[CS 11] Prac 0m – Parallelepiped

oj.dcs.upd.edu.ph/problem/cs11prac0m

Problem Statement

In n-dimensional space, what is the "volume" of the parallelogram determined by n vectors?

It turns out that the answer to this is mathematically satisfying! It can be derived by studying something called the "exterior algebra".

Basically, a consequence of this is that the volume can be obtained by picking *any* nontrivial alternating multilinear function $V^n \to F$ then multiplying by a constant (dependent on that function), then taking the absolute value.

Anyway, you don't need to worry about those details for now—that's just motivating background.

In 3-dimensional space, one such function is the so-called **triple product**. If v_1, v_2 and v_3 are 3D vectors, then their triple product is the quantity

$$v_1 \cdot (v_2 \times v_3)$$

where "$\cdot$" denotes the dot product and "$\times$" denotes the cross product. One can easily check that this is nontrivial, multilinear, and alternating, so it can measure the volume of the parallelepiped determined by the three vectors up to a constant. And as can be easily checked, that constant is 1, which means that the volume of the parallelepiped has the formula

$$|v_1 \cdot (v_2 \times v_3)|.$$

Given three vectors, determine the volume of the parallelepiped determined by the three vectors.

As a reminder:

- A 3D vector v can be represented as a triple of real numbers $v = \langle x, y, z \rangle$.

- The **dot product** of two 3D vectors v1=⟨x1,y1,z1⟩$v_1 = \langle x_1, y_1, z_1 \rangle$ and v2=⟨x2,y2,z2⟩$v_2 = \langle x_2, y_2, z_2 \rangle$, denoted v1⋅v2$v_1 \cdot v_2$, is a real number and is given by the formula x1x2+y1y2+z1z2$x_1 x_2 + y_1 y_2 + z_1 z_2$

- The **cross product** of two 3D vectors v1=⟨x1,y1,z1⟩$v_1 = \langle x_1, y_1, z_1 \rangle$ and v2=⟨x2,y2,z2⟩$v_2 = \langle x_2, y_2, z_2 \rangle$, denoted v1×v2$v_1 \times v_2$, is a 3D vector and is given by the formula ⟨y1z2−y2z1,z1x2−z2x1,x1y2−x2y1⟩. $\langle y_1 z_2 - y_2 z_1, z_1 x_2 - z_2 x_1, x_1 y_2 - x_2 y_1 \rangle$.

In code, we will represent a 3D vector as a 33-tuple, i.e., a triple `(x, y, z)`.

Task Details

Your task is to implement a function called `parallelepiped_volume`. This function has three parameters, each of which is a 33-tuple of `int`s, denoting the three 3D vectors.

The function must return an `int` denoting the volume of the parallelepiped determined by the three vectors.

For this problem, binding/assignment statements are allowed.

Example Calls

Example 1 Function Call

Copy

```
parallelepiped_volume((2, 0, 0), (0, 2, 0), (0, 0, 2))
```

Example 1 Return Value

Copy

```
8
```

Example 2 Function Call

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```
parallelepiped_volume((1, 1, 0), (1, 0, 1), (0, 1, 1))
```

Example 2 Return Value

Copy

```
2
```

Constraints

When the program is run:

- The function `parallelepiped_volume` will be called at most 2,0002,000 times.
- Each coordinate of each vector has absolute value at most 10610^6.

Scoring

You get 150150 ❤️ points if you solve all test cases correctly.

Report an issue

Clarifications

No clarifications have been made at this time.