[CS 11] Prac 1b – Divisor sum, but not really

oj.dcs.upd.edu.ph/problem/cs11prac1b

Problem Statement

The following is an interesting equation concerning the divisor-count function $d(k)$ (defined as the number of divisors of k, e.g., $d(6) = 4$ because 6 has 4 divisors: 1, 2, 3 and 6), *true for any* $n \geq 0$:

$$d(1) + d(2) + \ldots + d(n) = \lfloor \frac{n}{1} \rfloor + \lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{3} \rfloor + ... + \lfloor \frac{n}{n} \rfloor .$$

The proof is surprisingly straightforward!

Let's explore sums like the one on the right.

Given three integers n, a and b with $a \leq b$, compute

$$\lfloor \frac{n}{a} \rfloor + \lfloor \frac{n}{a+1} \rfloor + \lfloor \frac{n}{a+2} \rfloor + ... + \lfloor \frac{n}{b} \rfloor .$$

Formally, compute the following sum:

$$\sum_{k=a}^{b} \lfloor \frac{n}{k} \rfloor .$$

Task Details

Your task is to implement a function called `floor_sum`. This function has three parameters `n`, `a` and `b` in that order, all `int`s, whose meanings are described in the problem statement.

The function must return an `int` denoting the required sum.

Restrictions

For this problem:

- Assignment is allowed.
- Recursion is allowed.
- Up to 66 function definitions are allowed.
- Comprehensions are **disallowed**.
- `range` is **disallowed**.
- The `abs` symbol is now allowed.

- The source code limit is 10001000.

Example Calls

Example 1 Function Call

Copy

```
floor_sum(7, 1, 7)
```

Example 1 Return Value

Copy

```
16
```

Example 2 Function Call

Copy

```
floor_sum(7, 2, 5)
```

Example 2 Return Value

Copy

```
7
```

Constraints

- The function `floor_sum` will be called at most 10,00010,000 times.

- $1 \le a \le b \le n \le 100{,}000$.

Scoring

- You get 100 ❤️ points if you solve all test cases where:
  - $n \le 300$
- You get 150 💔 points if you solve all test cases.

Extra Fun

Here are some extra questions you may want to ponder upon, for fun. (But please do them after you've finished the practice session!)

- Can you find a proof of the equality above?
- Can you recast the proof in terms of bipartite graphs?
- It turns out that our variant sum $\lfloor \frac{n}{a} \rfloor + \ldots + \lfloor \frac{n}{b} \rfloor$ also has an equivalent sum analogous to the sum on the left—one only needs to define a modified divisor count function $d_{a,b}(n)$. Can you define such a function and prove an analogous equation in terms of this function?

[Report an issue](https://oj.dcs.upd.edu.ph/problem/cs11prac1b)

Clarifications

No clarifications have been made at this time.