University of Toronto – Faculty of Arts & Science – MAT235Y1: Multivariable Calculus

Term Test 1 (Practice Test) – Fall 2024/Winter 2025

Family Name (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number: ______________________________

U of T Email: ______________________________

This exam contains **8** pages (including this cover page) and **6** problems. Once the exam begins, check to see if any pages are missing. There are **50** possible points to be earned in this exam.

- Duration: **90 minutes**
- **No aids or calculators are permitted on the exam.**
- Power off all cell phones, smartphones, and other electronic devices, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
- **Do not tear any pages off this exam.**
- **One scrap page is provided at the end.** This page will not be graded unless specifically indicated. Please enter all of your answers in the space provided.
- Do not write in the page margins. Make sure that your writing is dark enough to be readable.
- **Unsupported answers to long answer questions will not receive full credit**. A correct answer without explanation will receive no credit unless otherwise noted; an incorrect answer supported by substantially correct calculations and explanations may receive partial credit.
- **Organize your work** in a reasonably neat and coherent way.
- You must use the methods learned in this course to solve all of the problems.

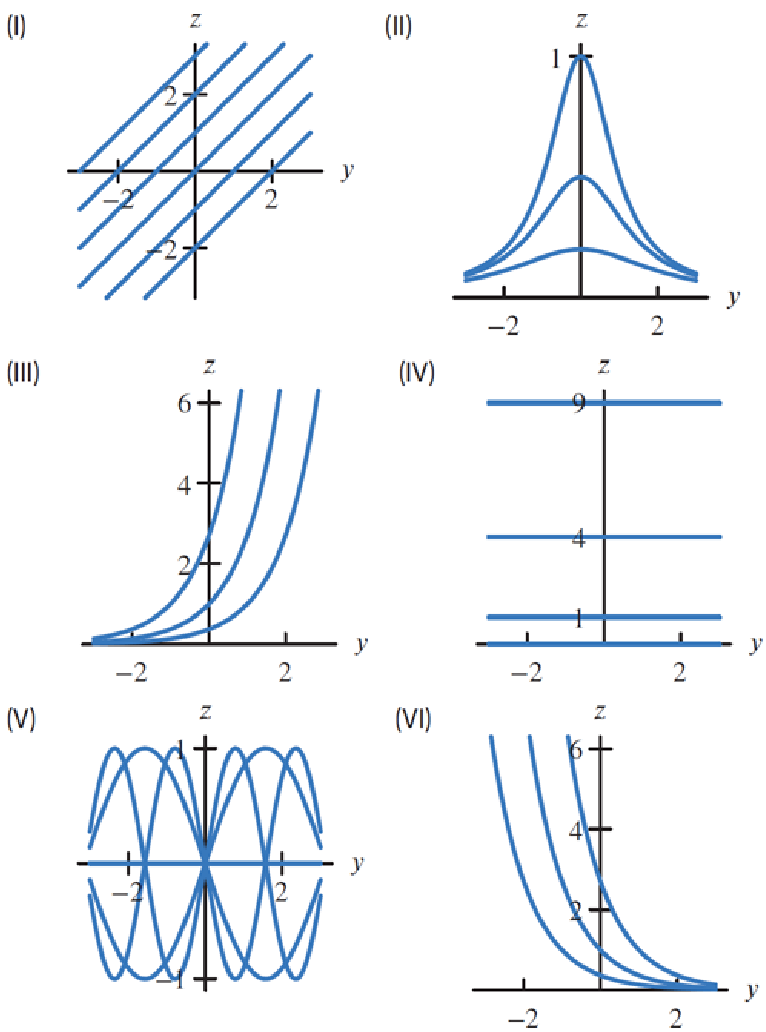
1. (6 points) Match each equation with one of the following cross-sections with x fixed. Write **(I), (II), (III), (IV), (V)** or **(VI)** for each equation. Only your final answer will be graded. Each part is worth 1.5 marks.

   1. $z = \sin(xy)$ Answer ________

   2. $z = e^{x-y}$ Answer ________

   3. $z = 1 + x + y$ Answer ________

   4. $z = x^2$ Answer ________



2. (8 points) Verify whether the following limits exist and, if they do, calculate their values. If they do not exist, provide a full justification for your conclusions.

   (a) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} 2x^2\, y^3 \sin(\cos(\ln(x^2+y^2)))$

   (b) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$

3. (9 points) (a) (5 points) Find the equation of the plane uniquely determined by the three points $P_1 = (1, 0, 0), P_2 = (0, 1, 0)$ and $P_3 = (0, 0, 1)$. Express your answer in the form $Ax + By + Cz = D$. Show all your work.

(b) (4 points) Consider a different set of three provided points: $A = (1, 1, 0), B = (2, 2, 0)$, and $C = (4, 4, 0)$. Show that it is not possible to find a unique plane containing these three points. Give **two** justifications for this fact: One using the cross product, and the other using the dot product. (Hint: What does the dot product tell you about the angle between two vectors?)

4. (9 points) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function given by

$$f(x, y) = \ln(x^2 + y^2 - 4).$$

(a) (3 points) Find the domain and range of f.

(b) (2 points) Describe the level curve $f(x, y) = c$ when $c = 1$.

(c) (1 point) Describe the cross-section of $f(x, y)$ when $y = 2$.

(d) (3 points) Is the graph of f a level surface of some function $g(x, y, z)$? Justify your answer.

5. (8 points) The pressure of gas in a storage container, in atmospheres, is given by

$$P = f(n, T, V) = \frac{82nT}{V}$$

where n is the amount of gas, in kilomoles, T is the temperature of the gas, in Kelvin, and V is the volume of the storage container, in liters.

(a) (4 points) Find a formula for the level surface of f containing the point $(n, T, V) = (1, 200, 41)$ and explain the significance of this surface in terms of pressure.

(b) (4 points) Describe the level surfaces of $P = f(n, T, V)$ algebraically for $P > 0$. Using your formula and viewing V as a function of n and T, what is the general shape of the cross-sections of the form $n = c$, where c is a constant?

6. (10 points) Suppose $T = f(x, y)$ is a function which gives the temperature at point (x, y) in a room. Suppose we have the following table of values representing data collected about this function:

$x\backslash y$	1	3	5	6
0	2	4	6	8
2	3	5	7	9
4	4	6	8	10
6	5	7	9	11

(a) (4 points) Based on the above data, could T be represented by a linear function? If so, find an expression for T. If not, give a detailed explanation.

(b) (6 points) Approximate the partial derivatives $f_x(0, 1)$ and $f_y(2, 3)$ using the above table of values. Then give a practical interpretation of these values.

Do not tear this page off. This page is for additional work and will not be graded unless you clearly indicate it on the original question page.