Lecture 17: Fri Oct 10th

How to take partial derivatives: $f(x,y) = x^2y^3$

$f_x = 2xy^3, \quad f_y = x^2(3y^2)$

Treat one variable as a constant and diff.

Recap: Find the partial derivatives.

1) $f(x,y) = xe^{\sqrt{xy}}$

A:

More examples (if you need it!)

2) $f(x,y) = z\ln(y\cos x)$

A: $f_x = \dfrac{z}{y\cos x}(-y\sin x) = -z\tan x$

$f_y = \dfrac{z}{y\cos x}(\cos x) = \dfrac{z}{y}$

$f_z = \ln(y\cos x)$

3) $f(x,y) = x^7 + 2^y + x^y$

A: Recall: $a^b = e^{\ln(a^b)} = e^{b\ln(a)}$

$f(x,y) = x^7 + e^{y\ln(2)} + e^{y\ln(x)}$

$f_x = 7x^6 + \dfrac{y}{x}e^{y\ln(x)} = 7x^6 + yx^{y-1}$

$f_y = \ln(2)e^{y\ln(2)} + \ln(x)e^{y\ln(x)}$

$= \ln(2)\,2^y + \ln(x)\,x^y$

Review:

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12.1

12.2 Graphs and cross-sections in $x+y$ (definition, drawing)

- common surfaces (spheres, paraboloid, cones, planes)
- general equations of circles, ellipses, hyperbolas, trig, polynomial, exp, log

completing the □

12.3 Contours (definition, drawing, matching)

ellipses: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

12.4 Planes

- linear functions
- linear equation from table
- linear equation from contour diagram
- properties of tables of linear functions

hyperbolas: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$

§13

12.5 Level surface (definition, drawing, how do they vary)
- matching level surfaces
- interpretation

12.6 Limits
- DNE: show 2 paths with 2 different limits
- Exist: Squeeze Thm (helpful inequalities)

Continuity
- Find where f is ctns
- Find c that makes f ctns

13.1, 2 Vectors
- displacement vectors
- $\|\cdot\|$, unit vectors
- components of 2d vectors, simple physics situation

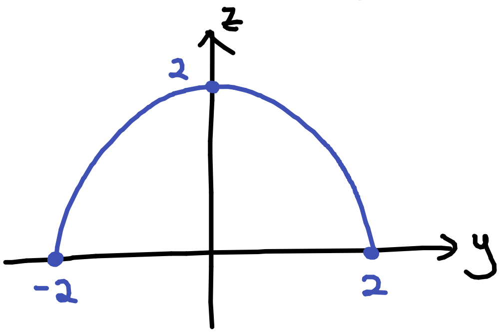
13.3 Dot product $\vec{v}\cdot\vec{w}$ (Geometric, algebraic)
- find θ
- find orthogonal/parallel vectors
- Work $W = \vec{F}\cdot\vec{d}$

13.4 Cross product $\vec{v}\times\vec{w}$ (Geometric, algebraic)
- 3 equations of plane
- Find plane from 3 pts
- Find area of parallelogram
- Find plane parallel to a plane + 1 pt
- Find plane $\perp$ to a plane + 2 pts
- Find plane containing a line + 1 pt
- When are planes $\perp$ or $\parallel$ or neither

Review:

1) Draw at least 3 cross-sections of $z = x^3 - \sin(y)$.

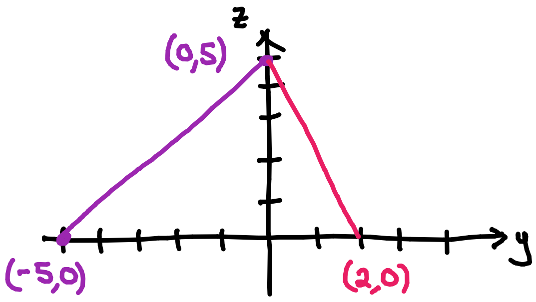
2a) An architect is designing a tunnel for cars.
If this is the cross-section for every fixed x, find the equation of the tunnel.



A:

b) Here are another plan for a tunnel.

If this is the cross-section for every fixed x, find the equation of the tunnel.



A:

c) Draw at least 4 contours of part (b).

A:

3) Find the set of points of continuity of $f(x,y) = \begin{cases} \frac{3x^2y^2}{x^2-5y^2}, & (x,y) \neq (0,0), (1,1) \\ 5 & (x,y) = (1,1) \\ 1 & (x,y) = (0,0) \end{cases}$

A:

4a) Which pt is furthest from $(1,2,3)$? $A = (1,5,1)$
$B = (0,0,0)$
$C = (2,1,2)$

A:

b) Find a vector in the direction of the longest displacement vector with length 4.

A:

5) Consider two temperature functions $f(x,y,z) = -\ln(x^2+z^2)$ and $g(x,y,z) = x^2+z^2$

a) Describe the level surfaces of f and explain their significance.

A:

b) Describe the level surfaces of g and explain their significance.

A:

c) As $c \to \infty$, how do these level surfaces of f and g change?

A:

d) Can these level surfaces be written as a graph of a 2-variable function?

A:

6) Find the plane perpendicular to $3x - y + 5z = 1$ and containing $(1,0,-1)$, $(2,1,0)$.