Lecture 16: Wed Oct 8th

Recap:

Q)

y|x	1	3	6
0	1	-1	-3
2	2	1	0
5	4	2	0

Estimate $f_x(3,2)$ and $f_y(3,2)$.

A: $f_x(3,2) \approx \frac{\Delta f}{\Delta x} = \frac{0-2}{6-1} = \frac{-2}{5}$ (keep $y=2$ fixed)

$f_y(3,2) \approx \frac{\Delta f}{\Delta y} = \frac{2-(-1)}{5-0} = \frac{3}{5}$ (keep $x=3$ fixed)

Q) Let $S = f(p,a)$ be sales of a product.

p = price ($/unit)

a = money spent on ads (thousands of $)

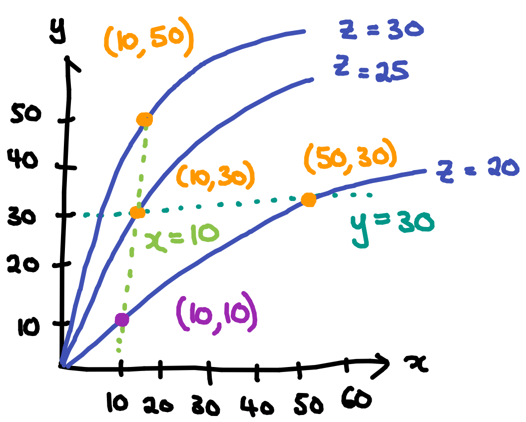
a) Is f_p positive or negative?

b) What does $f_a(8,12) = 150$ mean?

A: a) f_p is negative because the more expensive it is, the less sales there are.

b) When the price is $8 and $12000 is paid on ads, sales increase by 150 units for every $1000 spent on it.

Q) Contour diagrams: Estimate $f_x(10,30)$ and $f_y(10,30)$.



Start at $(10,30)$ and move along $y=30$ until you hit the next contour.

$$f_x(10,30) \approx \frac{\Delta f}{\Delta x} = \frac{20-25}{50-10} = \frac{-5}{40} = \frac{-1}{8}.$$

Similarly, move along $x=10$ until you hit the next contour.

$$f_y(10,30) \approx \frac{\Delta f}{\Delta y} = \frac{30-25}{50-30} = \frac{5}{20} = \frac{1}{4}.$$

Note: you can move in the negative x/y-direction but make sure you get the signs right in $\Delta x / \Delta y$.

If we use $(10,10)$ instead, $f_y(10,30) \approx \frac{20-25}{10-30} = \frac{-5}{-20} = \frac{1}{4}$.

Example: Give a possible contour diagram of $f(x,y)$:

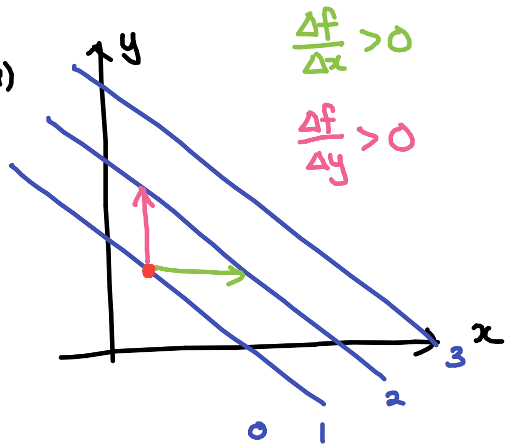
a) $f_x > 0, \quad f_y > 0$

b) $f_x > 0, \quad f_y < 0$

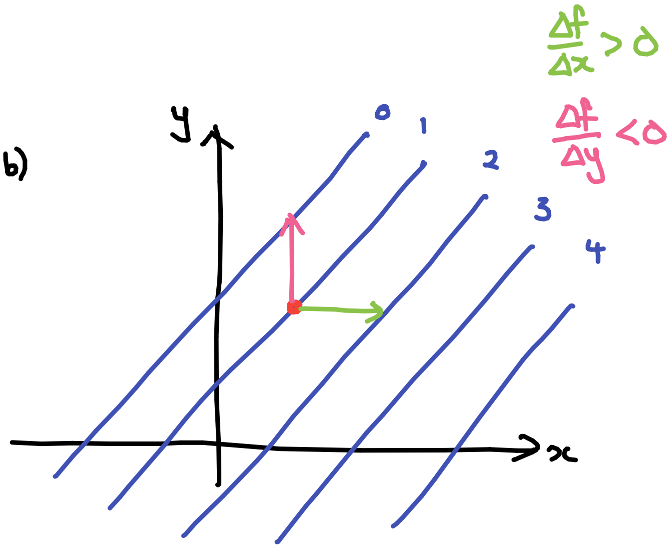
c) $f_x < 0, \quad f_y > 0$

d) $f_x < 0, \quad f_y < 0$

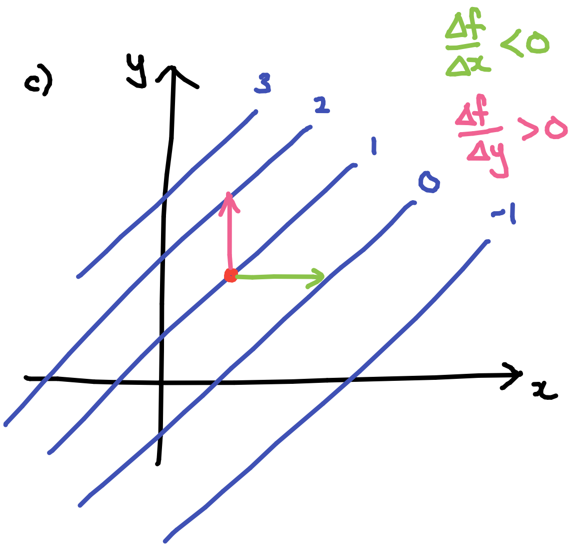
A: a)



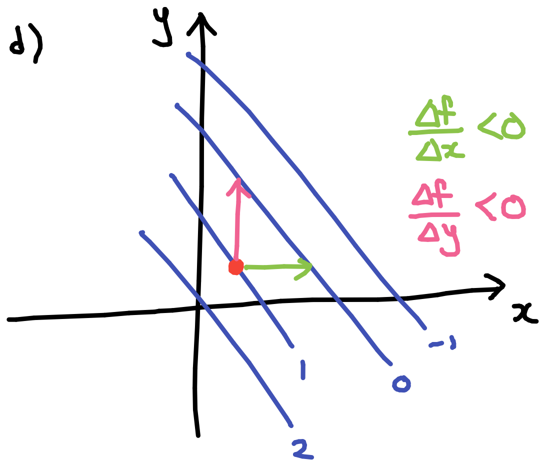
b)



c)



d)



§14.2: Calculating Partial Derivatives

Example: Let $f(x,y) = e^{x\ln(y)}$.

a) Use difference quotients with $h = 0.01$ to estimate $f_x(2,2)$ and $f_y(2,2)$.

A: Diff. quotients are $f_x(a,b) \approx \dfrac{f(a+h,b) - f(a,b)}{h}$ and $f_y(a,b) \approx \dfrac{f(a,b+h) - f(a,b)}{h}$

$$f_x(2,2) \approx \frac{f(2+0.01,2) - f(2,2)}{0.01} = \frac{e^{2.01\ln(2)} - e^{2\ln(2)}}{0.01} = \frac{2^{2.01} - 2^2}{0.01} \approx 2.78$$

$$f_y(2,2) \approx \frac{f(2,2+0.01) - f(2,2)}{0.01} = \frac{e^{2\ln(2.01)} - e^{2\ln 2}}{0.01} = \frac{2.01^2 - 2^2}{0.01} \approx 4.01$$

b) Compute $f_x(2,2)$ and $f_y(2,2)$ exactly.

A: $f_x(x,y) = \frac{\partial}{\partial x} e^{x\ln(y)} = \ln(y) e^{x\ln(y)}$

$f_y(x,y) = \frac{\partial}{\partial y} e^{x\ln(y)}$

$= \frac{dz}{du}\frac{du}{dy}$

$= e^u x/y$

$= e^{x\ln(y)} x/y$

$* \ u = x\ln(y)$

$\frac{du}{dy} = \frac{x}{y}$

$f_x(2,2) = \ln(2) e^{2\ln(2)} = \ln(2)\cdot 2^2 \approx 2.77$

$f_y(2,2) = e^{2\ln(2)} \, 2/2 = 2^2 = 4$