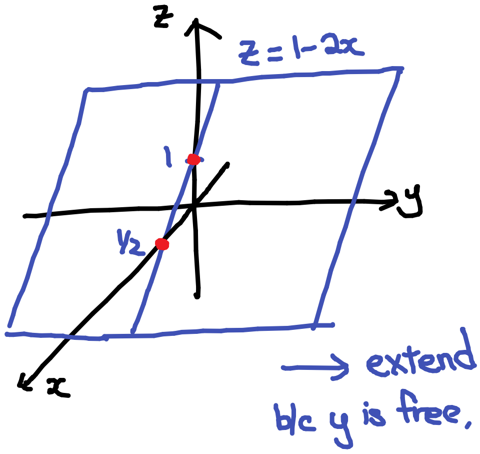
Lecture 7: Wed Sept 17th

Recap: linear equations $z = z_0 + m(x-x_0) + n(y-y_0)$

Q: How to draw planes with < 3 intercepts?



A: a) $z + 2x = 1$

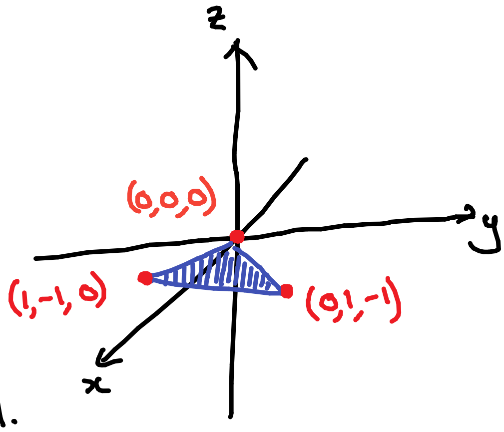
Only x & z-intercepts, which are

$(\frac{1}{2}, 0, 0)$ and $(0,0,1)$

In the lecture, I made a mistake and said $(\frac{1}{2}, y, 0)$ and $(0, y, 1)$. We are looking at where the plane meets x-axis & z-axis. So, set $y=0$ there.

Draw $z = 1-2x$ on xz-plane and extend along y-axis.

b) $x+y+z=0$



Only $(0,0,0)$ is the intercept.

Find 2 other points $(1,-1,0)$ and $(0,1,-1)$.

Plot them and $(0,0,0)$. Join them with lines.

You just need 3 pts on the plane to draw it.

So, $(3,-2,-1)$, $(2,-1,-1)$ and $(-1,-1,2)$ work as well.

Q: a) What values A and B make this table linear?

x\y	5	10	20	30
10	20	19	17	15
20	17	16	B	12
30	14	13	11	9
60	A	4	2	0

A: $B = 14$

$A = 5$

b) Find the linear equation.

A: $m = \frac{\Delta z}{\Delta x} = \frac{-3}{10}$

$n = \frac{\Delta z}{\Delta y} = \frac{-1}{5}$

Choose a pt $(x_0, y_0, z_0) = (10, 5, 20)$

$z = z_0 + m(x-x_0) + n(y-y_0)$

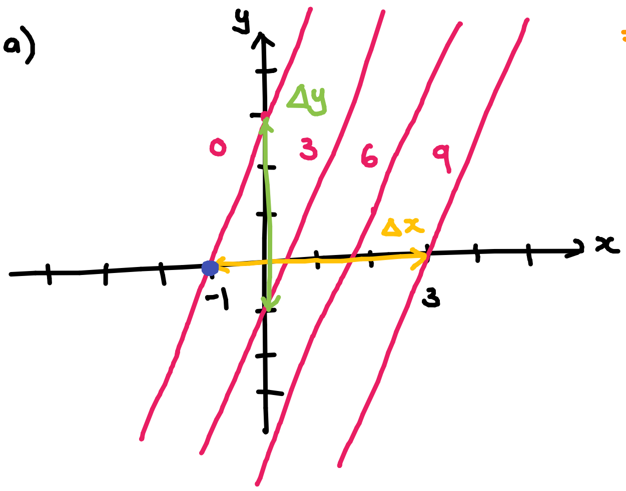
$= 20 - \frac{3}{10}(x-10) - \frac{1}{5}(y-5)$.

$= 20 - \frac{3}{10}x + 3 - \frac{1}{5}y + 1$

$= 24 - 0.3x - 0.2y$

Example: Find the linear equation from the contour diagram.

a)



A: Moving from $x=-1$ to $x=3$ takes us from $z=0$ contour to $z=9$ contour.

$$m = \frac{\Delta z}{\Delta x} = \frac{9-0}{3--1} = \frac{9}{4}.$$

Moving from $y=-1$ to $y=3$ takes us from $z=3$ contour to $z=0$ contour.

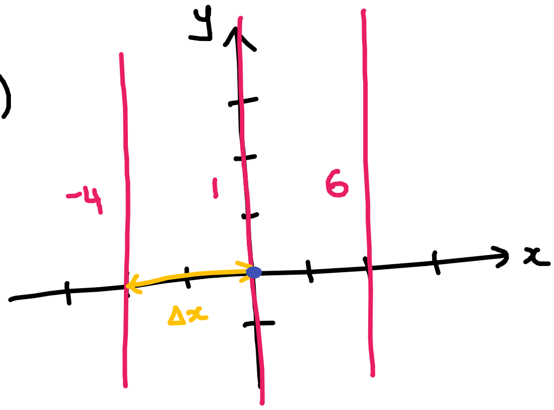
$$n = \frac{\Delta z}{\Delta y} = \frac{0-3}{3--1} = \frac{-3}{4}.$$

Choose point • $= (x_0, y_0, z_0) = (-1, 0, 0)$

$\therefore\ z = 0 + 9/4(x+1) + 3/4\, y$

$= 9/4 + 9/4\, x + 3/4\, y.$

b)



A: $m = \frac{\Delta z}{\Delta x} = \frac{1--4}{0--2} = \frac{5}{2}$

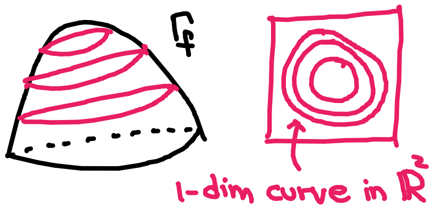
$n = \frac{\Delta z}{\Delta y} = \frac{0}{\Delta y} = 0$

• $= (x_0, y_0, z_0) = (0, 0, 1)$

$\therefore\ z = 1 + 5/2\, x$

§12.5: Level surfaces

Recall: The graph of $z = f(x,y)$ is a 2d surface in $\mathbb{R}^3$. It can be described by a family of level curves/contours $c = f(x,y)$ in $\mathbb{R}^2$ for varying c.



The graph of $w = g(x,y,z)$ is a 3d surface in $\mathbb{R}^4$. It can be described by a family of level surfaces/sets in $\mathbb{R}^3$ for varying c.

Defn: A level surface/set of a 3-variable function $w = g(x,y,z)$ is of the form $g(x,y,z) = c$, for a constant c.

Example: a) $w = g(x,y,z) = 5x + 2y + 3z$

For each c, the level surface $5x+2y+3z=c$ is a plane.

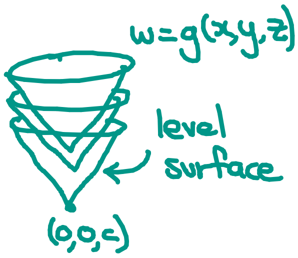
$\therefore\ w = g(x,y,z)$ can be described by a family of planes.

b) $w = g(x,y,z) = z - \sqrt{x^2+y^2}$

For each c, the level surface $c = z - \sqrt{x^2+y^2}$ is a cone whose vertex is at $(0,0,c)$.

$$z = \sqrt{x^2+y^2} + c$$

$\therefore$ $w = g(x,y,z)$ can be described by a family of cones moving up vertically as c increases.



Stacked icecream cones