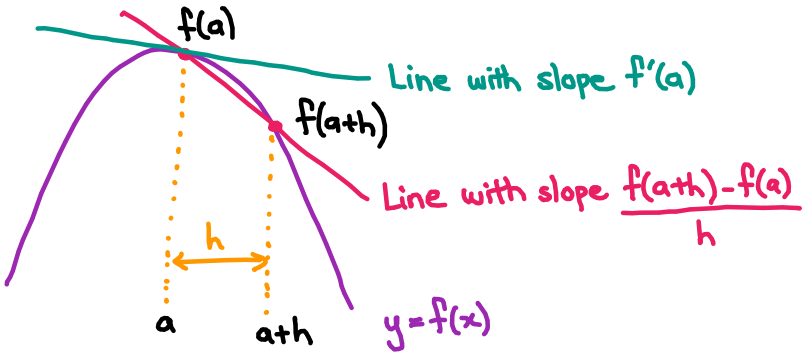
# Lecture 15: Mon Oct 6th

## §14.1: Partial Derivatives

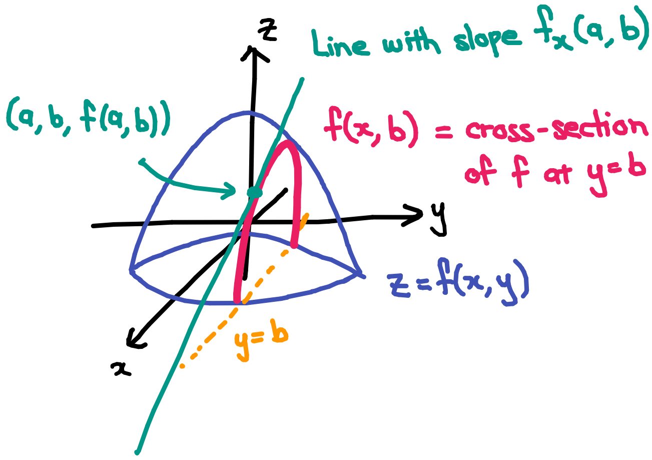
### Single-Variables:




$$f'(a) = \left.\frac{dy}{dx}\right|_a = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

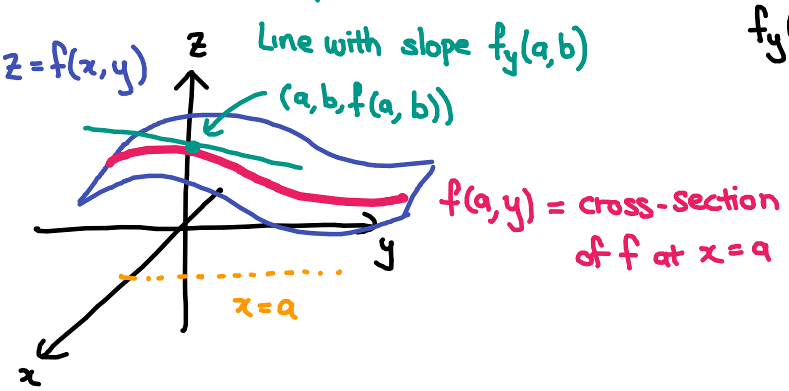
$= $ change of $f$ wrt $x$ at $a$.

### Two-variables:




$$f_x(a,b) = \left.\frac{\partial z}{\partial x}\right|_{(a,b)} = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h}$$

$= $ change of $f$ wrt $x$ at $(a,b)$




$$f_y(a,b) = \left.\frac{\partial z}{\partial y}\right|_{(a,b)} = \lim_{h\to 0} \frac{f(a,b+h)-f(b)}{h}$$

$= $ change of $f$ wrt $y$ at $(a,b)$.

### > Interpreting tables of values

$T(x,y) = $ temperature at $(x,y)$ (°C)

$x, y = $ position (m)

Estimate $\left.\frac{\partial T}{\partial x}\right|_{(3,-1)}$ and $\left.\frac{\partial T}{\partial y}\right|_{(3,-1)}$.

| y\x | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| -3 | 50 | 47 | 42 | 35 |
| -2 | 55 | 52 | 47 | 40 |
| -1 | 58 | 55 | 50 | 43 |
| 0 | 59 | 56 | 51 | 44 |

← keep y=-1 constant

↑ keep x=3 constant

$$\left.\frac{\partial T}{\partial x}\right|_{(3,-1)} \approx \left.\frac{\Delta T}{\Delta x}\right|_{(3,-1)} \approx \frac{T(4,-1)-T(2,-1)}{4-2} = \frac{43-55}{2} = \frac{-12}{2} = -6 \ °C/m$$

$$\left.\frac{\partial T}{\partial y}\right|_{(3,-1)} \approx \left.\frac{\Delta T}{\Delta y}\right|_{(3,-1)} \approx \frac{T(3,0)-T(3,-2)}{0-(-2)} = \frac{51-47}{2} = \frac{4}{2} = 2 \ °C/m$$

## Interpreting partial derivatives

**Example:** $P = f(A, r, N)$ = monthly payment
$A$ = initial amount borrowed ($)
$r$ = annual interest rate (%)
$N$ = # years to pay off loan

b) Interpret $f_r(92000, 14, 30) = 72.82$. $\approx \frac{\Delta P}{\Delta r}$ ($/%)

A: If your loan is $92000 over a fixed period of 30 years and interest rate is 14%, your monthly payment increases by $72.82 for every % increase in the interest rate.

c) Is $\partial P / \partial N$ positive or negative? Justify.

A: As the loan period ($N$) increases, the monthly payment $P$ should decrease. $\therefore \frac{\partial P}{\partial N} < 0$.

**Example:** You are riding your bike at speed $v$ (m/s).
Let $T$ be the actual air temperature (°C)
Let $W = f(T, v)$ be windchill temperature (°C)

Match the practical statement to mathematical statement.

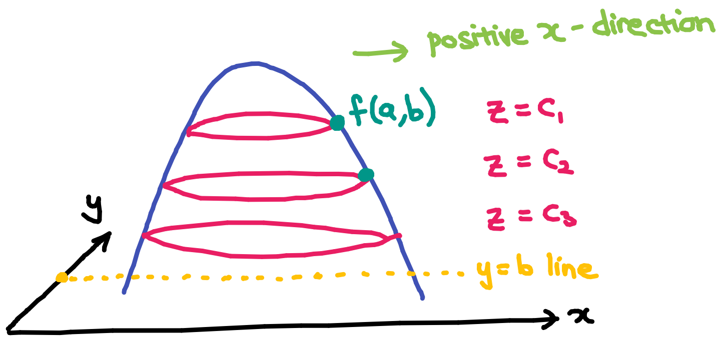
(i) The faster you ride, the colder you'll feel.
(ii) The warmer the day, the warmer you'll feel.

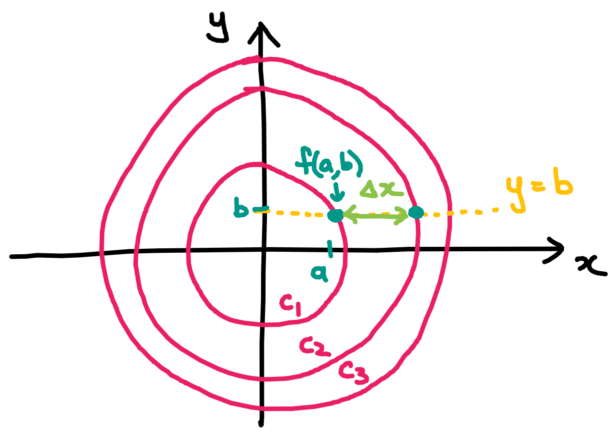
(a) $f_T(T, v) > 0$ — $f$ increases as $T$ increases
(b) $f(0, v) \leq 0$
(c) $f_v(T, v) < 0$ — $f$ decreases as $v$ increases

For the remaining statement, write the practical statement.

(b) When $T = 0$°C, you will feel at most 0°C no matter how fast you are cycling.

## Interpreting contour diagrams

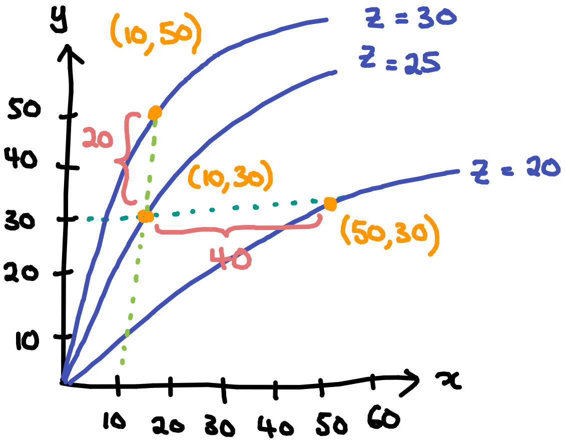



Estimate $f_x(a, b) \approx \frac{\Delta z}{\Delta x} \approx \frac{c_2 - c_1}{\Delta x}$.

Start at $f(a,b)$. Go in the positive $x$-direction while keeping $y = b$ constant until you hit the next contour.

**Example:** Estimate $f_x(10,30)$ and $f_y(10,30)$.




$$f_x(10,30) \approx \frac{\Delta z}{\Delta x} = \frac{20-25}{50-10} = \frac{-5}{40} = \frac{-1}{8}.$$

$$f_y(10,30) \approx \frac{\Delta z}{\Delta y} = \frac{30-25}{50-30} = \frac{5}{20} = \frac{1}{4}.$$