Lecture 1 : Sept 3rd

§12.1: What is a function?

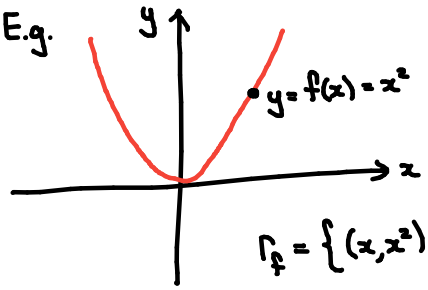
In single variables, we study $\mathbb{R}$-valued functions of 1 variable

$$f : X = \mathbb{R} \longrightarrow Y = \mathbb{R}$$

Functions assign one element of X to exactly one element of Y.
Here, X, Y = sets in general.

They can be visually represented by a graph:

E.g.

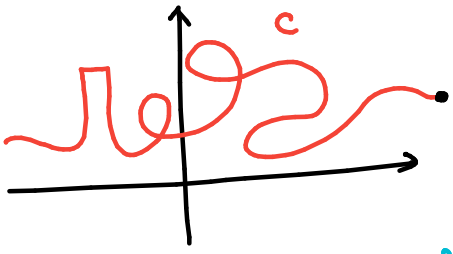


Graph $\Gamma_f = \{(x, f(x)) \in X \times Y\}$

(curves in $\mathbb{R}^2$ for single variable functions f)

$$\Gamma_f = \{(x, x^2) \in \mathbb{R} \times \mathbb{R}\}$$

Consider a particle moving in a plane.



Is C the graph of a function?

No. One x-value is mapped to 2 different y-values.

Fix: view C as the graph of $f : \mathbb{R} \longrightarrow \mathbb{R}^2$, where each coordinate is a function of t.

$$t \longmapsto (x(t), y(t))$$

E.g. $f(t) = (t^2, \sin(t))$.

Such curves are called parametric curves. (We'll study these later.)

> Multivariable functions:

Consider $f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$.

$$z = f(x, y)$$

(function; dependant variable; independant variables)

can be represented graphically, algebraically or numerically.

Example: (Numerical)

$f(w,T)$ = wind chill temp. (°C)

w = wind speed (km/h)

T = temp. (°C)

w \ T	20	15	10	5
5	19	13	7	1
10	15	9	3	-4
15	13	6	0	-7
20	11	4	-2	-9

← $f(w,T)$-values

a) Find and interpret $f(20, 5)$.

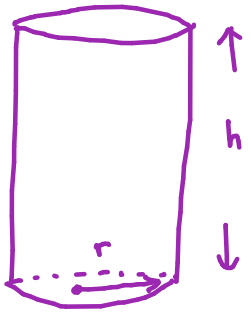
A: $f(20, 5) = -9$°C.

When the wind speed is 20 km/h and the temperature is 5°C, the wind chill is -9°C.

b) How fast does the wind have to blow for it to feel like 9°C when the actual temp. is 15°C?

A: $w = 10$ km/h

Example: (Algebraic)

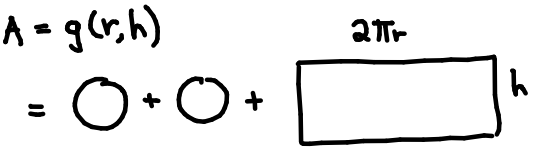


Volume $V = f(r,h)$

= Area of base × height

$= \pi r^2 h$

Surface area $A = g(r,h)$



$= 2\pi r^2 + 2\pi rh$