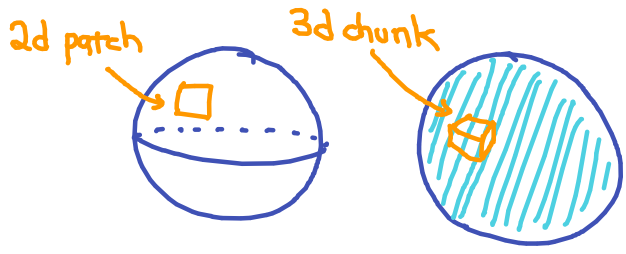
Lecture 3: Mon Sept 8th

§12.2 Functions of 2 variables

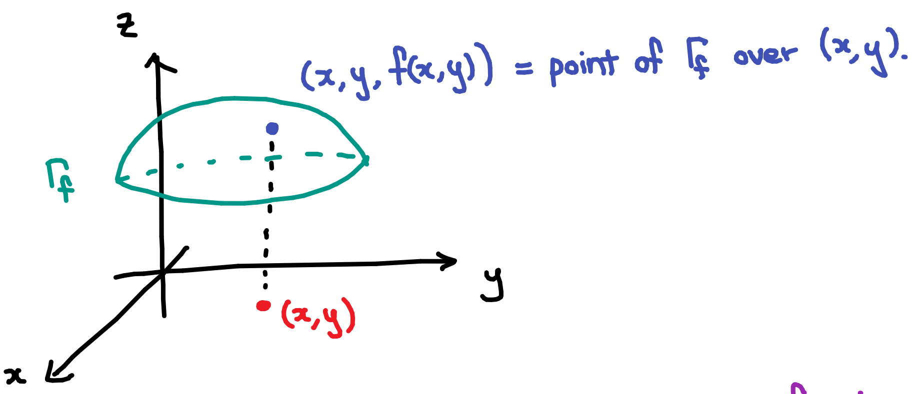


For a function of 2 variables,

the graph $\Gamma_f = \{(x,y,z) \in \mathbb{R}^3 : z = f(x,y)\}$

= surface in $\mathbb{R}^3$

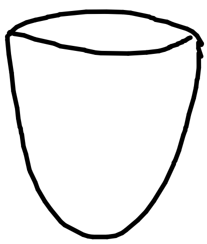
↪ 2-dimensional shape ("locally looks like 2d paper")



Examples: (graphs)

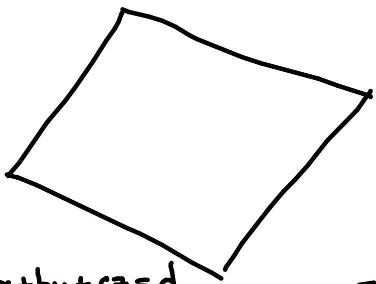
constant function 0

The xy-plane is the graph $\{(x,y,z) \in \mathbb{R}^3 : z = 0\} = \{(x,y,0) \in \mathbb{R}^3\}$.



$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

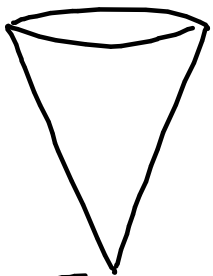
paraboloid



$ax + by + cz = d$

linear equation

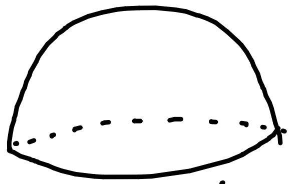
plane



$z = \sqrt{x^2+y^2}$

cone

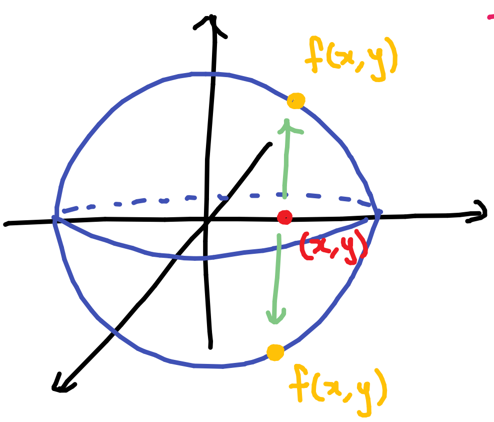
($z = -\sqrt{x^2+y^2}$ is bottom cone)



upper hemisphere

$z = \sqrt{r^2 - (x-a)^2 - (y-b)^2} + c$

Subtle pt: Technically speaking, the sphere as a whole is not a graph of a function.



There are 2 pts lying over (x,y). A function f cannot map (x,y) to 2 different pts.

You need to describe the sphere with two graphs.

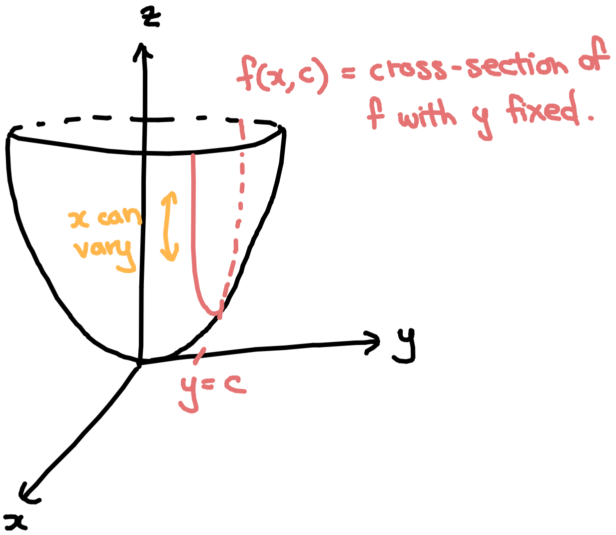
$z = \sqrt{r^2 - x^2 - y^2}$

$\cup$

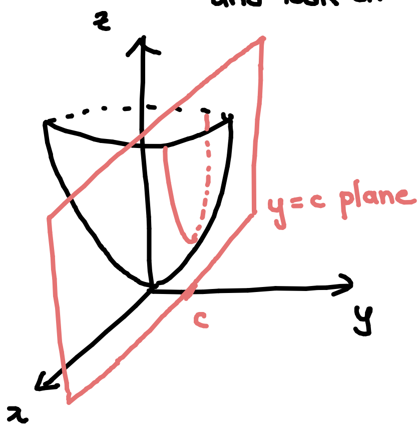
$z = -\sqrt{r^2 - x^2 - y^2}$

The function we get by holding x constant and letting y vary is called:
a cross-section of f with x fixed $= f(c,y)$
$(x=c)$

The function we get by holding y constant and letting x vary is called:
a cross-section of f with y fixed $= f(x,c)$
$(y=c)$



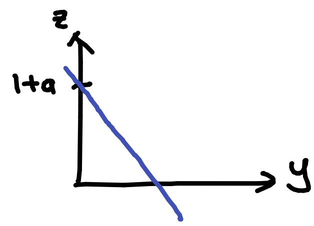
The graph of the cross-section $f(x,c)$
= slice graph of f with the plane $x=c$
and look at the intersection



Example 12.2.4: $f(x,y) = 1+x-y$

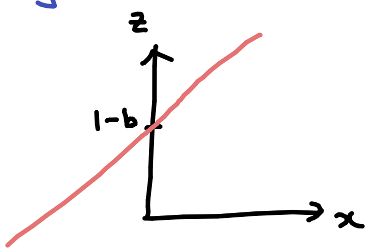
Cross-sections with $x=a$: $z = 1+a-y$
$= (1+a)-y$

lines sloping down in y



Cross-sections with $y=b$: $z = 1+x-b$
$= (1-b)+x$

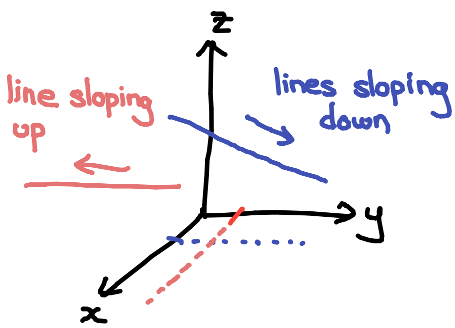
lines sloping up in x



All cross-sections are lines
$\Rightarrow$ the graph of f is a plane.

To see what the shape looks like, you can try and put the cross-sections together.

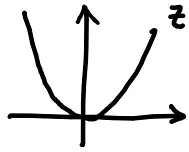
At $x=1$, you should see a line $z=(1+1)-y = 2-y$.
At $x=2$, you should see a line $z=(1+2)-y = 3-y$.
As a increases, the lines move further up in z.
Repeat for $y=b$.
As b increases, the lines move down in z.



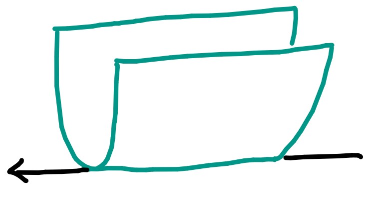
Refer to 3d Desmos. Get the cross-section = intersection between graph of $z=1+x-y$ and the plane $x=a$ (or $y=b$).

Cylinders (when a variable is missing)

What does $z = x^2$ look like? All cross-sections with y fixed is



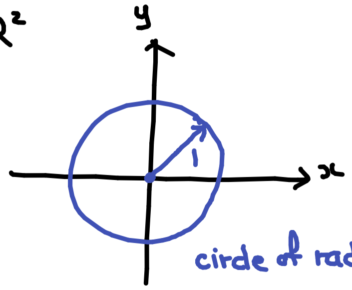
while y can be anything.



parabolic cylinder

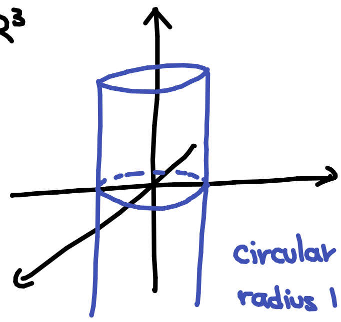
What does $x^2 + y^2 = 1$ look like in 2d v.s. 3d?

$\mathbb{R}^2$



circle of radius 1

$\mathbb{R}^3$



circular cylinder of radius 1