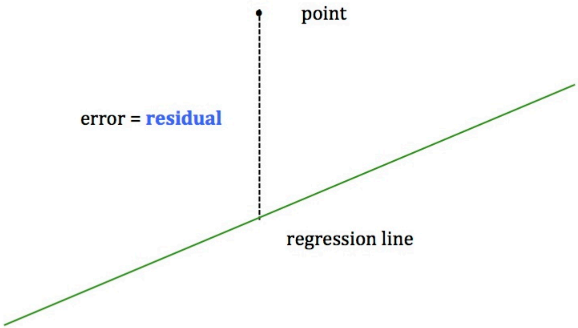
Residuals



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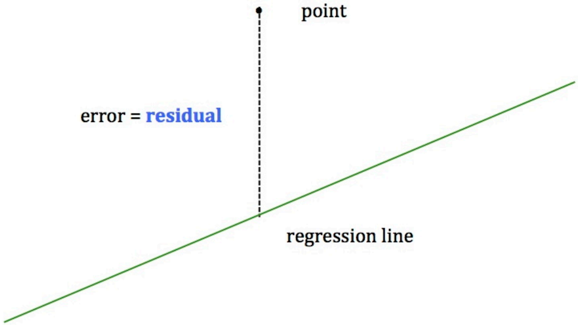

r.m.s. error of regression = r.m.s. of residuals

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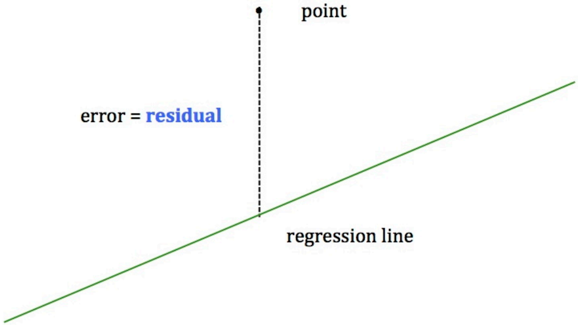

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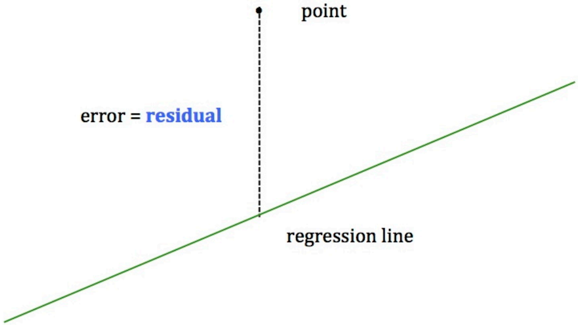

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no matter what the shape of the scatter diagram

Comparing two estimates

Estimating chick weight

egg size:	average 23.12 mm	SD = 0.45 mm	
chick weight:	average 6.43 gm	SD = 0.4 gm	$r = 0.6$

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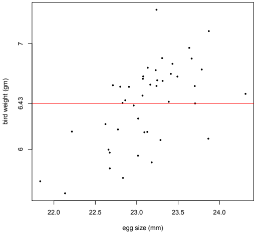

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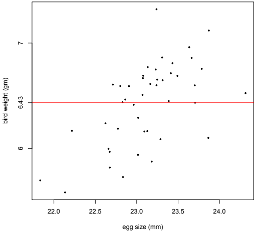

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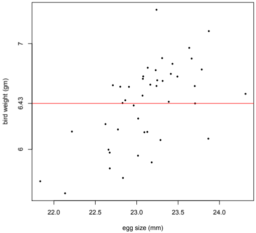

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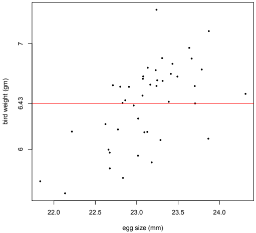

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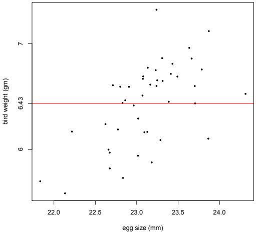

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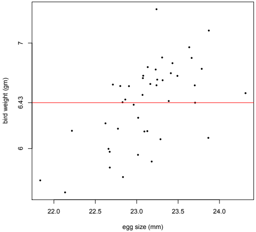

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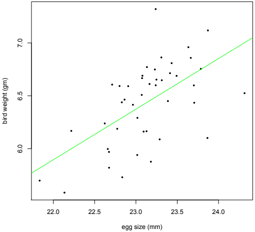

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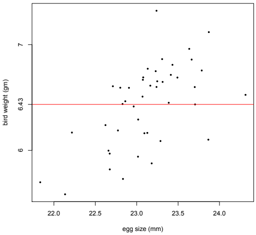

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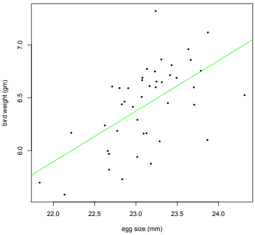

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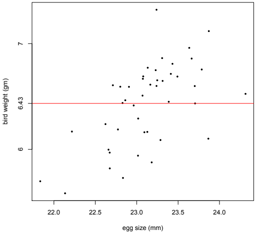

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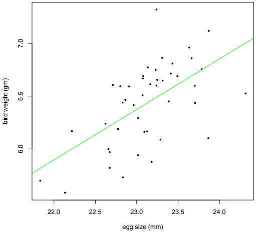

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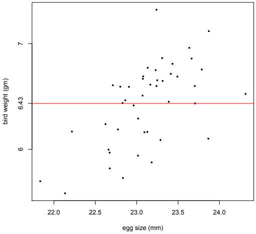

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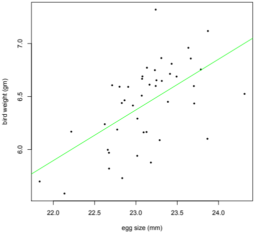

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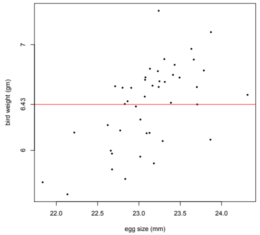

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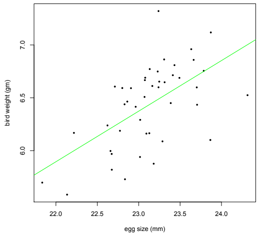

r.m.s. error

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Formula gets the basics right

Rough size of error in regression

r.m.s. error of regression $= \sqrt{1 - r^2} \cdot \text{SD of } y$

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Some useful analogies

one variable **two variables**

Some useful analogies

one variable	**two variables**
• normal curve	

Some useful analogies

one variable	**two variables**
• normal curve	football shaped scatter diagram

Some useful analogies

one variable	**two variables**
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• average	

Some useful analogies

one variable	**two variables**
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• average	regression line

Some useful analogies

one variable	**two variables**
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Some useful analogies

one variable	**two variables**
• normal curve	football shaped scatter diagram
• average	regression line
• SD	r.m.s. error of regression

Using the analogies

Using the analogies

Football shaped scatter diagram

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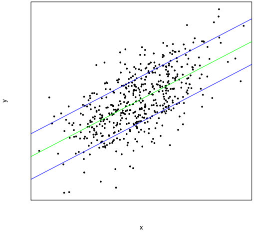
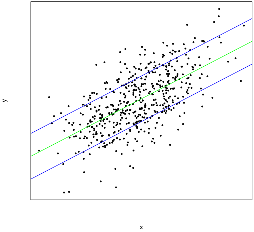


Football shaped scatter diagram



regression line $\pm$ 1 r.m.s. error
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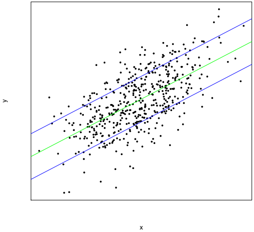

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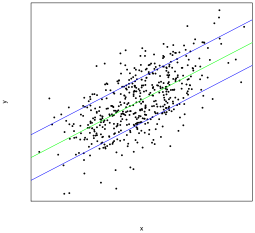

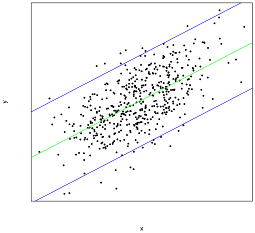

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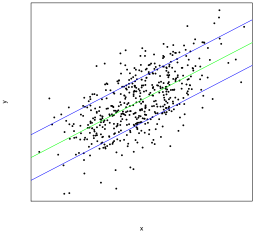

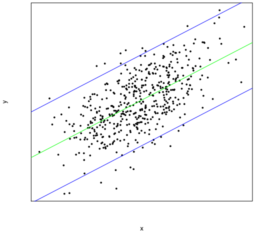

regression line $\pm$ 1 r.m.s. error contains about 68% of the points

regression line $\pm$ 2 r.m.s. errors contains about 95% of the points

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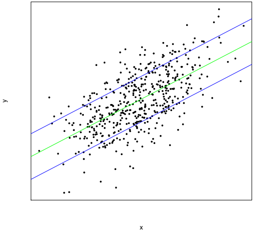

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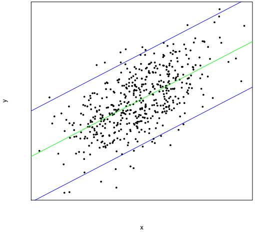

regression line $\pm$ 2 r.m.s. errors contains about 95% of the points

For about 95% of the points, the regression estimate is correct to within 2 r.m.s. errors.

A numerical example

Heights: average 67 inches, SD 3 inches
Weights: average 160 pounds, SD 20 pounds
scatter diagram is roughly football shaped $\qquad r = 0.5$

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Answer: Area between -2.31 and 2.31 under the standard normal curve

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Same statement: For **roughly 2%** of the people, the regression estimate of weight based on height is **off by more than 40 pounds.**

The average and SD of a vertical strip

Estimating weight based on height: football shaped scatter; $r = 0.5$
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Vertical strip: weights of people who are 72 inches tall

- **The average of these weights is roughly the regression estimate:**

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$(72 - 67)/3 = 1.67$; $0.5 \times 1.67 = 0.835$; $0.835 \times 20 + 160 =$ **176.7 lb**

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In a football shaped scatter diagram, the vertical spread is about the same throughout. **So the r.m.s. error of regression is the rough size of the deviations from average in any vertical strip.**

The average and SD of a vertical strip

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$(72 - 67)/3 = 1.67$; $0.5 \times 1.67 = 0.835$; $0.835 \times 20 + 160 =$ **176.7 lb**

• **Roughly what is the SD of these weights?**
In a football shaped scatter diagram, the vertical spread is about the same throughout. **So the r.m.s. error of regression is the rough size of the deviations from average in any vertical strip.**

The SD of the weights of the people who are 72 inches tall is about 17.32 lb.

Vertical strips and the normal curve

Estimating weight based on height: football shaped scatter; $r = 0.5$
Heights: average 67 in, SD 3 in
Weights: average 160 lb, SD 20 lb

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Answer: All the area to the right of 1.34 on the standard normal curve

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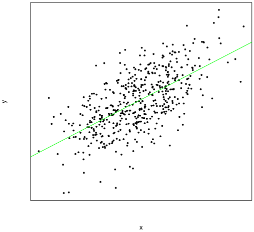
Answer: All the area to the right of 1.34 on the standard normal curve
$=$ **9%**, roughly

Plotting the residuals

Scatter and regression line



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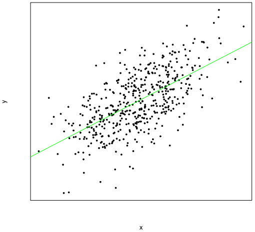
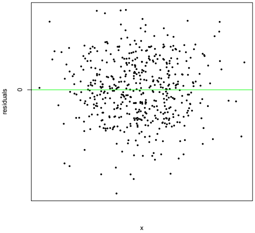


Residual plot



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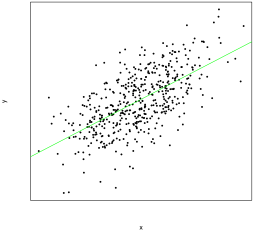

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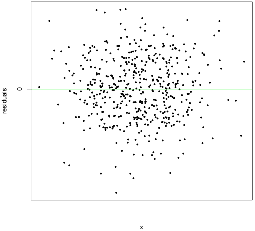
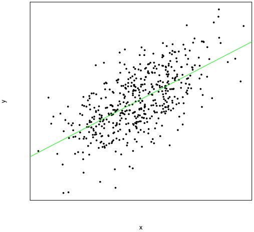


Math facts: Regardless of the shape of the scatter diagram,

Scatter and regression line



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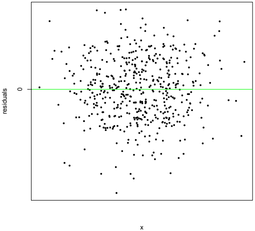

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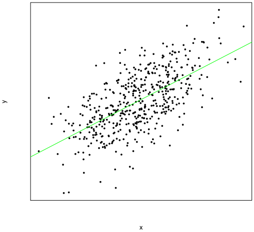
Plotting the residuals

Scatter and regression line



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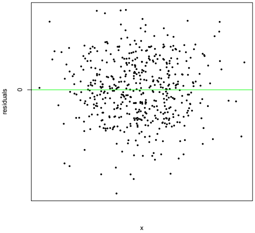

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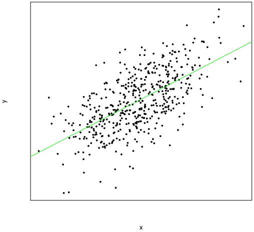
Plotting the residuals

Scatter and regression line



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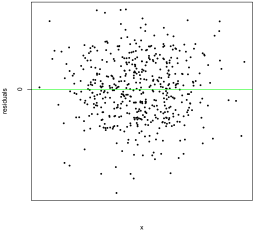

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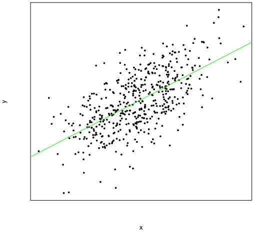
The residual plot cannot show any trend or linear relation.

Scatter and regression line



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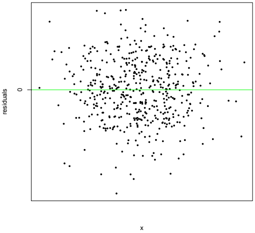

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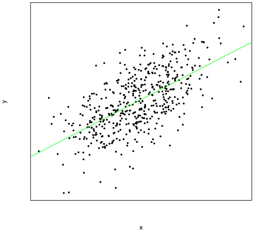
Plotting the residuals

Scatter and regression line



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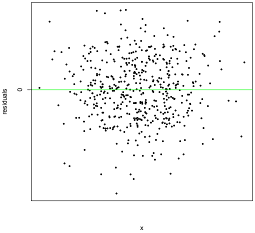

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Good regression:

Residual plot looks like a formless blob around the horizontal axis.

The residual plot as a diagnostic tool

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average weight (in pounds) of women of different heights (in inches)

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$r = 0.995$

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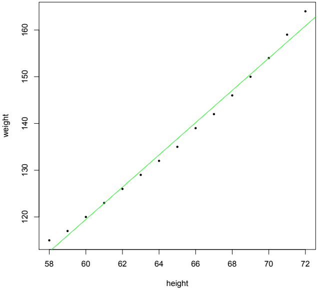

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