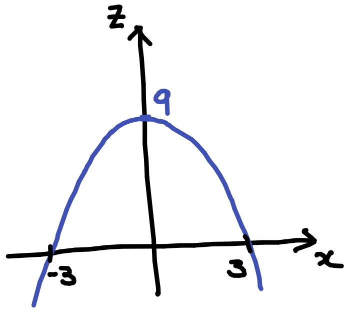
Lecture 18: TT1 Review Day 1

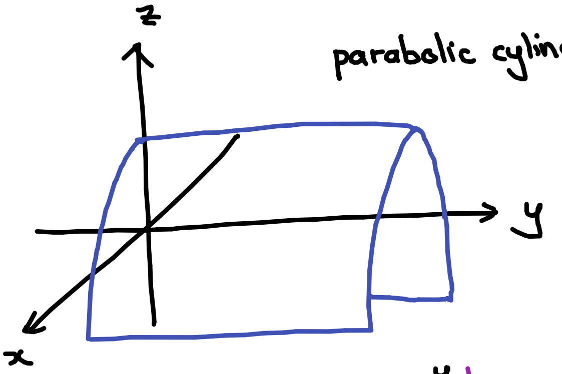
★ See Lecture 17 for a table of what to know & some review questions.

Question 1: The cross-section of a surface for every fixed y is $z = 9 - x^2$.

a) Describe the surface.

A:





parabolic cylinder along y-axis

b) Label the contours (on the right) with the appropriate z-value.

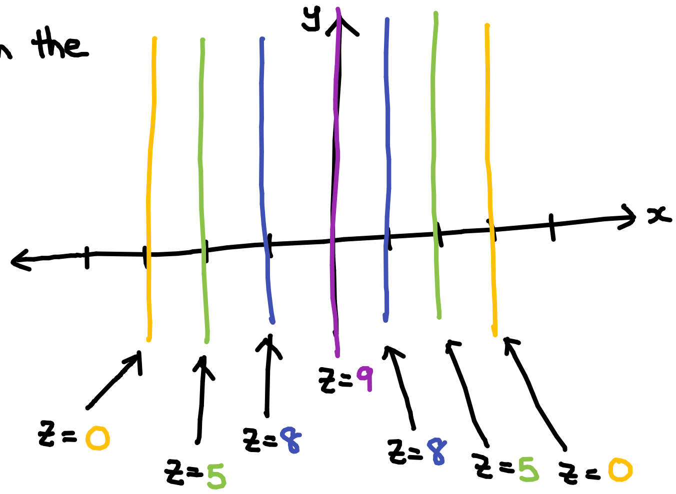
A: Use equation $z = 9 - x^2$.

For $x = \pm 3$, $z = 9 - (\pm 3)^2 = 0$

$x = \pm 2$, $z = 9 - (\pm 2)^2 = 5$

$x = \pm 1$, $z = 9 - (\pm 1)^2 = 8$

$x = 0$, $z = 9 - (0)^2 = 9$



Question 2: The temperature of an iron ball is given by $T(x,y,z) = (x^2+y^2+z^2)^{1/4}$ in °C, where x, y, z are in mm.

a) Describe the level surfaces and explain their practical meaning.

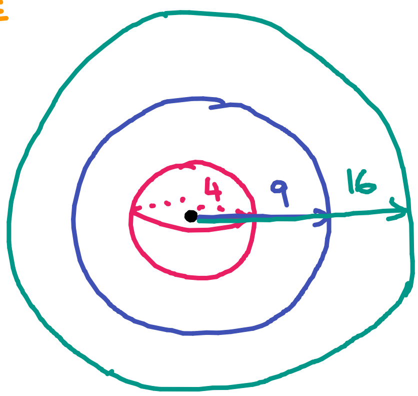
A: Level surfaces of T are given by $c = T(x,y,z) = (x^2+y^2+z^2)^{1/4}$

$c^4 = x^2+y^2+z^2$.

Each level surface is a sphere centred at the origin of radius c^2. It contains all points in the ball with temp. c °C.

b) An ant on the iron plate is at a location that is 3°C. It prefers a location that is 2°C or 4°C. Which of these locations is closest to the ant?

A:



Location with 2°C: sphere of $r = 2^2 = 4$ $\Big\}$ 5

Current location: sphere of $r = 3^2 = 9$ $\Big\}$ 7

Location with 4°C: sphere of $r = 4^2 = 16$

$\therefore$ Location with 2°C is closer.

Question 3: Find values c_1, c_2 so that $f(x,y) = \begin{cases} \dfrac{xy^3}{x^3+y^6}, & (x,y) \neq (0,0), (1,1), \\ c_1, & (x,y) = (1,1), \\ c_2, & (x,y) = (0,0), \end{cases}$
is continuous if they exist.

A: Over $(x,y) \neq (0,0)$, f is ctns b/c it is well-defined and is quotient of ctns functions.

To be ctns at $(1,1)$, $c_1 = f(1,1) = \lim\limits_{(x,y)\to(1,1)} \dfrac{xy^3}{x^3+y^6} = \dfrac{1}{1+1} = \dfrac{1}{2}$.

To be ctns at $(0,0)$, $c_2 = f(0,0) = \lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3}{x^3+y^6}$.

However, Path along $x=0$: $\lim\limits_{(0,y)\to(0,0)} f(x,y) = 0$

Path along $x=y^3$, $\lim\limits_{(y^3,y)\to(0,0)} f(x,y) = \lim\limits_{y\to 0} \dfrac{y^5}{2y^6} = \lim\limits_{y\to 0} \dfrac{1}{2y} = \infty$. DNE

$\therefore$ c_2 DNE.

Question 4: a) Which pt is furthest away from $A = (1,1,1)$:

$B = (2,3,2)$, $C = (3,3,4)$, $D = (2,1,3)$.

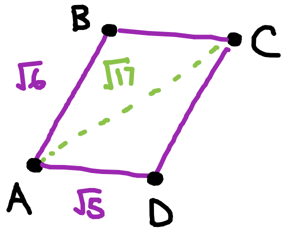
A: $\vec{AB} = (2,3,2) - (1,1,1) = (1,2,1)$, $\|\vec{AB}\| = \sqrt{1+4+1} = \sqrt{6}$
$\vec{AC} = (3,3,4) - (1,1,1) = (2,2,3)$, $\|\vec{AC}\| = \sqrt{4+4+9} = \sqrt{17}$
$\vec{AD} = (2,1,3) - (1,1,1) = (1,0,2)$, $\|\vec{AD}\| = \sqrt{1+0+4} = \sqrt{5}$

$\therefore$ C.

b) Fill in the blanks. The area of the parallelogram with vertices A, B, C, D is given by

$\| \_\_ \times \_\_ \|$

A:



Since C is the furthest, we use the other pts.

Indeed, $\vec{AC} = (2,2,3) = \vec{AB} + \vec{AD}$
$= (1,2,1) + (1,0,2)$ ✓

$\therefore$ The area $= \|\vec{AB} \times \vec{AD}\|$

Question 5: a) The wind is pushing a sailboat with a force $\vec{F_1} = (2,-3)$ while the river is pushing it with a force $\vec{F_2} = (5,1)$. What is the net force?

A: $\vec{F} = \vec{F_1} + \vec{F_2} = (7,-2)$

b) If the boat starts at position $(1,1)$ and it travels 10m, what is its position now?

A: The boat travels in the direction of $\vec{F}$.

$$\vec{u} = \frac{(7,-2)}{\sqrt{49+4}} = \frac{1}{\sqrt{53}}(7,-2)$$

Its position is at $(1,1) + \frac{10}{\sqrt{53}}(7,-2) = \left(\frac{\sqrt{53}+70}{\sqrt{53}}, \frac{\sqrt{53}-20}{\sqrt{53}}\right)$.

Question 6: (The many ways to find a plane) Find the plane that is:

a) perpendicular to $\vec{v}$ and containing a pt P_0.

A: Use $\vec{n} = \vec{v}$ and P_0 in Point-Normal form.

b) parallel to a plane and containing a pt P_0.

A: Use $\vec{n}$ of the plane and P_0 in Point-Normal form.

c) containing 3 pts A, B, C.

A: Use $\vec{n} = \vec{AB} \times \vec{AC}$ and P_0 in Point-Normal form.

d) perpendicular to a plane and containing pts A, B.

A: Use $\vec{n} = \vec{n_0} \times \vec{AB}$ and $P_0 = A$ in Point-Normal form.

e) containing a line and a pt P.

A: Find 2 pts A, B on the line.

Use $\vec{n} = \vec{AP} \times \vec{BP}$ and $P_0 = P$ in Point-Normal form.

★ When are 2 planes $\parallel$, $\perp$ or neither?