

University of Toronto – Faculty of Arts & Science –
MAT235Y1: Multivariable Calculus

Term Test 1 – Fall 2024/Winter 2025

Family Name (PRINT): _____

Given Name(s) (PRINT): _____

Student Number: _____

U of T Email: _____

This exam contains **8** pages (including this cover page) and **6** problems. Once the exam begins, check to see if any pages are missing. There are **50** possible points to be earned in this exam.

- Duration: **90 minutes**
- **No aids or calculators are permitted on the exam.**
- Power off all cell phones, smartphones, and other electronic devices, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
- **Do not tear any pages off this exam.**
- **One scrap page is provided at the end.** This page will not be graded unless specifically indicated. Please enter all of your answers in the space provided.
- Do not write in the page margins. Make sure that your writing is dark enough to be readable.
- **Unsupported answers to long answer questions will not receive full credit.** A correct answer without explanation will receive no credit unless otherwise noted; an incorrect answer supported by substantially correct calculations and explanations may receive partial credit.
- **Organize your work** in a reasonably neat and coherent way.
- You must use the methods learned in this course to solve all of the problems.

1. (6 points) Match the given equations with one of the following contour diagrams. Write **(I)**, **(II)**, **(III)**, **(IV)** or **(V)** for each equation. Only your final answer will be graded. Each part is worth 1.5 marks.

(a) $f(x, y) = 2 - x^2 - y^2$

Answer _____

(b) $f(x, y) = 6 - x^2$

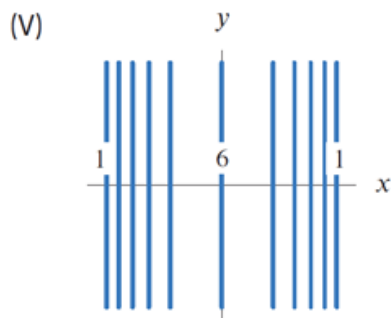
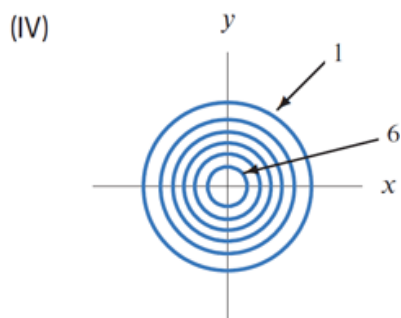
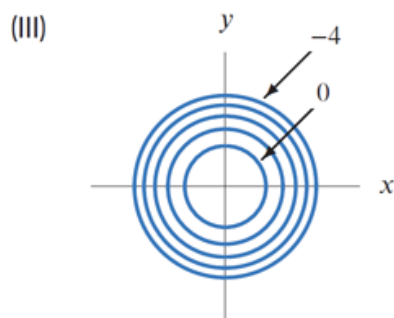
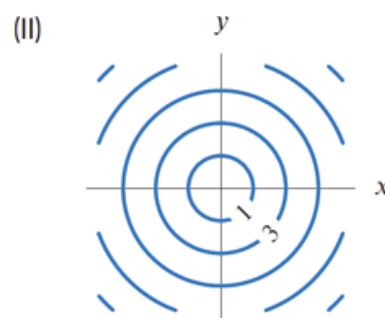
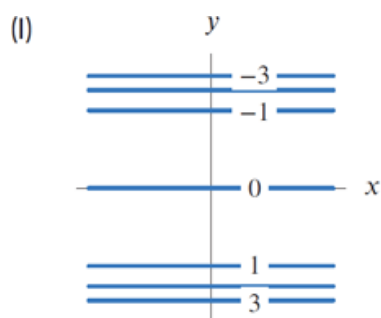
Answer _____

(c) $f(x, y) = \sqrt{x^2 + y^2}$

Answer _____

(d) $f(x, y) = -y^3$

Answer _____



2. (7 points) An oceanographer is studying the temperature of a coastal region. They collect data on the ocean temperature (T) in $^{\circ}\text{C}$ at various depths (d) in metres and distances from the shore (x) in kilometres.

- (a) (3 points) The collected data is presented in the following table of values. However, two data points T_1 and T_2 are missing:

$x \backslash d$	5	10	20	30
10	18	17	15	13
20	15	14	T_1	10
30	12	11	9	7
60	T_2	2	0	-2

The oceanographer suspects that temperature is a linear function of depth and distance. Find values for T_1 and T_2 which make the above table linear. Put your final answer in the indicated box. Only your final answers will be graded for this part.

$$T_1 = \boxed{}$$

$$T_2 = \boxed{}$$

- (b) (4 points) The oceanographer decides to incorporate time into their model, as they noticed temperature variations throughout the day. They now consider temperature as a function of depth (d), distance from shore (x), and time (t) in hours since midnight:

$$T(d, x, t) = 20 - 0.2d - 0.1x + 2 \sin(\pi t/12).$$

For a fixed time of day t , describe the general shape of the level surfaces of this function when $T = 15^{\circ}\text{C}$. How do they change throughout the day? Give a brief description.

3. (10 points) (a) (6 points) Find the set of points of continuity of the function

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0), (1, 1) \\ 1, & (x, y) = (1, 1) \\ 2, & (x, y) = (0, 0) \end{cases}$$

Justify your answer.

- (b) (4 points) Use the Squeeze Theorem to evaluate the limit

$$\lim_{(x,y) \rightarrow (0,0)} 2x^2 y^4 \sin(\ln(x^2 + y^2)).$$

Give a full justification.

4. (9 points) Alice pushes a heavy crate across a warehouse floor. The force applied to the crate is represented by the vector $\vec{F} = \langle 4, 3, -2 \rangle$, and the crate moves along the path represented by the displacement vector $\vec{d} = \langle 2, 1, 0 \rangle$.

(a) (3 points) Calculate the work done by Alice in moving the crate. (Hint: Recall that work is given by the dot product of force with displacement.)

(b) (3 points) Find an expression for the angle between \vec{F} and \vec{d} . Your final expression should not involve the terms \vec{F} or \vec{d} . (You do not need to fully simplify your answer.)

(c) (3 points) Suppose Bob decides to apply a force of $\vec{G} = \langle 4, 3, c \rangle$, where c is an unknown constant. The crate moves along the same path as before, given by the displacement vector $\vec{d} = \langle 2, 1, 0 \rangle$. Bob wants to do the same amount of work as Alice but wants to minimize the magnitude of \vec{G} . Which value of c should Bob choose? Give a brief justification.

5. (9 points) Consider a plane P in three-dimensional space. Assume that we have the following information about P :

- Two points on the plane P are given: $A(1, 2, -1)$ and $B(3, 0, 2)$.
- There is another plane Q with equation $2x - y + 3z = 6$ which is perpendicular to P .

(a) (2 points) Find a vector \vec{v} on the plane P using the two given points.

(b) (4 points) Find a normal vector \vec{n} to the plane P .

(c) (3 points) Using the above information, find an equation of the plane P . Express your final answer in the form $ax + by + cz = d$ for some constants a, b, c and d .

6. (9 points) A meteorologist wants to model the relationship between altitude (h , in kilometres), latitude (θ , in degrees north of the equator), and average annual temperature (T , in $^{\circ}\text{C}$) in a particular region. Assuming $T = f(h, \theta)$, the collected data is presented in the following table:

$h \setminus \theta$	30	35	40	45	50
0	25.0	22.5	20.0	17.5	15.0
1	19.0	16.5	15.0	11.5	9.0
2	12.0	10.5	8.0	5.5	3.0
3	6.0	4.5	2.0	-0.5	-3.0
4	1.0	-1.5	-4.0	-6.5	-9.5

- (a) (2 points) Give the definition of the partial derivative $f_{\theta}(a, b)$ for a general point (a, b) .
- (b) (4 points) Estimate the values of the partial derivatives $f_{\theta}(4, 40)$ and $f_h(4, 40)$ using the given data.
- (c) (3 points) Using your answer from part (b), give a practical interpretation of the value $f_h(4, 40)$. Include units in your interpretation.

Do not tear this page off. This page is for additional work and will not be graded unless you clearly indicate it on the original question page.