Lecture 2: Functions of 2 Variables & Graphs

Last Time: (1) Road Map of The Course

(2) Described planes in 3D

↳ Sets

(3) Defined the fct

↳ Pyth.

(4) Functions of 1 Var.

↳ Vertical Line Test.

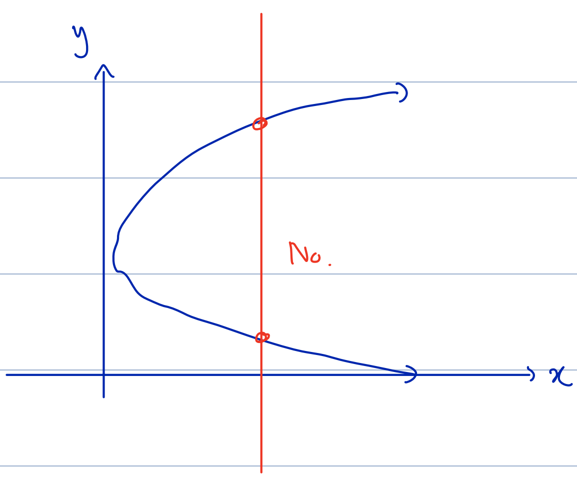
Today: (1) V.L.T

(2) Domains / Ranges of M.V. Functions

(3) Graphs.

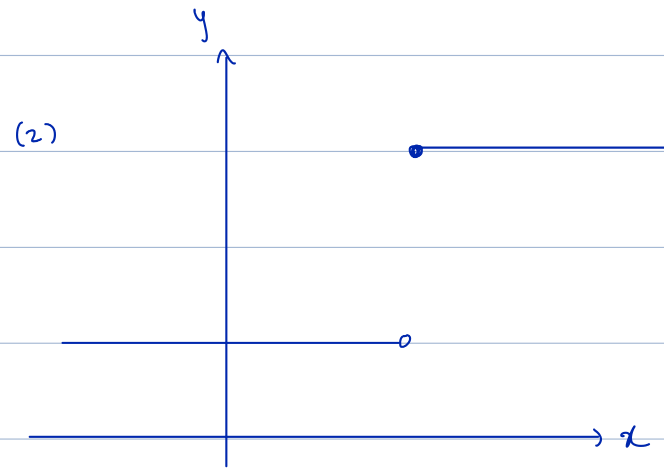
(1) V.L.T

(a)



$y = f(x)$

Aside: To make this a "fct" let t be a parameter $t \in I$ then define $x(t)$, $y(t)$

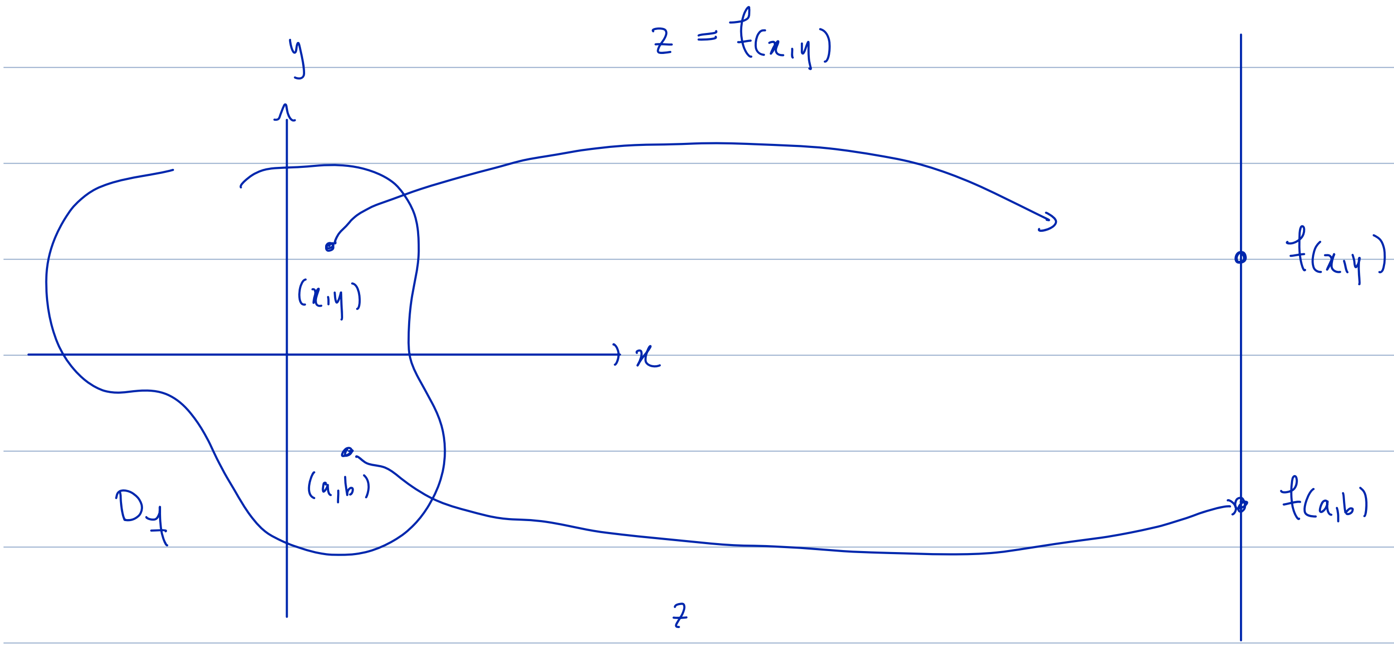


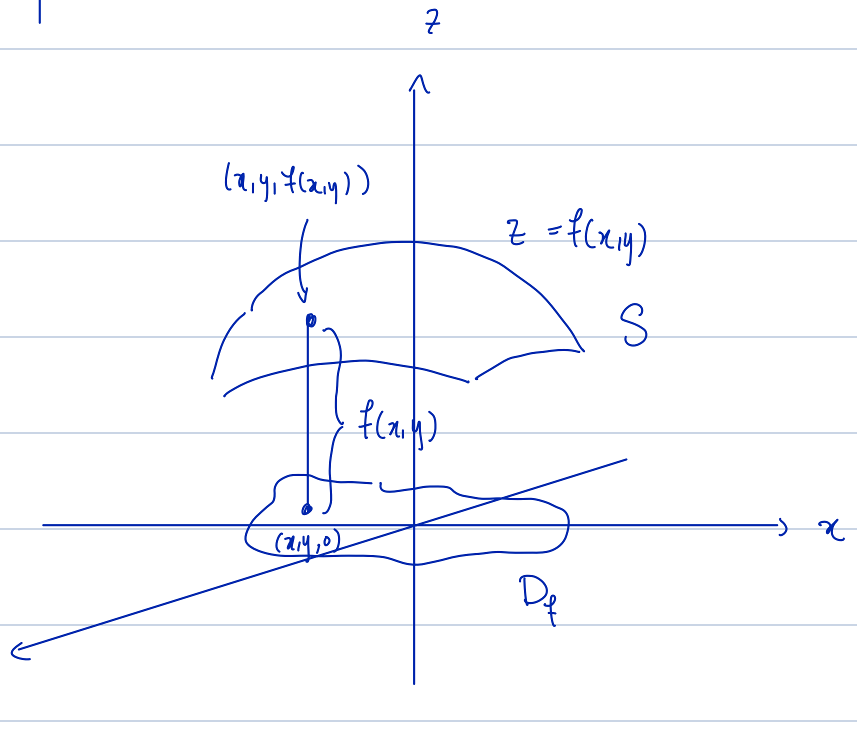
Question: What is the def of a fct with two variables

Def: A fct of two variables is a rule that assigns to each ordered pair of real numbers (x,y) in a set of D a unique real number denoted $f(x,y)$

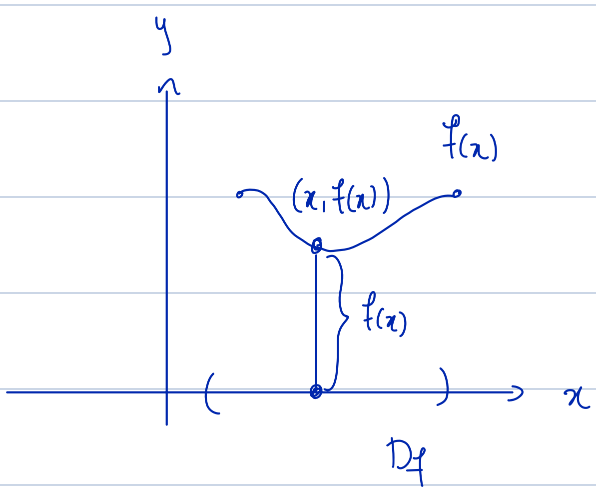
Note: The set D is the domain of f and its range is the set of values that f takes on, that is $R = \{ f(x,y) \mid (x,y) \in D \}$

Remark: We often write $z = f(x,y)$ to make explicit the value taken by f at the general pt (x,y)





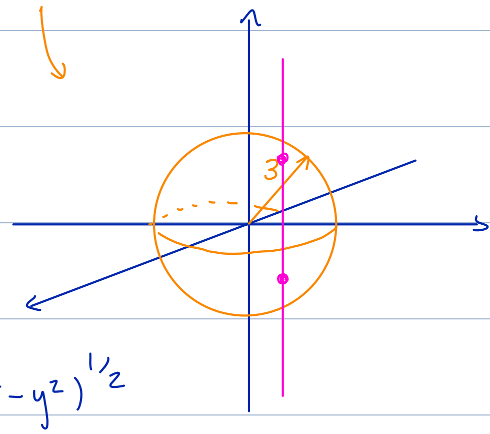
Analogue for 1D functions:



Ex. Sketch the graph of $f(x,y) = (9 - x^2 - y^2)^{1/2}$

$\rightarrow \quad z = (9 - x^2 - y^2)^{1/2} \quad \Rightarrow \quad z^2 = 9 - x^2 - y^2$

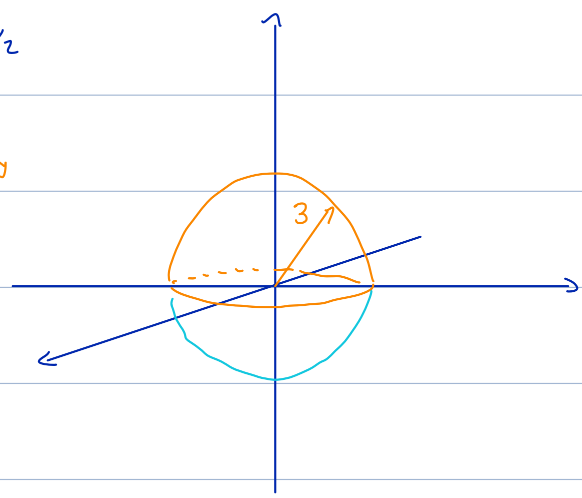
$\Rightarrow \quad x^2 + y^2 + z^2 = 9$



Reversing $z^2 = 9 - x^2 - y^2 \Rightarrow z$

we'll going to get $z = \pm (9 - x^2 - y^2)^{1/2}$

$z = (9 - x^2 - y^2)^{1/2}$

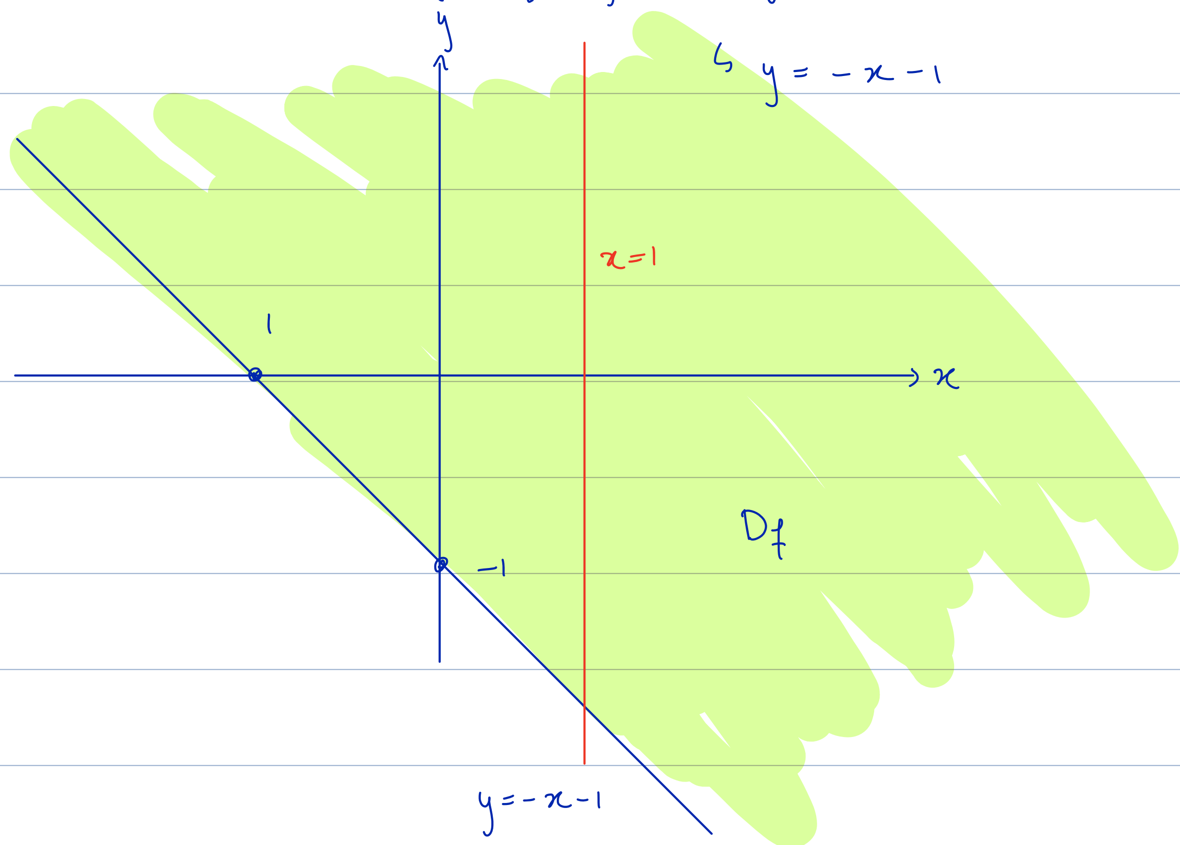


$z = -(9 - x^2 - y^2)^{1/2}$

For each of the following functions find the domain

(a) $f : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x,y) = \dfrac{(x+y+1)^{1/2}}{x-1}$

The function $\frac{1}{x-1}$ is only defined when $x \neq 1$, and $(x+y+1)^{1/2}$ is only defined over $\mathbb{R}$ when $x+y+1 \geq 0$. Thus the admissable domain is $D_f = \{(x,y) \mid x+y+1 \geq 0,\ x \neq 1\}$

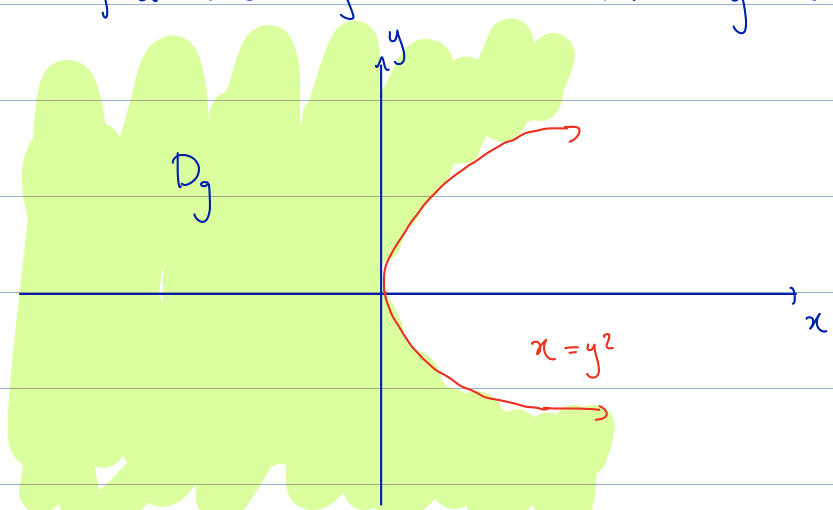


(b) $g: \mathbb{R}^2 \to \mathbb{R}$ given by $g(x,y) = x \ln(y^2 - x)$

This is defined when $y^2 - x > 0$ i.e. $D_g = \{(x,y) \mid x,y \in R,\ x < y^2\}$

$\downarrow$

$x = y^2$



Question: How can we visualize the graphs of fcts of two variables

Answer: Two Methods

(1) Sketch the surface $z = f(x,y)$ in 3D space as before

(2) Draw & label curves in the domain on which f has a value which is constant (level curve / cross sections)

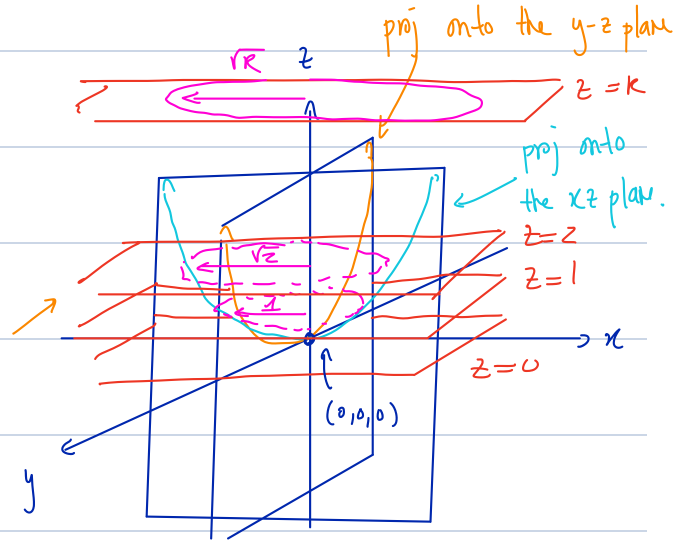
Def: The curves formed by intersecting a surface with planes are called cross sections

Note: The graph of the cross section of f with the plane $x = c$ is the curve we get by intersecting the graph of f with $x = c$. Similarly we can consider the plane $y = c$.

Ex. Sketch the graph of the following functions.

(a) $f(x,y) = x^2 + y^2$

Set $z = f(x,y) \Rightarrow z = x^2 + y^2$



Choose various values for $z = \text{const.}$

$z=0 \Rightarrow 0 = x^2+y^2 \Rightarrow x=y=0 \Rightarrow (0,0,0)$

$z=1 \Rightarrow 1 = x^2+y^2 \Rightarrow$ unit circle in the $z=1$ plane.

$z=2 \Rightarrow 2 = x^2+y^2 \Rightarrow$ circle of radius $\sqrt{2}$ in the $z=2$ plane

$\vdots$

$z=k \Rightarrow k = x^2+y^2 \Rightarrow$ circle of radius $\sqrt{k}$ in the $z=k$ plane.