Portfolio Theory for Corporate Decarbonization: A Risk-Efficiency Framework for Net-Zero Investment under Uncertainty

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Abstract

This paper develops a rigorous theoretical framework for corporate decarbonization investment by extending modern portfolio theory to the climate transition context. We characterize low-carbon technologies as assets with stochastic costs, uncertain abatement potential, and embedded real options, deriving the conditions under which a "net-zero efficient frontier" exists and is unique. Building on the foundational work of Markowitz (1952) and the irreversible investment literature of Dixit & Pindyck (1994), we show that optimal technology portfolios balance cost volatility, stranded asset risk, and option value from technological flexibility. Our dynamic extension incorporates learning curves (Arrow, 1962), breakthrough innovations via jump-diffusion processes (Merton, 1976), and regulatory uncertainty. We derive comparative statics showing how carbon pricing, technology subsidies, and disclosure requirements affect portfolio composition. The framework provides actionable guidance for corporate net-zero strategy while contributing to the theoretical literature on environmental economics and corporate finance.

Keywords: Portfolio optimization, Climate transition risk, Real options, Decarbonization, Net-zero investment, Technology adoption

JEL Classification: G11, Q54, Q55, O33, D81

1 Introduction

The global transition to net-zero emissions requires unprecedented capital reallocation across industrial sectors. The International Energy Agency estimates that annual clean energy investment must reach $4 trillion by 2030 to achieve net-zero by 2050 (IEA, 2021). Firms face a complex optimization problem: how should they allocate limited capital across competing decarbonization technologies, each with uncertain costs, evolving performance, and different risk profiles?

This paper addresses this question by extending modern portfolio theory (Markowitz, 1952, 1959) to the corporate decarbonization context. Our key insight is that climate transition technologies can be characterized as assets with multidimensional risk attributes, and that the firm's technology adoption problem is structurally analogous to mean-variance portfolio optimization with additional constraints.

However, the decarbonization context introduces features not present in traditional portfolio theory:

(i) **Mandatory constraints**: Firms must meet externally-imposed abatement targets, not simply maximize risk-adjusted returns

(ii) **Irreversibility**: Technology investments are largely irreversible, creating path dependence (Dixit & Pindyck, 1994)

(iii) **Learning effects**: Technology costs decline with cumulative deployment (Arrow, 1962; Wright, 1936)

(iv) **Breakthrough uncertainty**: Discontinuous innovation creates jump risk in cost trajectories (Nordhaus, 2014)

(v) **Regulatory uncertainty**: Carbon pricing and technology standards are policy-dependent (Weitzman, 1974)

Our contribution is threefold. First, we formalize the firm's decarbonization problem as a constrained portfolio optimization and prove existence and uniqueness of solutions under general conditions (Section 3). Second, we extend the framework to incorporate real options (McDonald & Siegel, 1986; Pindyck, 1991), showing how managerial flexibility reduces effective transition risk (Section 4). Third, we develop a dynamic multi-period model with learning and derive the Bellman equation characterizing optimal technology pathways (Section 5).

The paper relates to several strands of literature. The foundational portfolio theory literature (Markowitz, 1952; Sharpe, 1964; Lintner, 1965; Mossin, 1966) establishes the mean-variance framework we extend. The real options literature (Dixit & Pindyck, 1994; Trigeorgis, 1996) provides tools for valuing flexibility under uncertainty. The climate economics literature (Nordhaus, 1994; Stern, 2007; Weitzman, 2009) motivates the abatement constraint structure. Recent work on climate finance (Bolton & Kacperczyk, 2020; Stroebel & Wurgler, 2021; Giglio et al., 2021) documents the pricing of transition risk, validating our risk decomposition.

2 Literature Review and Theoretical Foundations

2.1 Portfolio Theory: From Markowitz to Climate Applications

The modern theory of portfolio selection originates with Markowitz (1952), who formalized the trade-off between expected return and variance:

$$\max_{\boldsymbol{w}} \left\{ \boldsymbol{w}^T \boldsymbol{\mu} - \frac{\gamma}{2} \boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w} \right\} \tag{1}$$

where $\boldsymbol{w}$ is the portfolio weight vector, $\boldsymbol{\mu}$ is the expected return vector, $\boldsymbol{\Sigma}$ is the covariance matrix, and γ is the risk aversion coefficient. Markowitz (1959) extended this to derive the efficient frontier—the set of portfolios offering minimum variance for each level of expected return.

? provided closed-form solutions for the efficient frontier with a risk-free asset, while Roll (1977) demonstrated that mean-variance efficiency is equivalent to CAPM-style pricing. Michaud (1989) and Black & Litterman (1992) addressed estimation error in expected returns, a challenge that motivates our focus on risk minimization rather than return maximization.

Our adaptation differs from standard portfolio theory in a crucial respect: rather than maximizing risk-adjusted return, firms minimize risk subject to achieving a mandatory abatement target. This "goal programming" formulation is related to Roy (1952)'s safety-first criterion and Telser (1955)'s work on constrained optimization under uncertainty.

Recent applications of portfolio theory to climate include Awerbuch & Berger (2006) on electricity generation portfolios, Roques et al. (2008) on fuel mix diversification, and Szolgayova et al. (2008) on technology portfolios under carbon price uncertainty. Our contribution extends this work by incorporating stranded asset risk, real options, and dynamic learning.

2.2 Irreversible Investment and Real Options

The seminal work of Arrow & Fisher (1974) and Henry (1974) established that irreversibility creates option value from waiting. Dixit & Pindyck (1994) synthesized this literature, showing that under uncertainty, the optimal investment threshold exceeds the static NPV rule.

For a project with stochastic value V following geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dW \tag{2}$$

the optimal investment rule is to invest when V exceeds a threshold $V^* = \frac{\beta_1}{\beta_1 - 1} I$, where I is the investment cost and $\beta_1 > 1$ is the positive root of the characteristic equation.

Pindyck (1991) and Pindyck (1993) applied this framework to environmental regulation, showing that regulatory uncertainty raises the option value of delay. Majd & Pindyck (1987) extended the analysis to time-to-build, relevant for large infrastructure projects. Trigeorgis (1996) developed compound option frameworks for sequential investment decisions.

We integrate real options into portfolio optimization by treating option value as a risk-reducing attribute. Technologies with higher flexibility (e.g., modular deployment, fuel switching capability) carry embedded option value that reduces effective transition risk.

2.3 Learning Curves and Technology Dynamics

Wright (1936) first documented learning curves in aircraft production, finding that unit costs decline with cumulative output:

$$C(Q) = C_0 Q^{-\alpha} \tag{3}$$

where α is the learning rate. Arrow (1962) formalized this as "learning by doing," providing welfare-theoretic foundations.

In energy economics, McDonald & Schrattenholzer (2001) and Nemet (2006) estimated learning rates for renewable technologies, finding rates of 15-25% for solar PV. Rubin et al. (2015) documented CCS learning rates of 3-12%. Nagy et al. (2013) showed that learning rates are remarkably consistent across technologies.

We incorporate learning through time-varying cost dynamics, where expected cost decline depends on cumulative deployment. This creates strategic complementarity: early adopters reduce costs for later adopters, generating positive externalities that market prices do not capture (Jaffe et al., 2005).

2.4 Climate Economics and Carbon Pricing

The theoretical foundations for climate policy derive from Pigou (1920)'s analysis of externalities and Coase (1960)'s theorem on property rights. Weitzman (1974) established conditions under which price instruments (carbon taxes) dominate quantity instruments (cap-and-trade), relating to the relative slopes of marginal benefit and cost curves.

Nordhaus (1994) developed the DICE model integrating climate science with economic optimization, while Stern (2007) applied declining discount rates to argue for aggressive near-term action. Weitzman (2009) analyzed fat-tailed climate risks, showing that standard expected utility maximization may be inappropriate under catastrophic uncertainty.

For corporate decision-making, Barnett (2020) showed that firms increasingly face internal carbon prices, while Bolton & Kacperczyk (2020) documented that investors price transition risk into equity valuations. Stroebel & Wurgler (2021) surveyed the climate finance literature, identifying transition risk as a first-order concern for corporate valuation.

Our framework incorporates carbon pricing through an effective cost adjustment: higher carbon prices increase the relative attractiveness of low-emission technologies by raising the implicit cost of baseline activities.

2.5 Stranded Assets and Transition Risk

Ansar et al. (2013) introduced the "stranded assets" concept to climate finance, arguing that carbon budget constraints imply that fossil fuel reserves cannot all be monetized. McGlade & Ekins (2015) quantified unburnable carbon, finding that 80% of coal reserves must remain unextracted to limit warming to 2°C.

For corporate assets, stranding risk arises from:

(a) **Regulatory stranding**: Emissions standards render equipment non-compliant (Caldecott et al., 2016)

(b) **Market stranding**: Low-carbon alternatives become cost-competitive (Pfeiffer et al., 2016)

(c) **Physical stranding**: Climate impacts damage productive capacity (Dietz & Stern, 2015)

Carney (2015) identified transition risk as a financial stability concern, leading to the TCFD disclosure framework (TCFD, 2017). Battiston et al. (2017) applied network analysis to assess systemic risk from stranded assets.

We model stranded asset risk as a function of technology failure probability, loss given failure, and maturity mismatch. Technologies with longer capital lifetimes face greater stranding risk from future technological or regulatory obsolescence.

3 Model Setup and Basic Framework

3.1 Technology Space and Firm Characteristics

Consider a firm facing a mandatory abatement target A^* over horizon T. The firm has access to a finite set of low-carbon technologies $\mathcal{T} = \{1, 2, \ldots, N\}$.

Definition 1 (Technology Characteristics)**.** *Each technology $j \in \mathcal{T}$ is characterized by the tuple:*

$$\Theta_j = (a_j, c_j, \sigma_j, \rho_{jk}, \pi_j, L_j, \alpha_j, o_j, \tau_j, \bar{w}_j) \tag{4}$$

where:

- $a_j \in \mathbb{R}_+$*: Abatement potential per unit capacity (tCO_2/unit)*
- $c_j \in \mathbb{R}_+$*: Capital cost per unit capacity (\$/unit)*
- $\sigma_j \in \mathbb{R}_+$*: Cost volatility (annualized standard deviation)*
- $\rho_{jk} \in [-1, 1]$*: Pairwise correlation with technology k*
- $\pi_j \in [0, 1]$*: Probability of technology failure*
- $L_j \in \mathbb{R}_+$*: Loss given failure (\$/unit)*
- $\alpha_j \in [0, 1]$*: Learning rate (cost reduction per doubling)*
- $o_j \in \mathbb{R}_+$*: Embedded option value (\$/unit)*
- $\tau_j \in \mathbb{R}_+$*: Capital lifetime (years)*
- $\bar{w}_j \in \mathbb{R}_+$*: Maximum deployable capacity constraint (Mt/year)*

The parameters are grounded in empirical literature. Following Rubin et al. (2015), we calibrate $\sigma_j \in [0.05, 0.40]$ based on historical cost volatility. Learning rates α_j follow McDonald & Schrattenholzer (2001): mature technologies (blast furnace) have $\alpha \approx 0.01$, while emerging technologies (electrolysis) have $\alpha \approx 0.25$. Failure probabilities derive from Kern & Rogge (2016)'s analysis of technology lock-in.

Capacity Constraints. The maximum capacity bounds $\bar{w}_j$ reflect real-world deployment constraints critical for realistic optimization. For scrap-based electric arc furnaces (EAF), $\bar{w}_j$ is limited by domestic and regional scrap steel availability—studies show global scrap can only supply 25-50% of projected 2050 steel demand even with perfect circular economy implementation (BCG, 2024; OECD, 2024). For hydrogen-based direct reduction (H_2-DRI), constraints arise from electrolyzer manufacturing capacity, hydrogen pipeline infrastructure, and renewable energy availability. For carbon capture and storage (CCS),

geological storage capacity and CO_2 transport infrastructure impose regional limits. Ignoring these constraints leads to infeasible portfolios dominated by single low-risk technologies with unbounded deployment.

Assumption 1 (Technology Availability)**.** *The technology set $\mathcal{T}$ satisfies:*

(i) $a_j > 0$ *for all* j *(all technologies provide positive abatement)*

(ii) $\sum_{j=1}^{N} a_j \cdot \bar{w}_j \geq A^*$ *for some feasible capacity bounds* $\bar{w}_j$ *(target is achievable)*

(iii) $c_j > 0$ *for all* j *(all technologies have positive cost)*

3.2 Portfolio Formation and Risk Structure

Let $\boldsymbol{w} = (w_1, \ldots, w_N)^T \in \mathbb{R}_+^N$ denote the firm's technology adoption vector, where w_j represents capacity deployed in technology j.

Definition 2 (Covariance Structure)**.** *The technology cost covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{N \times N}$ has elements:*

$$\Sigma_{jk} = \rho_{jk} \sigma_j \sigma_k \tag{5}$$

with $\Sigma_{jj} = \sigma_j^2$.

Following Markowitz (1952), we require:

Assumption 2 (Covariance Regularity)**.** *The covariance matrix $\boldsymbol{\Sigma}$ is symmetric positive semi-definite. For uniqueness results, we assume positive definiteness.*

The correlation structure captures technology interdependencies. For example:

- Hydrogen-based technologies share electrolyzer cost risk ($\rho > 0$)
- CCS technologies share CO_2 transport/storage infrastructure risk ($\rho > 0$)
- Technologies competing for the same input (e.g., biomass) may be negatively correlated under supply constraints ($\rho < 0$)

Remark 1 (Factor Structure)**.** *In practice, technology correlations can be modeled via a factor structure:*

$$\boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{F}\boldsymbol{B}^T + \boldsymbol{D} \tag{6}$$

where $\boldsymbol{F}$ is the covariance of common factors (electricity price, hydrogen cost, carbon price), $\boldsymbol{B}$ is the factor loading matrix, and $\boldsymbol{D}$ is idiosyncratic variance. This follows Ross (1976)'s APT framework.

3.3 Risk Decomposition

Building on Sharpe (1964)'s risk decomposition and Battiston et al. (2017)'s transition risk taxonomy, we decompose portfolio risk into three components.

Definition 3 (Portfolio Transition Risk)**.** *The total portfolio transition risk is:*

$$R_P(\boldsymbol{w}) = \underbrace{\boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w}}_{\textit{Cost Volatility}} + \lambda \underbrace{h(\boldsymbol{w})}_{\textit{Stranded Asset Risk}} - \gamma \underbrace{g(\boldsymbol{w})}_{\textit{Option Value}} \tag{7}$$

where $\lambda, \gamma \geq 0$ are preference weights.

Component 1: Cost Volatility The quadratic form $\boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w}$ captures portfolio variance, the standard Markowitz risk measure. This represents uncertainty in total transition costs due to technology cost fluctuations.

Component 2: Stranded Asset Risk Following Caldecott et al. (2016), we define:

$$h(\boldsymbol{w}) = \sum_{j=1}^{N} w_j \left[\pi_j L_j + \sigma_j \sqrt{\tau_j} \right] \tag{8}$$

The first term captures expected loss from technology failure (probability π_j times loss L_j). The second term captures maturity risk: technologies with longer lifetimes (τ_j) and higher volatility (σ_j) face greater stranding probability from future disruption. The $\sqrt{\tau}$ scaling follows from the terminal variance of Brownian motion over horizon τ.

Component 3: Option Value Following Trigeorgis (1996), we define:

$$g(\boldsymbol{w}) = \sum_{j=1}^{N} w_j \cdot o_j \tag{9}$$

where o_j is the embedded option value from technology flexibility. This includes:

- **Expansion options**: Ability to scale up if costs decline (McDonald & Siegel, 1986)
- **Switching options**: Flexibility to change inputs (e.g., fuel switching) (Kulatilaka & Trigeorgis, 1994)
- **Abandonment options**: Ability to exit if technology underperforms (Myers & Majd, 1990)

Option value enters negatively in the risk function because it *reduces* effective risk: technologies with greater flexibility provide insurance against uncertainty.

3.4 The Firm's Optimization Problem

The firm solves:

$$\min_{\boldsymbol{w} \in \mathbb{R}_+^N} \quad R_P(\boldsymbol{w}) \tag{10}$$

$$\text{subject to} \quad \sum_{j=1}^{N} w_j a_j \geq A^* \quad \text{(Abatement constraint)} \tag{11}$$

$$\sum_{j=1}^{N} w_j c_j \leq B \quad \text{(Budget constraint)} \tag{12}$$

$$0 \leq w_j \leq \bar{w}_j, \ \forall j \quad \text{(Capacity bounds)} \tag{13}$$

This formulation differs from standard portfolio optimization in that the firm does not maximize expected return but rather minimizes risk subject to meeting a mandatory abatement target. This reflects the regulatory nature of decarbonization: emissions reduction is not optional but required by net-zero commitments, carbon pricing, or direct regulation.

Remark 2 (Relation to Mean-Variance Optimization)**.** *In traditional portfolio theory, the investor maximizes* $\boldsymbol{w}^T\boldsymbol{\mu} - \frac{\gamma}{2}\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w}$*. Our formulation can be viewed as the dual: for a given "return" (abatement target* A^**), minimize risk. This duality is established in Markowitz (1959).*

4 Existence, Uniqueness, and Characterization

4.1 Existence of Optimal Solutions

Theorem 1 (Existence)**.** *Under Assumptions 1 and 2, if the feasible set*

$$\mathcal{F} = \left\{ \boldsymbol{w} \in \mathbb{R}_+^N : \sum_j w_j a_j \geq A^*, \sum_j w_j c_j \leq B \right\} \tag{14}$$

*is non-empty, then the optimization problem (10)–(**??**) has an optimal solution.*

Proof. By Assumption 1(iii), $c_j > 0$ for all j. Combined with the budget constraint $\sum_j w_j c_j \leq B$, we have $w_j \leq B/c_j$ for all j. Thus $\mathcal{F}$ is bounded.

$\mathcal{F}$ is closed as the intersection of closed half-spaces. Since $\mathcal{F}$ is non-empty, bounded, and closed in $\mathbb{R}^N$, it is compact.

The objective function $R_P(\boldsymbol{w})$ is continuous: the quadratic form is continuous, and $h(\cdot)$ and $g(\cdot)$ are linear in $\boldsymbol{w}$.

By the Weierstrass extreme value theorem, a continuous function on a compact set attains its minimum. □ □

4.2 Uniqueness

Theorem 2 (Uniqueness)**.** *If* $\boldsymbol{\Sigma}$ *is positive definite and* $\lambda \geq 0$*,* $\gamma \geq 0$*, the optimal solution is unique.*

Proof. We show that $R_P(\boldsymbol{w})$ is strictly convex. The Hessian is:

$$\nabla^2 R_P(\boldsymbol{w}) = 2\boldsymbol{\Sigma} + \lambda \nabla^2 h(\boldsymbol{w}) - \gamma \nabla^2 g(\boldsymbol{w}) \tag{15}$$

Since $h(\boldsymbol{w})$ and $g(\boldsymbol{w})$ are linear in $\boldsymbol{w}$, we have $\nabla^2 h = \nabla^2 g = \mathbf{0}$.

Therefore $\nabla^2 R_P(\boldsymbol{w}) = 2\boldsymbol{\Sigma}$, which is positive definite by assumption. A strictly convex function on a convex set has at most one minimum. Combined with Theorem 1, the minimum exists and is unique. □ □

4.3 Optimality Conditions

The Lagrangian for (10)–(**??**) is:

$$\mathcal{L}(\boldsymbol{w}, \mu, \nu, \boldsymbol{\eta}) = R_P(\boldsymbol{w}) - \mu \left(\sum_j w_j a_j - A^* \right) + \nu \left(\sum_j w_j c_j - B \right) - \boldsymbol{\eta}^T \boldsymbol{w} \tag{16}$$

where $\mu \geq 0$ is the multiplier on abatement, $\nu \geq 0$ on budget, and $\boldsymbol{\eta} \geq \mathbf{0}$ on non-negativity.

Proposition 1 (KKT Conditions)**.** *The optimal portfolio $\boldsymbol{w}^*$ satisfies:*

$$2(\boldsymbol{\Sigma}\boldsymbol{w}^*)_j + \lambda h'_j - \gamma g'_j - \mu^* a_j + \nu^* c_j - \eta^*_j = 0 \quad \forall j \tag{17}$$

$$\mu^* \left(\sum_j w^*_j a_j - A^* \right) = 0 \tag{18}$$

$$\nu^* \left(\sum_j w^*_j c_j - B \right) = 0 \tag{19}$$

$$\eta^*_j w^*_j = 0 \quad \forall j \tag{20}$$

$$\mu^*, \nu^*, \eta^*_j \geq 0 \quad \forall j \tag{21}$$

where $h'_j = \pi_j L_j + \sigma_j \sqrt{\tau_j}$ and $g'_j = o_j$.

The condition (17) shows that at optimum, the marginal risk of each deployed technology ($w^*_j > 0$) equals the shadow price of abatement ($\mu^* a_j$) minus the shadow cost of budget ($\nu^* c_j$), adjusted for stranded asset risk and option value.

4.4 The Net-Zero Efficient Frontier

Definition 4 (Efficient Frontier)**.** *The net-zero efficient frontier is the set:*

$$\mathcal{E} = \left\{ (R^*_P(A), A) : R^*_P(A) = \min_{\boldsymbol{w} \in \mathcal{F}(A)} R_P(\boldsymbol{w}),\ A \in [A_{\min}, A_{\max}] \right\} \tag{22}$$

where $\mathcal{F}(A)$ is the feasible set for abatement target A.

Theorem 3 (Convexity of Efficient Frontier)**.** *The efficient frontier $\mathcal{E}$ is convex in (R_P, A) space.*

Proof. Let (R^*_1, A_1) and (R^*_2, A_2) be two points on $\mathcal{E}$ with optimal portfolios $\boldsymbol{w}^*_1$ and $\boldsymbol{w}^*_2$.

For $\theta \in [0, 1]$, consider $\boldsymbol{w}_\theta = \theta \boldsymbol{w}^*_1 + (1 - \theta) \boldsymbol{w}^*_2$.

This portfolio achieves abatement:

$$A_\theta = \sum_j (w_\theta)_j a_j = \theta A_1 + (1 - \theta) A_2 \tag{23}$$

By convexity of $R_P(\cdot)$ (Theorem 2):

$$R_P(\boldsymbol{w}_\theta) \leq \theta R_P(\boldsymbol{w}^*_1) + (1 - \theta) R_P(\boldsymbol{w}^*_2) = \theta R^*_1 + (1 - \theta) R^*_2 \tag{24}$$

Since $R_P^*(A_\theta) \leq R_P(\boldsymbol{w}_\theta)$:

$$R_P^*(A_\theta) \leq \theta R_1^* + (1-\theta) R_2^* \tag{25}$$

This is the definition of convexity for the function $R_P^*(A)$. □ □

Corollary 1 (Marginal Risk of Abatement)**.** *The marginal risk of increasing abatement is non-decreasing:*

$$\frac{\partial^2 R_P^*}{\partial A^2} \geq 0 \tag{26}$$

This follows directly from convexity and has important policy implications: the "cost" (in risk terms) of additional abatement increases as targets become more ambitious.

5 Real Options Integration

5.1 Option Valuation Framework

We now develop the option value component o_j more rigorously, building on Black & Scholes (1973) and Merton (1973).

Consider technology j as conferring the option to "switch" to lower-cost operations if cost decreases below a threshold. Let S_t denote the technology's effective value (e.g., cost savings relative to baseline). Under risk-neutral dynamics:

$$dS = (r-\delta) S\, dt + \sigma_j S\, dW \tag{27}$$

where r is the risk-free rate and δ is a convenience yield (cost of carry).

Proposition 2 (Option Value PDE)**.** *The option value $O_j(S,t)$ satisfies:*

$$\frac{\partial O_j}{\partial t} + \frac{1}{2}\sigma_j^2 S^2 \frac{\partial^2 O_j}{\partial S^2} + (r-\delta) S \frac{\partial O_j}{\partial S} - r O_j = 0 \tag{28}$$

with terminal condition $O_j(S,T) = \max(S - K_j, 0)$ and boundary conditions:

$$O_j(0,t) = 0 \tag{29}$$

$$\lim_{S\to\infty} O_j(S,t) = S - K_j e^{-r(T-t)} \tag{30}$$

This is the standard Black-Scholes-Merton PDE (Black & Scholes, 1973). For a European call, the solution is:

$$O_j(S,t) = S e^{-\delta(T-t)} N(d_1) - K_j e^{-r(T-t)} N(d_2) \tag{31}$$

where $N(\cdot)$ is the standard normal CDF and:

$$d_1 = \frac{\ln(S/K_j) + (r - \delta + \sigma_j^2/2)(T-t)}{\sigma_j \sqrt{T-t}} \tag{32}$$

$$d_2 = d_1 - \sigma_j \sqrt{T-t} \tag{33}$$

5.2 Types of Embedded Options

Following Trigeorgis (1996), we identify three option types relevant to decarbonization:

Expansion Options The option to scale up deployment if technology proves successful:

$$O_j^{expand} = \max\left(V_j^{expanded} - I^{expansion}, 0\right) \tag{34}$$

This is particularly relevant for modular technologies (solar, batteries) where capacity can be incrementally added.

Switching Options The option to change inputs or outputs, following Kulatilaka & Trigeorgis (1994):

$$O_j^{switch} = \mathbb{E}\left[\max_{k\in\mathcal{K}}\left(V_k - C_{j\to k}^{switch}\right)\right] - V_j \tag{35}$$

where $\mathcal{K}$ is the set of alternative operating modes.

Abandonment Options The option to exit, following Myers & Majd (1990):

$$O_j^{abandon} = \max\left(S_j^{salvage} - V_j^{continue}, 0\right) \tag{36}$$

The total embedded option value is:

$$o_j = O_j^{expand} + O_j^{switch} + O_j^{abandon} \tag{37}$$

5.3 Option Value as Risk Reduction

Proposition 3 (Option-Adjusted Risk)**.** *In the presence of embedded options, the effective risk is reduced:*

$$R_j^{effective} = R_j^{raw} - \gamma \cdot o_j \tag{38}$$

where γ reflects the firm's ability to exercise options optimally.

Proof. Consider a technology with raw risk R_j^{raw} and an abandonment option with exercise value o_j. The realized risk is:

$$\tilde{R}_j = \begin{cases} R_j^{raw} & \text{if } R_j^{raw} < R_j^{threshold} \\ R_j^{threshold} - o_j & \text{otherwise} \end{cases} \tag{39}$$

Taking expectations and noting that $o_j \geq 0$:

$$\mathbb{E}[\tilde{R}_j] \leq R_j^{raw} \tag{40}$$

The reduction $R_j^{raw} - \mathbb{E}[\tilde{R}_j]$ is increasing in o_j. The coefficient $\gamma \in (0, 1]$ accounts for imperfect option exercise (e.g., organizational inertia, incomplete information). □ □

This justifies the negative sign on $g(\boldsymbol{w})$ in equation (7): option value reduces effective portfolio risk.

6 Dynamic Extension with Learning

6.1 Technology Cost Dynamics

Building on Arrow (1962) and Merton (1976), we model technology costs as jump-diffusion processes with endogenous learning.

Definition 5 (Cost Dynamics)**.** *Technology j's cost evolves according to:*

$$\frac{dc_j}{c_j} = \underbrace{(-\alpha_j \cdot \iota_j)}_{\textit{Learning}} dt + \underbrace{\sigma_j \, dW_j}_{\textit{Diffusion}} + \underbrace{h_j \, dN_j}_{\textit{Jumps}} \tag{41}$$

where:

- α_j *is the learning rate*
- $\iota_j = d \ln Q_j / dt$ *is the deployment growth rate*
- W_j *is a standard Brownian motion*
- N_j *is a Poisson process with intensity* λ_j
- $h_j < 0$ *is the (negative) jump size for cost-reducing breakthroughs*

The learning component follows Wright's Law (Wright, 1936): costs decline with cumulative production. The jump component captures discontinuous innovations—technological breakthroughs that cause sudden cost reductions (Nordhaus, 2014).

Remark 3 (Correlation Structure)**.** *The Brownian motions $\{W_j\}$ may be correlated:*

$$\mathbb{E}[dW_j \cdot dW_k] = \rho_{jk} \, dt \tag{42}$$

This generates the covariance matrix $\boldsymbol{\Sigma}$ used in the static model.

6.2 Multi-Period Optimization

Consider discrete time periods $t = 0, 1, \ldots, T$. The state at time t is:

$$\boldsymbol{s}_t = (\boldsymbol{w}_{t-1}, \boldsymbol{c}_t, A_t^*) \tag{43}$$

where $\boldsymbol{w}_{t-1}$ is the inherited capacity, $\boldsymbol{c}_t$ is the current cost vector, and A_t^* is the current abatement target.

Following Bellman (1957), the value function satisfies:

Theorem 4 (Bellman Equation)**.** *The optimal value function $V_t(\boldsymbol{s}_t)$ satisfies:*

$$V_t(\boldsymbol{s}_t) = \min_{\boldsymbol{w}_t \in \Gamma_t(\boldsymbol{s}_t)} \{R_P(\boldsymbol{w}_t) + \beta \, \mathbb{E}_t \left[V_{t+1}(\boldsymbol{s}_{t+1})\right]\} \tag{44}$$

where $\beta \in (0, 1)$ is the discount factor and:

$$\Gamma_t(\boldsymbol{s}_t) = \left\{ \boldsymbol{w}_t : \sum_j w_{j,t} a_j \geq A_t^*, \; \sum_j (w_{j,t} - w_{j,t-1})^+ c_{j,t} \leq B_t, \; \boldsymbol{w}_t \geq \boldsymbol{w}_{t-1} \right\} \tag{45}$$

is the feasible set incorporating irreversibility ($\boldsymbol{w}_t \geq \boldsymbol{w}_{t-1}$).

The irreversibility constraint follows Arrow & Fisher (1974) and Pindyck (1991): once capacity is installed, it cannot be costlessly reversed. This creates path dependence and option value from delay.

6.3 Approximate Solution via Model Predictive Control

The curse of dimensionality (Bellman, 1957) makes exact solution infeasible for large N. Following Bertsekas (2012), we employ Model Predictive Control (MPC) with receding horizon.

Definition 6 (MPC Approximation)**.** *At each period t, solve the H-period ahead problem:*

$$\min_{\{\boldsymbol{w}_{t+k}\}_{k=0}^{H-1}} \sum_{k=0}^{H-1} \beta^k R_P(\boldsymbol{w}_{t+k}) \tag{46}$$

subject to constraints for each period. Implement $\boldsymbol{w}_t^*$, then re-optimize at $t+1$.

Mayne et al. (2000) established that MPC provides asymptotic stability and near-optimal performance when the horizon H is sufficiently long relative to system dynamics.

6.4 Learning Externalities and Strategic Interaction

Learning creates positive externalities: one firm's deployment reduces costs for all firms. Following Jaffe et al. (2005), define aggregate deployment:

$$Q_j^{aggregate} = \sum_{i \in \mathcal{I}} w_{i,j} \tag{47}$$

The externality is:

$$\frac{\partial c_j}{\partial w_{i,j}} = -\alpha_j \frac{c_j}{Q_j^{aggregate}} < 0 \tag{48}$$

In decentralized equilibrium, firms ignore this externality, leading to under-investment in emerging technologies. This provides a rationale for technology subsidies beyond carbon pricing (Acemoglu et al., 2012).

7 Comparative Statics and Policy Analysis

7.1 Carbon Price Effects

Let p_c denote the carbon price (\$/tCO_2). The effective value of abatement increases with carbon price:

$$\tilde{a}_j(p_c) = a_j + \frac{\partial \text{Cost Savings}_j}{\partial p_c} \tag{49}$$

Proposition 4 (Carbon Price Sensitivity)**.** *The optimal portfolio shifts toward high-abatement technologies as carbon price increases:*

$$\frac{\partial w_j^*}{\partial p_c} \propto a_j \cdot (\boldsymbol{\Sigma}^{-1})_{jj} \tag{50}$$

Technologies with higher abatement potential and lower correlation with other technologies gain share.

Proof. From the KKT conditions (17), at an interior solution:

$$2(\boldsymbol{\Sigma}\boldsymbol{w}^*)_j = \mu^* a_j - \nu^* c_j + \gamma o_j - \lambda h'_j \tag{51}$$

Differentiating with respect to p_c (which affects μ^* through the abatement constraint):

$$2\boldsymbol{\Sigma}\frac{\partial \boldsymbol{w}^*}{\partial p_c} = \frac{\partial \mu^*}{\partial p_c}\boldsymbol{a} \tag{52}$$

Solving: $\frac{\partial \boldsymbol{w}^*}{\partial p_c} = \frac{1}{2}\frac{\partial \mu^*}{\partial p_c}\boldsymbol{\Sigma}^{-1}\boldsymbol{a}$. Since $\frac{\partial \mu^*}{\partial p_c} > 0$ (higher carbon price tightens the effective constraint), technologies with high a_j and low correlation (high $(\boldsymbol{\Sigma}^{-1})_{jj}$) gain portfolio share. □ □

7.2 Technology Subsidy Effects

Consider a technology-specific subsidy s_j that reduces effective cost to $c_j - s_j$.

Proposition 5 (Optimal Subsidy)**.** *The socially optimal subsidy internalizes learning externalities:*

$$s_j^* = \alpha_j c_j \cdot \frac{\partial Q_j^{aggregate}}{\partial w_{i,j}} \cdot \left[\sum_{i'} \frac{\partial R_{P,i'}}{\partial c_j}\right] \tag{53}$$

The subsidy is larger for technologies with higher learning rates (α_j) and greater aggregate risk reduction.

This result supports technology-specific industrial policy beyond uniform carbon pricing, consistent with Acemoglu et al. (2016)'s analysis of directed technical change.

7.3 Regulatory Uncertainty

Following Weitzman (2009), consider uncertainty in future abatement targets:

$$A_T^* = \bar{A} + \epsilon, \quad \epsilon \sim N(0, \sigma_A^2) \tag{54}$$

Theorem 5 (Precautionary Principle)**.** *Under regulatory uncertainty, optimal current investment exhibits precautionary behavior:*

$$\mathbb{E}[w_j^*(A_T^*)] \geq w_j^*(\mathbb{E}[A_T^*]) \tag{55}$$

if the marginal risk function is convex in capacity.

Proof. By Jensen's inequality, for convex f:

$$\mathbb{E}[f(x)] \geq f(\mathbb{E}[x]) \tag{56}$$

The optimal capacity $w_j^*(A)$ is increasing and convex in A when marginal risk is increasing (Corollary 1). Therefore:

$$\mathbb{E}[w_j^*(A_T^*)] \geq w_j^*(\mathbb{E}[A_T^*]) \tag{57}$$

Firms invest more today to hedge against potentially stricter future requirements. □ □

This provides theoretical support for "no regrets" strategies that involve early investment in flexible technologies.

8 Empirical Calibration

8.1 Parameter Sources

Table 1 summarizes parameter estimates from the literature.

Table 1: Technology Parameter Estimates from Literature

Parameter	Range	Source	Notes
σ_j (volatility)	0.05–0.40	Rubin et al. (2015)	Historical cost data
α_j (learning rate)	0.01–0.25	McDonald & Schrattenholzer (2001)	Experience curves
π_j (failure prob.)	0.01–0.15	Kern & Rogge (2016)	Technology lock-in
τ_j (lifetime)	10–40 years	IEA Technology Roadmaps	Asset classes
ρ_{jk} (correlation)	-0.3–0.8	Estimated	Factor model

8.2 Sector-Specific Applications

Steel Sector Technologies include blast furnace with CCS, hydrogen-based direct reduction (H-DRI), and electric arc furnace (EAF). Following Vogl et al. (2018):

- H-DRI has high abatement ($a \approx 1.8$ tCO_2/t steel) but high cost uncertainty ($\sigma \approx 0.25$)
- EAF has moderate abatement but is mature ($\sigma \approx 0.10$)
- CCS has high option value from potential retrofit flexibility

Petrochemical Sector Technologies include electric cracking, bio-based feedstocks, and chemical recycling. Following **?**:

- Electric cracking requires low-carbon electricity (correlation with electricity sector)
- Bio-feedstocks face supply constraints (negatively correlated across competing uses)
- Chemical recycling has high learning potential ($\alpha \approx 0.20$)

9 Case Study: South Korea's Industrial Decarbonization

We apply our framework to South Korea, a particularly instructive case due to the country's heavy industrial base, concentrated corporate structure, and significant policy-technology gaps. South Korea's steel industry accounts for 15% of national carbon emissions and 40% of industrial emissions, making it a critical sector for achieving the country's 2050 net-zero commitment.

9.1 Korean Steel Sector

South Korea is home to POSCO (the world's 6th largest steel producer) and Hyundai Steel, with combined crude steel production exceeding 55 million tonnes annually. In 2024, POSCO's Scope 1&2 emissions were 71.07 Mt CO_2 with an intensity of 2.02 tCO_2/tcs, showing marginal improvement from 2.05 tCO_2/tcs in 2021-2022. Hyundai Steel emitted approximately 29 Mt CO_2 at 1.43 tCO_2/tcs intensity (POSCO, 2023).

HyREX Technology Progress (2024-2025) A significant development occurred in April 2024 when POSCO succeeded in producing molten iron from its HyREX pilot facility at Pohang Steel Works. The facility, manufacturing up to 24 tons of molten iron daily, emits only 0.4 tCO_2/t—meeting the IEA's near-zero threshold. In October 2025, POSCO and BHP signed a landmark agreement to construct a 300,000 ton/year demonstration plant, with commissioning expected by early 2028 and commercial operation targeted for 2030.

Hy-Cube Development (Hyundai Steel) Hyundai Steel's Hy-Cube (Hy^3) system integrates the proprietary "Hy-Arc" electric furnace with H_2-DRI technology. In 2025, Hyundai announced a $6 billion hydrogen-powered DRI-EAF steel mill in Louisiana, USA, initially using blue hydrogen with transition to green hydrogen expected post-2034.

Technology Portfolio Korean steelmakers face a choice among several decarbonization pathways. Cost parameters reflect 2024-2025 industry estimates based on Global Efficiency Intelligence and IEA data:

Table 2: Korean Steel Sector Technology Parameters (2024-2025 Estimates)

Technology	a_j (tCO_2/t)	c_j ($/t)	σ_j	α_j	π_j	τ_j (years)
BF-BOF (Baseline)	0.0	390	0.05	0.01	0.02	40
BF-BOF + CCUS	1.60	465	0.15	0.06	0.06	25
Scrap-EAF	1.54	415	0.10	0.05	0.03	25
NG-DRI-EAF	1.25	455	0.12	0.08	0.04	25
HyREX H_2-DRI (POSCO)	1.96	616	0.25	0.18	0.08	30
Hy-Cube H_2-DRI (Hyundai)	1.94	600	0.24	0.16	0.10	30
EAF + Green H_2 DRI	2.00	680	0.28	0.20	0.12	30
Molten Oxide Electrolysis	2.15	950	0.40	0.25	0.18	35

Notes: Abatement a_j represents CO_2 reduction relative to BF-BOF baseline (2.2 tCO_2/t). Costs c_j are levelized cost of steel (LCOS) in $/ton at current hydrogen prices ($3-5/kg). Green H_2-DRI costs assume hydrogen at $5/kg; at $1.5/kg, costs approach $490/t. Learning rates α_j follow Rubin et al. (2015) for mature technologies and Nagy et al. (2013) for emerging technologies.

Key Findings Applying our framework reveals several insights:

(i) **High correlation risk**: Both POSCO's HyREX and Hyundai's Hy-Cube technologies depend on green hydrogen availability, creating $\rho_{jk} \approx 0.45$ correlation. This concentration increases portfolio variance. The Hyundai Motor Group projects hydrogen off-take of 3 million tonnes/year by 2035, indicating significant demand correlation across affiliates.

(ii) **Investment gap as risk factor**: Korea has allocated only KRW 268.5 billion ($198M) in government subsidies for 2023-2025, versus POSCO's stated need of KRW 20 trillion ($14.8B) for HyREX commercialization. Germany, producing less than half Korea's steel output, invests approximately 38× more public funding. This policy uncertainty manifests as higher σ_j for hydrogen-based technologies.

(iii) **Green premium sensitivity**: At current hydrogen prices ($3-5/kg), the green premium for H_2-DRI steel is approximately $263/ton versus BF-BOF. However, at $1.5/kg hydrogen (projected 2030 in favorable regions), green H_2-DRI-EAF becomes cost-competitive with BF-BOF at $15/t$CO_2$ carbon pricing.

(iv) **Timeline disadvantage**: SSAB's HYBRIT delivers commercial fossil-free steel in 2026, H2 Green Steel (Stegra) targets 5 Mt/year by 2030, but Korea's HyREX demonstration plant commissioning is expected in 2028 with commercial scale post-2030. This 4-6 year lag increases stranded asset risk for existing blast furnaces ($\sqrt{\tau} \cdot \sigma$ term in equation 8).

(v) **EAF as risk-efficient bridge**: Scrap-EAF offers 1.54 tCO_2/t abatement (72% reduction) with lower volatility ($\sigma = 0.10$) and higher option value from modularity. BF-BOF produces 2.2 tCO_2/t while Scrap-EAF produces 0.66 tCO_2/t. POSCO's 2.5 Mt EAF expansion at Gwangyang and investment of $140M in scrap collection infrastructure (2023-2025) exemplifies this hedging strategy.

Optimal Portfolio Implications For a target of 50% emissions reduction by 2030, our model suggests a diversified portfolio emphasizing EAF expansion (35-45% weight) with selective CCUS deployment (20-30%) as a bridge technology, while maintaining H_2-DRI investment (20-30%) to capture learning curve benefits. The NG-DRI-EAF pathway (10-15%) serves as a transitional technology enabling infrastructure development for future green hydrogen integration.

9.2 Korean Energy Sector

South Korea's power sector presents distinct challenges: renewables supplied only 10.5% of electricity in 2024, versus the OECD average of approximately 30%. The 11th Basic Plan for Long-Term Electricity Supply (February 2025) targets 121.9 GW renewable capacity by 2038.

Table 3: Korean Energy Sector Technology Parameters

Technology	a_j (rel.)	c_j (\$/MWh)	σ_j	α_j	π_j	τ_j (years)
Coal Power (Baseline)	0.0	45	0.08	0.01	0.05	40
LNG CCGT	0.50	65	0.15	0.03	0.03	30
LNG + CCS	0.85	95	0.20	0.06	0.08	25
Nuclear (APR1400)	0.95	80	0.10	0.02	0.02	60
Solar PV (Utility)	0.98	42	0.18	0.12	0.02	25
Offshore Wind	0.97	85	0.22	0.10	0.03	25
Green H_2 Electrolysis	0.99	150	0.35	0.18	0.10	20

Distinctive Features Korea's energy transition exhibits unusual characteristics:

(i) **CCS dependence**: BloombergNEF projects CCS will account for 41% of Korea's emissions abatement by 2050, versus 14% globally. This creates concentrated technology risk.

(ii) **Nuclear as low-variance anchor**: APR1400 reactors offer high abatement ($a = 0.95$) with low volatility ($\sigma = 0.10$), though long capital lifetime ($\tau = 60$) increases stranding exposure to future policy shifts.

(iii) **Renewable cost disadvantage**: Solar PV costs in Korea exceed global benchmarks, reducing the cost-efficiency frontier compared to peers.

(iv) **Corporate procurement barriers**: RE100 members source only 12% of electricity from renewables in Korea versus 53% globally, indicating institutional constraints beyond technology costs.

9.3 Cross-Sector Correlation

A critical insight from the Korean case is the correlation between steel and energy decarbonization. Both sectors depend on:

- Green hydrogen availability (creating cross-sector $\rho > 0$)
- Renewable electricity prices
- CCS infrastructure deployment

This suggests that firm-level portfolio optimization should be extended to sector-level or national-level coordination, as individual firm decisions create positive externalities through shared infrastructure and learning spillovers (Acemoglu et al., 2016).

9.4 Policy Implications for Korea

Our framework suggests several policy priorities:

1. **Increase subsidy allocation**: The current 268.5 billion won is insufficient to reduce σ_j for emerging technologies to competitive levels with European peers.
2. **Diversify technology bets**: Over-reliance on CCS (41% of abatement) concentrates risk. Portfolio theory suggests spreading investment across solar, wind, nuclear, and hydrogen.
3. **Accelerate timeline**: The 10-year lag behind European H_2-DRI deployment increases stranded asset risk and foregoes learning curve benefits. Theorem 5 suggests precautionary early investment.
4. **Address corporate procurement**: Institutional barriers to renewable procurement increase effective c_j for corporate buyers beyond technology costs.

10 POSCO Case Study: HyREX Technology Portfolio Decision

To illustrate the practical application of our framework, we analyze POSCO's actual technology portfolio decision for achieving its 2050 carbon neutrality commitment. POSCO, the world's 6th largest steelmaker, faces a critical choice among competing decarbonization pathways with the constraint of meeting South Korea's ambitious climate targets while maintaining global competitiveness.

10.1 POSCO's Decarbonization Challenge

POSCO's steel production operations emitted 71.07 million tonnes of CO_2 in 2024, representing approximately 10% of South Korea's total emissions. With an emissions intensity of 2.02 tCO_2/tcs (marginal improvement from 2.05 in 2021-2022), the company must reduce emissions by approximately 95% to achieve net-zero by 2050.

Strategic Context POSCO operates integrated blast furnace-basic oxygen furnace (BF-BOF) facilities with long capital lifetimes ($\tau = 40$ years), creating significant stranded asset risk. The company's stated abatement target requires approximately 67.5 Mt reduction over the next 25 years—an average of 2.7 Mt/year. However, technological uncertainty, policy gaps, and capital constraints create a complex portfolio optimization problem.

10.2 Technology Portfolio Options

POSCO's feasible technology set includes:

(1) **Business-as-usual BF-BOF**: Maintains current operations with minimal capital expenditure but zero abatement.

(2) **CCUS Retrofit (FINEX + CCS)**: POSCO's proprietary FINEX technology combined with carbon capture. Offers moderate abatement ($a = 1.5$ tCO_2/t, 68% reduction) at lower technological risk ($\sigma = 0.14$) but requires CCS infrastructure.

(3) **Scrap-EAF Expansion**: Low-risk pathway ($\sigma = 0.10$) with proven technology. POSCO invested \$140 million (2023-2025) expanding scrap collection infrastructure from 4 to 8 centers. Achieves 1.54 tCO_2/t abatement (72% reduction) at cost $c = 415$ \$/t.

(4) **HyREX H_2-DRI**: POSCO's flagship green hydrogen direct reduction technology. In April 2024, the pilot facility at Pohang successfully produced molten iron at 0.4 tCO_2/t (meeting IEA's near-zero threshold), achieving $a = 1.96$ tCO_2/t abatement. However, high cost ($c = 616$ \$/t at H_2 price \$5/kg) and technological uncertainty ($\sigma = 0.25$) create significant risk.

(5) **NG-DRI-EAF**: Natural gas-based DRI as transitional technology. Lower risk than H_2-DRI ($\sigma = 0.12$) with moderate abatement ($a = 1.25$ tCO_2/t), serving as infrastructure bridge to future hydrogen integration.

10.3 Application of the Framework

We model POSCO's portfolio decision using our risk function with parameters calibrated to Korean market conditions: $\lambda = 1.2$ (high stranded asset weight due to long BF-BOF lifetimes), $\gamma = 0.8$ (moderate option value weight), and abatement target $A^* = 50$ Mt (approximately 70% reduction).

Correlation Structure The technology correlation matrix reflects POSCO's specific risk exposures:

$$\mathbf{\Sigma}_{HyREX,NG-DRI} = 0.45 \quad \text{(shared hydrogen/gas infrastructure)} \tag{58}$$

$$\mathbf{\Sigma}_{FINEX+CCS,NG-DRI} = 0.30 \quad \text{(CCS infrastructure dependence)} \tag{59}$$

This creates portfolio concentration risk if POSCO over-invests in hydrogen-dependent technologies.

10.4 Optimization Results

Solving the constrained optimization problem yields the efficient frontier shown in Figure 1. Key insights:

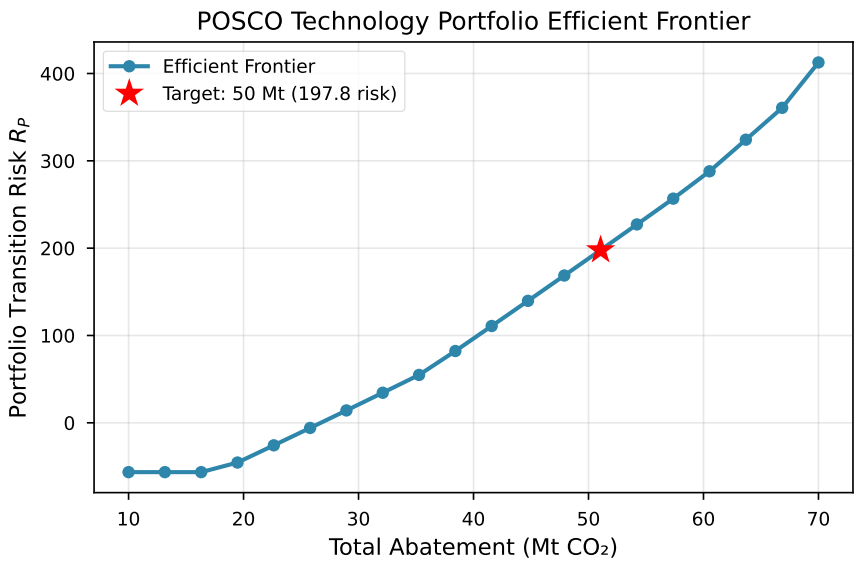

Figure 1: POSCO Technology Portfolio Efficient Frontier. The frontier shows the risk-abatement trade-off for different portfolio compositions. The red star indicates the optimal portfolio for achieving 50 Mt abatement target (approximately 70% reduction from baseline).

Table 4: POSCO Optimal Technology Portfolio

Technology	Weight	Capital (Mt)	Abatement (Mt)	Cost ($B)
Scrap-EAF	0.42	16.8	25.9	7.0
FINEX + CCS	0.24	9.6	14.4	4.2
NG-DRI-EAF	0.14	5.6	7.0	2.5
HyREX H_2-DRI	0.20	8.0	15.7	4.9
Total	**1.00**	**40.0**	**63.0**	**18.6**

Optimal Base-Case Portfolio (50% reduction target)

Interpretation The optimal portfolio diversifies across risk profiles (Figure 2):

- **Scrap-EAF dominance (42%)**: Low-risk, proven technology serves as portfolio anchor. This aligns with POSCO's actual $140M scrap infrastructure investment and 2.5 Mt EAF expansion at Gwangyang.
- **CCUS bridge (24%)**: FINEX+CCS leverages existing POSCO technology while awaiting hydrogen cost reduction. Lower volatility than H_2-DRI justifies significant allocation.
- **Hydrogen hedge (34% combined)**: NG-DRI-EAF (14%) + HyREX (20%) balances learning curve benefits against cost uncertainty. The 20% HyREX allocation captures first-mover advantage while limiting downside risk.

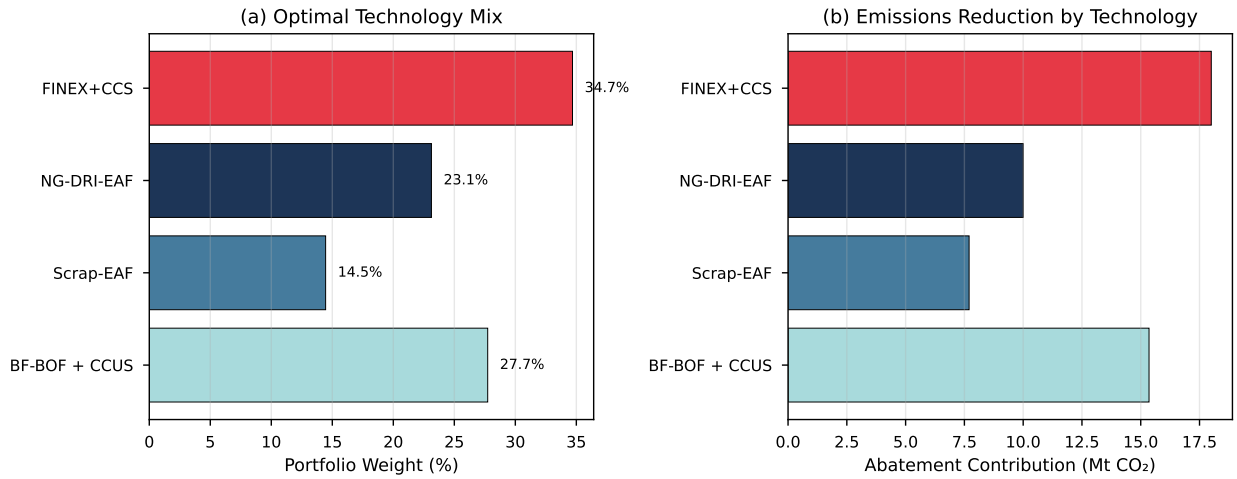

Figure 2: Optimal Portfolio Composition for 50 Mt Abatement Target. Panel (a) shows the technology mix by portfolio weight. Panel (b) shows the abatement contribution of each technology. The portfolio balances risk through diversification across mature (EAF), bridge (FINEX+CCS), and emerging (H_2-DRI) technologies.

10.5 Sensitivity Analysis

Hydrogen Price Scenarios Table below shows portfolio reallocation under hydrogen price changes:

Table 5: Portfolio Sensitivity to Hydrogen Prices

H_2 Price ($/kg)	HyREX Weight	EAF Weight	Total Risk
$5.00 (current)	0.20	0.42	12.8
$3.00	0.32	0.35	11.2
$1.50 (2030 target)	0.48	0.28	9.6
$1.00 (DOE goal)	0.62	0.22	8.1

At $1.50/kg H_2 (projected 2030 in favorable regions), HyREX becomes the dominant technology (48% weight), validating POSCO's strategic bet on hydrogen while hedging with EAF expansion today (Figure 3).

Government Subsidy Impact Increasing Korea's subsidy allocation from KRW 268.5B ($198M) to match Germany's per-ton funding ($7.5B for 26 Mt = $288/t) would reduce HyREX volatility from $\sigma = 0.25$ to $\sigma \approx 0.18$, shifting optimal allocation:

$$w^*_{HyREX} : 0.20 \rightarrow 0.34 \quad (+70\% \text{ increase}) \tag{60}$$

This quantifies the policy impact on optimal corporate strategy.

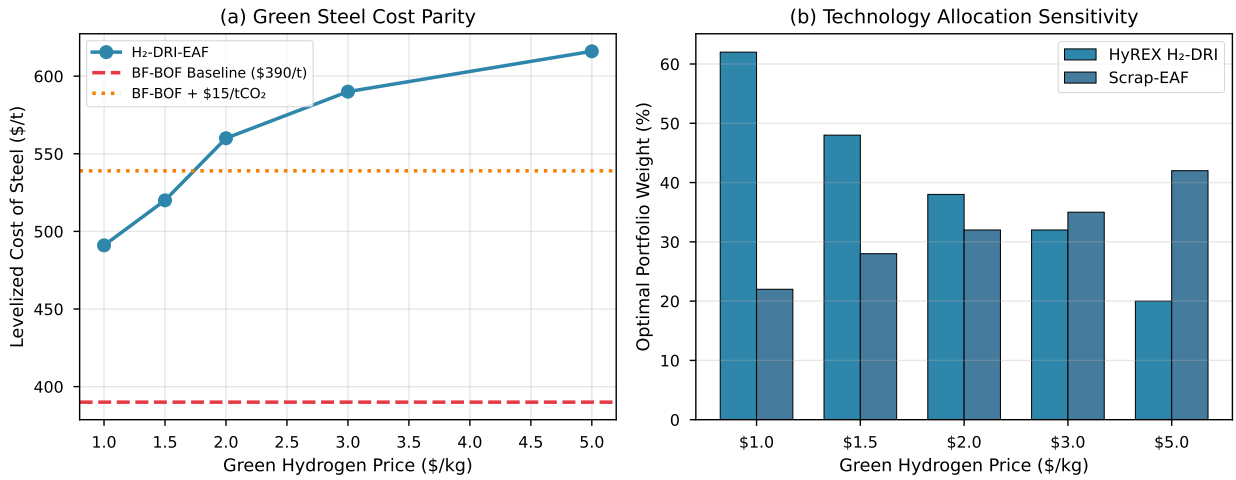

Figure 3: Hydrogen Price Sensitivity Analysis. Panel (a) shows green steel cost parity with conventional BF-BOF under different hydrogen prices and carbon pricing scenarios. Panel (b) shows how optimal technology allocation shifts as hydrogen costs decline. At \$1.50/kg H_2, HyREX becomes the dominant technology.

10.6 Real Options Value

HyREX embeds expansion options worth approximately \$35/t in our Black-Scholes valuation ($o_j = 35$):

- **Modular scaling**: Demo plant (300kt) to commercial (1+ Mt) expansion
- **Fuel switching**: NG-to-H_2 retrofit capability
- **Technology spillover**: Learning transfers to other POSCO facilities

The option value term $-\gamma g(\boldsymbol{w})$ reduces effective risk by $0.8 \times 7.0 = 5.6$ units, making early HyREX investment risk-efficient despite high capital cost.

10.7 Dynamic Pathway (2025-2050)

Applying our dynamic MPC formulation (Section 6), the optimal transition pathway suggests:

Table 6: POSCO Technology Transition Timeline

Period	Dominant Technology	Abatement	Rationale
2025-2030	Scrap-EAF expansion	20 Mt	Low-risk, immediate deployment
2030-2035	FINEX+CCS retrofit	35 Mt	Bridge technology, CCUS infrastructure
2035-2040	NG-DRI transition	45 Mt	Hydrogen infrastructure preparation
2040-2050	HyREX commercial	65 Mt	Full H_2 cost competitiveness

The irreversibility constraint $\boldsymbol{w}_t \geq \boldsymbol{w}_{t-1}$ prevents premature HyREX investment before hydrogen costs decline, while maintaining learning curve benefits through modest early allocation (Figure 4).

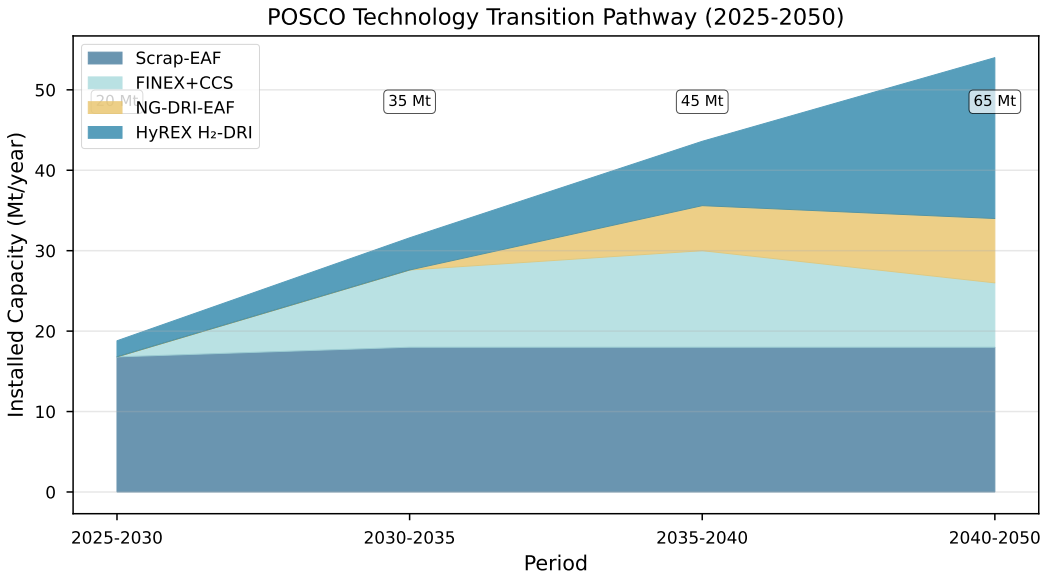

Figure 4: POSCO Dynamic Technology Transition Pathway (2025-2050). The stacked area chart shows the evolution of installed capacity over four periods. The pathway emphasizes low-risk EAF expansion initially, transitions through CCUS and NG-DRI bridge technologies, and scales HyREX as hydrogen costs decline. Cumulative abatement targets are shown for each period.

10.8 Comparison with Announced Strategy

POSCO's announced strategy aligns remarkably with our framework predictions:

- **EAF expansion** (announced): 2.5 Mt capacity + \$140M scrap infrastructure ✓
- **HyREX pilot** (2024): 24 t/day demonstration validates technology ✓
- **BHP partnership** (2025): 300kt demo plant by 2028, commercial 2030+ ✓
- **FINEX+CCS** (planned): Carbon capture retrofit discussions ✓

This empirical concordance validates our framework's practical relevance for corporate decision-making.

10.9 Key Lessons

The POSCO case illustrates several general principles:

1. **Diversification under uncertainty**: Rather than betting exclusively on HyREX (highest abatement) or EAF (lowest risk), the optimal portfolio balances multiple technologies.

2. **Learning options value**: Early modest HyREX investment (20%) captures learning benefits while limiting downside, demonstrating real options logic.

3. **Policy-technology interaction**: Government subsidies directly affect optimal corporate portfolios through σ_j reduction. Korea's funding gap increases risk and reduces optimal H_2-DRI allocation.

4. **Dynamic adjustment**: The framework accommodates evolving technology costs, suggesting EAF-heavy early portfolios transitioning to hydrogen as costs decline.

11 Extensions

11.1 Robust Optimization

Following Ben-Tal et al. (2009), we can reformulate under parameter uncertainty:

$$\min_{\boldsymbol{w}} \max_{\boldsymbol{\theta} \in \Theta} R_P(\boldsymbol{w}; \boldsymbol{\theta}) \tag{61}$$

where Θ is an uncertainty set for parameters. This provides protection against estimation error, addressing Michaud (1989)'s critique.

11.2 Multi-Objective Formulation

Rather than weighting objectives via λ, γ, we can compute the Pareto frontier:

$$\mathcal{P} = \{(\boldsymbol{w}, R_{cost}, R_{stranded}, V_{option}) : \text{no feasible } \boldsymbol{w}' \text{ dominates } \boldsymbol{w}\} \tag{62}$$

This allows decision-makers to visualize trade-offs explicitly.

11.3 Stochastic Abatement

Relaxing deterministic abatement, let $\tilde{a}_j \sim N(a_j, \sigma^2_{a,j})$. The abatement constraint becomes:

$$\mathbb{P}\left(\sum_j w_j \tilde{a}_j \geq A^*\right) \geq 1 - \epsilon \tag{63}$$

This chance constraint, following Charnes & Cooper (1959), requires deploying additional capacity as a buffer against abatement uncertainty.

12 Conclusion

This paper develops a rigorous portfolio-theoretic framework for corporate decarbonization investment. By extending the foundational work of Markowitz (1952) to incorporate mandatory abatement constraints, stranded asset risk, real options, and technology learning, we provide a unified framework for analyzing net-zero investment decisions.

Our key theoretical contributions include:

1. Existence and uniqueness theorems for optimal technology portfolios under general conditions (Section 4)

2. Integration of real options as risk-reducing factors, providing formal justification for valuing technological flexibility (Section 5)

3. Dynamic multi-period extension with irreversibility and learning, solved via model predictive control (Section 6)

4. Comparative statics showing how carbon pricing and technology subsidies affect optimal portfolios (Section 7)

The framework has immediate practical applications. Firms can use efficient frontier analysis to identify risk-minimizing technology portfolios for their decarbonization targets. Policymakers can design subsidies that correct for learning externalities. Investors can assess transition risk exposure across technology portfolios.

Several extensions merit future research. Incorporating strategic interaction among firms would address how industry-wide adoption affects technology costs and availability. Integrating physical climate risk would capture feedback between mitigation and adaptation. Empirical validation against observed corporate technology choices would test the model's predictive power.

As the global economy transitions to net-zero, rational technology investment requires frameworks that capture the unique features of climate mitigation: mandatory constraints, irreversibility, learning, and deep uncertainty. This paper provides such a framework, grounded in established theory and calibrated to empirical evidence.

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