Lecture 3: Graphs & Cylinders.

Last Time: (1) D_f & R_f

↳ Q: What is the admissable domain.

(2) Graphs (Representations)

↳ Two Ways – Sketch the surface $z = f(x,y)$

– Draw curves of f for which z, x, y are const

(level / cross sections).

(3) Elliptic Paraboloid.

↳ level curves.

Today: (1) Formalize This

(2) A few more examples

(3) Cylinders.

Def: A level curve $z = f(x,y)$ is obtained by setting z = const.

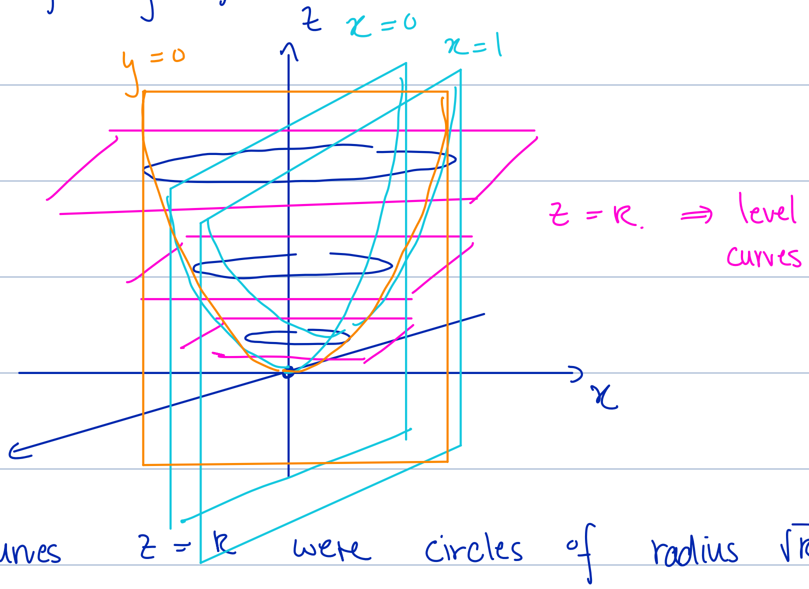
(think topographical map)

Def: A cross section (trace) is obtained by intersecting $z = f(x,y)$ with planes parallel to the coordinate axes i.e. x = const or y = const.

Ex. Sketch the graph of the following functions

(a) $f(x,y) = x^2 + y^2$.

$z = x^2 + y^2$.



We saw that level curves $z = k$ were circles of radius $\sqrt{k}$.

Cross sections (traces) are obtained by setting $x = \text{const}$ or $y = \text{const}$.

$x = 0 \Rightarrow z = y^2$

$x = 1 \Rightarrow z = 1 + y^2$

$\vdots$

$x = l \Rightarrow z = l^2 + y^2$

parabola's opening upwards and translated up by l^2.

Recall: $x = 0 \Rightarrow S = \{ (x,y,z) \mid y,z \in \mathbb{R},\ x = 0 \}$

$y = 0 \Rightarrow z = x^2$

$y = 1 \Rightarrow z = x^2 + 1$

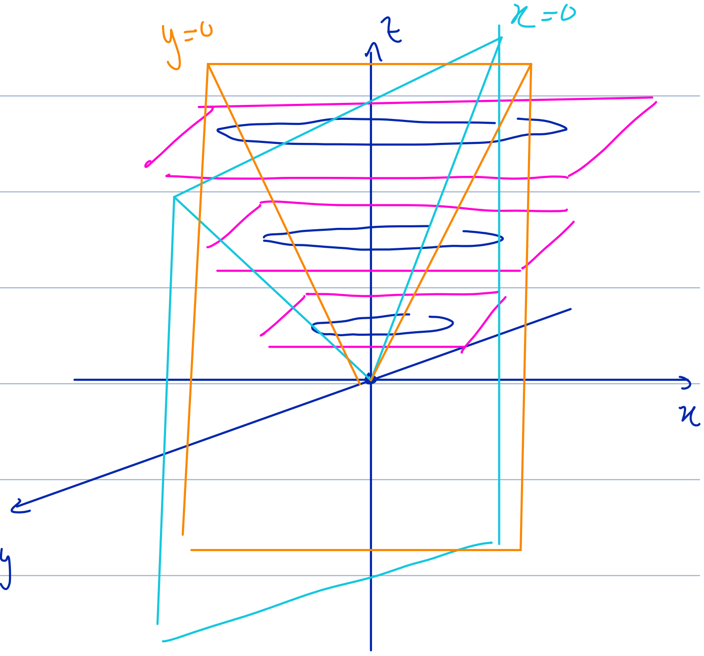
$\vdots$

$y = m \Rightarrow z = x^2 + m^2$

parabolas opening upwards and translated up by m^2

This is called an "elliptic paraboloid."

b) $g(x,y) = (x^2+y^2)^{1/2}$



$z = (x^2+y^2)^{1/2} \Rightarrow z^2 = (x^2+y^2)$

Level Curves $\Rightarrow$ $z = k$

$z = k = (x^2+y^2)^{1/2} \Rightarrow k^2 = x^2+y^2 \Rightarrow$ circles of radius k.

$z = 0 \Rightarrow 0 = (x^2+y^2)^{1/2} \Rightarrow x = y = 0$ pt.

$\vdots$

$k^2 = x^2+y^2 \Rightarrow$ circles of radius k.

Cross Sections $\Rightarrow$ $x = l$ or $y = m$

$x = l \Rightarrow z = (l^2+y^2)^{1/2} \Rightarrow z^2 = l^2+y^2$

$x = 0 \Rightarrow z = (y^2)^{1/2} = \pm y$ linear lines through $\underset{\text{origin}}{O}$ $\underset{\text{with}}{\overline{w}}$ slope ± 1

$\vdots$

$x = l \Rightarrow z^2 = l^2 + y^2 \rightarrow$ hyperbola opening up

$y = m \Rightarrow z^2 = m^2 + x^2$

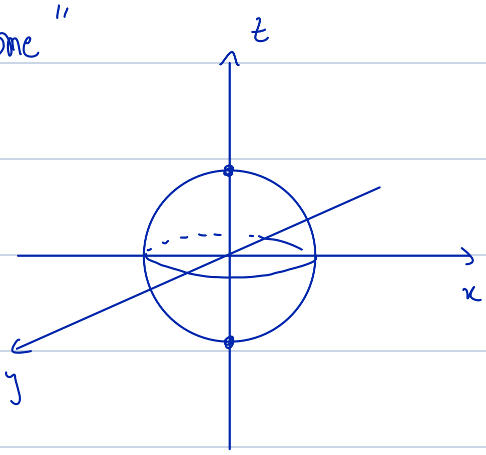
$y = 0 \Rightarrow z^2 = x^2 \Rightarrow z = \pm x$ linear lines through $\vec{0}$ $\bar{w}$ slope ± 1

$\vdots$

$y = m \Rightarrow z^2 = m^2 + x^2 \longrightarrow$ hyperbola opening up.

This is a "cone"

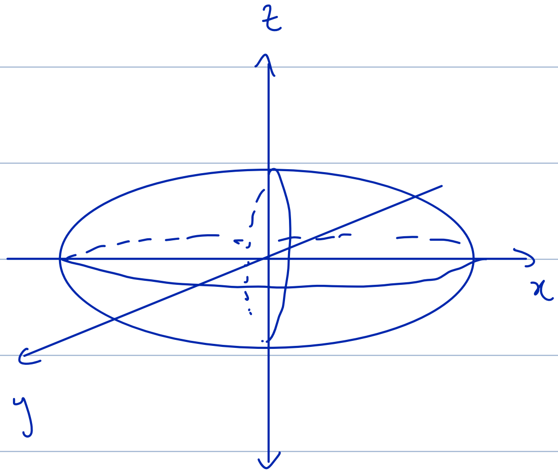
Intuition.



$z = k \Rightarrow$ circles

$x = l \Rightarrow$ circles

$y = m \Rightarrow$ circles



$z = k \Rightarrow$ ellipses

$x = l \Rightarrow$ ellipses / circles

$y = m \Rightarrow$ ellipses / circle.

Better Textbook $\rightarrow$ Stewart Multi. Var Calculus.

Exercise: Use traces (cross sections) to clasify the surface $x^2 + 2z^2 - 6x - y + 10 = 0$

Cylinders

Def: A cylinder is a surface that consists of all lines which a parallel to a given lines and passing through a given plane curve.

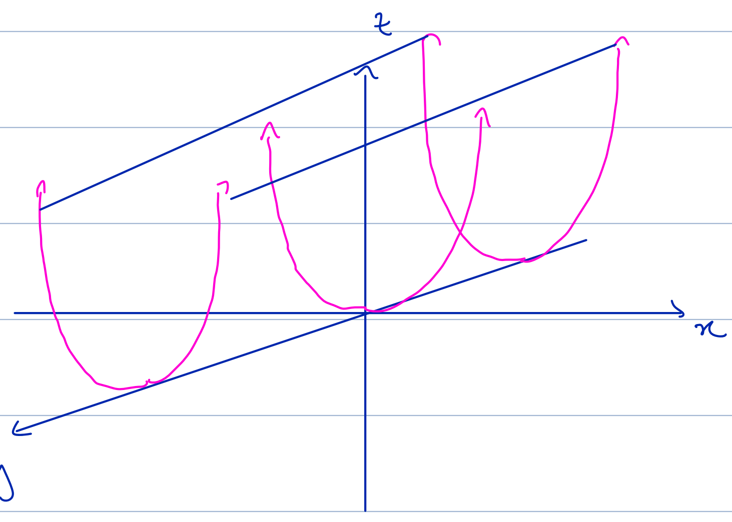
Ex.. Sketch the graph of the surface $z = x^2$.

$z = 0 \Rightarrow S = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$

Q: What is this as a set.

A: $S = \{(x, y, z) \mid z = x^2\} \sim \{(x, y, x^2) \mid x, y \in \mathbb{R}\}$

Parabolic Cylinder



Note: The eq does not include $y \Rightarrow$ any plane w eq $y = k$ (parallel to the xz plane) intersects the graph in a curve with eq $z = x^2$.

Ex. Sketch $x^2+y^2=1$

Since z is "missing", $z=k$ represents a circle with radius 1 in the plane $z=k$