Lecture 7: Linear Functions & Functions of 3 Variables

Last Time: (1) Discussed linear functions of 1 variables

(2) Started linear functions of 2 variables

(3) OH → Today 10:10 – 11:00 Baker Math Grad Lounge (6^{th} floor).

(4) → Dove' Liana

Today: (1) Finish linear functions of 2 variables

(2) Functions of 3 variables.

Recall: A linear fct of 2 variables is of the form $f(x,y) = z = mx + ny + c$ where $m, n, c \in \mathbb{R}$. $m = \frac{\Delta z}{\Delta x}$ and $n = \frac{\Delta z}{\Delta y}$ and $c = z_0$.

Class 1: To consider a table of values.

x \ y	1	2	3	4
1	2	6	10	14
2	-3	1	5	9
3	-8	-4	0	4
4	-13	-9	-5	-1

Question: How can we obtain the rule of the function from the table of values.

Step 1: Compute Slopes: $m = \dfrac{\Delta z}{\Delta x} = \dfrac{1-6}{2-1} = \dfrac{-5}{1} = -5$

$n = \dfrac{\Delta z}{\Delta y} = \dfrac{6-2}{2-1} = \dfrac{4}{1} = 4.$

Step 2: Sub into $f(x,y) = mx + ny + c$

$\rightarrow f(x,y) = -5x + 4y + c = z.$

Step 3: Find the value c

$\rightarrow$ Sub in any pt. $(1,1,2)$

$2 = -5 + 4 + c \Rightarrow c = 3.$

Step 4: Sub c back into f.

$\rightarrow f(x,y) = -5x + 4y + 3.$

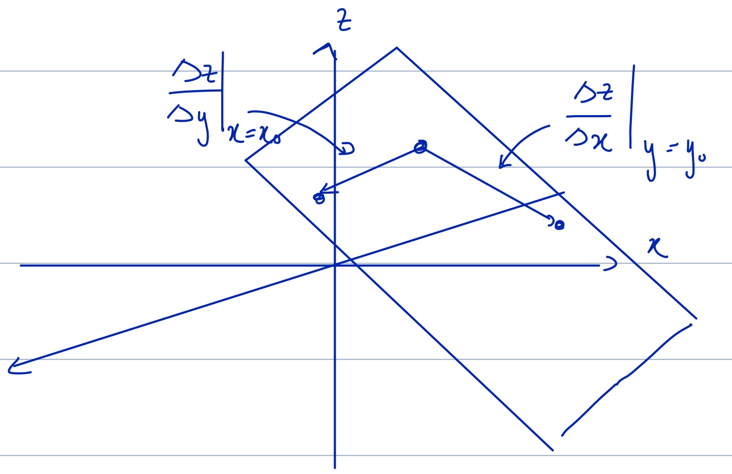
$\blacksquare$

Class 2: Given 3 pts find the eq. & sketch

Ex. $A = (1,0,0)$, $B = (0,2,0)$, $C = (0,0,3)$ be pts in $\mathbb{R}^3$.

Step 1: Find the slopes.

Here you have to be careful since in order to compute $m = \frac{\Delta z}{\Delta x}$ you need y to be the same. and similarly to compute $n = \frac{\Delta z}{\Delta y}$ you need x to be the same.



To compute $m = \frac{\Delta z}{\Delta x}$ Choose the pts A & C. then

$$m = \frac{3-0}{0-1} = -3$$

And similarly to compute $n = \frac{\Delta z}{\Delta y}$ Choose the pts B and C then

$$n = \frac{\Delta z}{\Delta y} = \frac{3-0}{0-2} = -\frac{3}{2}.$$

Step 2: Sub into $f = mx + ny + c$

$\rightarrow f = -3x - \frac{3}{2}y + c.$

Step 3: Find c by using any pt.

$3 = c$

Step 4: Sub c back into f

$\rightarrow f = -3x - \frac{3}{2}y + 3.$

Aside: $w = f(x,y,z) = x^2 + 3y + 4z + 9$ not a hyperplane since it is quadratic in x.

Ex. Let $f(x,y) = x + 2y + 1.$

1. Is f a linear function?

Yes.

2. The following table is the table that contains the values of f. Complete the table.

$x \backslash y$	0	1	2	3	4	5	6
0	1	3	5	.	.	.	.
1	2	4	6				

3. Compare $\frac{\Delta z}{\Delta y}$ for each row

$\frac{3-1}{1-0} = 2$ for pts $(3,1)$ & $(1,0)$

$\frac{5-3}{2-1} = 2$ for pts $(5,2)$ & $(3,1)$

$\vdots$

$\frac{\Delta z}{\Delta y} = 2$ for any two consecutive pts.

4. Compare $\frac{\Delta z}{\Delta x}$ for each row

$\frac{\Delta z}{\Delta x} = 1$ for any two consecutive pts.

5. Takeaways.

For every linear fct all the rows have the same slope and all the columns have the same slope.

Ex. Which of the following are the tables of values for a linear function.

(a)

$x \backslash y$	0	1	2
0	4	1	4
1	1	0	1
2	4	1	4

not a linear function since neither row/column increases linearly.

(b)

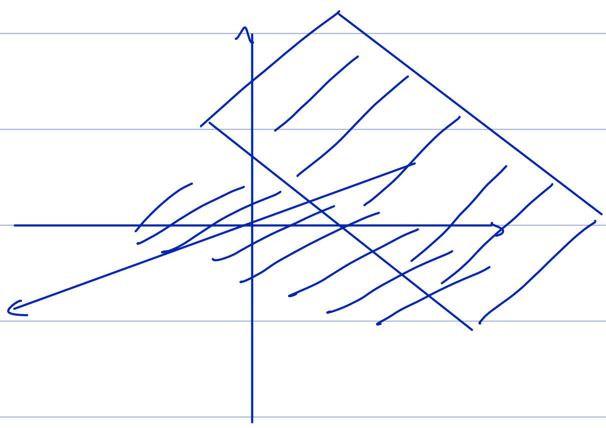
$x \backslash y$	0	1	2
0	10	13	16
1	6	9	12
2	2	5	8

this is a linear function.

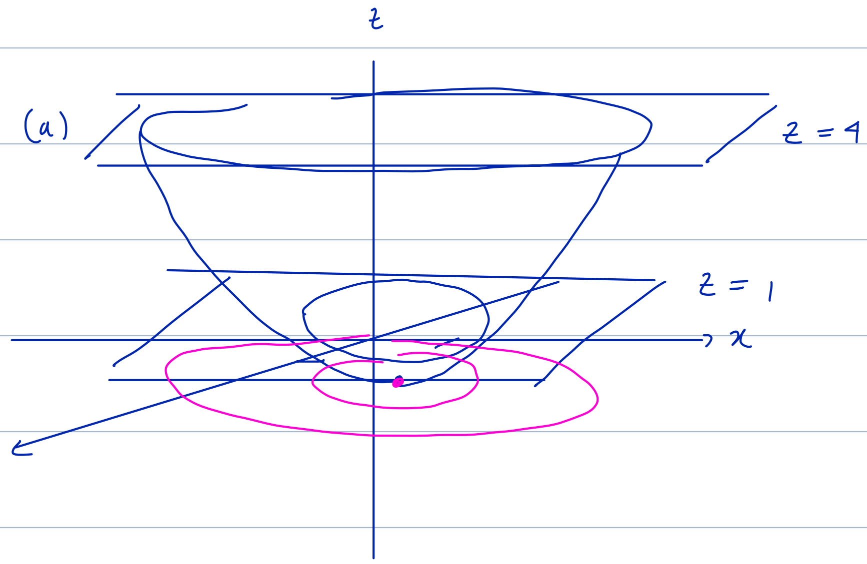
(c)

$x \backslash y$	0	1	2
0	0	5	10
1	2	7	12
2	4	9	14

this is a linear fct.



Lets draw (a)



Summary: How to recognize linear fcts from a table.

- each row/column is linear
- all rows have the same slope.
- all columns have the same slope.

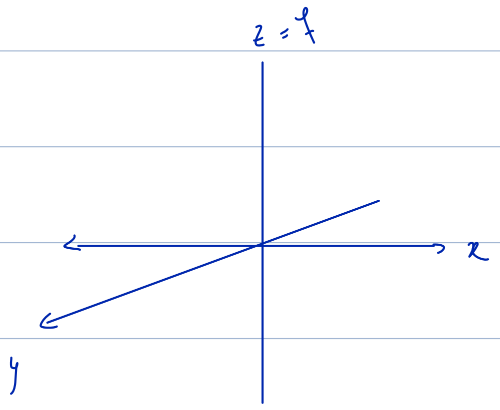
Functions of 3 Variables

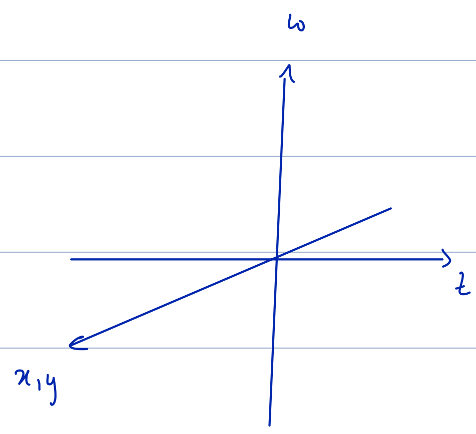
Def: A fct of 3 variables, f, is a rule that assigns to each ordered triple (x,y,z) in the domain $D \subseteq \mathbb{R}^3$ a unique (single) real number denoted by $w = f(x,y,z)$.

Ex. The temp T at a pt on the surface of the Earth depends on the longitude x, the latitude y, and the time t. i.e.

$T = f(x,y,t)$

Remark: Let $f: \mathbb{R}^3 \to \mathbb{R}$ be fct of 3 variables. Then the graph of f is $\{ (x,y,z,w) \in \mathbb{R}^4 \mid w = f(x,y,z) \} \subseteq \mathbb{R}^4$.





Q: If we want to depict fcts of 3 variables what can we look at?

A: Its 3d level surfaces

Def: Let $f: \mathbb{R}^3 \to \mathbb{R}$. The set of pts $(x,y,z) \in \mathbb{R}^3$ where f has a constant value $f(x,y,z) = c$ are level surfaces. In other words

$$f^{-1}(c) = \{ (x,y,z) \in D_f \mid c = f(x,y,z) \}.$$