Lecture 12: Mon Sept 29th

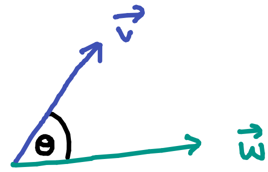
§13.3 Dot product:

Defn: The dot product of $\vec{v}$ and $\vec{w}$ is given by

vector • vector = scalar

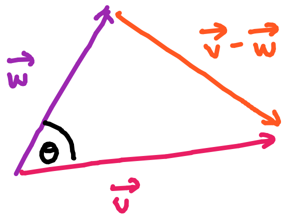
$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta \qquad (0 \le \theta \le \pi) \qquad \boxed{\text{Geometric defn}}$$

$$= v_1w_1 + v_2w_2 + v_3w_3, \quad \text{where } \vec{v} = (v_1, v_2, v_3),\ \vec{w} = (w_1, w_2, w_3). \qquad \boxed{\text{Algebraic defn}}$$



$$\vec{v}\cdot\vec{v} = \|\vec{v}\|^2 = v_1^2 + v_2^2 + v_3^2 \qquad (\text{Here, } \theta = 0. \text{ So, } \cos(0) = 1)$$

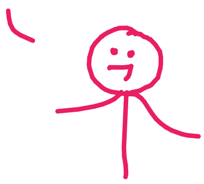
Why: Geometric defn = Algebraic defn? $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\|\cos\theta$ (Law of cosines)



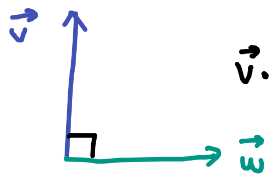
Properties of the dot product:

- $\vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{v}$ (commutativity of •)
- $\vec{v}\cdot(\lambda\vec{w}) = \lambda(\vec{v}\cdot\vec{w}) = (\lambda\vec{v})\cdot\vec{w}$ (compatibility with scalar mult)
- $(\vec{v}+\vec{w})\cdot\vec{u} = \vec{v}\cdot\vec{u} + \vec{w}\cdot\vec{u}$ (distributivity)

How would you prove this?



Defn: Two non-zero vectors are orthogonal or perpendicular iff $\vec{v}\cdot\vec{w} = 0$



$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos(\pi/2) = 0$$

Example: Let $\vec{v} = 3\vec{i} + 3\vec{j} + 4\vec{k}$

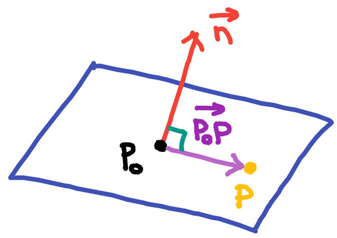
$\vec{w} = 3\vec{i} + 4\vec{k}$

(a) Find angle θ between $\vec{v}$ and $\vec{w}$

(b) Find the value "a" so that $a\vec{i} + 2a\vec{j} + 3\vec{k}$ is perpendicular to $\vec{v}$.

A:

Defn: A normal vector to a plane is a vector perpendicular to it. Fix $P_0 = (x_0, y_0, z_0)$ on the plane. Any point $P = (x,y,z)$ on the plane satisfies $\vec{n} \perp \overrightarrow{P_0P}$.



Point-Normal form of a plane	$\vec{n} \cdot \overrightarrow{P_0P} = 0$ $(a,b,c) \cdot (x-x_0, y-y_0, z-z_0) = 0$ $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

Set $d = \vec{n} \cdot \overrightarrow{OP_0} = ax_0 + by_0 + cz_0$,

General Equation of a plane	$ax + by + cz = d$

Set $m = a/c$ and $n = b/c$,

Point-slope form of a plane	$z = z_0 + m(x-x_0) + n(y-y_0)$

Example: What is the normal vector to the planes:

(a) $2x + 3y - 5z = 4$

(b) $z = x - 2y$

(c) $z = 1 - 2y$

Example: a) Find the plane perpendicular to $(1,4,-7)$ and passing through $(2,-3,5)$.

b) Find a vector parallel to this plane.

A: a)

b)

Example: Find the equation of the plane parallel to $z = 1 - x + 6y$ that contains the point $(1,1,1)$.

A: