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Units of the average: same as the units of the list

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For example, a list whose entries are whole numbers need not have a whole number as its average. **Don't round off the average!**

The average as a smoothing operation

List: \$1, \$2, \$3, \$4

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$$\$1/4 = \$0.25 \quad \$2/4 = \$0.50 \quad \$3/4 = \$0.75 \quad \$4/4 = \$1$$

Step 2. Give each person a piece from everyone:

$$\$0.25 + \$0.50 + \$0.75 + \$1 = \mathbf{\$2.50}$$

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The new average is the old average plus the change:

new average = \$2.50 + \$25 = \$27.50

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- You didn't need to know which person got richer.

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Observe:

- You didn't need to know which person got richer.
- You didn't need to know how much money each person had.

Change multiple entries, change the average

A class of 30 students has an average score of 65 on the midterm. Two students ask for their papers to be regraded. After the regrading, one student's score increases by 10 points and the other comes down by 4 points. What happens to the class average?

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- This change gets split evenly among the 30 students: $6/30 = 0.2$.
- The class average becomes $65 + 0.2 = 65.2$.