Almadrones, Kurt Justine M. COM282 Assignment 3

1.) Compute Probabilities

Customer	Time on Site(X_1)	Pages Viewed(X_2)	Purchase(y)	$\bar{y}$
A	1	4	0	0.168
B	2	3	0	0.231
C	3	7	1	0.769
D	5	2	1	0.690
E	6	6	1	0.961

$= m_1 = 0.8 \quad e = 2.718$

$m_2 = 0.4$

$b = 4.0$

1.) $z = 0.8(1) + 0.4(4) + (-4.0) = -1.6$

$\bar{y} = \frac{1}{1+2.718^{-(-1.6)}} = \boxed{0.168}$

2.) $z = 0.8(2) + 0.4(3.0) + (-4.0) = -1.2$

$\bar{y} = \frac{1}{1+2.718^{-(-1.2)}} = \boxed{0.231}$

3.) $z = 0.8(3) + 0.4(7) + (-4.0) = 1.2$

$\bar{y} = \frac{1}{1+2.718^{-(1.2)}} = \boxed{0.769}$

4.) $z = 0.8(5) + 0.4(2) + (-4.0) = 0.8$

$\bar{y} = \frac{1}{1+2.718^{-(0.8)}} = \boxed{0.690}$

5.) $z = 0.8(6) + 0.4(6) + (-4.0) = 3.2$

$\bar{y} = \frac{1}{1+2.718^{-(3.2)}} = \boxed{0.961}$

2.) Compute Average Loss

Customer	Time on site(X_1)	Pages Viewed(X_2)	Purchase(y)	$\bar{y}$	Loss
A	1	4	0	0.168	0.1839
B	2	3	0	0.231	0.2627
C	3	7	1	0.769	0.2627
D	5	2	1	0.690	0.3711
E	6	6	1	0.961	0.0398

Avg Loss = 0.2240

1.) Loss = $-(0 \times \ln(0.168) + (1-0) \times \ln(1-0.168))$ = 0.1839

2.) Loss = $-(0 \times \ln(0.231) + (1-0) \times \ln(1-0.231))$ = 0.2627

3.) Loss = $-(1 \times \ln(0.769) + (1-1) \times \ln(1-0.769))$ = 0.2627

4.) $-(1 \times \ln(0.690) + (1-1) \times \ln(1-0.690))$ = 0.3711

5.) Loss = $-(1 \times \ln(0.961) + (1-1) \times \ln(1-0.961))$ = 0.0398

Average Loss = $\frac{.1839 + .2627 + .2627 + .3711 + .0398}{5} = 0.2240$

3.) Update the slope and intercept using Gradient Descent

Customer	X_1	X_2	Y	$\bar{Y}$	$\bar{y}-y$	$(\bar{y}-y) \times X_1$	$(\bar{y}-y) \times X_2$
A	1	4	0	0.168	0.168	0.168	0.672
B	2	3	0	0.231	0.231	0.462	0.693
C	3	7	1	0.769	-0.769	-0.693	-1.617
D	5	2	1	0.690	-0.690	-1.55	-0.620
E	6	6	1	0.961	-0.961	-0.234	-0.234

$= \frac{0.168 + 0.231 + (-0.769) + (-0.690) + (-0.961)}{5}$

Avg $(\bar{y}-y) = -0.181$

$= \frac{0.168 + 0.462 + (-0.693) + (-1.55) + (-0.234)}{5}$

Avg $(\bar{y}-y) \times X_1 = -1.847$

$= \frac{0.672 + 0.693 + (-1.617) + (-0.620) + (-0.234)}{5}$

Avg $(\bar{y}-y) \times X_2 = -1.106$

$\frac{d\,loss}{db} = \frac{-0.181}{5} = -0.03362$ → new b = $-4.0 - 0.1(-0.03362) = \boxed{-3.996}$

$\frac{d\,loss}{dm_1} = \frac{-1.847}{5} = -0.3694$ → new m_1 = $0.8 - 0.1(-0.3694) = \boxed{0.8369}$

$\frac{d\,loss}{dm_2} = \frac{-1.106}{5} = -0.2212$ → new m_2 = $0.4 - 0.1(-0.2212) = \boxed{0.4221}$

4.) Compute new probabilities using the new slopes and intercept

Customer	X_1	X_2	Y	new $\bar{y}$
A	1	4	0	0.187
B	2	3	0	0.258
C	3	7	1	0.813
D	5	2	1	0.737
E	6	6	1	0.972

$= m_1 = -3.996$

$m_2 = 0.8369$

$b = 0.4221$

1.) $z = 0.8369(1) + 0.4221(4) + (-3.996) = 1.4707$

$\bar{y} = \frac{1}{1 + 2.718^{-(-1.4707)}} = 0.187$

2.) $z = 0.8369(2) + 0.4221(3) + (-3.996) = -1.0559$

$\bar{y} = \frac{1}{1 + 2.718^{-(-1.0559)}} = 0.258$

3.) $z = 0.8369(3) + 0.4221(7) + (-3.996) = 1.4694$

$\bar{y} = \frac{1}{1 + 2.718^{-(1.4694)}} = 0.813$

4.) $z = 0.8369(5) + 0.4221(2) + (-3.996) = 1.0327$

$\bar{y} = \frac{1}{1 + 2.718^{-(1.0327)}} = 0.737$

5.) $z = 0.8369(6) + 0.4221(6) + (-3.996) = 3.558$

$\bar{y} = \frac{1}{1 + 2.718^{-(3.558)}} = 0.972$

5.) Compute new average loss

Customer	X_1	X_2	Y	new $\hat{Y}$	new loss
A	1	4	0	0.187	0.2070
B	2	3	0	0.258	0.2984
C	3	7	1	0.813	0.2070
D	5	2	1	0.757	0.3052
E	6	6	1	0.972	0.0284

1.) Loss = $-(0 \times \ln(0.187) + (1-0) \times \ln(1-0.187))$

= 0.2070

2.) Loss = $-(0 \times \ln(0.258) + (1-0) \times \ln(1-0.258))$

= 0.2984

3.) Loss = $-(1 \times \ln(0.813) + (1-1) \times \ln(1-0.813))$

= 0.2070

4.) Loss = $-(1 \times \ln(0.757) + (1-1) \times \ln(1-0.757))$

= 0.3052

5.) Loss = $-(1 \times \ln(0.972) + (1-1) \times \ln(1-0.972))$

= 0.0284

Average Loss = $\frac{0.2070 + 0.2984 + 0.2070 + 0.3052 + 0.0284}{5}$

= 0.2092