



香港科技大學

THE HONG KONG UNIVERSITY OF
SCIENCE AND TECHNOLOGY

Principles of Programming Languages

COMP 3031, Fall 2025

Lionel Parreaux

Principles of Programming Languages (PPL)

Instructor

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The materials in this course were largely adapted from similar EPFL courses by Martin Odersky and Viktor Kuncak, most notably CS-210.

What is This Course About?

*A tour of important programming language paradigms and constructs,
with an emphasis on functional programming and Scala.*

Goals of the Course (ambitiously)

To *expand your mind* and give you **new tools** enabling you
to solve **hard problems** in **easier and better** ways.

Expected Learning Outcomes

Become better at modeling problems and using the right programming
tools to solve them, using powerful programming language concepts.

Organization of the Course (1)

Lectures (L1)

Tuesday & Thursday, 12:00–13:20.

Rm G009B, CYT Bldg

Labs (LA1)

Mondays, 15:00–16:20, Rm 4214.

(Alternates between computer labs and pen & paper sessions.)

Rm 4214

Office Hours

Up to 40 min after the lectures.

Rm 3547

Midterm Exam

mid October (**TENTATIVE**)

Organization of the Course (2)

Assignments

There will be three programming assignments. (Details to be given later.)

Exams

There will be a midterm exam and a final exam. The final exam will cover all course material, with emphasis on post-midterm topics.

We will give you some material to prepare.

Grading

Each assignment counts for 10%; the midterm 30%; and the final 40%.

All assignments should be done independently by individual students. You can discuss among peers, but the hand-ins should be your own work.

No late hand-ins will be accepted.

Programming Paradigms

Paradigm: In science, a *paradigm* describes distinct concepts or thought patterns in some scientific discipline.

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Main *programming* paradigms:

- ▶ imperative

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Paradigm: In science, a *paradigm* describes distinct concepts or thought patterns in some scientific discipline.

Main *programming* paradigms:

- ▶ imperative
- ▶ functional
- ▶ logic
- ▶ object-oriented
- ▶ concurrent
- ▶ parallel
- ▶ dependently-typed
- ▶ etc.

Review: Imperative programming

Imperative programming is about

- ▶ modifying mutable variables and memory locations using assignments
- ▶ control structures such as if-then-else, loops, break, continue, return

The most common informal way to understand imperative programs is as instruction sequences for a von Neumann computer.

Imperative Programs and Computers

There's a strong correspondence between

Variables	\approx	registers
Mutable fields	\approx	memory cells
Variable dereferences	\approx	load instructions
Variable assignments	\approx	store instructions
Control structures	\approx	jumps

Problem: Scaling up. How can we avoid conceptualizing programs word by word?

Reference: John Backus, Can Programming Be Liberated from the von Neumann Style?, Turing Award Lecture 1978.

Scaling Up

In the end, pure imperative programming is limited by the “von Neumann” bottleneck:

One tends to conceptualize data structures word-by-word.

We need other techniques for defining high-level abstractions such as collections, polynomials, geometric shapes, strings, documents.

Ideally: Develop *theories* of collections, shapes, strings, ...

What is a Theory?

A theory consists of

- ▶ one or more *data types*
- ▶ *operations* on these types
- ▶ *laws* that describe the relationships between values and operations

Normally, a theory does not describe mutations!

Theories without Mutation

For instance the theory of polynomials defines the sum of two polynomials by laws such as:

$$(a*x + b) + (c*x + d) = (a + c)*x + (b + d)$$

But it does not define an operator to change a coefficient while keeping the polynomial the same!

Theories without Mutation

For instance the theory of polynomials defines the sum of two polynomials by laws such as:

$$(a*x + b) + (c*x + d) = (a + c)*x + (b + d)$$

But it does not define an operator to change a coefficient while keeping the polynomial the same!

Whereas in an imperative program one *can* write:

```
class Polynomial { double[] coefficient; }
Polynomial p = ...;
p.coefficient[0] = 42;
```

Theories without Mutation

Other example:

The theory of strings defines a concatenation operator `++` which is associative:

$$(a \text{ } ++ \text{ } b) \text{ } ++ \text{ } c = a \text{ } ++ \text{ } (b \text{ } ++ \text{ } c)$$

But it does not define an operator to change a sequence element while keeping the sequence the same!

(This one, some languages *do* get right; e.g. Java's strings are immutable)

Consequences for Programming

If we want to implement high-level concepts following their mathematical theories, there's no place for mutation.

- ▶ The theories do not admit it.
- ▶ Mutation can destroy useful laws in the theories.

Therefore, let's

- ▶ concentrate on defining theories for operators expressed as functions,
- ▶ avoid mutations,
- ▶ have powerful ways to abstract and compose functions.

Functional Programming

- ▶ In a *restricted* sense, functional programming (FP) means programming without mutable variables, assignments, loops, and other imperative control structures.
- ▶ In a *wider* sense, functional programming means focusing on the functions and immutable data.
- ▶ In particular, functions can be values that are produced, consumed, and composed.
- ▶ All this becomes easier in a functional language.

Functional Programming Languages

- ▶ In a *restricted* sense, a functional programming language is one which does not have mutable variables, assignments, or imperative control structures.
- ▶ In a *wider* sense, a functional programming language enables the construction of elegant programs that focus on functions and immutable data structures.
- ▶ In particular, functions in a FP language are first-class citizens. This means
 - ▶ they can be defined anywhere, including inside other functions
 - ▶ like any other value, they can be passed as parameters to functions and returned as results
 - ▶ as for other values, there exists a set of operators to compose functions

Some functional programming languages

In the restricted sense:

- ▶ Pure Lisp, XSLT, XPath, XQuery, FP
- ▶ Haskell (without UnsafePerformIO & related escape hatches)

In the wider sense:

- ▶ (Lisp, Scheme), Racket, Clojure
- ▶ SML, Ocaml, F#
- ▶ Haskell (full language)
- ▶ Scala
- ▶ (Smalltalk, Ruby, JavaScript)

(...): languages with first class functions but incomplete support for immutable data

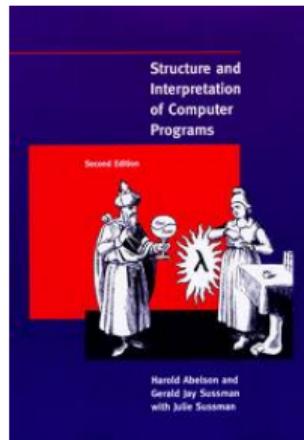
History of FP languages

1959	(Lisp)	2003	Scala
1975-77	ML, FP, Scheme	2005	F#
1978	(Smalltalk)	2007	Clojure
1986	Standard ML	2012	Elixir
1990	Haskell, Erlang	2014	Swift
2000	OCaml	2017	Idris
		2020	Scala 3

Scala 3 is the language we use in this course.

Recommended Book (1)

Structure and Interpretation of Computer Programs. Harold Abelson and Gerald J. Sussman. 2nd edition. MIT Press 1996.



A classic. Parts of the course and quizzes are based on it, but we change the language from Scheme to Scala.

The full text [can be downloaded here](#).

Recommended Book (2)

Programming in Scala. Martin Odersky, Lex Spoon, and Bill Venners. 3rd edition. Artima 2016.

A comprehensive step-by-step guide

Programming in

Scala

Second Edition



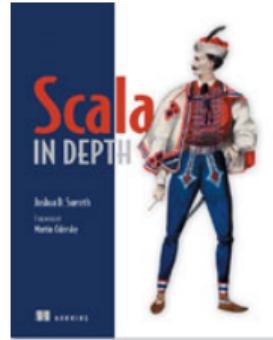
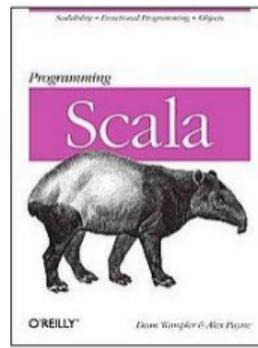
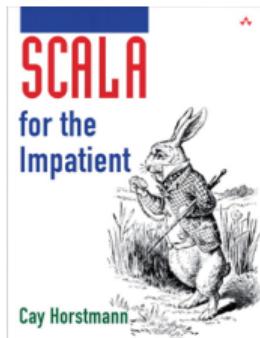
Updated for Scala 2.8

Martin Odersky
Lex Spoon
Bill Venners

artima

The standard language introduction and reference.

Other Recommended Books



Elements of Programming

Elements of Programming

Every non-trivial programming language provides:

- ▶ primitive expressions representing the simplest elements
- ▶ ways to *combine* expressions
- ▶ ways to *abstract* expressions, which introduce a name for an expression by which it can then be referred to.

The Read-Eval-Print Loop

Functional programming is a bit like using a calculator

An interactive shell (or REPL, for Read-Eval-Print-Loop) lets one write expressions and responds with their value.

The Scala REPL can be started by simply typing

```
> scala
```

Expressions

Here are some simple interactions with the REPL

```
scala> 87 + 145  
res0: Int = 232
```

Functional programming languages are more than simple calculators because they let one define values and functions:

```
scala> def size = 2  
size: Int
```

```
scala> 5 * size  
res1: Int = 10
```

Evaluation

A non-primitive expression is evaluated as follows.

1. Take the leftmost operator
2. Evaluate its operands (left before right)
3. Apply the operator to the operands

A name is evaluated by replacing it with the right hand side of its definition

The evaluation process stops once it results in a value

A value is a number (for the moment)

Later on we will consider also other kinds of values

Example

Here is the evaluation of an arithmetic expression:

```
def pi = 3.14159
```

```
def radius = 10
```

```
(2 * pi) * radius
```

Example

Here is the evaluation of an arithmetic expression:

```
(2 * pi) * radius
```

```
(2 * 3.14159) * radius
```

Example

Here is the evaluation of an arithmetic expression:

```
(2 * pi) * radius
```

```
(2 * 3.14159) * radius
```

```
6.28318 * radius
```

Example

Here is the evaluation of an arithmetic expression:

(2 * pi) * radius

(2 * 3.14159) * radius

6.28318 * radius

6.28318 * 10

Example

Here is the evaluation of an arithmetic expression:

(2 * pi) * radius

(2 * 3.14159) * radius

6.28318 * radius

6.28318 * 10

62.8318

Parameters

Definitions can have parameters. For instance:

```
scala> def square(x: Double) = x * x  
square: (x: Double)Double
```

```
scala> square(2)  
4.0
```

```
scala> square(5 + 4)  
81.0
```

```
scala> square(square(4))  
256.0
```

```
scala> def sumOfSquares(x: Double, y: Double) = square(x) + square(y)  
sumOfSquares: (x: Double, y: Double)Double
```

Parameter and Return Types

Function parameters come with their type, which is given after a colon

```
def power(x: Double, y: Int): Double = ...
```

If a return type is given, it follows the parameter list.

Primitive types are as in Java, but are written capitalized, e.g:

Int 32-bit integers

Double 64-bit floating point numbers

Boolean boolean values true and false

Evaluation of Function Applications

Applications of parameterized functions are evaluated in a similar way as operators:

1. Evaluate all function arguments, from left to right
2. Replace the function application by the function's right-hand side, and, at the same time
3. Replace the formal parameters of the function by the actual arguments.

Example

```
sumOfSquares(3, 2+2)
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

```
3 * 3 + square(4)
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

```
3 * 3 + square(4)
```

```
9 + square(4)
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

```
3 * 3 + square(4)
```

```
9 + square(4)
```

```
9 + 4 * 4
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

```
3 * 3 + square(4)
```

```
9 + square(4)
```

```
9 + 4 * 4
```

```
9 + 16
```

Example

```
sumOfSquares(3, 2+2)
```

```
sumOfSquares(3, 4)
```

```
square(3) + square(4)
```

```
3 * 3 + square(4)
```

```
9 + square(4)
```

```
9 + 4 * 4
```

```
9 + 16
```

```
25
```

The substitution model

This scheme of expression evaluation is called the *substitution model*.

The idea underlying this model is that all evaluation does is *reduce an expression to a value*.

It can be applied to all expressions, as long as they have no side effects.

The substitution model is formalized in the λ -*calculus*, which gives a foundation for functional programming.

Evaluation Strategies and Termination

Termination

- ▶ *Does every expression reduce to a value (in a finite number of steps)?*

Termination

- ▶ Does every expression reduce to a value (in a finite number of steps)?
- ▶ No. Here is a counter-example

```
def loop: Int = loop
```

```
loop
```

Changing the evaluation strategy

The interpreter reduces function arguments to values before rewriting the function application.

One could alternatively apply the function to unreduced arguments.

For instance:

```
sumOfSquares(3, 2+2)
```

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For instance:

```
sumOfSquares(3, 2+2)
square(3) + square(2+2)
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
```

Changing the evaluation strategy

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For instance:

```
sumOfSquares(3, 2+2)
square(3) + square(2+2)
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
9 + 4 * (2+2)
```

Changing the evaluation strategy

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For instance:

```
sumOfSquares(3, 2+2)
square(3) + square(2+2)
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
9 + 4 * (2+2)
9 + 4 * 4
```

Changing the evaluation strategy

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For instance:

```
sumOfSquares(3, 2+2)
square(3) + square(2+2)
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
9 + 4 * (2+2)
9 + 4 * 4
```

Call-by-name and call-by-value

The first evaluation strategy is known as *call-by-value*, the second is known as *call-by-name*.

Both strategies reduce to the same final values as long as

- ▶ the reduced expression consists of pure functions, and
- ▶ both evaluations terminate.

Call-by-value has the advantage that it evaluates every function argument only once.

Call-by-name has the advantage that a function argument is not evaluated if the corresponding parameter is unused in the evaluation of the function body.

Call-by-name vs call-by-value

Question: Say you are given the following function definition:

```
def test(x: Int, y: Int) = x * x
```

For each of the following function applications, indicate which evaluation strategy is fastest (has the fewest reduction steps)

CBV fastest	CBN fastest	same #steps	
0	0	0	test(2, 3)
0	0	0	test(3+4, 8)
0	0	0	test(7, 2*4)
0	0	0	test(3+4, 2*4)

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CBV fastest	CBN fastest	same #steps	
0	0	X	test(2, 3)
0	0	0	test(3+4, 8)
0	0	0	test(7, 2*4)
0	0	0	test(3+4, 2*4)

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0	0	X	test(2, 3)
X	0	0	test(3+4, 8)
0	0	0	test(7, 2*4)
0	0	0	test(3+4, 2*4)

Call-by-name vs call-by-value

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def test(x: Int, y: Int) = x * x
```

For each of the following function applications, indicate which evaluation strategy is fastest (has the fewest reduction steps)

CBV	CBN	same	
fastest	fastest	#steps	
0	0	X	test(2, 3)
X	0	0	test(3+4, 8)
0	X	0	test(7, 2*4)
0	0	0	test(3+4, 2*4)

Call-by-name vs call-by-value

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def test(x: Int, y: Int) = x * x
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X	0	0	test(3+4, 8)
0	X	0	test(7, 2*4)
0	0	X	test(3+4, 2*4)

Call-by-name, Call-by-value and termination

The call-by-name and call-by-value evaluation strategies reduce an expression to the same value *as long as both evaluations terminate.*

But what if termination is not guaranteed?

We have:

- ▶ If CBV evaluation of an expression e terminates, then CBN evaluation of e terminates, too.
- ▶ The other direction is not true

Non-termination example

Question: Find an expression that terminates under CBN but not under CBV.

Non-termination example

Let's define

```
def first(x: Int, y: Int) = x
```

and consider the expression `first(1, loop)`.

Under CBN:

```
first(1, loop)
```

Under CBV:

```
first(1, loop)
```

Scala's evaluation strategy

Scala normally uses call-by-value.

But if the type of a function parameter starts with => it uses call-by-name.

Example:

```
def constOne(x: Int, y: => Int) = 1
```

Let's trace the evaluations of

```
constOne(1+2, loop)
```

and

```
constOne(loop, 1+2)
```

Trace of constOne(1 + 2, loop)

```
constOne(1 + 2, loop)
```

Trace of constOne(1 + 2, loop)

```
constOne(1 + 2, loop)
```

```
constOne(3, loop)
```

Trace of constOne(1 + 2, loop)

constOne(1 + 2, loop)

constOne(3, loop)

1

Trace of constOne(loop, 1 + 2)

```
constOne(loop, 1 + 2)
```

Trace of constOne(loop, 1 + 2)

```
constOne(loop, 1 + 2)
```

```
constOne(loop, 1 + 2)
```

```
constOne(loop, 1 + 2)
```

```
...
```

Conditionals and Value Definitions

Conditional Expressions

To express choosing between two alternatives, Scala has a conditional expression `if-then-else`.

It resembles an `if-else` in Java, but is used for expressions, not statements.

Example:

```
def abs(x: Int) = if x >= 0 then x else -x
```

`x >= 0` is a *predicate*, of type Boolean.

Boolean Expressions

Boolean expressions b can be composed of

```
true  false      // Constants  
!b                // Negation  
b && b          // Conjunction  
b || b           // Disjunction
```

and of the usual comparison operations:

```
e <= e, e >= e, e < e, e > e, e == e, e != e
```

Rewrite rules for Booleans

Here are reduction rules for Boolean expressions (e is an arbitrary expression):

```
!true      -->  false
!false     -->  true
true && e  -->  e
false && e -->  false
true || e   -->  true
false || e  -->  e
```

Note that $\&\&$ and $\| \|$ do not always need their right operand to be evaluated.

We say that these expressions use “short-circuiting evaluation”.

Exercise: Formulate rewrite rules for if-then-else

Value Definitions

We have seen that function parameters can be passed by value or be passed by name.

The same distinction applies to definitions.

The def form is “by-name”, its right hand side is evaluated on each use.

There is also a val form, which is “by-value”. Example:

```
val x = 2  
val y = square(x)
```

The right-hand side of a val definition is evaluated at the point of the definition itself.

Afterwards, the name refers to the value.

For instance, y above refers to 4, not square(2).

Value Definitions and Termination

The difference between `val` and `def` becomes apparent when the right hand side does not terminate. Given

```
def loop: Boolean = loop
```

A definition

```
def x = loop
```

is OK, but a definition

```
val x = loop
```

will lead to an infinite loop.

Exercise

Write functions and and or such that for all argument expressions x and y:

$$\begin{aligned}\text{and}(x, y) &==& x \ \&\& y \\ \text{or}(x, y) &==& x \ ||\| y\end{aligned}$$

(do not use `||` and `&&` in your implementation)

What are good operands to test that the equalities hold?

Example Task: Square roots with Newton's method

We will define a function

```
/** Calculates the square root of parameter x */  
def sqrt(x: Double): Double = ...
```

The classical way to achieve this is by successive approximations using Newton's method.

Method

To compute \sqrt{x} :

- ▶ Start with an initial *estimate* y (let's pick $y = 1$).
- ▶ Repeatedly improve the estimate by taking the mean of y and x/y .

Example:

Estimation	Quotient	Mean
1	$2 / 1 = 2$	1.5
1.5	$2 / 1.5 = 1.333$	1.4167
1.4167	$2 / 1.4167 = 1.4118$	1.4142
1.4142

Implementation in Scala (1)

First, define a function which computes one iteration step

```
def sqrtIter(guess: Double, x: Double): Double =  
    if isGoodEnough(guess, x) then guess  
    else sqrtIter(improve(guess, x), x)
```

Note that `sqrtIter` is *recursive*: its right-hand side calls itself.

Recursive functions need an explicit return type in Scala.

For non-recursive functions, the return type is optional

Implementation in Scala (2)

Second, define a function `improve` to improve an estimate and a test to check for termination:

```
def improve(guess: Double, x: Double) =  
  (guess + x / guess) / 2
```

```
def isGoodEnough(guess: Double, x: Double) =  
  abs(guess * guess - x) < 0.001
```

Implementation in Scala (3)

Third, define the sqrt function:

```
def sqrt(x: Double) = sqrtIter(1.0, x)
```

Exercise

1. The `isGoodEnough` test is not very precise for small numbers and can lead to non-termination for very large numbers. Explain why.
2. Design a different version of `isGoodEnough` that does not have these problems.
3. Test your version with some very very small and large numbers, e.g.

0.001

0.1e-20

1.0e20

1.0e50

Blocks and Lexical Scope

Nested functions

It's good functional programming style to split up a task into many small functions.

But the names of functions like `sqrtIter`, `improve`, and `isGoodEnough` matter only for the *implementation* of `sqrt`, not for its *usage*.

Normally we would not like users to access these functions directly.

We can achieve this and at the same time avoid “name-space pollution” by putting the auxiliary functions inside `sqrt`.

The sqrt Function, Take 2

```
def sqrt(x: Double) = {
    def sqrtIter(guess: Double, x: Double): Double =
        if isGoodEnough(guess, x) then guess
        else sqrtIter(improve(guess, x), x)

    def improve(guess: Double, x: Double) =
        (guess + x / guess) / 2

    def isGoodEnough(guess: Double, x: Double) =
        abs(square(guess) - x) < 0.001

    sqrtIter(1.0, x)
}
```

The sqrt Function, Take 2

```
def sqrt(x: Double) =  
    def sqrtIter(guess: Double, x: Double): Double =  
        if isGoodEnough(guess, x) then guess  
        else sqrtIter(improve(guess, x), x)  
  
    def improve(guess: Double, x: Double) =  
        (guess + x / guess) / 2  
  
    def isGoodEnough(guess: Double, x: Double) =  
        abs(square(guess) - x) < 0.001  
  
    sqrtIter(1.0, x)
```

In Scala 3, braces are optional for indented code.

Blocks in Scala

- ▶ A block is delimited by braces { ... }.

```
{ val x = f(3)  
    x * x  
}
```

- ▶ It contains a sequence of definitions or expressions.
- ▶ The last element of a block is an expression that defines its value.
- ▶ This return expression can be preceded by auxiliary definitions.
- ▶ Blocks are themselves expressions; a block may appear everywhere an expression can.
- ▶ In Scala 3, braces are optional (i.e. implied) around a correctly indented expression that appears after =, then, else,

Blocks and Visibility

```
val x = 1
def f(y: Int) = y + x
val result =
  val x = f(3)
  x * x
```

- ▶ The definitions inside a block are only visible from within the block.
- ▶ The definitions inside a block *shadow* definitions of the same names outside the block.

Exercise: Scope Rules

Question: What is the value of result in the following program?

```
val x = 1
def f(y: Int) = y + x
val result = {
    val x = f(3)
    x * x
} + x
```

Possible answers:

- 0 1
- 0 17
- 0 20
- 0 reduction does not terminate

Lexical Scoping

Definitions of outer blocks are visible inside a block unless they are shadowed.

Therefore, we can simplify `sqrt` by eliminating redundant occurrences of the `x` parameter, which means everywhere the same thing:

The sqrt Function, Take 3

```
def sqrt(x: Double) =  
    def sqrtIter(guess: Double): Double =  
        if isGoodEnough(guess) then guess  
        else sqrtIter(improve(guess))  
  
    def improve(guess: Double) =  
        (guess + x / guess) / 2  
  
    def isGoodEnough(guess: Double) =  
        abs(square(guess) - x) < 0.001  
  
sqrtIter(1.0)
```

End Markers

With heavily indented code it is sometimes hard to see where a construct ends.

End markers are a tool to make this explicit.

```
def f() =
```

```
    ...  
    ...  
    ...
```

```
end f
```

- ▶ An end marker is followed by the name that's defined in the definition that ends at this point.
- ▶ It must align with the opening keyword (def in this case).

The sqrt Function, Take 4

```
def sqrt(x: Double) =  
    def sqrtIter(guess: Double): Double =  
        if isGoodEnough(guess) then guess  
        else sqrtIter(improve(guess))  
  
    def improve(guess: Double) =  
        (guess + x / guess) / 2  
  
    def isGoodEnough(guess: Double) =  
        abs(square(guess) - x) < 0.001  
  
    sqrtIter(1.0)  
end sqrt
```

Semicolons

In Scala, semicolons at the end of lines are in most cases optional

You could write

```
val x = 1;
```

but most people would omit the semicolon.

On the other hand, if there are more than one statements on a line, they need to be separated by semicolons:

```
val y = x + 1; y * y
```

Summary

You have seen simple elements of functional programming in Scala.

- ▶ arithmetic and boolean expressions
- ▶ conditional expressions if-then-else
- ▶ functions with recursion
- ▶ nesting and lexical scope

You have learned the difference between the call-by-name and call-by-value evaluation strategies.

You have learned a way to reason about program execution: reduce expressions using the substitution model.

This model will be an important tool for the coming sessions.