Project 1 Qu, Botong

1. **(Image decolorization)** for an input image I of NxN pixels, compute the intensity of each pixel based on two different formulas and compare the results. Save the result in an image. Discuss in the report which formula you like better and why? Be sure to try out all images before reaching a conclusion.

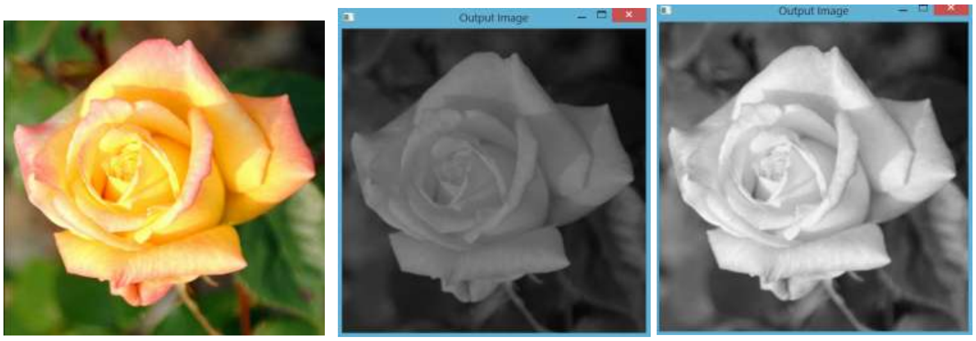


Figure 1.2 Left: origianl image center: average intensity right: colormetric intensity

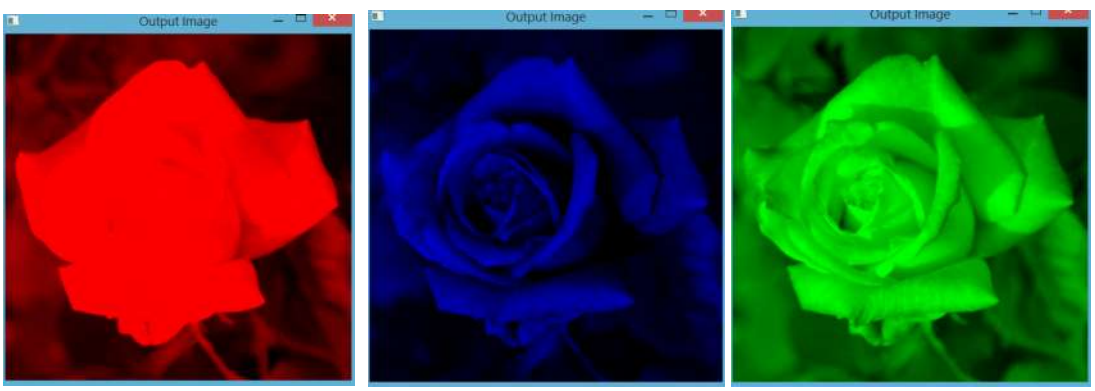


Figure 1.3 Three channels of the image

Solution: From above pictures, we can observed that the colormetric intensity calculation method is much better. Since it leads to our eyes with the more realistic greyscale version of the image which obeyed the luminence information of the original image better, i.e. the flower look like much brighter than the background, this property successfully perserveed by the colormetric tranformation. Also, the average method make the picture little darker than the colormetric one, so we can deduce that this transition will make some dark parts lose more details information like the leaf of the rose.

2. **(Color maps)** Treat the intensity function (use your favorite scheme from part 1) as a scalar function and use the eight different color maps mentioned in the class notes to visualize this scalar function. Which schemes are the most efficient for understanding the images? Now do the same for the red, green, and blue channels of the images, i.e., treating them as scalar functions and visualize them with color maps. Again, which schemes are most useful? Moreover, which channel is best aligned with the intensity of the image?

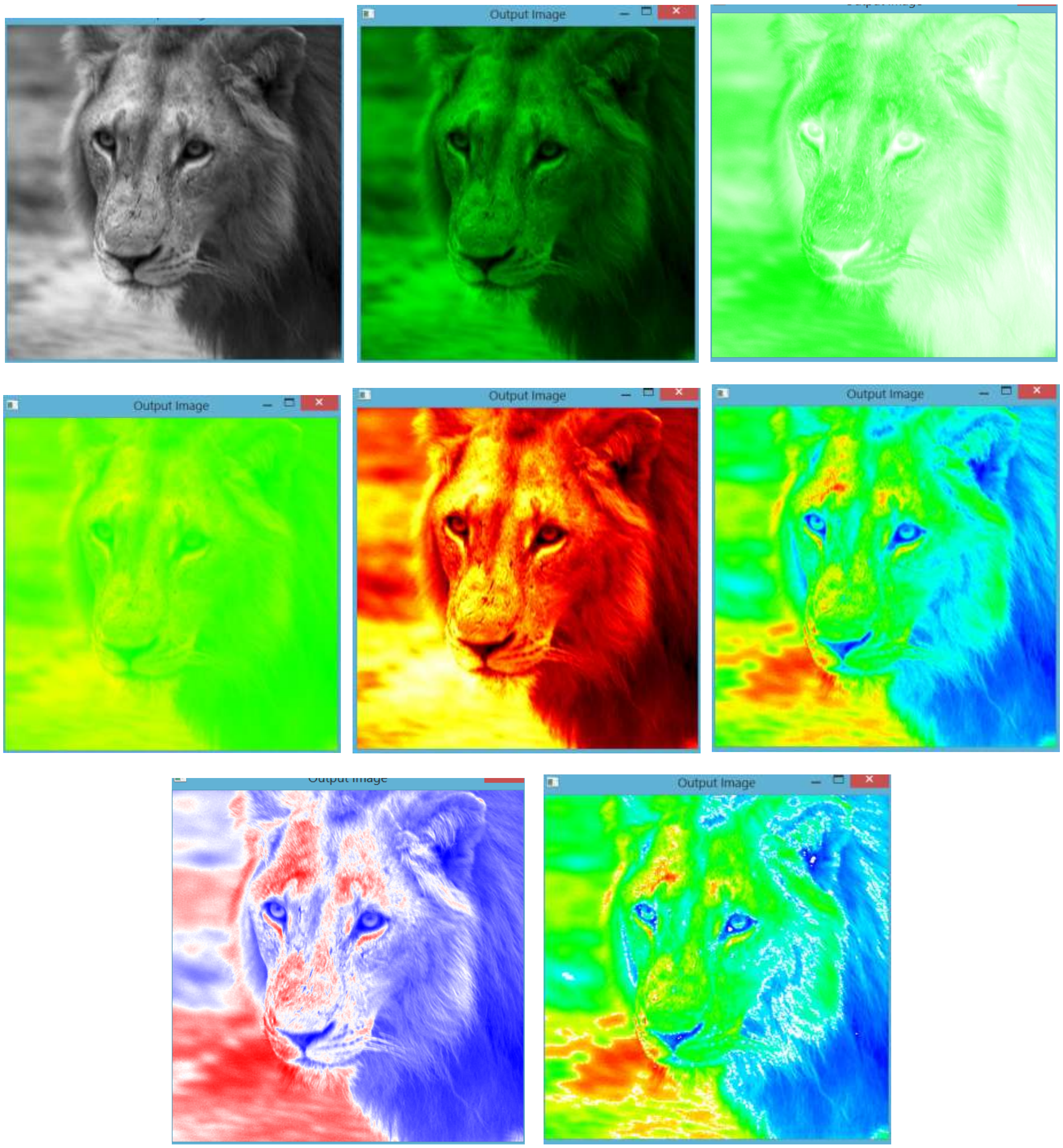


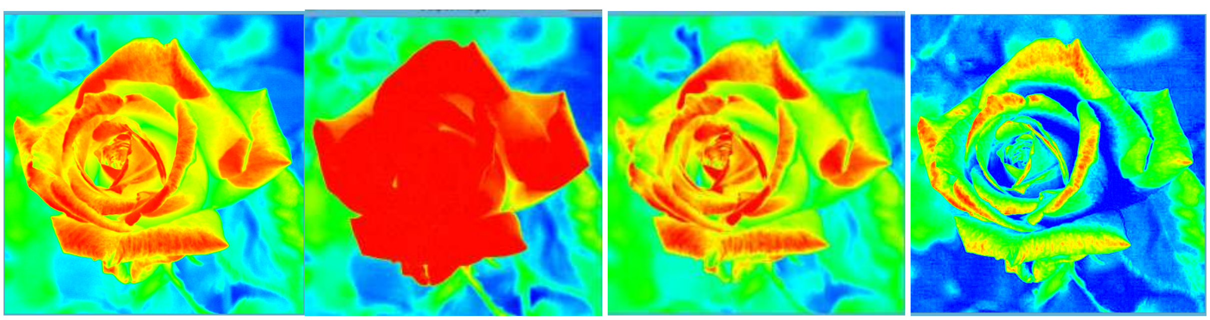
Figure 2.1 eight color schemes

Solution:

All above eight schemes can somehow give the reviewer some information of the intensity of the original image, but we can see from figure 2.1, there are some flaws in some schemes. The third (green and white) make the observation harder because people usually don't used to use white (light color) to render the high intensity part. This is the same reason I don't like the blue white red image, the white part is somehow un-natural, but I think this scheme is also good for user to separate the image in to upper intensity regions and lower intensity regions. And fourth (green and yellow) image make the observation harder because the yellow and green is so close to make some divisions. So for me, I think the green-black, heat and the rainbow schemes are good for observations. And among these three schemes, the rainbow will be the most efficient because of the multiple colors will be helpful for distinguishing the corresponding intensity region.

When the image is composed by the multiple colors and doesn't oblique to any channel, like Mona Lisa, the all channel will be good enough to preserve the intensity. But when the image is composed by some pure pigment, which will make the other two channel lose a lot of reality. For instance, the duck's mouth is composed by a lot of red, when we calculate the intensity with using blue channel, the mouse will become so dark and just opposite with the original image. But I found that the green channel often saves the most details and aligned with the image best.

Figure 2.2 flowers intensity in different channel, the green channel save the base details



When I use blue channel to calculate the intensity, as said before, the duck's red mouse will become a problem. And using different schemes will also make the observation harder because the intensity map already lose a lot of details information. But I observed that the heat scheme is good for make difference between objects. And green-black scheme is good for the details. The rainbow this time make the water and the duck body is near to each other and then difficult to perceive the original image, but good for the intensity observation.

Figure 2.3 blue channel make the dark's red mouse darker, which will not be observed in some color scheme

3. **(Image smoothing)** Apply the image blur operator on your images. You can use Adobe Photoshop or Irfanview for this purpose. Apply the intensity computation to the blurred images? What do you observe using the same color map visualization? Provide justifications to your findings.

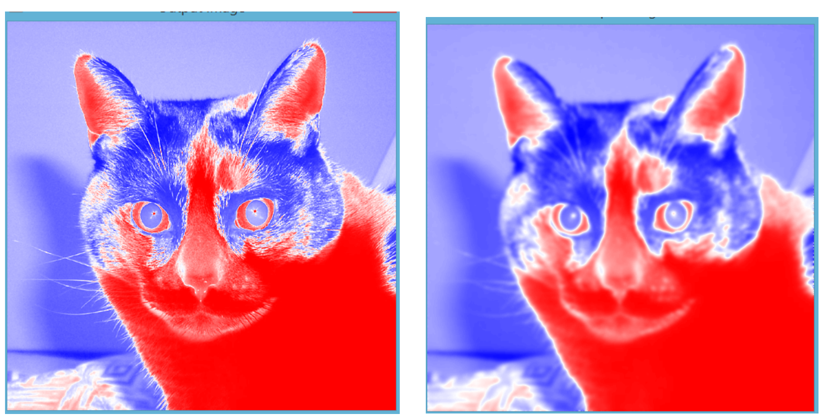


Figure 3.1 blurred image make the mustache information lost

When I blurred the image and I found that I obviously will lose a lot of the details, like figure 3.1 shows that the mustache and hair of the cat is not as clear as before. Also when I use the blurred image and the red and blue color scheme, it is obviously that the white region which is the center of the intensity range is become bigger and wider.

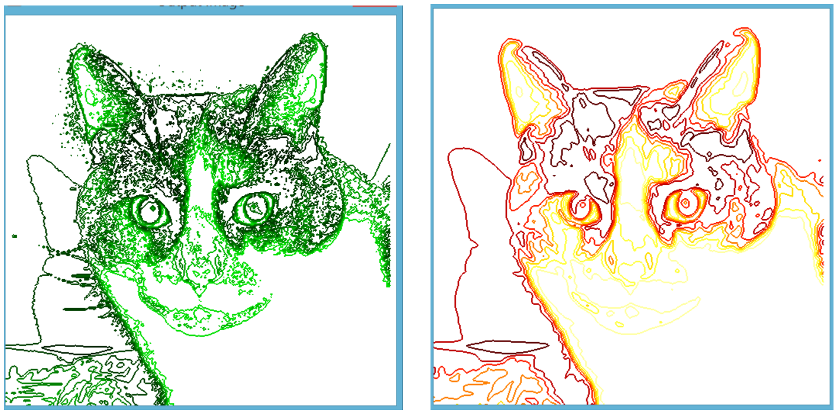


Figure 3.2 blurred image make the noise in contour disappear

Figure 3 blurred image make the green channel intensity not much different

But somehow, like Mona Lisa, the blurred green channel is not much different with the original one, and I think it caused by all the details lost are kind of similar when using blur or just calculate the green channel intensity.

Also, the most important is I found the noise points in the contour lines disappear! This I think is caused by the blurring make the color of one pixel similar to the nearby pixels, so the intensity will also be continuous with nearby pixels.

4. **(Image contours)** For an input image I of NxN pixels, compute the contour plot for its intensity function as well as for the R, G, and B channels, respectively. Do these contours align with one another? Why or why not? You can do this with the Marching Square algorithm that we discussed the class. Superimpose the color maps on the contour lines. How does image blurring impact the visualization?

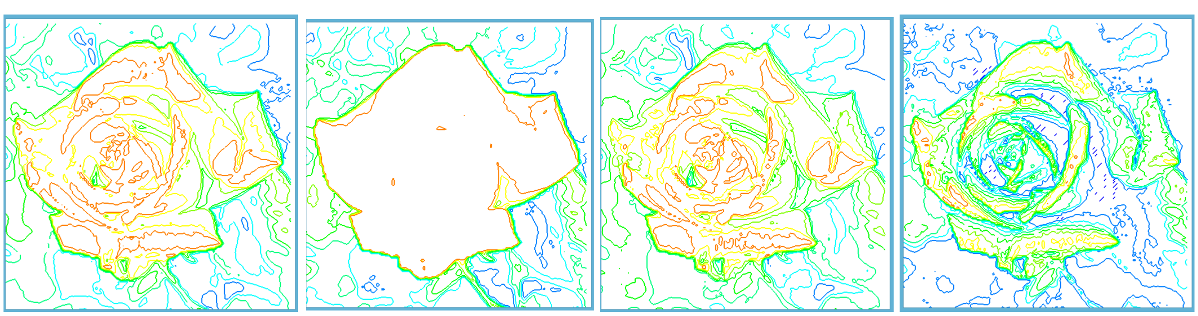


Figure 4.1 Colormetric contour, red channel, green channel, blue channel

Solution:

First of all, in this question, I changed my cell length with to different numbers, from figure 4.2, we can see with the bigger the cells, the noise in the contours can be decrease and also the contours become more continuous when change length of cell's edge from 2 to 4. But when the cell's edge length is too big, like 16 pixels, the contours will become no more smooth curves and also too much details will miss.

So change the cell's length from 1 to 4 has some strength, first it will make the calculation much more efficient since less cells is calculated. Second, the noise will somehow decrease and the curve will become much smoother.

With using different channel to calculate the contour, I found that the contours are not aligned with each other since the intensity of each pixel is different when using different methods. Besides, the blue channel make the flowers a lot of straight line contours which is obviously some kind of noise, which is also undesirable.

As solution to question 2 said (as figure 3.2 shows), the blur image can decrease the noise in the contours. In the program, on the output window press 7 and then input number from 1 to 7 to select the color map will lead the user the color mapped contours.

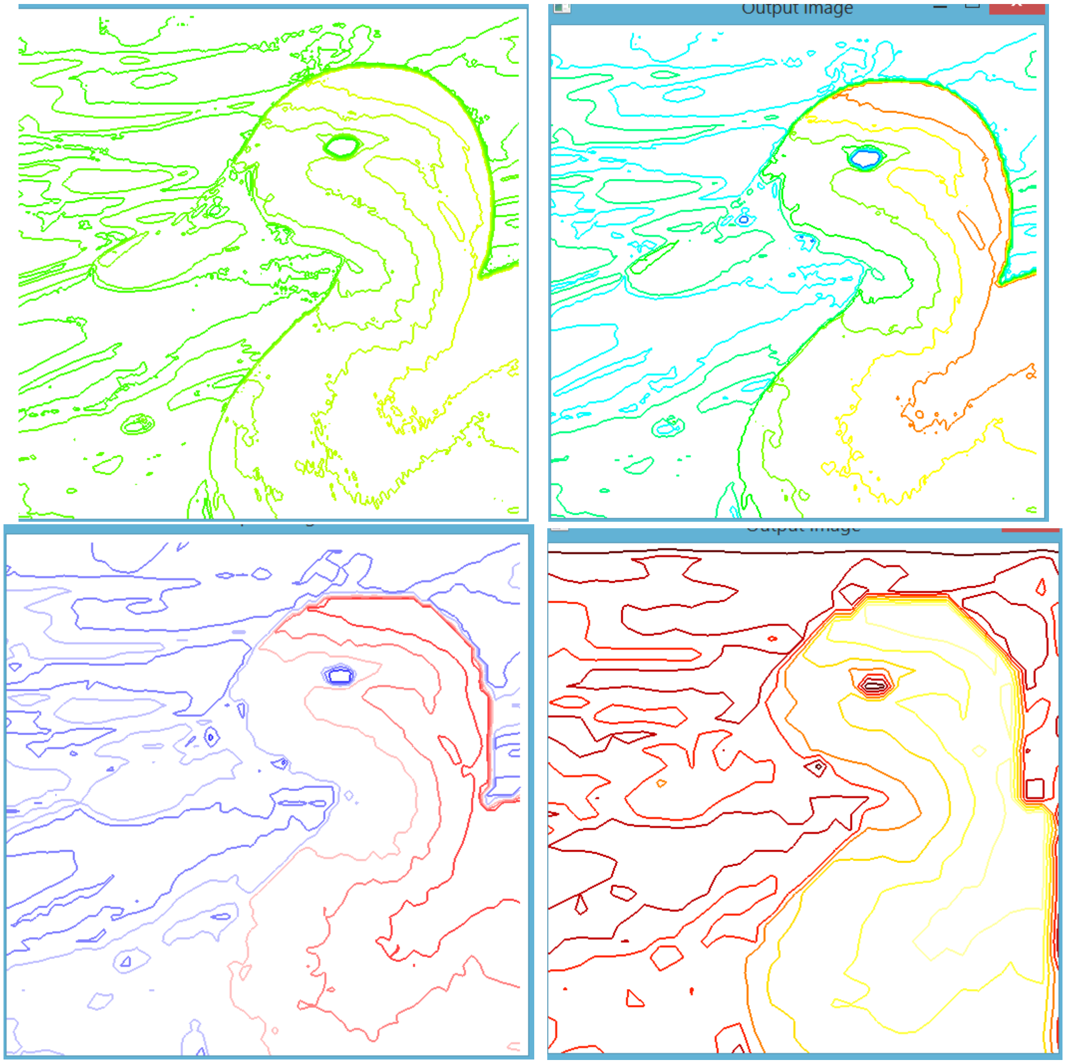


Figure 4.2 contours in cell's edge length is 2, 4, 8, 16

5. (**Images as height fields)** Display the image intensity function, the red, green, and blue channels as a height field. Color the height fields with your favorite color map from part 2. Draw the contour lines on the height field. Is this visualization helpful?

Solution: The results as below shows that the height field is much more easily for the viewers to catch the scalar value different between each pixels. We can also see the contour lines on the height field give more easy ways to compare the intensity between each pixels.

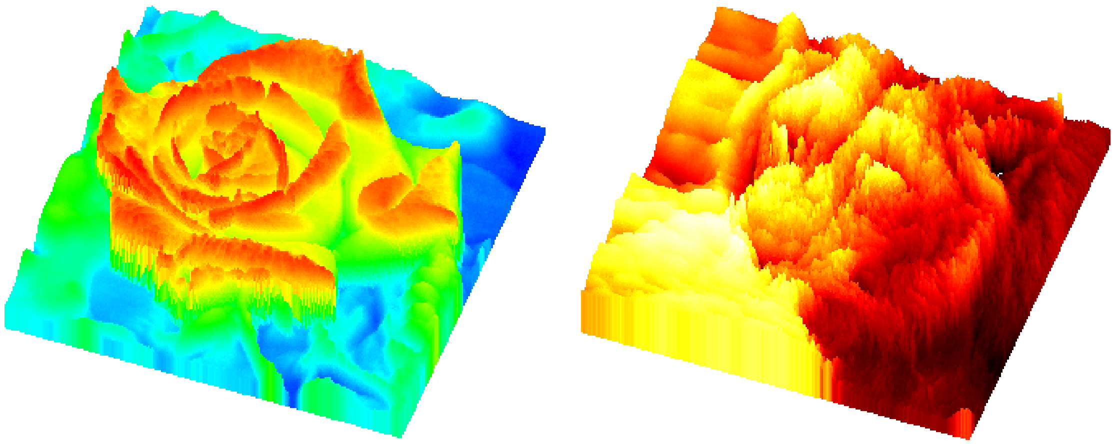


Figure 5.1 height field of rose and lion

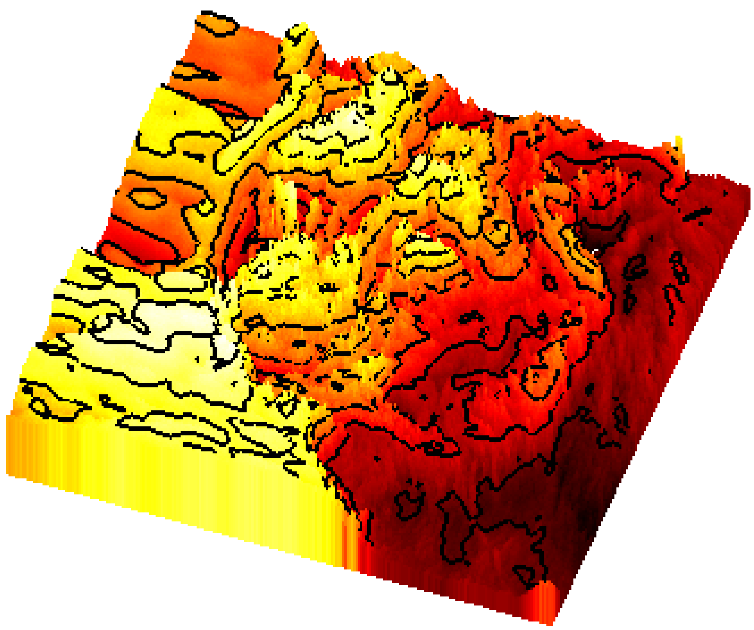


Figure 5.2 height field of lion with contours

6. **(Scalar field topology)** For each of the following function defined on the unit sphere in 3D, i.e., decide whether it is Morse and provide your justification. If the function is Morse, find all of its critical points, classify them, and compute their Reeb graphs and Morse-Smale complexes. You will need to draw the Reeb graphs and Morse-Smale complexes using a drawing tool such as Adobe illustrator, or by hand and scan the handdrawing.

Solution: From the definition, we know that the Morse theory said that the point will be a critical point if the coordinate of this point make all the partial derivative of the scalar function vanish. And also the critical points' matrix of second partials derivative should be non-singular to make the critical point non-degenerate. In this case the function will be Morse.

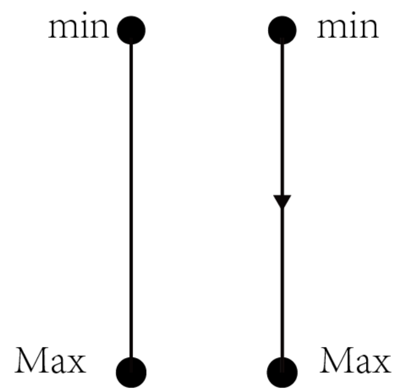
But for this question, since the function is defined on the unit sphere, we can not only analysis the first order and second order derivative to get the result. We should also analysis the existence of maximum, minimum and saddles to make the analysis. I draw all the graphs with using Adobe Illustrator.

a. $\mathrm{f(x, y, z)} = 1$

This function is not Morse, since all points on the unit sphere have the same scalar 1, so there is no critical points.

b. $\mathrm{f(x, y, z)} = x$

Since we can get the maximum of this function at $(1,0,0)$, and get the minimum at $(-1,0,0)$. So the function is Morse. And the Reeb graph (left) and Morse-Smale complex graph (right) as following:

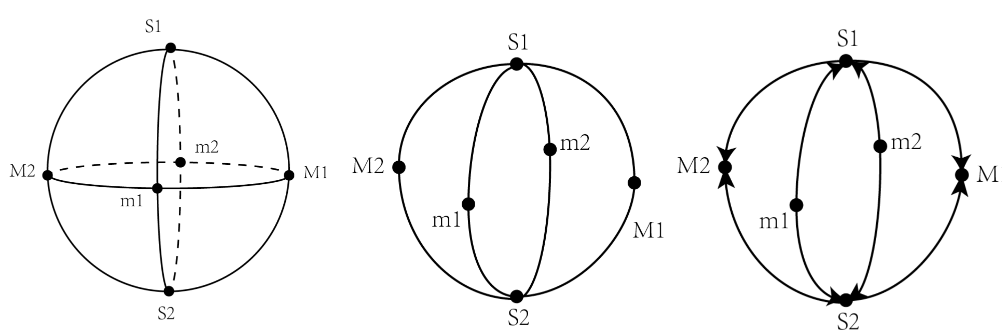


Left: Reeb right: Morse-Smale complex

c. $\mathrm{f(x, y, z)} = x^2$ is not Morse, since the minima can be got when x equals to 0, so all the points on the longitude of the sphere where x is 0 is the critical points. The critical points are not isolated. That is the function is not Morse.

d. $\mathrm{f(x, y, z)} = x^2 - y^2$

This function is Morse. With calculate the first derivative, we can get the critical points in the space must on the z axis. So on the unit sphere, the top and bottom points are two critical points with scalar value is 0. Also we can found this function's maximum and minimum can be got at each intersection point of the axis and the unit sphere (as following graph shows).



Left: scalar field, center: Reeb graph, right: Morse-Smale complex

e. $\mathrm{f}_N(\mathrm{x},\mathrm{y},\mathrm{z}) = \sum_0^N x^{\mathrm{k}} y^{N-k} \cos(\frac{N-k}{2}\pi) = \mathrm{Re}(\mathrm{x}+\mathrm{yi})^N$

$$\mathrm{f}_0(\mathrm{x},\mathrm{y},\mathrm{z}) = 1$$

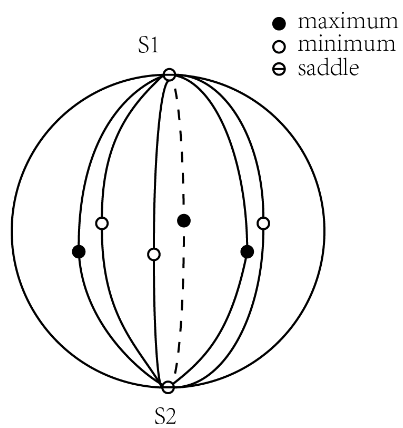
$$\mathrm{f}_1(\mathrm{x},\mathrm{y},\mathrm{z}) = x$$

$$\mathrm{f}_2(\mathrm{x},\mathrm{y},\mathrm{z}) = \mathrm{x}^2 - y^2$$

$$\mathrm{f}_3(\mathrm{x},\mathrm{y},\mathrm{z}) = \mathrm{x}^3 - 3xy^2$$

$$\mathrm{f}_4(\mathrm{x},\mathrm{y},\mathrm{z}) = \mathrm{x}^4 - 6x^2y^2 + y^4$$

So we can find that when N is 0, as the a said, this function is not morse. When N is 1, the funciton is morse and the image as same as b. When N is 2, the function as same as the d and so it is morse. When N is 3, we found that there will have three maxima and three minima on the latitude when y is 0, and there will be one monkey saddle on each pole point. In this case, I found as N increase, the maxima and minima on the “equator” of the unit sphere increase, and we will have N maxima and N minima on the equator of the sphere, and at each pole we will have a N saddle. And the saddle will become more and more unstable.

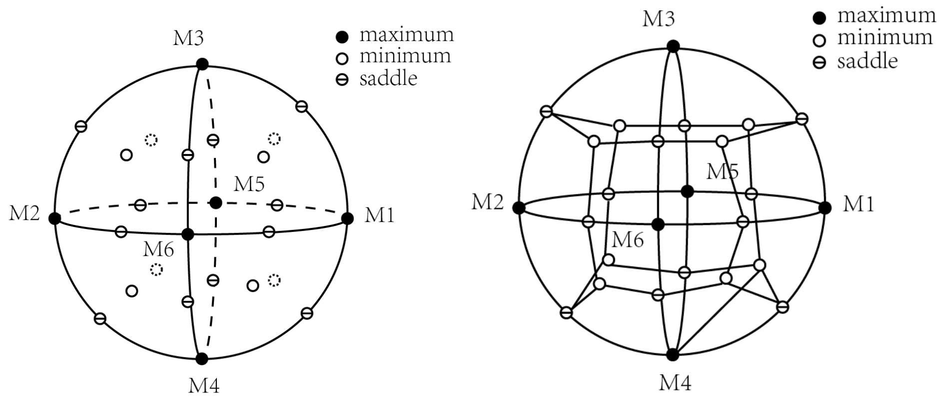


The Reeb Graph and Morse-Smale graph when N = 3

f. $\mathrm{f}(\mathrm{x},\mathrm{y},\mathrm{z}) = x^4 + y^4 + z^4$

This function is Morse, with $x^2 + y^2 + z^2 = 1$, we know that the maximum of f(x,y,z) is 1, so we can get this maximum on each intersection point of the axes and the unit sphere.

So (0, 0, 1), (0, 0,-1), (0, 1, 0), (0, -1, 0), (1, 0, 0), (-1, 0, 0) are the maximum of the function. The minimum is $\frac{1}{3}$ and can be got at $(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$, $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$, $(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})$, $(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$, $(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})$ $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})$, $(-\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$ and $(-\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})$. So we will have 8 minimum and 6 maximum, to obey with the Euler Characteristic, we should also have 12 saddle points and these saddle should exist on the middle of the maximum and minimum. As following image show. The Reeb graph is as same as the Morse-Smale complex.



Left: the position of critical points right: Reeb and Mors-Smale complex graph