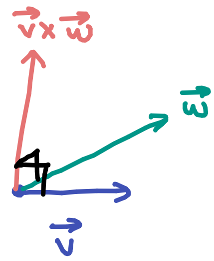
Lecture 14: Fri Oct 4th

Recap: $\vec{v} \times \vec{w} = (\text{Area of parallelogram})\, \vec{n}$ ← unit vector $\perp \vec{v}, \vec{w}$

$= (\|\vec{v}\| \|\vec{w}\| \sin\theta)\, \vec{n}$

$\boxed{\text{Geometric Defn}}$



For the algebraic definition, we need determinants.

① Determinant of a 2x2 matrix

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

E.g. $\begin{vmatrix} 4 & -3 \\ 9 & 7 \end{vmatrix} = 4 \cdot 7 - (-3) \cdot 9$
$= 28 - (-27)$
$= 55$

② Determinant of a 3x3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$\boxed{\text{Algebraic defn}}$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

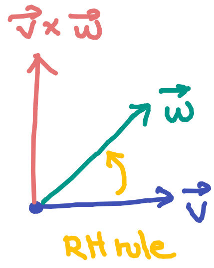
$$= \vec{i} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} - \vec{j} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} + \vec{k} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

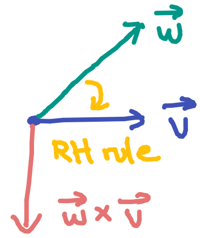
$$= (v_2 w_3 - v_3 w_2)\,\vec{i} - (v_1 w_3 - v_3 w_1)\,\vec{j} + (v_1 w_2 - v_2 w_1)\,\vec{k}.$$

Example: Find $(2, -3, 5) \times (4, 1, -3)$. (Will probably skip this in the lecture)

Properties of $\times$

1) $\vec{v} \times \vec{w} = -(\vec{w} \times \vec{v})$
2) $(\lambda \vec{v}) \times \vec{w} = \lambda(\vec{v} \times \vec{w}) = \vec{v} \times (\lambda \vec{w})$
3) $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$





Example: Find unit vector perpendicular to both $\vec{v} = 3\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{w} = 3\vec{i} + 4\vec{k}$.

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Example: Find the equation of the plane containing 3 points: $P = (1, 2, -1)$, $Q = (2, 3, 0)$ and $R = (3, -1, 2)$.

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Example: Find the area of the parallelogram with vertices

$P=(2,1,1)$, $Q=(3,3,0)$, $R=(4,0,2)$, $S=(5,2,1)$.

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Example: Find the plane that contains $(1,0,-1)$ and the line $x-2y=4$ in the xy-plane.

A:// Method 1: Using the cross product (More steps but works for general lines)

For more examples like this, see Questions 9-12 of 13.3-13.4 practice worksheet.

Method 2: Using the fact that the line is in the xy-plane

Bonus: Determinant of 4x4 matrices

$$
\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}
= a \begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} & \cancel{d} \\ \cancel{e} & f & g & h \\ \cancel{i} & j & k & l \\ \cancel{m} & n & o & p \end{vmatrix}
- b \begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} & \cancel{d} \\ e & \cancel{f} & g & h \\ i & \cancel{j} & k & l \\ m & \cancel{n} & o & p \end{vmatrix}
+ c \begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} & \cancel{d} \\ e & f & \cancel{g} & h \\ i & j & \cancel{k} & l \\ m & n & \cancel{o} & p \end{vmatrix}
$$

$$
- d \begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} & \cancel{d} \\ e & f & g & \cancel{h} \\ i & j & k & \cancel{l} \\ m & n & o & \cancel{p} \end{vmatrix}
$$

$$
= a \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix}
- b \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix}
+ c \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix}
- d \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}
$$

$= \; \ldots$ Now compute each 3x3 determinant