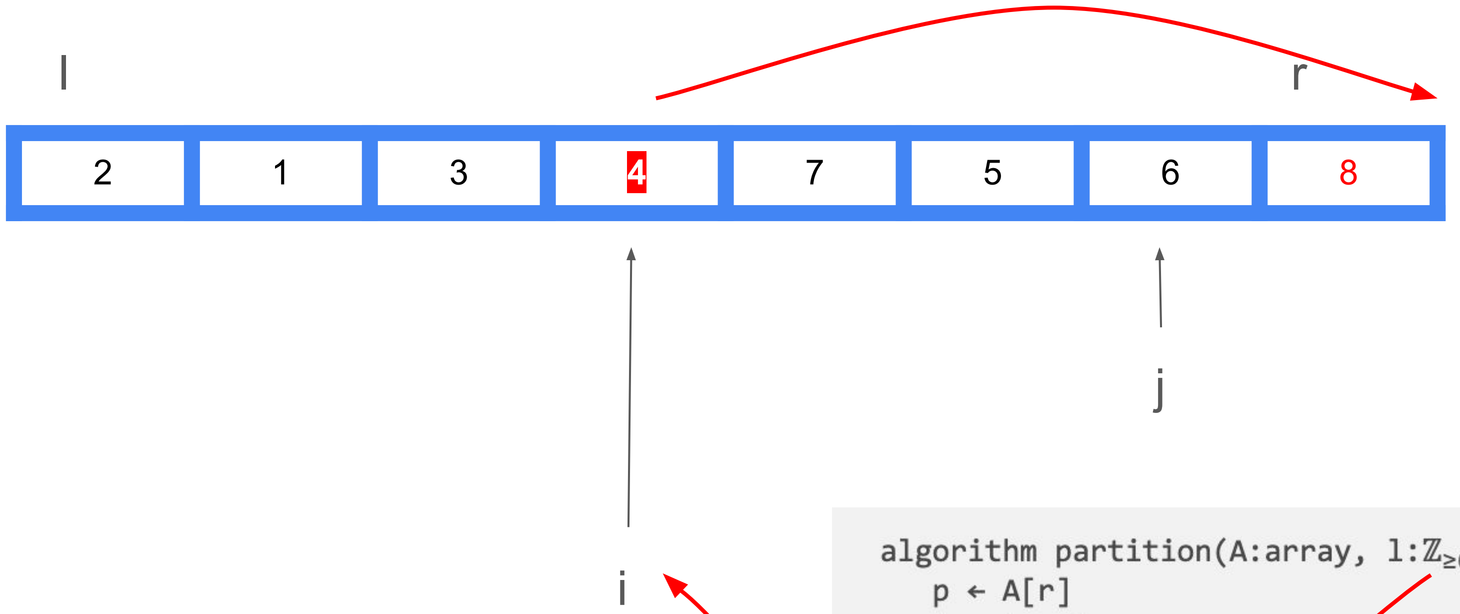


That was the last for loop iteration

Everything up to i is less than the pivot

**pivot index = i + 1**

```
algorithm partition(A:array, l:ℤ≥0, r:ℤ≥0) → ℤ≥0
   p ← A[r]
   i ← l - 1
   for j from l to r - 1 do
      if A[j] < p then
         i ← i + 1
         swap(A, i, j)
      end if
   end for
   i ← i + 1
   swap(A, i, r)
   return i
end algorithm
```

## Question 2

**(Quick sort)**

(1) Illustrate the operation of the **Partition** step in Quick sort on $A = [2, 8, 7, 1, 3, 5, 6, 4]$.

(2) Can we understand the average-case runtime of Quick sort? What is the best policy for selecting the pivot value in the quick sort?

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How does our recurrence cost look like?

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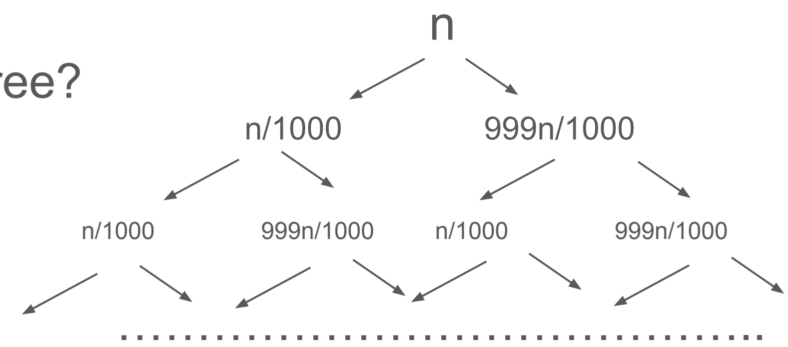

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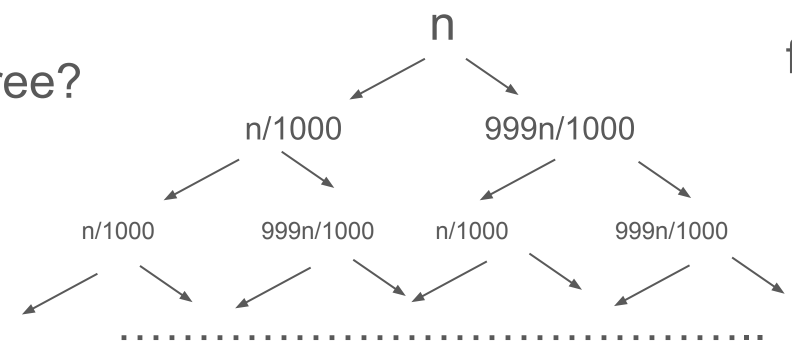


Takeaway: any constant fraction split is O(n log n)