

# Mathematical Framework for Quantifying Climate Risk Premium in Infrastructure Finance

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November 18, 2025

## Abstract

We develop a rigorous mathematical framework for quantifying the Climate Risk Premium (CRP) in project finance for carbon-intensive infrastructure. The model integrates transition risk (policy-driven constraints and carbon pricing) and physical risk (climate hazards affecting operations) into a coherent expected loss framework. We prove that climate risks systematically increase the cost of capital through three channels: reduced cash flows, increased volatility, and shortened asset lifetimes. The framework is applied to the Samcheok coal-fired power plant case study, demonstrating that properly priced climate risks can increase financing costs by 50-200 basis points and reduce project NPV by 30-50%.

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# 1 Introduction

## 1.1 Motivation

Climate change introduces two distinct but interconnected risks to infrastructure investments:

1. **Transition Risk:** Policy interventions (carbon pricing, dispatch restrictions, early retirement mandates) that reduce asset utilization and profitability.
2. **Physical Risk:** Climate hazards (wildfires, droughts, extreme temperatures) that disrupt operations and increase costs.

Traditional project finance models do not systematically incorporate these risks into cost of capital calculations. This paper develops a formal framework to quantify the *Climate Risk Premium* (CRP)—the increase in weighted average cost of capital required to compensate investors for climate-related losses.

## 1.2 Contribution

Our contributions are threefold:

1. A formal definition of CRP as a function of expected climate-induced losses relative to capital at risk.
2. Analytical results linking climate risk parameters to financing spreads through project finance metrics (DSCR, LLCR, NPV).
3. Empirical implementation demonstrating CRP magnitudes for a representative coal power plant.

# 2 Model Setup

## 2.1 Notation and Preliminaries

**Definition 1** (Time Horizon). *Let  $T \in \mathbb{N}$  denote the project's design operating lifetime (years). The actual operating lifetime under climate risk is  $T^* \leq T$ , determined by transition risk scenarios.*

**Definition 2** (State Variables). *Define the following state variables for year  $t \in \{1, 2, \dots, T^*\}$ :*

- $C \in \mathbb{R}_+$ : Installed capacity (MW)
- $\rho_t \in [0, 1]$ : Capacity factor (fraction of time operating)
- $p_t \in \mathbb{R}_+$ : Electricity price (\$/MWh)
- $\tau_t \in \mathbb{R}_+$ : Carbon price (\$/tCO<sub>2</sub>)
- $\lambda_t \in [0, 1]$ : Outage rate (probability of forced shutdown)
- $\delta_t \in [0, 1]$ : Capacity derating factor

## 2.2 Cash Flow Model

**Definition 3** (Annual Generation). *Effective annual electricity generation (MWh) in year  $t$  is:*

$$Q_t = C \cdot 8760 \cdot \rho_t \cdot (1 - \delta_t) \cdot (1 - \lambda_t) \quad (1)$$

where 8760 is hours per year.

**Definition 4** (Revenue). *Annual revenue in year  $t$ :*

$$R_t = p_t \cdot Q_t \quad (2)$$

**Definition 5** (Operating Costs). *Total operating costs comprise:*

$$\text{Fuel Cost: } FC_t = Q_t \cdot h \cdot f_t \quad (3)$$

$$\text{Variable O&M: } VC_t = Q_t \cdot v_t \quad (4)$$

$$\text{Fixed O&M: } FC_t = C \cdot 1000 \cdot f_t^{fix} \quad (5)$$

$$\text{Carbon Cost: } CC_t = Q_t \cdot e \cdot \tau_t \quad (6)$$

$$\text{Outage Penalty: } OC_t = Q_t \cdot \lambda_t \cdot p_t \quad (7)$$

where:

- $h$ : heat rate (MMBtu/MWh)
- $f_t$ : fuel price (\$/MMBtu)
- $v_t$ : variable O&M cost (\$/MWh)
- $f_t^{fix}$ : fixed O&M cost (\$/kW-year)
- $e$ : emissions intensity ( $tCO_2/MWh$ )

**Definition 6** (EBITDA and Free Cash Flow).

$$EBITDA_t = R_t - FC_t - VC_t - FC_t - CC_t - OC_t \quad (8)$$

$$FCF_t = EBITDA_t - CAPEX_t \quad (9)$$

where  $CAPEX_t$  is sustaining capital expenditure in year  $t$ .

## 3 Climate Risk Integration

### 3.1 Transition Risk

**Definition 7** (Transition Risk Parameters). *A transition risk scenario  $\mathcal{T}$  is characterized by:*

$$\mathcal{T} = (\Delta\rho, T^*, \{\tau_t\}_{t=1}^{T^*}) \quad (10)$$

where:

- $\Delta\rho \in [0, 1]$ : dispatch penalty (reduction in capacity factor)
- $T^* \leq T$ : enforced retirement date

- $\{\tau_t\}$ : carbon price trajectory

**Assumption 1** (Carbon Price Interpolation). *Carbon prices follow a piecewise linear trajectory with anchor points at years {2025, 2030, 2040, 2050}:*

$$\tau_t = \tau_{t_i} + \frac{t - t_i}{t_{i+1} - t_i}(\tau_{t_{i+1}} - \tau_{t_i}), \quad t \in [t_i, t_{i+1}] \quad (11)$$

**Proposition 1** (Capacity Factor Adjustment). *Under transition scenario  $\mathcal{T}$ , the adjusted capacity factor is:*

$$\rho_t^{\mathcal{T}} = \max(0, \rho_t^{baseline} - \Delta\rho) \quad (12)$$

## 3.2 Physical Risk

**Definition 8** (Physical Risk Parameters). *A physical risk scenario  $\mathcal{P}$  is characterized by:*

$$\mathcal{P} = (\lambda, \delta, \epsilon) \quad (13)$$

where:

- $\lambda \in [0, 1]$ : wildfire-induced outage rate (annual probability)
- $\delta \in [0, 1]$ : water stress capacity derating
- $\epsilon \in [0, 1]$ : thermal efficiency loss from cooling constraints

**Assumption 2** (Independence of Physical Hazards). *We assume  $\lambda$ ,  $\delta$ , and  $\epsilon$  are independently distributed. For small probabilities, the compound effect on generation is approximately additive:*

$$Q_t \approx Q_t^{baseline} \cdot (1 - \lambda) \cdot (1 - \delta) \cdot (1 - \epsilon) \quad (14)$$

## 3.3 Combined Risk Scenario

**Definition 9** (Combined Scenario). *A combined climate risk scenario is  $\mathcal{S} = (\mathcal{T}, \mathcal{P})$ , affecting cash flows through:*

$$FCF_t^{\mathcal{S}} = f(\mathcal{T}, \mathcal{P}, \text{plant parameters}) \quad (15)$$

## 4 Financial Metrics

### 4.1 Net Present Value

**Definition 10** (NPV). *The net present value of scenario  $\mathcal{S}$  at discount rate  $r$  is:*

$$NPV^{\mathcal{S}}(r) = \sum_{t=1}^{T^*} \frac{FCF_t^{\mathcal{S}}}{(1+r)^t} \quad (16)$$

### 4.2 Internal Rate of Return

**Definition 11** (IRR). *The internal rate of return is the rate  $r^*$  satisfying:*

$$NPV^{\mathcal{S}}(r^*) = 0 \quad (17)$$

### 4.3 Debt Service Coverage Ratio

**Definition 12** (Annual Debt Service). *For debt amount  $D$ , interest rate  $r_d$ , and tenor  $n$ , the level annual debt service is:*

$$DS = D \cdot \frac{r_d(1 + r_d)^n}{(1 + r_d)^n - 1} \quad (18)$$

**Definition 13** (DSCR). *The debt service coverage ratio in year  $t$  is:*

$$DSCR_t = \frac{EBITDA_t}{DS} \quad (19)$$

Lenders typically require  $\min_t DSCR_t \geq 1.25$ .

### 4.4 Loan Life Coverage Ratio

**Definition 14** (LLCR). *The loan life coverage ratio at inception is:*

$$LLCR = \frac{PV(\text{Cash flows available for debt service})}{D} \quad (20)$$

where the present value uses the debt interest rate  $r_d$ .

## 5 Expected Loss Framework

### 5.1 Definition of Expected Loss

**Definition 15** (Expected Loss). *Let  $\mathcal{S}_0$  denote the baseline scenario (no climate risk). The expected loss under scenario  $\mathcal{S}$  is:*

$$EL(\mathcal{S}) = NPV^{\mathcal{S}_0}(r) - NPV^{\mathcal{S}}(r) \quad (21)$$

**Definition 16** (Expected Loss Percentage). *Relative to total capital at risk  $K$  (typically total CAPEX):*

$$EL\%(\mathcal{S}) = \frac{EL(\mathcal{S})}{K} \times 100\% \quad (22)$$

**Theorem 2** (Monotonicity of Expected Loss). *For transition scenarios  $\mathcal{T}_1 \subset \mathcal{T}_2$  (i.e.,  $\mathcal{T}_2$  has stricter constraints), we have:*

$$EL(\mathcal{T}_2) \geq EL(\mathcal{T}_1) \quad (23)$$

*Proof.* Stricter transition constraints imply:

- $\Delta\rho_2 \geq \Delta\rho_1 \Rightarrow \rho_t^{\mathcal{T}_2} \leq \rho_t^{\mathcal{T}_1}$
- $T_2^* \leq T_1^*$
- $\tau_t^2 \geq \tau_t^1$

Each condition reduces cash flows:  $FCF_t^{\mathcal{T}_2} \leq FCF_t^{\mathcal{T}_1}$  for all  $t$ , hence:

$$NPV^{\mathcal{T}_2} \leq NPV^{\mathcal{T}_1} \Rightarrow EL(\mathcal{T}_2) \geq EL(\mathcal{T}_1) \quad (24)$$

□

## 5.2 Statistical Extension

For a probability distribution over scenarios  $\mathbb{P}(\mathcal{S})$ :

**Definition 17** (Statistical Expected Loss).

$$\mathbb{E}[EL] = \int_{\mathcal{S}} EL(\mathcal{S}) d\mathbb{P}(\mathcal{S}) \quad (25)$$

# 6 Climate Risk Premium

## 6.1 Financing Cost Structure

**Definition 18** (Weighted Average Cost of Capital). *For debt fraction  $w_d$  and equity fraction  $w_e = 1 - w_d$ :*

$$WACC = w_d \cdot r_d + w_e \cdot r_e \quad (26)$$

where  $r_d$  is the debt rate and  $r_e$  is the required equity return.

## 6.2 Spread Mapping

**Assumption 3** (Linear Spread Response). *Debt spreads respond linearly to expected loss:*

$$s_d(EL\%) = s_0 + \beta_d \cdot EL\% \quad (27)$$

where:

- $s_0$ : baseline spread (bps)
- $\beta_d$ : spread sensitivity (bps per 1% EL)

**Assumption 4** (Equity Premium Response). *Equity return premiums respond similarly:*

$$\pi_e(EL\%) = \beta_e \cdot EL\% \quad (28)$$

where  $\beta_e$  is equity premium sensitivity (% per 1% EL).

**Definition 19** (Risk-Adjusted Cost of Capital).

$$r_d^{\mathcal{S}} = r_f + \frac{s_d(EL\%^{\mathcal{S}})}{10000} \quad (29)$$

$$r_e^{\mathcal{S}} = r_e^0 + \frac{\pi_e(EL\%^{\mathcal{S}})}{100} \quad (30)$$

where  $r_f$  is the risk-free rate and  $r_e^0$  is baseline equity return.

**Definition 20** (Climate Risk Premium). *The CRP is the increase in WACC attributable to climate risks:*

$$CRP^{\mathcal{S}} = WACC^{\mathcal{S}} - WACC^{S_0} \quad (31)$$

In basis points:

$$CRP_{bps}^{\mathcal{S}} = 10000 \times CRP^{\mathcal{S}} \quad (32)$$

### 6.3 Main Result

**Theorem 3** (CRP Existence and Bounds). *For any climate risk scenario  $\mathcal{S}$  with  $EL(\mathcal{S}) > 0$ :*

1.  $CRP^{\mathcal{S}} > 0$  (*climate risks always increase cost of capital*)
2.  $CRP^{\mathcal{S}} \leq w_d \beta_d \frac{EL\%}{10000} + w_e \beta_e \frac{EL\%}{100}$

*Proof.* (1) From Assumption 5.1 and 5.2, both debt and equity components increase with  $EL\% > 0$ :

$$WACC^{\mathcal{S}} = w_d(r_f + s_d) + w_e(r_e^0 + \pi_e) > w_d r_f + w_e r_e^0 = WACC^{\mathcal{S}_0} \quad (33)$$

(2) The bound follows from the linear assumptions:

$$CRP^{\mathcal{S}} = w_d \frac{s_d(EL\%) - s_0}{10000} + w_e \frac{\pi_e(EL\%)}{100} \quad (34)$$

$$= w_d \frac{\beta_d \cdot EL\%}{10000} + w_e \frac{\beta_e \cdot EL\%}{100} \quad (35)$$

□

**Corollary 4** (CRP Scaling). *CRP scales linearly with expected loss:*

$$\frac{\partial CRP^{\mathcal{S}}}{\partial EL\%} = \frac{w_d \beta_d}{10000} + \frac{w_e \beta_e}{100} > 0 \quad (36)$$

## 7 Comparative Statics

### 7.1 Sensitivity to Carbon Pricing

**Proposition 5** (Carbon Price Impact). *Let  $\tau_t(\alpha) = \alpha \cdot \tau_t^{ref}$  be a scaled carbon price trajectory. Then:*

$$\frac{\partial NPV^{\mathcal{S}}}{\partial \alpha} = - \sum_{t=1}^{T^*} \frac{Q_t \cdot e \cdot \tau_t^{ref}}{(1+r)^t} < 0 \quad (37)$$

*Proof.* From the carbon cost term  $CC_t = Q_t \cdot e \cdot \tau_t(\alpha)$ :

$$\frac{\partial FCF_t}{\partial \alpha} = -Q_t \cdot e \cdot \tau_t^{ref} \quad (38)$$

Summing discounted impacts yields the result. □

### 7.2 Sensitivity to Physical Risk

**Proposition 6** (Outage Rate Impact).

$$\frac{\partial NPV^{\mathcal{S}}}{\partial \lambda} \approx - \sum_{t=1}^{T^*} \frac{C \cdot 8760 \cdot \rho_t \cdot p_t}{(1+r)^t} < 0 \quad (39)$$

### 7.3 Interaction Effects

**Theorem 7** (Subadditivity of Combined Risks). *For independent transition and physical scenarios:*

$$EL(\mathcal{T}, \mathcal{P}) \leq EL(\mathcal{T}, \emptyset) + EL(\emptyset, \mathcal{P}) \quad (40)$$

*with equality if risks affect disjoint cash flow components.*

*Proof.* Physical risks reduce generation  $Q_t$ , which reduces the base for carbon costs. Thus:

$$CC_t^{(\mathcal{T}, \mathcal{P})} = Q_t^{\mathcal{P}} \cdot e \cdot \tau_t < Q_t^{\text{baseline}} \cdot e \cdot \tau_t = CC_t^{\mathcal{T}} \quad (41)$$

This creates a negative interaction: physical risk partially offsets transition risk carbon costs, leading to subadditivity.  $\square$

## 8 Empirical Calibration

### 8.1 Parameter Estimates

Based on literature and market observations:

Parameter	Symbol	Value
Baseline spread	$s_0$	150 bps
Spread sensitivity	$\beta_d$	50 bps/%
Equity sensitivity	$\beta_e$	0.8%/%
Risk-free rate	$r_f$	3%
Baseline equity return	$r_e^0$	12%
Debt fraction	$w_d$	70%
Equity fraction	$w_e$	30%

Table 1: Calibrated financing parameters

### 8.2 Scenario Results

Application to Samcheok Power Plant (2,000 MW coal):

Scenario	NPV (M\$)	EL%	CRP (bps)
Baseline	3,736	0%	0
Moderate Transition	-855	143%	7,165
Aggressive Transition	-2,387	191%	9,575
Moderate Physical	3,416	10%	500
High Physical	2,935	25%	1,250
Combined Moderate	-998	148%	7,400
Combined Aggressive	-2,460	194%	9,700

Table 2: Climate risk premium by scenario

### 8.3 Key Findings

1. Transition risks dominate physical risks in magnitude.
2. CRP can exceed 100 bps even for moderate scenarios.
3. Aggressive decarbonization scenarios render projects NPV-negative.
4. Combined risks exhibit subadditivity ( $9,700 \text{ bps} < 9,575 + 1,250 \text{ bps}$ ).

## 9 Credit Rating Integration

### 9.1 Credit Rating Fundamentals

Traditional credit rating methodologies provide an alternative lens for quantifying climate-induced financing costs. While our CRP framework directly maps expected losses to spreads, credit ratings offer market-standard assessments that institutional investors rely upon.

**Definition 21** (Credit Rating). *A credit rating  $R \in \{\text{AAA}, \text{AA}, \text{A}, \text{BBB}, \text{BB}, \text{B}, \text{CCC}, \text{CC}, \text{C}, \text{D}\}$  represents an ordinal assessment of default probability and loss-given-default.*

**Definition 22** (Rating Numeric Scale). *For computational purposes, we map ratings to numeric values:*

$$N(R) : \{\text{AAA}, \text{AA}, \text{A}, \text{BBB}, \text{BB}, \text{B}\} \rightarrow \{1, 2, 3, 4, 5, 6\} \quad (42)$$

where higher values indicate higher credit risk.

**Definition 23** (Investment Grade). *A rating is investment grade if  $N(R) \leq 4$  (i.e., BBB or better). This threshold is critical as many institutional investors (pension funds, insurance companies) face regulatory or policy constraints prohibiting sub-investment grade holdings.*

### 9.2 Structural vs. Statistical Rating Approaches

#### 9.2.1 Merton-Moody's Structural Model

The seminal approach by Merton (1974) and operationalized by Moody's treats corporate debt as a European put option on firm assets.

**Assumption 5** (Firm Value Dynamics). *Let  $V_t$  denote firm value following geometric Brownian motion:*

$$dV_t = \mu V_t dt + \sigma V_t dW_t \quad (43)$$

where  $\mu$  is drift,  $\sigma$  is volatility, and  $W_t$  is a Wiener process.

**Definition 24** (Default Threshold). *Default occurs at maturity  $T$  if  $V_T < D$ , where  $D$  is face value of debt.*

**Definition 25** (Distance to Default). *The distance to default (DD) is:*

$$DD = \frac{\ln(V_0/D) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \quad (44)$$

**Theorem 8** (Merton Default Probability). *Under risk-neutral valuation, the probability of default is:*

$$PD = \Phi(-DD) \quad (45)$$

where  $\Phi$  is the standard normal CDF.

*Proof.* Default occurs if:

$$V_T = V_0 \exp \left[ \left( \mu - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} Z \right] < D \quad (46)$$

where  $Z \sim N(0, 1)$ . Rearranging:

$$Z < -\frac{\ln(V_0/D) + (\mu - \sigma^2/2)T}{\sigma \sqrt{T}} = -DD \quad (47)$$

Hence  $PD = P(Z < -DD) = \Phi(-DD)$ .  $\square$

**Proposition 9** (Rating-PD Mapping). *Moody's establishes a nonparametric mapping from default probabilities to ratings based on historical default frequencies:*

$$R = \mathcal{M}(PD) \quad (48)$$

where  $\mathcal{M}$  is the Moody's rating function, typically calibrated as:

Rating	PD Range (%)
AAA	< 0.01
AA	0.01 – 0.05
A	0.05 – 0.20
BBB	0.20 – 1.00
BB	1.00 – 5.00
B	5.00 – 20.00

### 9.2.2 Korea Investors Service (KIS) Quantitative Grid

For power generation projects, KIS employs a deterministic scoring grid based on six financial ratios. This approach is more transparent and directly applicable to project finance structures.

**Definition 26** (KIS Rating Metrics). *A project's creditworthiness is assessed via six metrics  $\mathbf{m} = (m_1, m_2, m_3, m_4, m_5, m_6)$ :*

1.  $m_1$ : Capacity (MW) — business scale
2.  $m_2$ : EBITDA / Fixed Assets (%) — profitability
3.  $m_3$ : EBITDA / Interest Expense ( $x$ ) — coverage
4.  $m_4$ : Net Debt / EBITDA ( $x$ ) — leverage
5.  $m_5$ : Total Debt / Total Equity (%) — capital structure

6.  $m_6$ : Total Debt / Total Assets (%) — asset leverage

**Definition 27** (KIS Component Rating Function). *For each metric  $m_i$ , define rating function  $R_i : \mathbb{R} \rightarrow \{\text{AAA}, \text{AA}, \text{A}, \text{BBB}, \text{BB}, \text{B}\}$  via threshold rules. For example, for capacity (metric 1):*

$$R_1(m_1) = \begin{cases} \text{AAA} & \text{if } m_1 \geq 2000 \\ \text{AA} & \text{if } 800 \leq m_1 < 2000 \\ \text{A} & \text{if } 400 \leq m_1 < 800 \\ \text{BBB} & \text{if } 100 \leq m_1 < 400 \\ \text{BB} & \text{if } 20 \leq m_1 < 100 \\ \text{B} & \text{if } m_1 < 20 \end{cases} \quad (49)$$

Similar piecewise functions apply to  $R_2, \dots, R_6$ .

**Definition 28** (KIS Overall Rating). *KIS uses a conservative aggregation approach:*

$$R_{\text{overall}} = \max_{i=1}^6 N(R_i(\mathbf{m})) \quad (50)$$

That is, the overall rating is determined by the worst component rating (highest numeric value).

**Assumption 6** (Rating Conservatism). *The max aggregation reflects regulatory prudence: a project cannot be stronger than its weakest financial dimension. This contrasts with weighted-average approaches used in corporate finance.*

### 9.3 Rating-Spread Relationship

**Definition 29** (Credit Spread). *The credit spread  $s(R)$  is the yield differential between debt rated  $R$  and risk-free government bonds of equivalent maturity.*

**Proposition 10** (Rating-Spread Mapping). *Empirical studies establish an approximately exponential relationship:*

$$s(R) = s_0 \exp(\alpha \cdot N(R)) \quad (51)$$

where  $s_0$  is baseline spread and  $\alpha$  is spread elasticity.

For KIS ratings, market-calibrated spreads (in basis points) are:

Rating	Numeric	Spread (bps)	Typical Sector
AAA	1	50	Government-backed
AA	2	100	Blue-chip utilities
A	3	150	Mature corporates
BBB	4	250	Investment-grade threshold
BB	5	400	Sub-investment grade
B	6	600	Distressed

Table 3: KIS rating-spread calibration for Korean power sector

**Theorem 11** (Spread Monotonicity). *Credit spreads are strictly increasing in rating numeric value:*

$$N(R_2) > N(R_1) \Rightarrow s(R_2) > s(R_1) \quad (52)$$

*Proof.* Higher credit risk (higher  $N(R)$ ) implies higher default probability and/or higher loss-given-default. Under risk-neutral pricing, the credit spread compensates for expected losses:

$$s(R) \approx \text{PD}(R) \times \text{LGD}(R) \quad (53)$$

where LGD is loss-given-default. Since both terms increase with  $N(R)$ , so does  $s(R)$ .  $\square$

## 9.4 Credit Rating Migration Under Climate Risks

**Definition 30** (Rating Migration). *A rating migration from baseline scenario  $\mathcal{S}_0$  to risk scenario  $\mathcal{S}$  is:*

$$\Delta R = N(R^{\mathcal{S}}) - N(R^{\mathcal{S}_0}) \quad (54)$$

where  $\Delta R > 0$  indicates a downgrade.

**Definition 31** (Spread Migration). *The spread increase due to rating migration is:*

$$\Delta s = s(R^{\mathcal{S}}) - s(R^{\mathcal{S}_0}) \quad (55)$$

**Theorem 12** (Climate Risk Induces Rating Downgrades). *For climate risk scenario  $\mathcal{S}$  with  $\text{EL}(\mathcal{S}) > 0$ :*

1. *At least one KIS metric deteriorates:  $\exists i$  such that  $m_i^{\mathcal{S}} < m_i^{\mathcal{S}_0}$  (for metrics where lower is worse).*
2. *If deterioration crosses a rating threshold,  $\Delta R \geq 1$  (at least one notch downgrade).*
3. *Consequently,  $\Delta s \geq 0$ .*

*Proof.* Climate risks reduce cash flows and profitability:

- Transition risks increase carbon costs  $CC_t$ , reducing EBITDA:  $\text{EBITDA}^{\mathcal{S}} < \text{EBITDA}^{\mathcal{S}_0}$
- Physical risks reduce generation  $Q_t$ , reducing revenue and EBITDA
- Lower EBITDA directly worsens metrics  $m_2$  (EBITDA/Assets),  $m_3$  (EBITDA/Interest), and  $m_4$  (Debt/EBITDA)

Since KIS uses max aggregation, if any  $R_i$  worsens, the overall rating may worsen. If the deterioration crosses a threshold in the piecewise function  $R_i$ , a downgrade occurs. By spread monotonicity,  $\Delta s \geq 0$ .  $\square$

**Corollary 13** (Investment Grade Loss). *If baseline rating is  $N(R^{\mathcal{S}_0}) = 4$  (BBB) and climate risks degrade any metric sufficiently:*

$$N(R^{\mathcal{S}}) \geq 5 \Rightarrow \text{Loss of investment grade} \quad (56)$$

*This has severe implications: many institutional investors face regulatory prohibition on holding sub-IG debt.*

## 9.5 Linking Credit Ratings to CRP

The credit rating approach and our CRP model are complementary:

**Proposition 14** (Dual Spread Quantification). *For a given climate scenario  $\mathcal{S}$ , two spread measures exist:*

1. **CRP-based spread:** Derived from expected loss via linear sensitivity:

$$s_{CRP}^{\mathcal{S}} = s_0 + \beta_d \cdot EL\%(\mathcal{S}) \quad (57)$$

2. **Rating-based spread:** Derived from KIS methodology:

$$s_{rating}^{\mathcal{S}} = s(R^{\mathcal{S}}) \quad (58)$$

**Proposition 15** (Spread Relationship). *In well-calibrated models:*

$$s_{CRP}^{\mathcal{S}} \approx s_{rating}^{\mathcal{S}} \quad (59)$$

However, differences arise from:

- CRP uses continuous expected loss; ratings use discrete thresholds
- CRP is scenario-specific; ratings aggregate multiple risk factors
- CRP is forward-looking (projected cash flows); ratings may use historical averages

**Theorem 16** (Combined Risk Premium). *The total financing cost increase combines both methodologies:*

$$\text{Total Risk Premium} = \max(s_{CRP}^{\mathcal{S}}, s_{rating}^{\mathcal{S}}) - s_0 \quad (60)$$

Taking the maximum reflects that lenders will price based on the more conservative assessment.

## 9.6 Empirical Results: Samcheok Power Plant

Applying KIS methodology to Samcheok case:

Scenario	Rating	Spread (bps)	$\Delta$ Notches	$\Delta$ Spread (bps)
Baseline	BBB	250	0	0
Moderate Transition	B	600	2	350
Aggressive Transition	B	600	2	350
Moderate Physical	BBB	250	0	0
High Physical	BBB	250	0	0
Combined Moderate	B	600	2	350
Combined Aggressive	B	600	2	350

Table 4: Rating migration under climate scenarios

**Key Findings:**

1. Baseline achieves BBB (investment grade), driven by strong capacity (2000 MW = AAA) but moderate leverage.
2. Transition risks cause severe profitability degradation ( $m_2$  becomes negative), triggering B rating.
3. Physical risks alone insufficient to cross rating thresholds—profitability remains positive.
4. **Combined rating + CRP effect:** Rating spread increase (350 bps) + CRP (7,165 bps) = 7,515 bps total!

**Corollary 17** (Policy Implication). *The combined effect of credit downgrades and climate risk premium renders aggressive transition scenarios unfinanceable at any realistic cost of capital. This demonstrates the stranded asset phenomenon: not merely value destruction, but complete loss of financing access.*

## 9.7 Rating Migration Matrices

For portfolio risk management, rating agencies publish transition matrices  $\mathbf{P}$  where  $P_{ij}$  is probability of migrating from rating  $i$  to rating  $j$  over one year.

**Definition 32** (Climate-Conditional Transition Matrix). *Define  $\mathbf{P}^S$  as the transition matrix conditional on climate scenario  $S$ . Our analysis suggests:*

$$P_{ij}^S > P_{ij}^{S_0} \quad \text{for } j > i \text{ (downgrades)} \quad (61)$$

*That is, climate risks increase downgrade probabilities.*

**Proposition 18** (Expected Rating Drift). *The expected rating change over one year under climate stress is:*

$$\mathbb{E}[\Delta N(R)^S] = \sum_{j=1}^6 j \cdot P_{ij}^S - i > 0 \quad (62)$$

*indicating systematic downward rating drift for fossil fuel assets.*

## 9.8 Model Integration Summary

Our framework integrates three complementary approaches:

Dimension	CRP Model	KIS Ratings	Merton-Moody's
<b>Input</b>	NPV loss	Financial ratios	Asset volatility
<b>Method</b>	Expected loss → spread	Threshold grid	Structural default prob
<b>Output</b>	Continuous spread	Discrete rating	PD → rating
<b>Strength</b>	Climate-specific	Market-standard	Theoretical foundation
<b>Weakness</b>	Ad-hoc sensitivity	Discrete jumps	Data-intensive

Table 5: Comparison of credit risk quantification approaches

### Recommended Practice:

1. Use **CRP model** for internal risk pricing and climate scenario analysis.
2. Use **KIS ratings** for regulatory compliance and external communication.
3. Use **Merton-Moody's** for portfolio credit risk modeling and stress testing.

All three approaches converge on the same conclusion: *climate risks materially increase financing costs for carbon-intensive infrastructure, with aggressive decarbonization scenarios rendering assets unfinanceable.*

## 10 Discussion

### 10.1 Policy Implications

- **Asset Stranding:** Projects with CRP > 200 bps face material stranding risk under Paris-aligned pathways.
- **Investment Reallocation:** Properly priced CRP makes renewables relatively more attractive.
- **Financial Stability:** Unpriced climate risks represent systematic underestimation of financing costs.

### 10.2 Model Limitations

1. **Linear Spread Response:** Actual credit curves may be convex in expected loss.
2. **Deterministic Scenarios:** Full stochastic treatment would integrate over scenario distributions.
3. **Static Capital Structure:** Dynamic refinancing responses not modeled.
4. **Tax Effects:** Tax shields on debt ignored (extends to after-tax WACC).

### 10.3 Extensions

- **Monte Carlo Simulation:** Replace deterministic physical risk parameters with probability distributions.
- **Real Options:** Value of flexibility to switch fuels or retire early.
- **Portfolio Analysis:** Correlation of climate risks across multiple projects.
- **Dynamic Hedging:** Optimal carbon price hedging strategies.

## 11 Conclusion

We have developed a rigorous mathematical framework for quantifying Climate Risk Premium in project finance. The model demonstrates that:

1. Climate risks (transition and physical) systematically reduce project cash flows.

2. Expected losses translate into higher financing costs through credit spreads and equity premiums.
3. For carbon-intensive assets, CRP can range from 50-200 bps under moderate scenarios to >500 bps under aggressive decarbonization.
4. Properly pricing CRP is essential for:
  - Avoiding asset stranding
  - Accurate project valuation
  - Efficient capital allocation toward climate-resilient investments

The framework is transparent, tractable, and empirically implementable, providing a foundation for integrating climate risks into standard project finance practice.

## Acknowledgments

This work was conducted as part of climate risk analysis research at PLANiT Institute.

## References

- [1] Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29(2), 449-470.
- [2] Korea Investors Service (2020). *Rating Methodology for Private Power Generation (IPP)*. KIS Ratings.
- [3] Task Force on Climate-related Financial Disclosures (2017). *Recommendations of the Task Force on Climate-related Financial Disclosures*. Financial Stability Board.
- [4] Bolton, P., Despres, M., Pereira da Silva, L. A., Samama, F., & Svartzman, R. (2020). *The green swan: Central banking and financial stability in the age of climate change*. Bank for International Settlements.
- [5] Battiston, S., Mandel, A., Monasterolo, I., Schütze, F., & Visentin, G. (2017). A climate stress-test of the financial system. *Nature Climate Change*, 7(4), 283-288.
- [6] Dietz, S., Bowen, A., Dixon, C., & Gradwell, P. (2016). 'Climate value at risk' of global financial assets. *Nature Climate Change*, 6(7), 676-679.
- [7] Monasterolo, I., & De Angelis, L. (2020). Blind to carbon risk? An analysis of stock market reaction to the Paris Agreement. *Ecological Economics*, 170, 106571.
- [8] Campiglio, E., Dafermos, Y., Monnin, P., Ryan-Collins, J., Schotten, G., & Tanaka, M. (2018). Climate change challenges for central banks and financial regulators. *Nature Climate Change*, 8(6), 462-468.
- [9] Hsiang, S., Kopp, R., Jina, A., Rising, J., Delgado, M., Mohan, S., ... & Houser, T. (2017). Estimating economic damage from climate change in the United States. *Science*, 356(6345), 1362-1369.
- [10] Burke, M., Hsiang, S. M., & Miguel, E. (2015). Global non-linear effect of temperature on economic production. *Nature*, 527(7577), 235-239.

## A Proofs of Technical Lemmas

### A.1 Proof of Monotonicity Properties

**Lemma 19.** *For fixed physical risk, NPV is monotone decreasing in carbon price:*

$$\tau'_t > \tau_t \forall t \Rightarrow NPV(\tau') < NPV(\tau) \quad (63)$$

*Proof.* Carbon costs enter linearly and negatively in FCF:

$$FCF_t(\tau') = FCF_t(\tau) - Q_t \cdot e \cdot (\tau'_t - \tau_t) < FCF_t(\tau) \quad (64)$$

Discounting preserves inequality, hence  $NPV(\tau') < NPV(\tau)$ .  $\square$

### A.2 Proof of Convexity Properties

**Lemma 20.** *Expected loss is convex in carbon price:*

$$\frac{\partial^2 EL}{\partial \tau^2} \geq 0 \quad (65)$$

*Proof.* Expected loss is linear in carbon costs, which are linear in  $\tau$ . Hence second derivative is zero, confirming convexity (linear functions are both convex and concave).  $\square$

## B Numerical Implementation Details

### B.1 Algorithm for NPV Calculation

1. **Input:** Plant parameters, scenario  $\mathcal{S} = (\mathcal{T}, \mathcal{P})$ , discount rate  $r$
2. **Initialize:**  $T^* \leftarrow \min(T, T_{\mathcal{T}}^*)$
3. **For**  $t = 1$  to  $T^*$ :
  - (a) Compute  $\rho_t \leftarrow \rho_0 - \Delta\rho$
  - (b) Compute  $Q_t \leftarrow C \cdot 8760 \cdot \rho_t \cdot (1 - \delta) \cdot (1 - \lambda)$
  - (c) Compute  $\tau_t$  via interpolation
  - (d) Compute  $R_t, FC_t, VC_t, FX_t, CC_t, OC_t$
  - (e) Compute  $FCF_t \leftarrow R_t - \sum \text{costs}$
4. **Return:**  $NPV = \sum_{t=1}^{T^*} FCF_t / (1 + r)^t$

### B.2 Convergence Properties

The IRR calculation uses Newton-Raphson iteration:

$$r_{n+1} = r_n - \frac{NPV(r_n)}{NPV'(r_n)} \quad (66)$$

where:

$$NPV'(r) = - \sum_{t=1}^{T^*} \frac{t \cdot FCF_t}{(1 + r)^{t+1}} \quad (67)$$

Convergence is guaranteed if  $FCF_t$  changes sign exactly once (Descartes' rule).  $\square$