Lecture 1: Intro & Multivariable Functions

Today: (1) Mikale Reddy

mikale.reddy@mail.utoronto.ca → content related questions

(2) All admin related questions → admin235@math.toronto.ca

(3) Section 0101 → MWF 9:10 - 10:00

MP103 SS1107 MP103

(4) Office Hrs → W 10:10 – 11:00 Room TBA.

(5) Fall Semester – Vector Functions

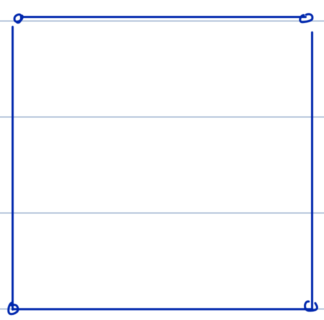
– Partial Derivatives

– Optimization

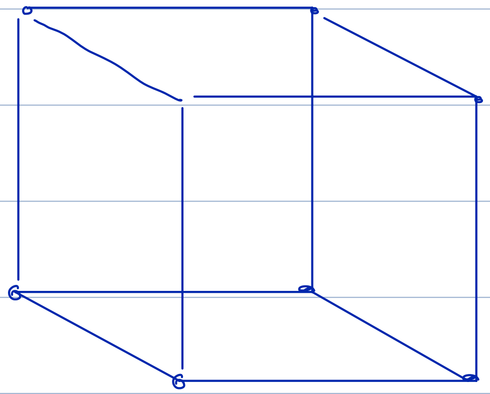
Question: How do we visualize n-dim space in our minds?

1D:  $\mathbb{R}$

2D: $\mathbb{R} \times \mathbb{R} \cong \mathbb{R}^2$

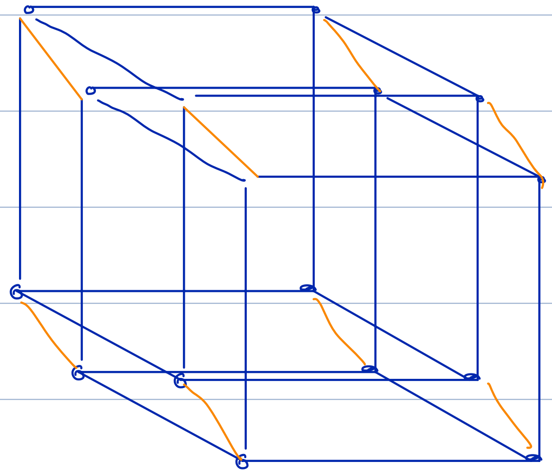


3D:



$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \cong \mathbb{R}^3$

4D:



$\mathbb{R}^4$.

5D . . .

The way we created these was by taking two copies of the n-1 dim space and identifying them.

	Single Var Calculus	Multi Var Calculus
1st Term	Functions (real valued) $f: \mathbb{R} \to \mathbb{R}$	Functions of Several Variables $f: \mathbb{R}^2 \to \mathbb{R}$, $f: \mathbb{R}^3 \to \mathbb{R}$
	Representations	Representations
	Limits of S.V. functions	Limits of M.V. functions
	Continuity	Continuity
	Derivatives – Chain Rule – Max / Min Values	Partial Derivatives – Chain Rule – Max / Min Values – Lagrange Multipliers
2nd Term	Integrals – Sub. Rule – F.T.O.C.	Double / Triple Integrals – Changing order of integration – Greens Theorem – Stokes Theorem – The Divergence Theorem.

Functions of Two Variables

Recall: we denote by $\mathbb{R}$ the set of all real numbers

Question : What is the set $\mathbb{R}^2$

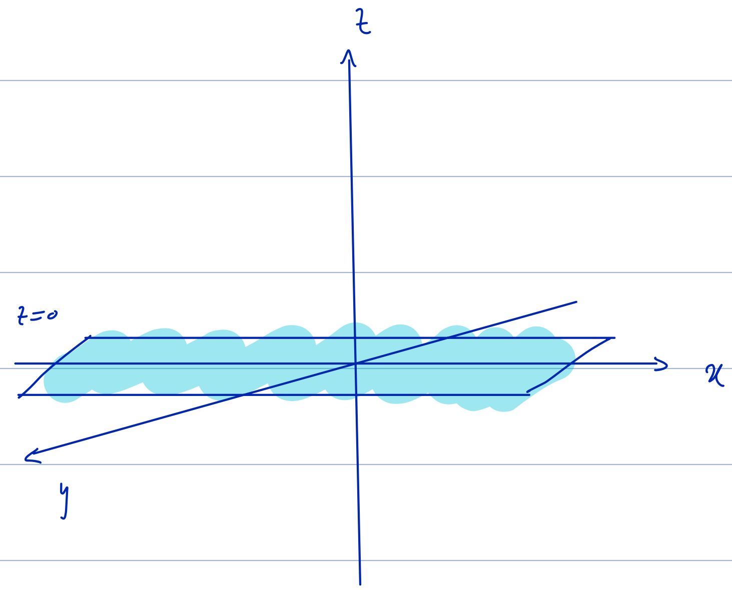
It's simply $\mathbb{R}^2 = \{(x,y) \mid x, y \in \mathbb{R}\}$

Question : What is the set $\mathbb{R}^3$

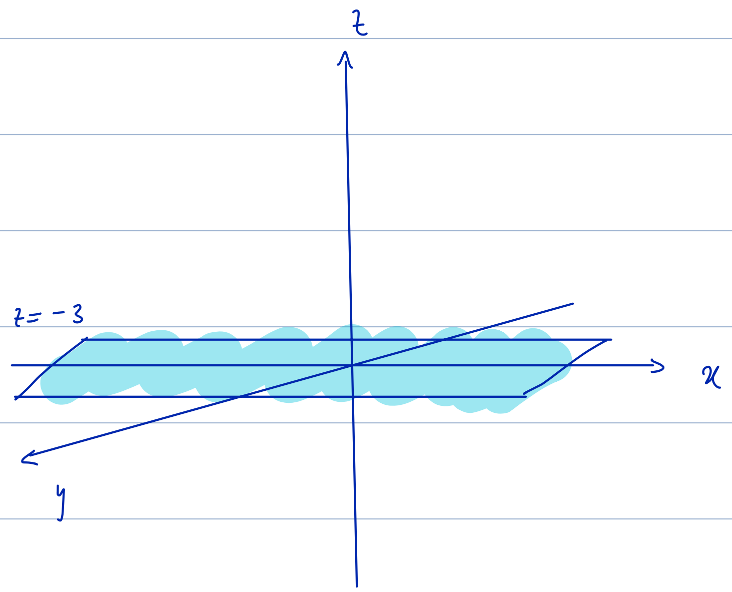
$\mathbb{R}^3 = \{(x,y,z) \mid x, y, z \in \mathbb{R}\}$

Remark : We can graph an eq involving variables x, y, z in $\mathbb{R}^3$ s.t. the graph is a picture of all pts (x,y,z) that satisfy the equation.

Ex. (1) $z = 0 \Rightarrow S = \{(x,y,z) \mid x, y \in \mathbb{R}, z = 0\}$



(2) $z = -3 \Rightarrow S = \{(x,y,z) \mid x, y \in \mathbb{R}, z = -3\}$



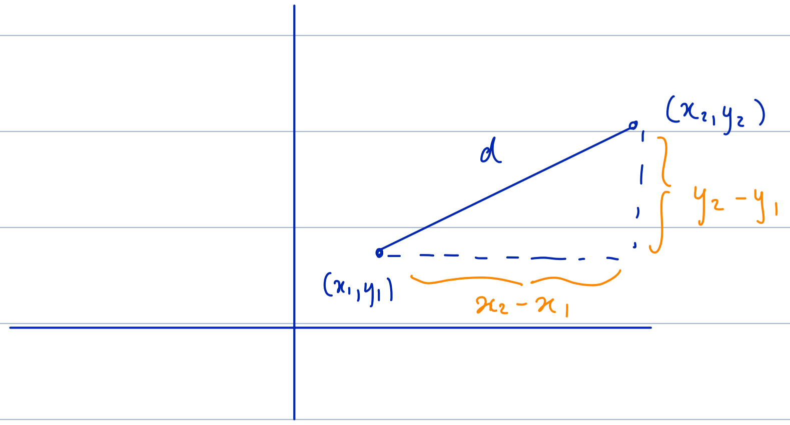
Ex. (1) yz-plane $\Rightarrow$ $S = \{(x,y,z) \mid y,z \in \mathbb{R},\ x=0\}$.

(2) xz-plane $\Rightarrow$ $S = \{(x,y,z) \mid x,z \in \mathbb{R},\ y=0\}$.

(3) 4 units below the xy-plane and in the yz-plane. $\Rightarrow$ $S = \{(x,y,z) \mid y \in \mathbb{R},\ x=0,\ z=-4\}$
$= \{(0,y,-4)\}$.

Distance Between Two Points

Suppose we have two pts (x_1,y_1) & (x_2,y_2) in $\mathbb{R}^2$.



Q: What is the distance

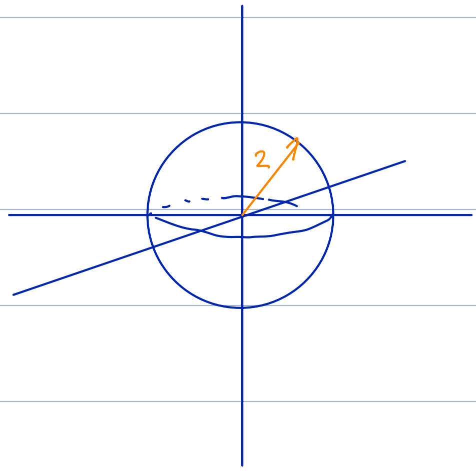
A: $d = \left((x_2 - x_1)^2 + (y_2 - y_1)^2 \right)^{1/2}$

Q: What about in $\mathbb{R}^3$ $\rightarrow$ $P_1 = (x_1, y_1, z_1)$

$P_2 = (x_2, y_2, z_2)$

A: $d = \left((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right)^{1/2}$

Ex. Find an eq for a sphere of radius 2 with centre at the origin.



Let $P = (x, y, z)$ $\quad O = (0, 0, 0)$

$$d = 2 = \left((x-0)^2 + (y-0)^2 + (z-0)^2 \right)^{1/2}$$

$$= (x^2 + y^2 + z^2)^{1/2}$$

$\Rightarrow \quad 4 = x^2 + y^2 + z^2$

The general formula for a sphere is $r^2 = x^2 + y^2 + z^2$.

Recall: Single Variable Functions.

We think of functions as machines that eat something and spit out something for a domain to a range.

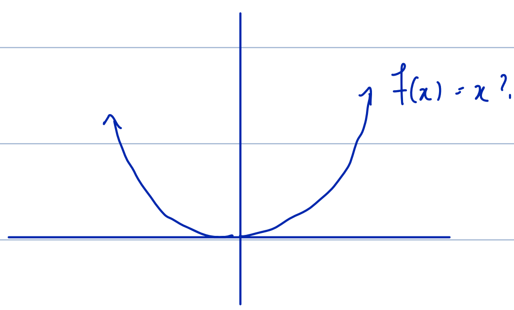
The most useful method for visualizing them is by there graphs.

Def: Let $f: \mathbb{R} \to \mathbb{R}$ be a fct with domain D. Then the graph of f, denoted G_f, is the set of ordered pairs $G_f = \{(x, f(x)) \mid x \in D\}$.

Note: The graph of a fct of one variable is the set of points in $\mathbb{R}^2$

Ex. The fct $f: \mathbb{R} \to \mathbb{R}$ given by $y = f(x) = x^2$

$G_f = \{(x, x^2) \mid x \in \mathbb{R}\}$.



These need to pass the vertical line test.