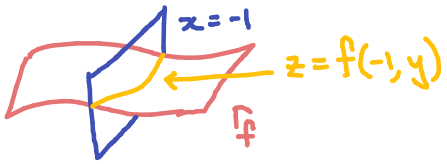
Lecture 4: Wed Sept 10th



Recap: Cross-sections

Example: 12.2.19: Let $z = f(x,y) = y^3 + xy$. Draw the cross-sections of f at

a) $x = -1, 0, 1$ — slice f with the $x=-1$ plane

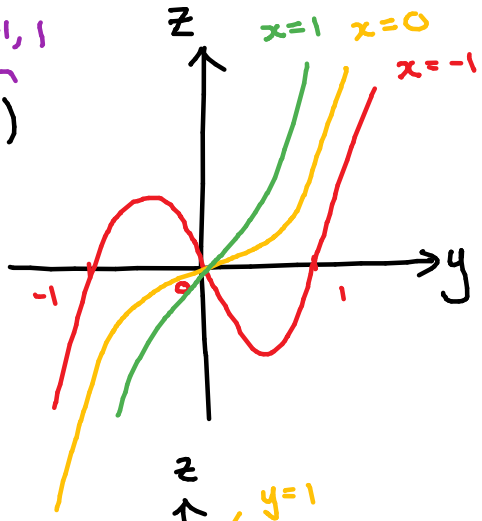
roots at $0, -1, 1$

A: $x=-1$: $z = f(-1,y) = y^3 - y = y(y^2-1) = y(y+1)(y-1)$

$x=0$: $z = f(0,y) = y^3$

$x=1$: $z = f(1,y) = y^3 + y = y(y^2+1)$

root at 0; no real roots

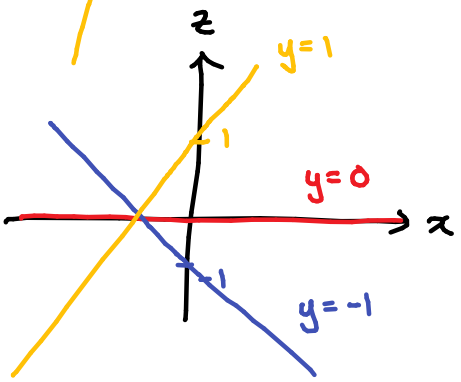


b) $y = -1, 0, 1$

A: $y=-1$: $z = f(x,-1) = -1 - x$

$y=0$: $z = f(x,0) = 0$

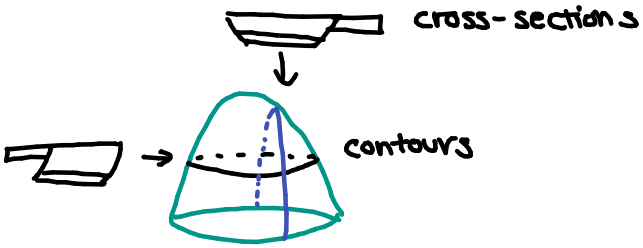
$y=1$: $z = f(x,1) = 1 + x$



§ 12.3 Contour diagrams

Contour lines or level curves $f(x,y) = c$

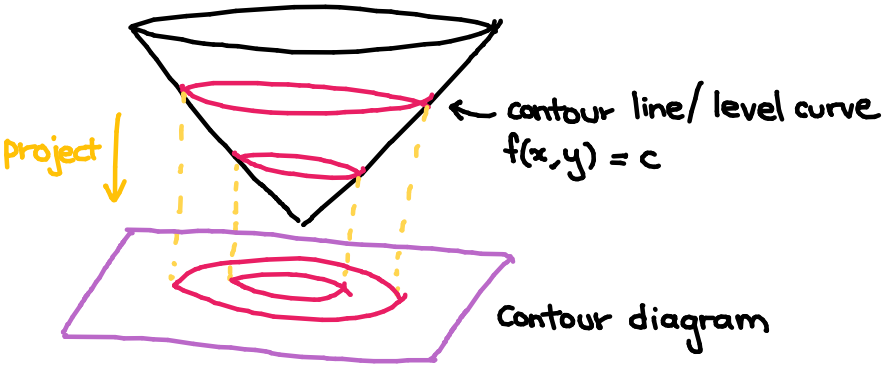
= obtained by slicing the graph of f with the plane $z = c$.



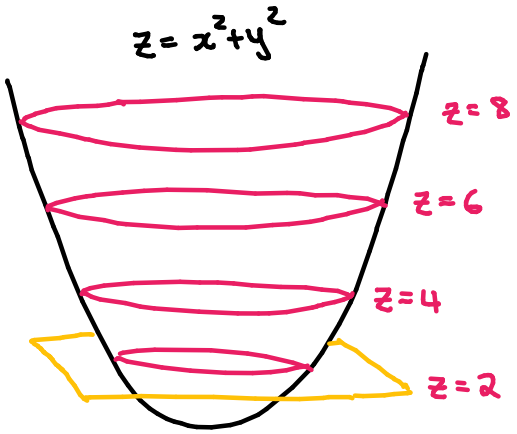
Contour map = collection of level curves

= obtained by plotting all of the level curves on the same page

OR projecting all of the level curves down to the same page.



Example:

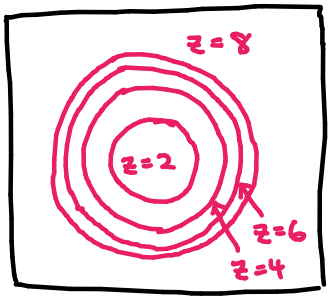


Equations for contours	Graphs
$f(x,y) = x^2 + y^2 = 8$	circle of radius $\sqrt{8} = 2.83$
$f(x,y) = x^2 + y^2 = 6$	circle of radius $\sqrt{6} = 2.45$
$f(x,y) = x^2 + y^2 = 4$	circle of radius 2
$f(x,y) = x^2 + y^2 = 2$	circle of radius $\sqrt{2} = 1.41\ldots$

diff = 0.38

diff = 0.45

diff = 0.69



contour diagram

As we can see,

distance between contours get smaller as z increases

$\Rightarrow$ surface gets steeper as z increases.

Slogan: contours growing closer = steeper

Why:

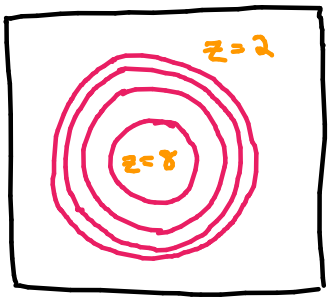
The vertical/z-distance between contours are the same. (Here, $\Delta z = 2$.)

Closer contours mean that less horizontal distance is being travelled for the same vertical distance. Δz ← constant

$\Rightarrow$ Slope $\approx \frac{\text{rise}}{\text{run}} = \frac{\Delta \text{vertical}}{\Delta \text{horizontal}}$ is getting bigger. The surface gets steeper.

growing smaller

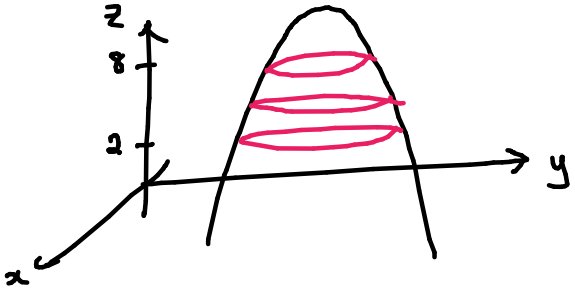
What about:



Contour diagram

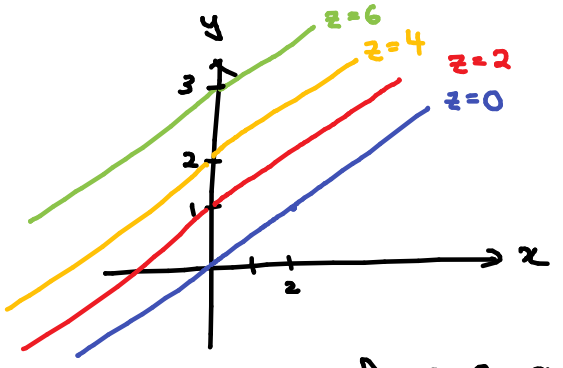
$\Rightarrow$ contours grow closer as z decreases.

$\Rightarrow$ Surface gets steeper as z decreases.



Example: Make a contour plot for $z = 2y - x$ with at least 3 contours.

$z = 0$	$0 = 2y - x$	$y = \frac{1}{2}x$
$z = 2$	$2 = 2y - x$	$y = \frac{1}{2}x + 1$
$z = 4$	$4 = 2y - x$	$y = \frac{1}{2}x + 2$
$z = 6$	$6 = 2y - x$	$y = \frac{1}{2}x + 3$



Contour diagram for $z = 2y - x$

It can be tricky but you can use the contours to visualise the surface.

At $z = 0$, place a line $y = \frac{1}{2}x$.

At $z = 2$, place a line $y = \frac{1}{2}x + 1$. This is just the previous line but it moves along the y-axis to $y = 1$.

At $z = 3$, place a line $y = \frac{1}{2}x + 2$.

And so on...

(See 3d Desmos)