Lecture 2: Fri Sept 5th

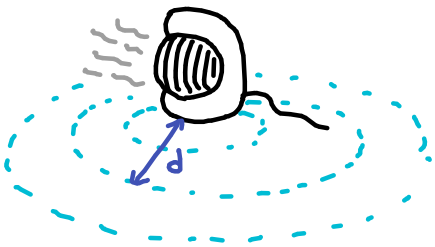
What we can do with multivariable functions is fix one variable and let the other vary.

For instance, if $y = c$ constant, $f(x,c)$ = function of x.

Example: Let $T = f(d,t)$ = temperature

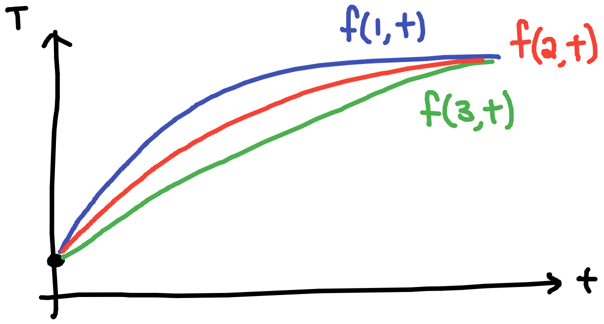
t = # mins after heater was turned on

d = distance from heater



a) Is T an increasing or decreasing function of t?
Sketch the graphs of $f(1,t)$, $f(2,t)$, $f(3,t)$.

A:

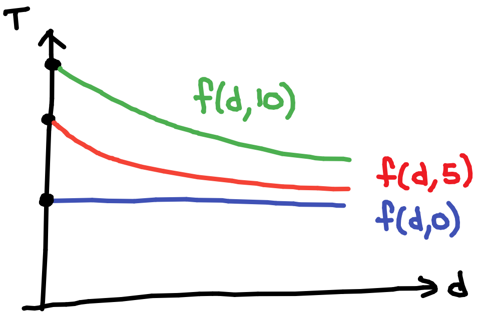


At a fixed distance d, the temperature gets warmer as time goes on. So, T is increasing.

The closer you are to the heater, that location will get heated earlier.

b) Is T an increasing or decreasing function of d?
Sketch the graphs of $f(d,0)$, $f(d,5)$, $f(d,10)$.

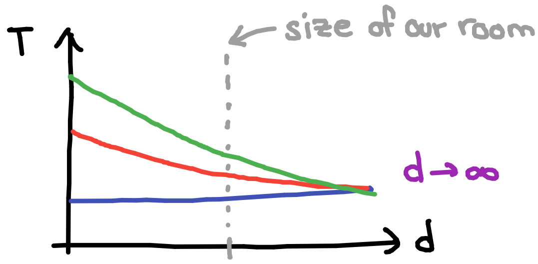
A:



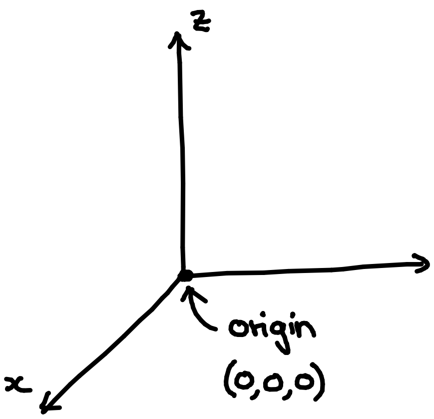
T is decreasing function of d. The further away you are from heater, the colder it is.

At $t=0$, the room is at the same temperature everywhere. As time goes on, more of the room will be heated.

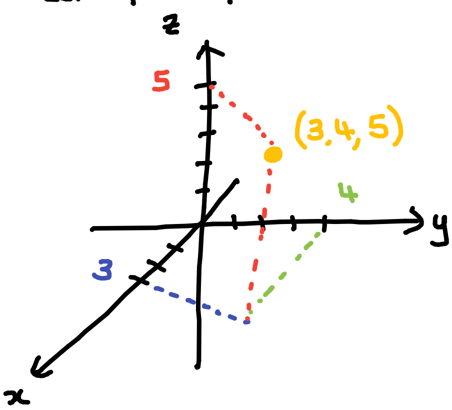
As $d \to \infty$, the curves should converge but since our room is finite, the graph gets "cuts off".



Three-dimensional space:

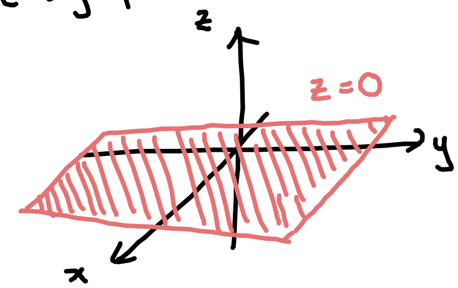


Let's plot a point $(3,4,5)$.



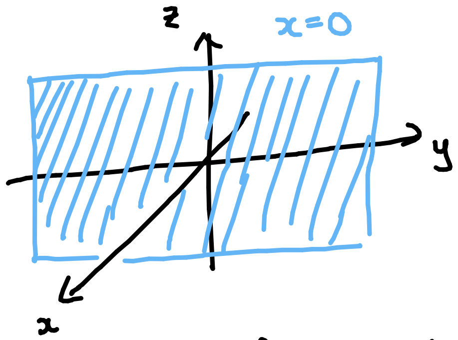
Planes in 3d space:

The xy-plane is defined by the equation $z=0$. I.e. it is the set



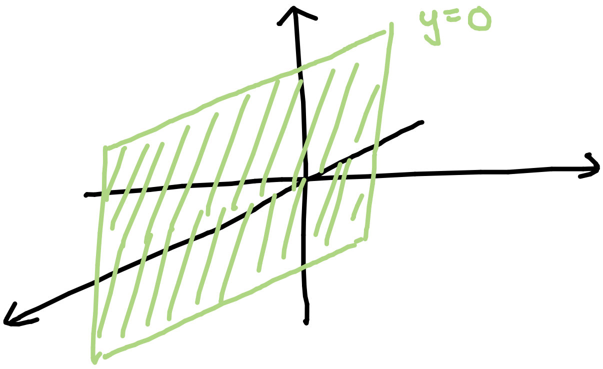
$$\{(x,y,z)\in\mathbb{R}^3 : z=0\}$$
$$=\{(x,y,0)\in\mathbb{R}^3\}.$$

The yz-plane is defined by the equation $x=0$. I.e. it is the set



$$\{(x,y,z)\in\mathbb{R}^3 : x=0\}$$
$$=\{(0,y,z)\in\mathbb{R}^3\}.$$

The xz-plane is defined by the equation $y=0$. I.e. it is the set

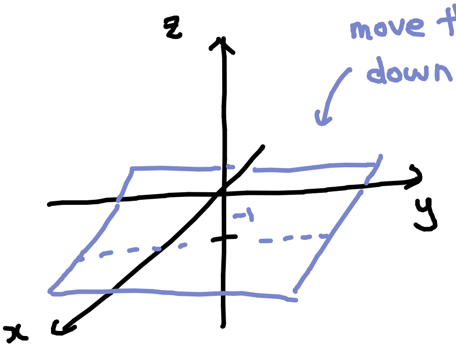


$$\{(x,y,z)\in\mathbb{R}^3 : y=0\}$$
$$=\{(x,0,z)\in\mathbb{R}^3\}.$$

Examples:

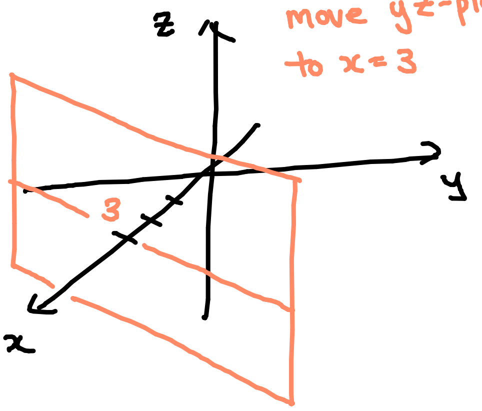
What is the graph of $z=-1$?

move the xy-plane down to $z=-1$



What is the graph of $x=3$?

move yz-plane to $x=3$



Does $(2,2,4)$ lie on the graph of

(a) $z=4$ — Yes

(b) $x+y+z=0$ — No

(c) $x-y=0$ — Yes

(d) $x^2+y^2+z^2=14$ — No

sphere w/ centre $(0,0,0)$ & radius $r=\sqrt{14}$

Sub $(2,2,4)$ into the equation and see if it is satisfied.

Example 12.1.7: Which of the pairs:

$A = (1,-1,0)$

$B = (0,3,4)$

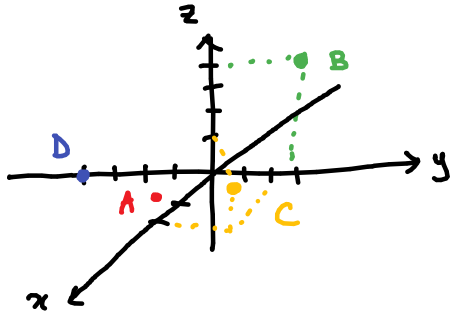
$C = (2,2,1)$

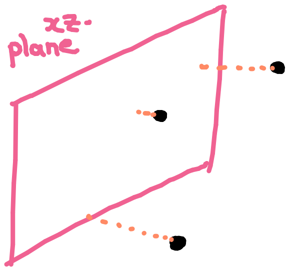
$D = (0,-4,0)$

lie closest to xz-plane?

Answer: The distance to xz-plane $(y=0)$ is given by the magnitude of y-coordinate.

$\therefore$ A is closest.





Longer explanation

Since we only care about the distance between the point and the plane $y=0$, it does not matter how high (z-direction) or how far along (x-direction) the point is.

Spheres in 3d space

By extended Pythagoras theorem,

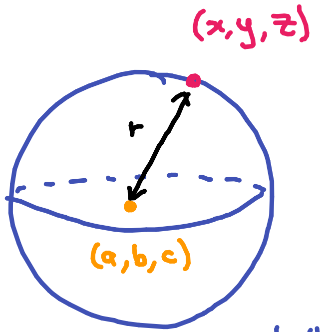
distance between (x,y,z) and (a,b,c) $= \sqrt{(x-a)^2+(y-b)^2+(z-c)^2}$

Let's describe spheres.

A sphere of centre (a,b,c) and radius r is the set of all points $(x,y,z) \in \mathbb{R}^3$ whose distance from (a,b,c) is r. It is the set

$$\left\{(x,y,z) \in \mathbb{R}^3 : \sqrt{(x-a)^2+(y-b)^2+(z-c)^2} = r\right\}$$

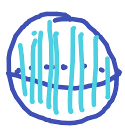
$$= \left\{(x,y,z) \in \mathbb{R}^3 : \underbrace{(x-a)^2+(y-b)^2+(z-c)^2 = r^2}_{\text{defining equation of sphere}}\right\}$$



sphere = outer shell

Remark:

$(x-a)^2+(y-b)^2+(z-c)^2 \leq r^2$ describes a filled sphere because it contains points closer than r away from the center.



$(x-a)^2+(y-b)^2+(z-c)^2 < r^2$ describes the filled sphere without shell.