

Percentiles directly from the data

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12 What is the median now?

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12 What is the median now?
“Somewhere between 4 and 7; let’s take the midpoint, which is 5.5”

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12 What is the median now?
“Somewhere between 4 and 7; let’s take the midpoint, which is 5.5”

What about the **25th percentile** of this list?

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12 What is the median now?
“Somewhere between 4 and 7; let’s take the midpoint, which is 5.5”

What about the **25th percentile** of this list?
25% of 6 entries is “1.5” entries

Percentiles directly from the data

- example list: 7, 2, 7, 4, 0
- increasing order: 0, 2, 4, 7, 7 **median** = 4
- new list: 0, 2, 4, 7, 7, 12 What is the median now?
“Somewhere between 4 and 7; let’s take the midpoint, which is 5.5”

What about the **25th percentile** of this list?

25% of 6 entries is “1.5” entries

25th percentile: somewhere between 0 and 2. But where, exactly?

Formal definition of percentile

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list.

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large at 25% of the list.

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large as 25% of the list.
- 2 is $2/6 = 33.33\%$ of the way into the list; so it is definitely at least as large as 25% of the list;

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large as 25% of the list.
- 2 is $2/6 = 33.33\%$ of the way into the list; so it is definitely at least as large as 25% of the list; **25th percentile = 2**

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large as 25% of the list.
- 2 is $2/6 = 33.33\%$ of the way into the list; so it is definitely at least as large as 25% of the list; **25th percentile = 2**
- 4 is $3/6 = 50\%$ of the way into the list; so it is definitely at least as large as 25% of the list,

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large as 25% of the list.
- 2 is $2/6 = 33.33\%$ of the way into the list; so it is definitely at least as large as 25% of the list; **25th percentile = 2**
- 4 is $3/6 = 50\%$ of the way into the list; so it is definitely at least as large as 25% of the list, *but it is not the smallest number with that property!*

Formal definition of percentile

The **p th percentile** of a list of numbers is the smallest number that is at least as large as $p\%$ of the list.

Find the 25th percentile of the list 0, 2, 4, 7, 7, 12.

- 0 is $1/6 = 16.67\%$ of the way into the list; so it is at least as large as 16.67% of the list. But it is not at least as large as 25% of the list.
- 2 is $2/6 = 33.33\%$ of the way into the list; so it is definitely at least as large as 25% of the list; **25th percentile = 2**
- 4 is $3/6 = 50\%$ of the way into the list; so it is definitely at least as large as 25% of the list, *but it is not the smallest number with that property!*

That honor goes to 2. Confirmed: **25th percentile = 2**

Applying the definition

Mechanics:

- put the list in increasing order

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p^0\%$ of the way from the bottom of the list

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p^0\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Find the median.

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Find the median.

50% of the way into the list is $0.5 \times 6 = 3$ places, so **median** = 4.

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Find the median.

50% of the way into the list is $0.5 \times 6 = 3$ places, so **median = 4**.

Compare with the “conventional” median of 5.5; the two are not the same!

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Find the median.

50% of the way into the list is $0.5 \times 6 = 3$ places, so **median = 4**.

Compare with the “conventional” median of 5.5; the two are not the same!

40th percentile: $0.4 \times 6 = 2.4$; take the 3rd entry: **40th percentile = 4**

Applying the definition

Mechanics:

- put the list in increasing order
- identify the place that is $p\%$ of the way from the bottom of the list
- if that's a place on the list, take the number in that place; if not, take the next one up

Practice: 0, 2, 4, 7, 7, 12

Find the median.

50% of the way into the list is $0.5 \times 6 = 3$ places, so **median = 4**.

Compare with the “conventional” median of 5.5; the two are not the same!

40th percentile: $0.4 \times 6 = 2.4$; take the 3rd entry: **40th percentile = 4**

Check that the 60th, 70th, and 80th percentiles are all equal to 7!

Why this definition?

Why this definition?

- clear, unambiguous definition for all lists and all percents p

Why this definition?

- clear, unambiguous definition for all lists and all percents p
- has useful properties in the abstract math world