# Lecture 6: Mon Sept 15th

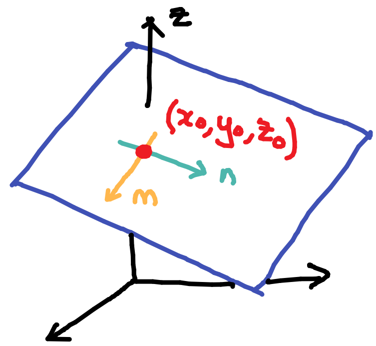
## §12.4: Linear functions

Plane = graph of linear function $f(x,y) = z = z_0 + m(x-x_0) + n(y-y_0)$,

where $m$ = slope in $x$-direction

$n$ = slope in $y$-direction

$(x_0, y_0, z_0)$ = point on the plane




If $c = z_0 - mx_0 - ny_0$, then $z = c + mx + ny$.

= constant

**How to draw a plane:** Let's draw $z = 2-2x+y$ (or $2x - y + z = 2$)

Plot $x, y, z$-intercepts. (where plane intercepts $x, y, z$-axes).

① $x$-intercept: set $y = z = 0$ and see what $x$ has to be.

$$2x - y + z = 2 \Rightarrow (1,0,0)$$

1 0 0

② $y$-intercept: set $x = z = 0$ and see what $y$ has to be.

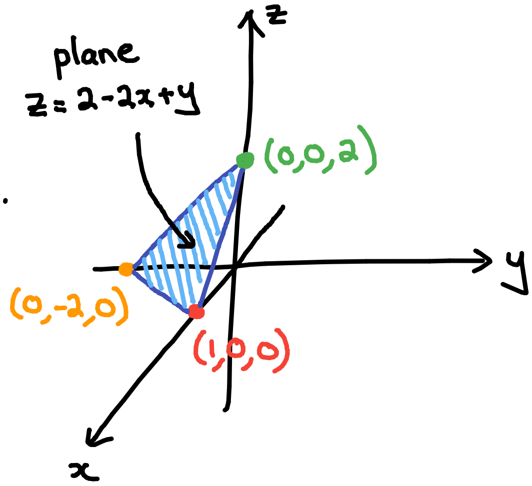
$$2x - y + z = 2 \Rightarrow (0,-2,0)$$

0 -2 0

③ $z$-intercept: set $x = y = 0$ and see what $z$ has to be.

$$2x - y + z = 2 \Rightarrow (0,0,2)$$

0 0 2




Draw lines between the intercepts ⇒ the resulting triangle is a portion of the infinite plane.

**Example:** (Find linear equation from a table) An airline sells discount and full fares.

$d$ = # of discount fares

$f$ = " " full "

$R$ = revenue (1000s of dollars)

Find a formula for $R = g(d,f)$.

| d \ f | 0 | 100 | 200 | 300 |
|---|---|---|---|---|
| 0 | 0 | 24 | 48 | 72 |
| 100 | 8 | 32 | 56 | 80 |
| 200 | 16 | 40 | 64 | 88 |
| 300 | 24 | 48 | 72 | 96 |

A: slope in $d$ direction $= \frac{\Delta R}{\Delta d} = \frac{8}{100} = 0.08$

slope in $f$ direction $= \frac{\Delta R}{\Delta f} = \frac{24}{100} = 0.24$.

Point on plane $= (d_0, f_0, R_0) = (0,0,0)$.

Then $R = R_0 + m(d-d_0) + n(f-f_0) = 0 + 0.08(d-0) + 0.24(f-0)$

$= 0.08d + 0.24f$.

**Example:** (Finding plane equation from 3 pts)

Find the equation of the plane through $(4,0,0)$, $(0,3,0)$ and $(0,0,2)$.

**A:** Using $(\underset{x_1}{0},0,\underset{z_1}{2})$ and $(\underset{x_2}{4},0,\underset{z_2}{0})$, $m = \dfrac{\Delta z}{\Delta x} = \dfrac{0-2}{4-0} = -\dfrac{1}{2}$ $\left(\dfrac{z_2 - z_1}{x_2 - x_1}\right)$

Using $(0,0,2)$ and $(0,3,0)$, $n = \dfrac{\Delta z}{\Delta y} = \dfrac{0-2}{3-0} = -\dfrac{2}{3}$

Choose a point on the plane, say $(4,0,0)$.

$$\begin{aligned} z &= z_0 + m(x-x_0) + n(y-y_0) \\ &= 0 - \tfrac{1}{2}(x-4) - \tfrac{2}{3}(y-0) \\ &= 2 - \tfrac{1}{2}x - \tfrac{2}{3}y. \end{aligned}$$

**Example:** Find the equation of the plane through $(-3,2,-4)$, $(-1,2,-1)$ and $(-1,-1,1)$.

**A:** For $m = x$-slope, use $(-3,2,-4)$ and $(-1,2,-1)$ b/c they have same $y$-coord.

$$m = \frac{\Delta z}{\Delta x} = \frac{-1--4}{-1--3} = \frac{3}{2}$$

For $n$, use $(-1,-1,1)$ and $(-1,2,-1)$ b/c they have same $x$-coord.

$$n = \frac{\Delta z}{\Delta y} = \frac{-1-1}{2--1} = \frac{-2}{3}$$

Choose $(-1,-1,1)$. $z = 1 + \tfrac{3}{2}(x+1) - \tfrac{2}{3}(y+1)$
$= \tfrac{11}{6} + \tfrac{3}{2}x - \tfrac{2}{3}y.$

**Remark:**

For general points, sub the 3 points into $z = c + mx + ny$ and solve a system of 3 eqns.

E.g. With the pts from the previous example,

$$\begin{aligned} -4 &= c + m(-3) + 2n \\ -1 &= c + m(-1) + 2n \\ 1 &= c + m(-1) - n \end{aligned}$$

Solve for $m, n, c$ and form $z = c + mx + ny$.

**How to recognize linear function from a table:**

Linear functions satisfy:

- each row & each column is linear.
- all rows have the same slope.
- all columns have the same slope.

**Example:** Which tables could represent linear functions?

| x\y | 0 | 5 | 10 |
|---|---|---|---|
| 1 | 2 | 4 | 6 |
| 5 | 4 | 8 | 12 |
| 9 | 8 | 16 | 32 |

No. Slope in $x$ is not constant.

| x\y | 0 | 3 | 6 |
|---|---|---|---|
| 0 | 1 | -4 | -9 |
| 2 | 4 | -1 | -6 |
| 4 | 7 | 2 | -3 |

Yes. $m = \frac{4-1}{2-0} = \frac{3}{2}$ is constant.

$n = \frac{-4-1}{3-0} = \frac{-5}{3}$ is constant.

| x\y | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 2 | 2 | 2 | 2 |

No.