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Height and Weight Dataset

Person	Height (cm)	Weight (kg)	r_{women}	r_{men}
A	158	52	0.98	0.02
B	162	56	0.95	0.05
C	166	60	1.00	0.00
D	175	72	0.10	0.90
E	180	78	0.05	0.95
F	185	84	0.02	0.98

Parameter	women	men
mean (μ)	$\begin{bmatrix}160\\55\end{bmatrix}$	$\begin{bmatrix}180\\78\end{bmatrix}$
mixing coefficient (π)	0.5	0.5
covariance (Σ)	$\begin{bmatrix}9&0\\0&16\end{bmatrix}$	$\begin{bmatrix}16&0\\0&25\end{bmatrix}$

1.) Compute the responsibility for Sample C

$= |\Sigma_{women}| = \begin{bmatrix}9&0\\0&16\end{bmatrix} = 144$ $\qquad |\Sigma_{men}| = \begin{bmatrix}16&0\\0&25\end{bmatrix} = 400$

$= 144^{1/2} = \sqrt{144} = 12$ $\qquad = 400^{1/2} = \sqrt{400} = 20$

$= 2(\pi)^{2/2}(12) = 75.39822 \approx \underline{75.39}$ $\qquad = 2(\pi)^{2/2}(20) = 125.6637 \approx \underline{125.66}$

$= (C - \mu_{women/men})^T \Sigma^{-1} (C - \mu_{women/men}) = \dfrac{(x_h - \mu_h)^2}{\Sigma} + \dfrac{(x_w - \mu_w)^2}{\Sigma}$

$= \text{women} = \dfrac{(166-160)^2}{9} + \dfrac{(60-55)^2}{16}$

$= 5.5625 \approx \underline{5.56}$

$= \text{men} = \dfrac{(166-180)^2}{16} + \dfrac{(60-78)^2}{25} = \underline{25.21}$

$\cdot\ N(A \mid \mu_{women/men}, \Sigma_{women/men}) = \dfrac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\dfrac{1}{2}(A - \mu_{women/men})^T \Sigma^{-1}_{women/men}(A - \mu_{women/men})\right)$

$= \text{women} - \dfrac{1}{75.39} \exp\left(-\dfrac{1}{2}(5.56)\right) = \dfrac{1}{75.39}\exp(-2.78) = \dfrac{1}{75.39}(0.0620) = \underline{8.22 \times 10^{-4}}$

$= \text{men} = \dfrac{1}{125.66} \exp\left(-\dfrac{1}{2}(25.21)\right) = \dfrac{1}{125.66}\exp(-12.61) = \dfrac{1}{125.66}(3.34 \times 10^{-6}) = \underline{2.67 \times 10^{-8}}$

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$= r_{ic} = \dfrac{\pi_c N(x_i \mid \mu_c, \Sigma_c)}{\sum_{j=1}^{k} \pi_j N(x_i \mid \mu_j, \Sigma_j)}$ $= r_{women} = \dfrac{0.5 \times 8.22 \times 10^{-4}}{0.5 \times 8.22 \times 10^{-4} + 0.5 \times 2.67 \times 10^{-8}} = 0.999 \approx 1.00$

$= r_{men} = \dfrac{0.5 \times 2.67 \times 10^{-8}}{0.5 \times 8.22 \times 10^{-4} + 0.5 \times 2.67 \times 10^{-8}} = 3.25 \times 10^{-5} = 0.0000325 \approx 0.00$

2. Compute the data points ~~of~~ for each cluster

$= m_c = \sum_{i=1} r_{ic}$

- $m_{women} = 0.98 + 0.95 + 1.00 + 0.10 + 0.05 + 0.02 = 3$
- $m_{men} = 0.02 + 0.05 + 0.00 + 0.90 + 0.95 + 0.98 = 3$

3. Compute the updated mixing coefficient for each cluster

$= \pi_c = \dfrac{m_c}{m}$

- $\pi_{women} = \dfrac{3}{6} = \dfrac{1}{2} = 0.5$

$\pi_{men} = \dfrac{3}{6} = \dfrac{1}{2} = 0.5$

4. Compute the updated means of each cluster

$\mu_c = \dfrac{1}{m_c} \sum_i r_{ic} x^i$

$\mu_{height} = \dfrac{158 + 162 + 166}{3} = 162$ $\mu_{men} = \dfrac{175 + 180 + 185}{3} = 180$

$\mu_{weight} = \dfrac{52 + 56 + 60}{3} = 56$ $\mu_{men} = \dfrac{72 + 78 + 84}{3} = 78$

$\mu_{women} = \begin{bmatrix} 162 \\ 56 \end{bmatrix}$ $\mu_{men} = \begin{bmatrix} 180 \\ 78 \end{bmatrix}$

5. Compute the updated covariance of each cluster

$= \mu_{women} = \begin{bmatrix} 162 \\ 56 \end{bmatrix} = A = \begin{bmatrix} 158 \\ 52 \end{bmatrix}$ $B = \begin{bmatrix} 162 \\ 56 \end{bmatrix}$ $C = \begin{bmatrix} 166 \\ 60 \end{bmatrix}$

- $\mu_{men} = \begin{bmatrix} 180 \\ 78 \end{bmatrix} = D = \begin{bmatrix} 175 \\ 72 \end{bmatrix}$ $E = \begin{bmatrix} 180 \\ 78 \end{bmatrix}$ $F = \begin{bmatrix} 185 \\ 84 \end{bmatrix}$

$= (A - \mu_{women})\left(\begin{bmatrix}162\\56\end{bmatrix} - \begin{bmatrix}158\\52\end{bmatrix}\right) = \begin{bmatrix}4\\4\end{bmatrix} \quad (A-\mu_{women})^T = [4, 4]$

$= (A-\mu_{women})(A-\mu_{women})^T = \begin{bmatrix}4\\4\end{bmatrix}[4, 4] = \begin{bmatrix}16 & 16\\16 & 16\end{bmatrix}$

$= \left(\begin{bmatrix}162\\56\end{bmatrix} - \begin{bmatrix}162\\56\end{bmatrix}\right) = [0]$

$= \left(\begin{bmatrix}162\\56\end{bmatrix} - \begin{bmatrix}166\\60\end{bmatrix}\right) = \begin{bmatrix}-4\\-4\end{bmatrix} \quad = (C-\mu_{women})^T = [-4, -4]$

$= (C-\mu_{women})(C-\mu_{women})^T = \begin{bmatrix}-4\\-4\end{bmatrix}[-4, -4] = \begin{bmatrix}16 & 16\\16 & 16\end{bmatrix}$

$= \Sigma_{women} = \frac{1}{3}\begin{bmatrix}16 & 16\\16 & 16\end{bmatrix} + 0 + \begin{bmatrix}16 & 16\\16 & 16\end{bmatrix} =$

$= \Sigma_{women} = \frac{1}{3}\begin{bmatrix}32 & 32\\32 & 32\end{bmatrix} = \begin{bmatrix}10.67 & 10.67\\10.67 & 10.67\end{bmatrix}$

$= (D - \mu_{men})\left(\begin{bmatrix}180\\78\end{bmatrix} - \begin{bmatrix}175\\72\end{bmatrix}\right) = \begin{bmatrix}5\\6\end{bmatrix} \quad (D-\mu_{men})^T = [5, 6]$

$= (D-\mu_{men})(D-\mu_{men})^T = \begin{bmatrix}5\\6\end{bmatrix}[5, 6] = \begin{bmatrix}25 & 30\\30 & 36\end{bmatrix}$

$= \left(\begin{bmatrix}180\\78\end{bmatrix} - \begin{bmatrix}180\\78\end{bmatrix}\right) = [0]$

$= (F - \mu_{men})\left(\begin{bmatrix}180\\78\end{bmatrix} - \begin{bmatrix}185\\84\end{bmatrix}\right) = \begin{bmatrix}-5\\-6\end{bmatrix} \quad (F-\mu_{men})^T = [-5, -6]$

$= (F-\mu_{men})(F-\mu_{men})^T = \begin{bmatrix}-5\\-6\end{bmatrix}[-5, -6] = \begin{bmatrix}25 & 30\\30 & 36\end{bmatrix}$

$= \Sigma_{men} = \frac{1}{3}\begin{bmatrix}25 & 30\\30 & 36\end{bmatrix} + 0 + \begin{bmatrix}25 & 30\\30 & 36\end{bmatrix}$

$= \Sigma_{men} = \frac{1}{3} = \begin{bmatrix}50 & 60\\60 & 72\end{bmatrix} = \begin{bmatrix}16.67 & 20\\20 & 24\end{bmatrix}$