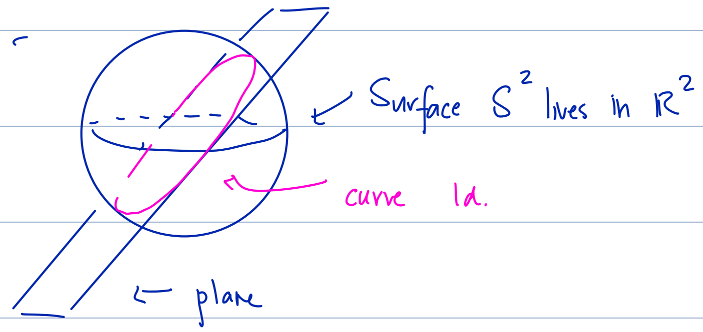
Lecture 4: Cylinders, Level Curves & Contour Diagrams

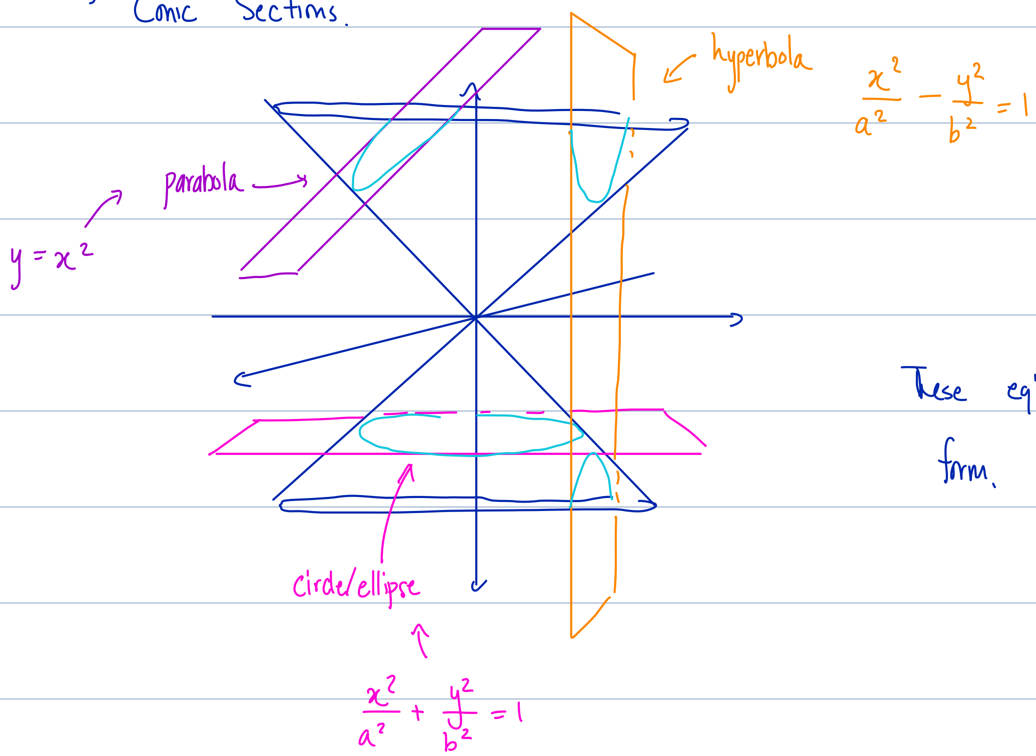
Last Time: (1) Finished traces $\to$ $f(x,y) = z$

↳ Dealing w/ surfaces in $\mathbb{R}^3$



Lets go back and classify some curves.

↳ Conic Sections.



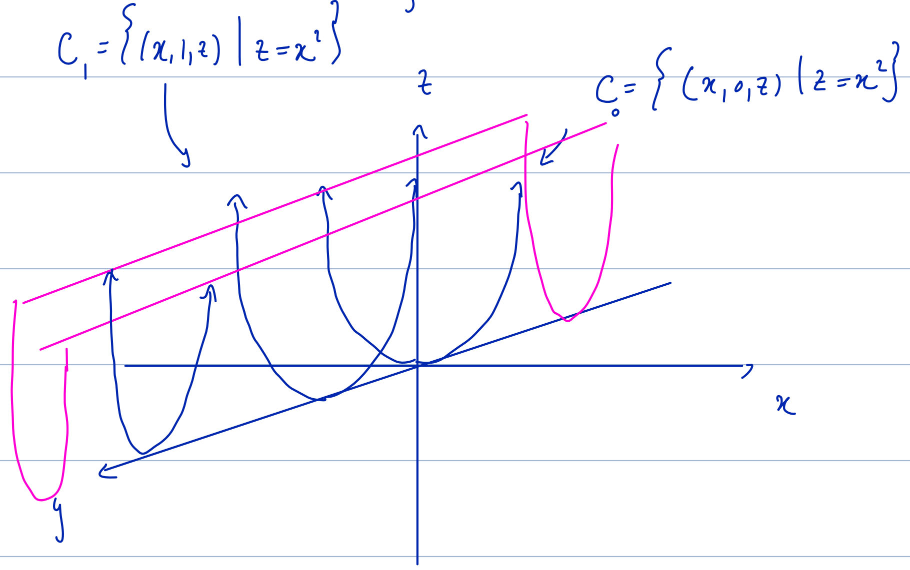
These eq's are in "standard" form.

(2) Cylinders.

↳ Def: A cylinder is a surface that consists of all lines which a parallel to a given plane curve.

Ex. Sketch the graph of the surface $z = x^2$

Viewed as a subset of $\mathbb{R}^3$ this set $S = \{(x,y,z) \mid x,y,z \in \mathbb{R},\ z = x^2\}$



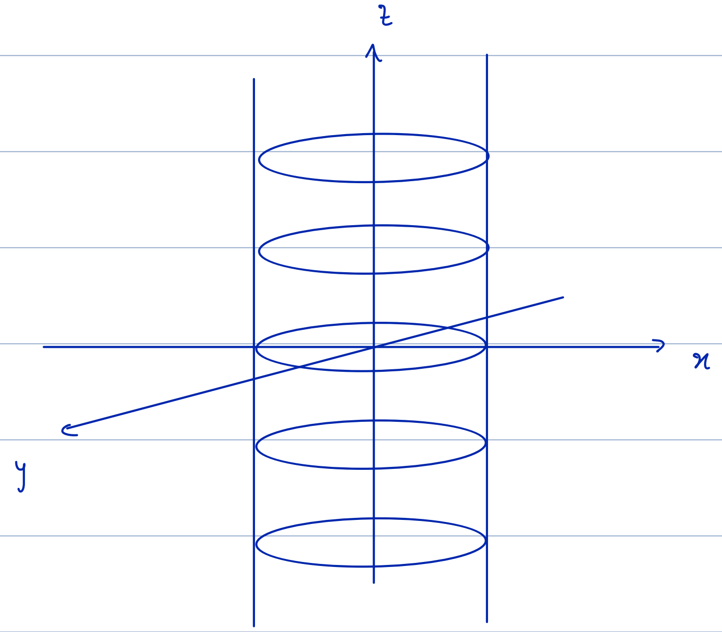
Parabolic Cylinder.

Note: This eq does not contain y ⇒ any plane curve contained within the plane $y = k$ has eq $z = x^2$.

Ex. $x^2 + y^2 = 1$

$$S = \{ (x,y,z) \mid x,y,z \in \mathbb{R},\ x^2+y^2=1 \}.$$

Since z is missing, the plane $z = k$ contains the curve $x^2+y^2=1$



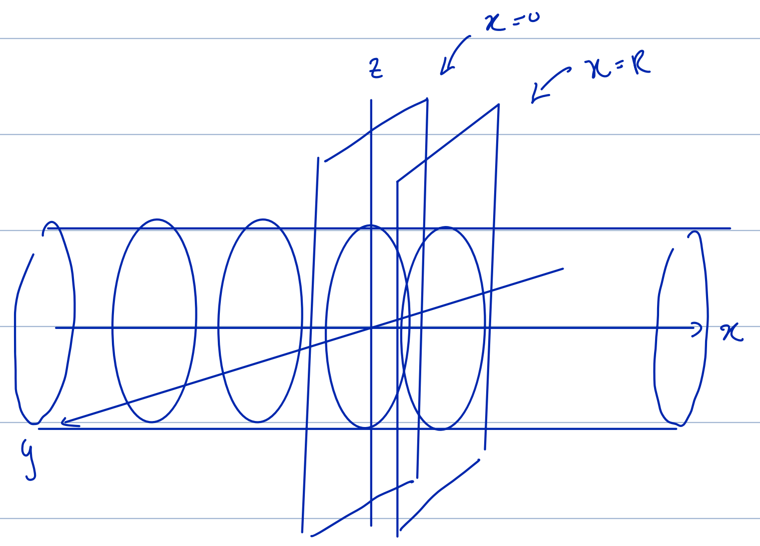
Today: (1) Finish Cylinders

(2) Level Sets / Contours.

Example: $y^2 + z^2 = 1$

In particular, in the plane $x=0$ this a circle.

—— " —— $x = k$ — " ——



Summary: (1) The graph of the fct $f(x,y)$ is the set of pts $(x,y,f(x,y))$ in 3-space

(2) A cross section (trace) of a fct is the one variable fcts obtained by setting x or y equal to a constant.

(3) A level curve of a fct $f(x,y)$ is obtained by setting z = constant.

(4) A cylinder is the result of having one of the variables unspecified such as $f(x,y) = x^2$.

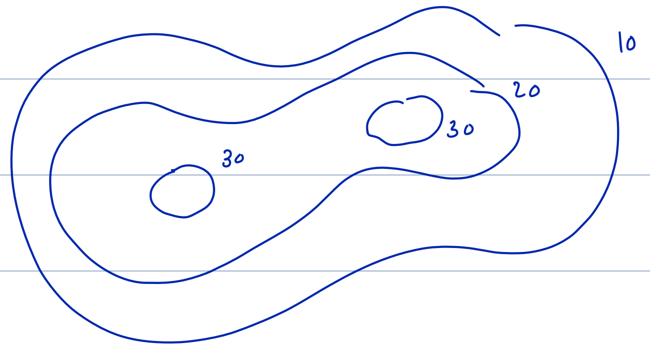
Note: Office Hrs Today 10:10 – 11 Bahen Grad Lounge (6^{th} floor)

Level Curves & Contour Diagrams

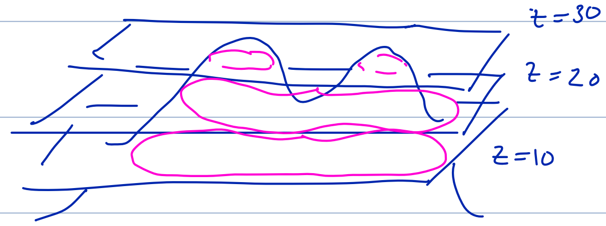
Def: The level curves of a fct f of two variables are the curves with eq's $f(x,y) = k$ where k is a const in the range of f and $f(x,y) = z$.

Intuition: The level curve $f(x,y) = k$ is the set of all pts in the domain of f at which f takes on a given value k.

↳ Think Topographical Map

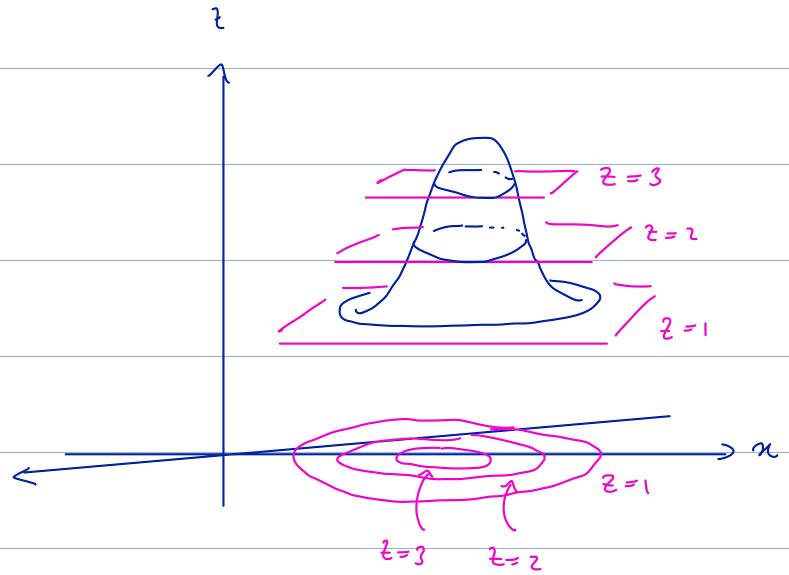


each one of these is a level curve.



Def: A collection of level curves is called a contour map / diagram.

Ex.



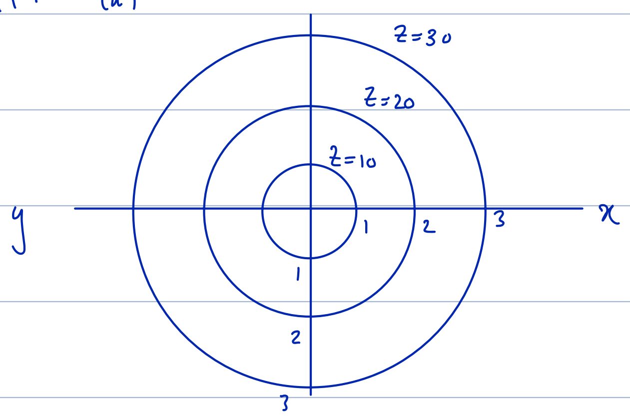
Q: what is the relationship between the level curves $f(x,y) = k$ and traces?

A: The level curves are the traces of f in the plane $z = k$

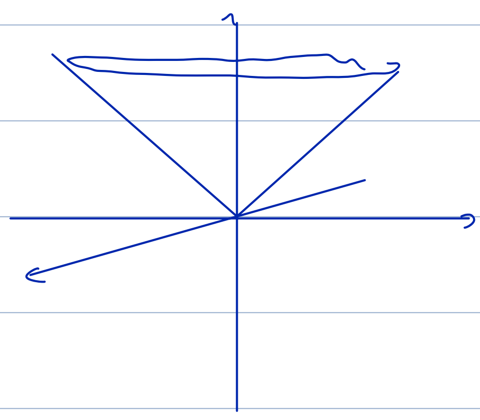
The contour diagram are the level curves projected onto the xy-plane.

Important: Contours are usually drawn for equally spaced values of z.

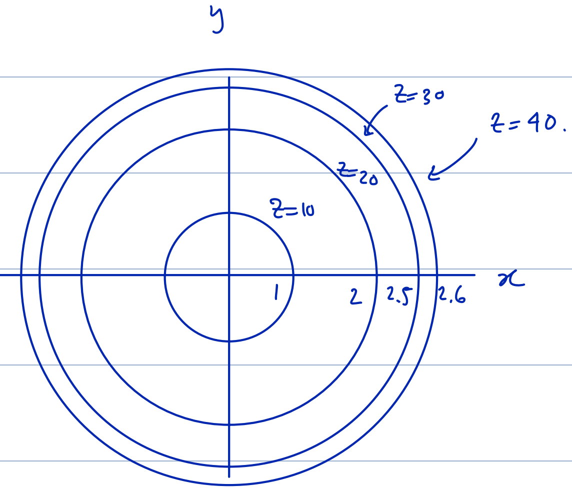
Why is this important: (a)



As a surface in $\mathbb{R}^3$ this is a cone



(b)



As a surface in $\mathbb{R}^3$ this is an elliptic paraboloid