Portfolio Theory for Corporate Decarbonization: A Risk-Efficiency Framework for Net-Zero Investment under Uncertainty

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Abstract

This paper develops a rigorous theoretical framework for corporate decarbonization investment by extending modern portfolio theory to the climate transition context. We characterize low-carbon technologies as assets with stochastic costs, uncertain abatement potential, and embedded real options, deriving the conditions under which a "net-zero efficient frontier" exists and is unique. Building on the foundational work of Markowitz (1952) and the irreversible investment literature of Dixit & Pindyck (1994), we show that optimal technology portfolios balance cost volatility, stranded asset risk, and option value from technological flexibility. Our dynamic extension incorporates learning curves (Arrow, 1962), breakthrough innovations via jump-diffusion processes (Merton, 1976), and regulatory uncertainty. We derive comparative statics showing how carbon pricing, technology subsidies, and disclosure requirements affect portfolio composition. The framework provides actionable guidance for corporate net-zero strategy while contributing to the theoretical literature on environmental economics and corporate finance.

Keywords: Portfolio optimization, Climate transition risk, Real options, Decarbonization, Net-zero investment, Technology adoption

JEL Classification: G11, Q54, Q55, O33, D81

1 Introduction

The global transition to net-zero emissions requires unprecedented capital reallocation across industrial sectors. The International Energy Agency estimates that annual clean energy investment must reach $4 trillion by 2030 to achieve net-zero by 2050 (IEA, 2021). Firms face a complex optimization problem: how should they allocate limited capital across competing decarbonization technologies, each with uncertain costs, evolving performance, and different risk profiles?

This paper addresses this question by extending modern portfolio theory (Markowitz, 1952, 1959) to the corporate decarbonization context. Our key insight is that climate transition technologies can be characterized as assets with multidimensional risk attributes, and that the firm's technology adoption problem is structurally analogous to mean-variance portfolio optimization with additional constraints.

However, the decarbonization context introduces features not present in traditional portfolio theory:

(i) **Mandatory constraints**: Firms must meet externally-imposed abatement targets, not simply maximize risk-adjusted returns

(ii) **Irreversibility**: Technology investments are largely irreversible, creating path dependence (Dixit & Pindyck, 1994)

(iii) **Learning effects**: Technology costs decline with cumulative deployment (Arrow, 1962; Wright, 1936)

(iv) **Breakthrough uncertainty**: Discontinuous innovation creates jump risk in cost trajectories (Nordhaus, 2014)

(v) **Regulatory uncertainty**: Carbon pricing and technology standards are policy-dependent (Weitzman, 1974)

Our contribution is threefold. First, we formalize the firm's decarbonization problem as a constrained portfolio optimization and prove existence and uniqueness of solutions under general conditions (Section 3). Second, we extend the framework to incorporate real options (McDonald & Siegel, 1986; Pindyck, 1991), showing how managerial flexibility reduces effective transition risk (Section 4). Third, we develop a dynamic multi-period model with learning and derive the Bellman equation characterizing optimal technology pathways (Section 5).

The paper relates to several strands of literature. The foundational portfolio theory literature (Markowitz, 1952; Sharpe, 1964; Lintner, 1965; Mossin, 1966) establishes the mean-variance framework we extend. The real options literature (Dixit & Pindyck, 1994; Trigeorgis, 1996) provides tools for valuing flexibility under uncertainty. The climate economics literature (Nordhaus, 1994; Stern, 2007; Weitzman, 2009) motivates the abatement constraint structure. Recent work on climate finance (Bolton & Kacperczyk, 2020; Stroebel & Wurgler, 2021; Giglio et al., 2021) documents the pricing of transition risk, validating our risk decomposition.

2 Literature Review and Theoretical Foundations

2.1 Portfolio Theory: From Markowitz to Climate Applications

The modern theory of portfolio selection originates with Markowitz (1952), who formalized the trade-off between expected return and variance:

$$\max_{\boldsymbol{w}} \left\{ \boldsymbol{w}^T \boldsymbol{\mu} - \frac{\gamma}{2} \boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w} \right\} \tag{1}$$

where $\boldsymbol{w}$ is the portfolio weight vector, $\boldsymbol{\mu}$ is the expected return vector, $\boldsymbol{\Sigma}$ is the covariance matrix, and γ is the risk aversion coefficient. Markowitz (1959) extended this to derive the efficient frontier—the set of portfolios offering minimum variance for each level of expected return.

? provided closed-form solutions for the efficient frontier with a risk-free asset, while Roll (1977) demonstrated that mean-variance efficiency is equivalent to CAPM-style pricing. Michaud (1989) and Black & Litterman (1992) addressed estimation error in expected returns, a challenge that motivates our focus on risk minimization rather than return maximization.

Our adaptation differs from standard portfolio theory in a crucial respect: rather than maximizing risk-adjusted return, firms minimize risk subject to achieving a mandatory abatement target. This "goal programming" formulation is related to Roy (1952)'s safety-first criterion and Telser (1955)'s work on constrained optimization under uncertainty.

Recent applications of portfolio theory to climate include Awerbuch & Berger (2006) on electricity generation portfolios, Roques et al. (2008) on fuel mix diversification, and Szolgayova et al. (2008) on technology portfolios under carbon price uncertainty. Our contribution extends this work by incorporating stranded asset risk, real options, and dynamic learning.

2.2 Irreversible Investment and Real Options

The seminal work of Arrow & Fisher (1974) and Henry (1974) established that irreversibility creates option value from waiting. Dixit & Pindyck (1994) synthesized this literature, showing that under uncertainty, the optimal investment threshold exceeds the static NPV rule.

For a project with stochastic value V following geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dW \tag{2}$$

the optimal investment rule is to invest when V exceeds a threshold $V^* = \frac{\beta_1}{\beta_1 - 1} I$, where I is the investment cost and $\beta_1 > 1$ is the positive root of the characteristic equation.

Pindyck (1991) and Pindyck (1993) applied this framework to environmental regulation, showing that regulatory uncertainty raises the option value of delay. Majd & Pindyck (1987) extended the analysis to time-to-build, relevant for large infrastructure projects. Trigeorgis (1996) developed compound option frameworks for sequential investment decisions.

We integrate real options into portfolio optimization by treating option value as a risk-reducing attribute. Technologies with higher flexibility (e.g., modular deployment, fuel switching capability) carry embedded option value that reduces effective transition risk.

2.3 Learning Curves and Technology Dynamics

Wright (1936) first documented learning curves in aircraft production, finding that unit costs decline with cumulative output:

$$C(Q) = C_0 Q^{-\alpha} \tag{3}$$

where α is the learning rate. Arrow (1962) formalized this as "learning by doing," providing welfare-theoretic foundations.

In energy economics, McDonald & Schrattenholzer (2001) and Nemet (2006) estimated learning rates for renewable technologies, finding rates of 15-25% for solar PV. Rubin et al. (2015) documented CCS learning rates of 3-12%. Nagy et al. (2013) showed that learning rates are remarkably consistent across technologies.

We incorporate learning through time-varying cost dynamics, where expected cost decline depends on cumulative deployment. This creates strategic complementarity: early adopters reduce costs for later adopters, generating positive externalities that market prices do not capture (Jaffe et al., 2005).

2.4 Climate Economics and Carbon Pricing

The theoretical foundations for climate policy derive from Pigou (1920)'s analysis of externalities and Coase (1960)'s theorem on property rights. Weitzman (1974) established conditions under which price instruments (carbon taxes) dominate quantity instruments (cap-and-trade), relating to the relative slopes of marginal benefit and cost curves.

Nordhaus (1994) developed the DICE model integrating climate science with economic optimization, while Stern (2007) applied declining discount rates to argue for aggressive near-term action. Weitzman (2009) analyzed fat-tailed climate risks, showing that standard expected utility maximization may be inappropriate under catastrophic uncertainty.

For corporate decision-making, Barnett (2020) showed that firms increasingly face internal carbon prices, while Bolton & Kacperczyk (2020) documented that investors price transition risk into equity valuations. Stroebel & Wurgler (2021) surveyed the climate finance literature, identifying transition risk as a first-order concern for corporate valuation.

Our framework incorporates carbon pricing through an effective cost adjustment: higher carbon prices increase the relative attractiveness of low-emission technologies by raising the implicit cost of baseline activities.

2.5 Stranded Assets and Transition Risk

Ansar et al. (2013) introduced the "stranded assets" concept to climate finance, arguing that carbon budget constraints imply that fossil fuel reserves cannot all be monetized. McGlade & Ekins (2015) quantified unburnable carbon, finding that 80% of coal reserves must remain unextracted to limit warming to 2°C.

For corporate assets, stranding risk arises from:

(a) **Regulatory stranding**: Emissions standards render equipment non-compliant (Caldecott et al., 2016)

(b) **Market stranding**: Low-carbon alternatives become cost-competitive (Pfeiffer et al., 2016)

(c) **Physical stranding**: Climate impacts damage productive capacity (Dietz & Stern, 2015)

Carney (2015) identified transition risk as a financial stability concern, leading to the TCFD disclosure framework (TCFD, 2017). Battiston et al. (2017) applied network analysis to assess systemic risk from stranded assets.

We model stranded asset risk as a function of technology failure probability, loss given failure, and maturity mismatch. Technologies with longer capital lifetimes face greater stranding risk from future technological or regulatory obsolescence.

3 Model Setup and Basic Framework

3.1 Technology Space and Firm Characteristics

Consider a firm facing a mandatory abatement target A^* over horizon T. The firm has access to a finite set of low-carbon technologies $\mathcal{T} = \{1, 2, \ldots, N\}$.

Definition 1 (Technology Characteristics)**.** *Each technology $j \in \mathcal{T}$ is characterized by the tuple:*

$$\Theta_j = (a_j, c_j, \sigma_j, \rho_{jk}, \pi_j, L_j, \alpha_j, o_j, \tau_j) \tag{4}$$

where:

- $a_j \in \mathbb{R}_+$*: Abatement potential per unit capacity (tCO_2/unit)*
- $c_j \in \mathbb{R}_+$*: Capital cost per unit capacity ($/unit)*
- $\sigma_j \in \mathbb{R}_+$*: Cost volatility (annualized standard deviation)*
- $\rho_{jk} \in [-1, 1]$*: Pairwise correlation with technology k*
- $\pi_j \in [0, 1]$*: Probability of technology failure*
- $L_j \in \mathbb{R}_+$*: Loss given failure ($/unit)*
- $\alpha_j \in [0, 1]$*: Learning rate (cost reduction per doubling)*
- $o_j \in \mathbb{R}_+$*: Embedded option value ($/unit)*
- $\tau_j \in \mathbb{R}_+$*: Capital lifetime (years)*

The parameters are grounded in empirical literature. Following Rubin et al. (2015), we calibrate $\sigma_j \in [0.05, 0.40]$ based on historical cost volatility. Learning rates α_j follow McDonald & Schrattenholzer (2001): mature technologies (blast furnace) have $\alpha \approx 0.01$, while emerging technologies (electrolysis) have $\alpha \approx 0.25$. Failure probabilities derive from Kern & Rogge (2016)'s analysis of technology lock-in.

Assumption 1 (Technology Availability)**.** *The technology set $\mathcal{T}$ satisfies:*

(i) $a_j > 0$ *for all j (all technologies provide positive abatement)*

(ii) $\sum_{j=1}^{N} a_j \cdot \bar{w}_j \geq A^*$ *for some feasible capacity bounds $\bar{w}_j$ (target is achievable)*

(iii) $c_j > 0$ *for all j (all technologies have positive cost)*

3.2 Portfolio Formation and Risk Structure

Let $\boldsymbol{w} = (w_1, \ldots, w_N)^T \in \mathbb{R}_+^N$ denote the firm's technology adoption vector, where w_j represents capacity deployed in technology j.

Definition 2 (Covariance Structure)**.** *The technology cost covariance matrix* $\boldsymbol{\Sigma} \in \mathbb{R}^{N \times N}$ *has elements:*

$$\Sigma_{jk} = \rho_{jk}\sigma_j\sigma_k \tag{5}$$

with $\Sigma_{jj} = \sigma_j^2$.

Following Markowitz (1952), we require:

Assumption 2 (Covariance Regularity)**.** *The covariance matrix* $\boldsymbol{\Sigma}$ *is symmetric positive semi-definite. For uniqueness results, we assume positive definiteness.*

The correlation structure captures technology interdependencies. For example:

- Hydrogen-based technologies share electrolyzer cost risk ($\rho > 0$)
- CCS technologies share CO_2 transport/storage infrastructure risk ($\rho > 0$)
- Technologies competing for the same input (e.g., biomass) may be negatively correlated under supply constraints ($\rho < 0$)

Remark 1 (Factor Structure)**.** *In practice, technology correlations can be modeled via a factor structure:*

$$\boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{F}\boldsymbol{B}^T + \boldsymbol{D} \tag{6}$$

where $\boldsymbol{F}$ *is the covariance of common factors (electricity price, hydrogen cost, carbon price),* $\boldsymbol{B}$ *is the factor loading matrix, and* $\boldsymbol{D}$ *is idiosyncratic variance. This follows Ross (1976)'s APT framework.*

3.3 Risk Decomposition

Building on Sharpe (1964)'s risk decomposition and Battiston et al. (2017)'s transition risk taxonomy, we decompose portfolio risk into three components.

Definition 3 (Portfolio Transition Risk)**.** *The total portfolio transition risk is:*

$$R_P(\boldsymbol{w}) = \underbrace{\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w}}_{\textit{Cost Volatility}} + \lambda \underbrace{h(\boldsymbol{w})}_{\textit{Stranded Asset Risk}} - \gamma \underbrace{g(\boldsymbol{w})}_{\textit{Option Value}} \tag{7}$$

where $\lambda, \gamma \geq 0$ *are preference weights.*

Component 1: Cost Volatility The quadratic form $\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w}$ captures portfolio variance, the standard Markowitz risk measure. This represents uncertainty in total transition costs due to technology cost fluctuations.

Component 2: Stranded Asset Risk Following Caldecott et al. (2016), we define:

$$h(\boldsymbol{w}) = \sum_{j=1}^{N} w_j \left[\pi_j L_j + \sigma_j \sqrt{\tau_j}\right] \tag{8}$$

The first term captures expected loss from technology failure (probability π_j times loss L_j). The second term captures maturity risk: technologies with longer lifetimes (τ_j) and higher volatility (σ_j) face greater stranding probability from future disruption. The $\sqrt{\tau}$ scaling follows from the terminal variance of Brownian motion over horizon τ.

Component 3: Option Value Following Trigeorgis (1996), we define:

$$g(\boldsymbol{w}) = \sum_{j=1}^{N} w_j \cdot o_j \tag{9}$$

where o_j is the embedded option value from technology flexibility. This includes:

- **Expansion options**: Ability to scale up if costs decline (McDonald & Siegel, 1986)
- **Switching options**: Flexibility to change inputs (e.g., fuel switching) (Kulatilaka & Trigeorgis, 1994)
- **Abandonment options**: Ability to exit if technology underperforms (Myers & Majd, 1990)

Option value enters negatively in the risk function because it *reduces* effective risk: technologies with greater flexibility provide insurance against uncertainty.

3.4 The Firm's Optimization Problem

The firm solves:

$$\min_{\boldsymbol{w} \in \mathbb{R}_+^N} \quad R_P(\boldsymbol{w}) \tag{10}$$

$$\text{subject to} \quad \sum_{j=1}^{N} w_j a_j \geq A^* \quad \text{(Abatement constraint)} \tag{11}$$

$$\sum_{j=1}^{N} w_j c_j \leq B \quad \text{(Budget constraint)} \tag{12}$$

$$\boldsymbol{w} \geq \boldsymbol{0} \quad \text{(Non-negativity)} \tag{13}$$

This formulation differs from standard portfolio optimization in that the firm does not maximize expected return but rather minimizes risk subject to meeting a mandatory abatement target. This reflects the regulatory nature of decarbonization: emissions reduction is not optional but required by net-zero commitments, carbon pricing, or direct regulation.

Remark 2 (Relation to Mean-Variance Optimization)**.** *In traditional portfolio theory, the investor maximizes $\boldsymbol{w}^T \boldsymbol{\mu} - \frac{\gamma}{2} \boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w}$. Our formulation can be viewed as the dual: for a given "return" (abatement target A^*), minimize risk. This duality is established in Markowitz (1959).*

4 Existence, Uniqueness, and Characterization

4.1 Existence of Optimal Solutions

Theorem 1 (Existence)**.** *Under Assumptions 1 and 2, if the feasible set*

$$\mathcal{F} = \left\{ \boldsymbol{w} \in \mathbb{R}^N_+ : \sum_j w_j a_j \geq A^*, \sum_j w_j c_j \leq B \right\} \tag{14}$$

is non-empty, then the optimization problem (10)–(13) has an optimal solution.

Proof. By Assumption 1(iii), $c_j > 0$ for all j. Combined with the budget constraint $\sum_j w_j c_j \leq B$, we have $w_j \leq B/c_j$ for all j. Thus $\mathcal{F}$ is bounded.

$\mathcal{F}$ is closed as the intersection of closed half-spaces. Since $\mathcal{F}$ is non-empty, bounded, and closed in $\mathbb{R}^N$, it is compact.

The objective function $R_P(\boldsymbol{w})$ is continuous: the quadratic form is continuous, and $h(\cdot)$ and $g(\cdot)$ are linear in $\boldsymbol{w}$.

By the Weierstrass extreme value theorem, a continuous function on a compact set attains its minimum. □ □

4.2 Uniqueness

Theorem 2 (Uniqueness)**.** *If $\boldsymbol{\Sigma}$ is positive definite and $\lambda \geq 0$, $\gamma \geq 0$, the optimal solution is unique.*

Proof. We show that $R_P(\boldsymbol{w})$ is strictly convex. The Hessian is:

$$\nabla^2 R_P(\boldsymbol{w}) = 2\boldsymbol{\Sigma} + \lambda \nabla^2 h(\boldsymbol{w}) - \gamma \nabla^2 g(\boldsymbol{w}) \tag{15}$$

Since $h(\boldsymbol{w})$ and $g(\boldsymbol{w})$ are linear in $\boldsymbol{w}$, we have $\nabla^2 h = \nabla^2 g = \boldsymbol{0}$.

Therefore $\nabla^2 R_P(\boldsymbol{w}) = 2\boldsymbol{\Sigma}$, which is positive definite by assumption. A strictly convex function on a convex set has at most one minimum. Combined with Theorem 1, the minimum exists and is unique. □ □

4.3 Optimality Conditions

The Lagrangian for (10)–(13) is:

$$\mathcal{L}(\boldsymbol{w}, \mu, \nu, \boldsymbol{\eta}) = R_P(\boldsymbol{w}) - \mu \left(\sum_j w_j a_j - A^* \right) + \nu \left(\sum_j w_j c_j - B \right) - \boldsymbol{\eta}^T \boldsymbol{w} \tag{16}$$

where $\mu \geq 0$ is the multiplier on abatement, $\nu \geq 0$ on budget, and $\boldsymbol{\eta} \geq \boldsymbol{0}$ on non-negativity.

Proposition 1 (KKT Conditions)**.** *The optimal portfolio* $\boldsymbol{w}^*$ *satisfies:*

$$2(\boldsymbol{\Sigma}\boldsymbol{w}^*)_j + \lambda h'_j - \gamma g'_j - \mu^* a_j + \nu^* c_j - \eta^*_j = 0 \quad \forall j \tag{17}$$

$$\mu^* \left(\sum_j w^*_j a_j - A^* \right) = 0 \tag{18}$$

$$\nu^* \left(\sum_j w^*_j c_j - B \right) = 0 \tag{19}$$

$$\eta^*_j w^*_j = 0 \quad \forall j \tag{20}$$

$$\mu^*, \nu^*, \eta^*_j \geq 0 \quad \forall j \tag{21}$$

where $h'_j = \pi_j L_j + \sigma_j \sqrt{\tau_j}$ *and* $g'_j = o_j$*.*

The condition (17) shows that at optimum, the marginal risk of each deployed technology ($w^*_j > 0$) equals the shadow price of abatement ($\mu^* a_j$) minus the shadow cost of budget ($\nu^* c_j$), adjusted for stranded asset risk and option value.

4.4 The Net-Zero Efficient Frontier

Definition 4 (Efficient Frontier)**.** *The net-zero efficient frontier is the set:*

$$\mathcal{E} = \left\{ (R^*_P(A), A) : R^*_P(A) = \min_{\boldsymbol{w} \in \mathcal{F}(A)} R_P(\boldsymbol{w}),\ A \in [A_{\min}, A_{\max}] \right\} \tag{22}$$

where $\mathcal{F}(A)$ *is the feasible set for abatement target* A*.*

Theorem 3 (Convexity of Efficient Frontier)**.** *The efficient frontier* $\mathcal{E}$ *is convex in* (R_P, A) *space.*

Proof. Let (R^*_1, A_1) and (R^*_2, A_2) be two points on $\mathcal{E}$ with optimal portfolios $\boldsymbol{w}^*_1$ and $\boldsymbol{w}^*_2$.

For $\theta \in [0, 1]$, consider $\boldsymbol{w}_\theta = \theta \boldsymbol{w}^*_1 + (1 - \theta)\boldsymbol{w}^*_2$.

This portfolio achieves abatement:

$$A_\theta = \sum_j (w_\theta)_j a_j = \theta A_1 + (1 - \theta) A_2 \tag{23}$$

By convexity of $R_P(\cdot)$ (Theorem 2):

$$R_P(\boldsymbol{w}_\theta) \leq \theta R_P(\boldsymbol{w}^*_1) + (1 - \theta) R_P(\boldsymbol{w}^*_2) = \theta R^*_1 + (1 - \theta) R^*_2 \tag{24}$$

Since $R^*_P(A_\theta) \leq R_P(\boldsymbol{w}_\theta)$:

$$R^*_P(A_\theta) \leq \theta R^*_1 + (1 - \theta) R^*_2 \tag{25}$$

This is the definition of convexity for the function $R^*_P(A)$. □ □

Corollary 1 (Marginal Risk of Abatement)**.** *The marginal risk of increasing abatement is non-decreasing:*

$$\frac{\partial^2 R^*_P}{\partial A^2} \geq 0 \tag{26}$$

This follows directly from convexity and has important policy implications: the "cost" (in risk terms) of additional abatement increases as targets become more ambitious.

5 Real Options Integration

5.1 Option Valuation Framework

We now develop the option value component o_j more rigorously, building on Black & Scholes (1973) and Merton (1973).

Consider technology j as conferring the option to "switch" to lower-cost operations if cost decreases below a threshold. Let S_t denote the technology's effective value (e.g., cost savings relative to baseline). Under risk-neutral dynamics:

$$dS = (r - \delta)S\,dt + \sigma_j S\,dW \tag{27}$$

where r is the risk-free rate and δ is a convenience yield (cost of carry).

Proposition 2 (Option Value PDE)**.** *The option value $O_j(S, t)$ satisfies:*

$$\frac{\partial O_j}{\partial t} + \frac{1}{2}\sigma_j^2 S^2 \frac{\partial^2 O_j}{\partial S^2} + (r - \delta)S\frac{\partial O_j}{\partial S} - rO_j = 0 \tag{28}$$

with terminal condition $O_j(S, T) = \max(S - K_j, 0)$ and boundary conditions:

$$O_j(0, t) = 0 \tag{29}$$

$$\lim_{S\to\infty} O_j(S, t) = S - K_j e^{-r(T-t)} \tag{30}$$

This is the standard Black-Scholes-Merton PDE (Black & Scholes, 1973). For a European call, the solution is:

$$O_j(S, t) = Se^{-\delta(T-t)}N(d_1) - K_j e^{-r(T-t)}N(d_2) \tag{31}$$

where $N(\cdot)$ is the standard normal CDF and:

$$d_1 = \frac{\ln(S/K_j) + (r - \delta + \sigma_j^2/2)(T - t)}{\sigma_j\sqrt{T - t}} \tag{32}$$

$$d_2 = d_1 - \sigma_j\sqrt{T - t} \tag{33}$$

5.2 Types of Embedded Options

Following Trigeorgis (1996), we identify three option types relevant to decarbonization:

Expansion Options The option to scale up deployment if technology proves successful:

$$O_j^{expand} = \max\left(V_j^{expanded} - I^{expansion}, 0\right) \tag{34}$$

This is particularly relevant for modular technologies (solar, batteries) where capacity can be incrementally added.

Switching Options The option to change inputs or outputs, following Kulatilaka & Trigeorgis (1994):

$$O_j^{switch} = \mathbb{E}\left[\max_{k\in\mathcal{K}}\left(V_k - C_{j\to k}^{switch}\right)\right] - V_j \tag{35}$$

where $\mathcal{K}$ is the set of alternative operating modes.

Abandonment Options The option to exit, following Myers & Majd (1990):

$$O_j^{abandon} = \max\left(S_j^{salvage} - V_j^{continue}, 0\right) \tag{36}$$

The total embedded option value is:

$$o_j = O_j^{expand} + O_j^{switch} + O_j^{abandon} \tag{37}$$

5.3 Option Value as Risk Reduction

Proposition 3 (Option-Adjusted Risk). *In the presence of embedded options, the effective risk is reduced:*

$$R_j^{effective} = R_j^{raw} - \gamma \cdot o_j \tag{38}$$

where γ reflects the firm's ability to exercise options optimally.

Proof. Consider a technology with raw risk R_j^{raw} and an abandonment option with exercise value o_j. The realized risk is:

$$\tilde{R}_j = \begin{cases} R_j^{raw} & \text{if } R_j^{raw} < R_j^{threshold} \\ R_j^{threshold} - o_j & \text{otherwise} \end{cases} \tag{39}$$

Taking expectations and noting that $o_j \geq 0$:

$$\mathbb{E}[\tilde{R}_j] \leq R_j^{raw} \tag{40}$$

The reduction $R_j^{raw} - \mathbb{E}[\tilde{R}_j]$ is increasing in o_j. The coefficient $\gamma \in (0, 1]$ accounts for imperfect option exercise (e.g., organizational inertia, incomplete information). □ □

This justifies the negative sign on $g(\boldsymbol{w})$ in equation (7): option value reduces effective portfolio risk.

6 Dynamic Extension with Learning

6.1 Technology Cost Dynamics

Building on Arrow (1962) and Merton (1976), we model technology costs as jump-diffusion processes with endogenous learning.

Definition 5 (Cost Dynamics)**.** *Technology j's cost evolves according to:*

$$\frac{dc_j}{c_j} = \underbrace{(-\alpha_j \cdot \iota_j)}_{\textit{Learning}}\, dt + \underbrace{\sigma_j\, dW_j}_{\textit{Diffusion}} + \underbrace{h_j\, dN_j}_{\textit{Jumps}} \tag{41}$$

where:

- α_j *is the learning rate*
- $\iota_j = d \ln Q_j / dt$ *is the deployment growth rate*
- W_j *is a standard Brownian motion*
- N_j *is a Poisson process with intensity* λ_j
- $h_j < 0$ *is the (negative) jump size for cost-reducing breakthroughs*

The learning component follows Wright's Law (Wright, 1936): costs decline with cumulative production. The jump component captures discontinuous innovations—technological breakthroughs that cause sudden cost reductions (Nordhaus, 2014).

Remark 3 (Correlation Structure)**.** *The Brownian motions* $\{W_j\}$ *may be correlated:*

$$\mathbb{E}[dW_j \cdot dW_k] = \rho_{jk}\, dt \tag{42}$$

This generates the covariance matrix $\boldsymbol{\Sigma}$ *used in the static model.*

6.2 Multi-Period Optimization

Consider discrete time periods $t = 0, 1, \ldots, T$. The state at time t is:

$$\boldsymbol{s}_t = (\boldsymbol{w}_{t-1}, \boldsymbol{c}_t, A_t^*) \tag{43}$$

where $\boldsymbol{w}_{t-1}$ is the inherited capacity, $\boldsymbol{c}_t$ is the current cost vector, and A_t^* is the current abatement target.

Following Bellman (1957), the value function satisfies:

Theorem 4 (Bellman Equation)**.** *The optimal value function* $V_t(\boldsymbol{s}_t)$ *satisfies:*

$$V_t(\boldsymbol{s}_t) = \min_{\boldsymbol{w}_t \in \Gamma_t(\boldsymbol{s}_t)} \{R_P(\boldsymbol{w}_t) + \beta\, \mathbb{E}_t\, [V_{t+1}(\boldsymbol{s}_{t+1})]\} \tag{44}$$

where $\beta \in (0, 1)$ *is the discount factor and:*

$$\Gamma_t(\boldsymbol{s}_t) = \left\{ \boldsymbol{w}_t : \sum_j w_{j,t} a_j \geq A_t^*,\ \sum_j (w_{j,t} - w_{j,t-1})^+ c_{j,t} \leq B_t,\ \boldsymbol{w}_t \geq \boldsymbol{w}_{t-1} \right\} \tag{45}$$

is the feasible set incorporating irreversibility ($\boldsymbol{w}_t \geq \boldsymbol{w}_{t-1}$*).*

The irreversibility constraint follows Arrow & Fisher (1974) and Pindyck (1991): once capacity is installed, it cannot be costlessly reversed. This creates path dependence and option value from delay.

6.3 Approximate Solution via Model Predictive Control

The curse of dimensionality (Bellman, 1957) makes exact solution infeasible for large N. Following Bertsekas (2012), we employ Model Predictive Control (MPC) with receding horizon.

Definition 6 (MPC Approximation)**.** *At each period t, solve the H-period ahead problem:*

$$\min_{\{\boldsymbol{w}_{t+k}\}_{k=0}^{H-1}} \sum_{k=0}^{H-1} \beta^k R_P(\boldsymbol{w}_{t+k}) \tag{46}$$

subject to constraints for each period. Implement $\mathbf{w}_t^*$, then re-optimize at $t+1$.

Mayne et al. (2000) established that MPC provides asymptotic stability and near-optimal performance when the horizon H is sufficiently long relative to system dynamics.

6.4 Learning Externalities and Strategic Interaction

Learning creates positive externalities: one firm's deployment reduces costs for all firms. Following Jaffe et al. (2005), define aggregate deployment:

$$Q_j^{aggregate} = \sum_{i\in\mathcal{I}} w_{i,j} \tag{47}$$

The externality is:

$$\frac{\partial c_j}{\partial w_{i,j}} = -\alpha_j \frac{c_j}{Q_j^{aggregate}} < 0 \tag{48}$$

In decentralized equilibrium, firms ignore this externality, leading to under-investment in emerging technologies. This provides a rationale for technology subsidies beyond carbon pricing (Acemoglu et al., 2012).

7 Comparative Statics and Policy Analysis

7.1 Carbon Price Effects

Let p_c denote the carbon price (\$/tCO_2). The effective value of abatement increases with carbon price:

$$\tilde{a}_j(p_c) = a_j + \frac{\partial \text{Cost Savings}_j}{\partial p_c} \tag{49}$$

Proposition 4 (Carbon Price Sensitivity)**.** *The optimal portfolio shifts toward high-abatement technologies as carbon price increases:*

$$\frac{\partial w_j^*}{\partial p_c} \propto a_j \cdot (\boldsymbol{\Sigma}^{-1})_{jj} \tag{50}$$

Technologies with higher abatement potential and lower correlation with other technologies gain share.

Proof. From the KKT conditions (17), at an interior solution:

$$2(\boldsymbol{\Sigma}\boldsymbol{w}^*)_j = \mu^* a_j - \nu^* c_j + \gamma o_j - \lambda h'_j \tag{51}$$

Differentiating with respect to p_c (which affects μ^* through the abatement constraint):

$$2\boldsymbol{\Sigma}\frac{\partial \boldsymbol{w}^*}{\partial p_c} = \frac{\partial \mu^*}{\partial p_c}\boldsymbol{a} \tag{52}$$

Solving: $\frac{\partial \boldsymbol{w}^*}{\partial p_c} = \frac{1}{2}\frac{\partial \mu^*}{\partial p_c}\boldsymbol{\Sigma}^{-1}\boldsymbol{a}$. Since $\frac{\partial \mu^*}{\partial p_c} > 0$ (higher carbon price tightens the effective constraint), technologies with high a_j and low correlation (high $(\boldsymbol{\Sigma}^{-1})_{jj}$) gain portfolio share. □ □

7.2 Technology Subsidy Effects

Consider a technology-specific subsidy s_j that reduces effective cost to $c_j - s_j$.

Proposition 5 (Optimal Subsidy)**.** *The socially optimal subsidy internalizes learning externalities:*

$$s_j^* = \alpha_j c_j \cdot \frac{\partial Q_j^{aggregate}}{\partial w_{i,j}} \cdot \left[\sum_{i'} \frac{\partial R_{P,i'}}{\partial c_j}\right] \tag{53}$$

The subsidy is larger for technologies with higher learning rates (α_j) and greater aggregate risk reduction.

This result supports technology-specific industrial policy beyond uniform carbon pricing, consistent with Acemoglu et al. (2016)'s analysis of directed technical change.

7.3 Regulatory Uncertainty

Following Weitzman (2009), consider uncertainty in future abatement targets:

$$A_T^* = \bar{A} + \epsilon, \quad \epsilon \sim N(0, \sigma_A^2) \tag{54}$$

Theorem 5 (Precautionary Principle)**.** *Under regulatory uncertainty, optimal current investment exhibits precautionary behavior:*

$$\mathbb{E}[w_j^*(A_T^*)] \geq w_j^*(\mathbb{E}[A_T^*]) \tag{55}$$

if the marginal risk function is convex in capacity.

Proof. By Jensen's inequality, for convex f:

$$\mathbb{E}[f(x)] \geq f(\mathbb{E}[x]) \tag{56}$$

The optimal capacity $w_j^*(A)$ is increasing and convex in A when marginal risk is increasing (Corollary 1). Therefore:

$$\mathbb{E}[w_j^*(A_T^*)] \geq w_j^*(\mathbb{E}[A_T^*]) \tag{57}$$

Firms invest more today to hedge against potentially stricter future requirements. □ □

This provides theoretical support for "no regrets" strategies that involve early investment in flexible technologies.

8 Empirical Calibration

8.1 Parameter Sources

Table 1 summarizes parameter estimates from the literature.

Table 1: Technology Parameter Estimates from Literature

Parameter	Range	Source	Notes
σ_j (volatility)	0.05–0.40	Rubin et al. (2015)	Historical cost data
α_j (learning rate)	0.01–0.25	McDonald & Schrattenholzer (2001)	Experience curves
π_j (failure prob.)	0.01–0.15	Kern & Rogge (2016)	Technology lock-in
τ_j (lifetime)	10–40 years	IEA Technology Roadmaps	Asset classes
ρ_{jk} (correlation)	-0.3–0.8	Estimated	Factor model

8.2 Sector-Specific Applications

Steel Sector Technologies include blast furnace with CCS, hydrogen-based direct reduction (H-DRI), and electric arc furnace (EAF). Following Vogl et al. (2018):

- H-DRI has high abatement ($a \approx 1.8$ tCO_2/t steel) but high cost uncertainty ($\sigma \approx 0.25$)
- EAF has moderate abatement but is mature ($\sigma \approx 0.10$)
- CCS has high option value from potential retrofit flexibility

Petrochemical Sector Technologies include electric cracking, bio-based feedstocks, and chemical recycling. Following **?**:

- Electric cracking requires low-carbon electricity (correlation with electricity sector)
- Bio-feedstocks face supply constraints (negatively correlated across competing uses)
- Chemical recycling has high learning potential ($\alpha \approx 0.20$)

9 Extensions

9.1 Robust Optimization

Following Ben-Tal et al. (2009), we can reformulate under parameter uncertainty:

$$\min_{\boldsymbol{w}} \max_{\boldsymbol{\theta} \in \Theta} R_P(\boldsymbol{w}; \boldsymbol{\theta}) \tag{58}$$

where Θ is an uncertainty set for parameters. This provides protection against estimation error, addressing Michaud (1989)'s critique.

9.2 Multi-Objective Formulation

Rather than weighting objectives via λ, γ, we can compute the Pareto frontier:

$$\mathcal{P} = \{(\boldsymbol{w}, R_{cost}, R_{stranded}, V_{option}) : \text{no feasible } \boldsymbol{w}' \text{ dominates } \boldsymbol{w}\} \tag{59}$$

This allows decision-makers to visualize trade-offs explicitly.

9.3 Stochastic Abatement

Relaxing deterministic abatement, let $\tilde{a}_j \sim N(a_j, \sigma^2_{a,j})$. The abatement constraint becomes:

$$\mathbb{P}\left(\sum_j w_j \tilde{a}_j \geq A^*\right) \geq 1 - \epsilon \tag{60}$$

This chance constraint, following Charnes & Cooper (1959), requires deploying additional capacity as a buffer against abatement uncertainty.

10 Conclusion

This paper develops a rigorous portfolio-theoretic framework for corporate decarbonization investment. By extending the foundational work of Markowitz (1952) to incorporate mandatory abatement constraints, stranded asset risk, real options, and technology learning, we provide a unified framework for analyzing net-zero investment decisions.

Our key theoretical contributions include:

1. Existence and uniqueness theorems for optimal technology portfolios under general conditions (Section 4)

2. Integration of real options as risk-reducing factors, providing formal justification for valuing technological flexibility (Section 5)

3. Dynamic multi-period extension with irreversibility and learning, solved via model predictive control (Section 6)

4. Comparative statics showing how carbon pricing and technology subsidies affect optimal portfolios (Section 7)

The framework has immediate practical applications. Firms can use efficient frontier analysis to identify risk-minimizing technology portfolios for their decarbonization targets. Policymakers can design subsidies that correct for learning externalities. Investors can assess transition risk exposure across technology portfolios.

Several extensions merit future research. Incorporating strategic interaction among firms would address how industry-wide adoption affects technology costs and availability. Integrating physical climate risk would capture feedback between mitigation and adaptation. Empirical validation against observed corporate technology choices would test the model's predictive power.

As the global economy transitions to net-zero, rational technology investment requires frameworks that capture the unique features of climate mitigation: mandatory constraints, irreversibility, learning, and deep uncertainty. This paper provides such a framework, grounded in established theory and calibrated to empirical evidence.

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