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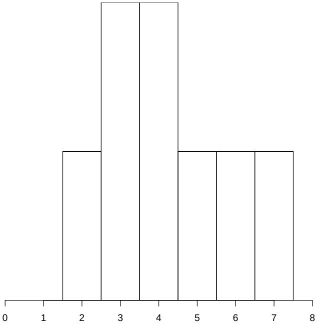

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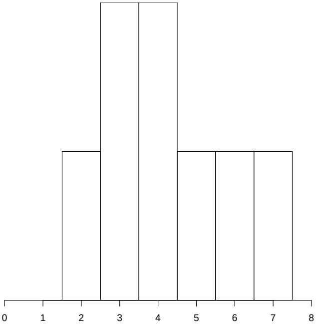


| value | deviation |
|---|---|
| 2 | $-2.25$ |
| 3 | $-1.25$ |
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| 4 | $-0.25$ |
| 4 | $-0.25$ |
| 5 | 0.75 |
| 6 | 1.75 |
| 7 | 2.75 |

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**deviation from average = value − average**

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# About how big are the deviations?

**deviation from average = value − average**

| | value | deviation | squared deviation |
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= root mean square of deviations from average = \$1.56

The average and the SD have the same units.

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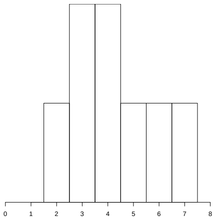

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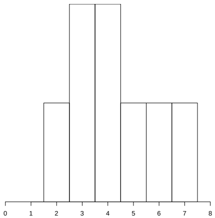


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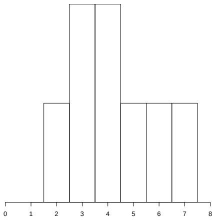


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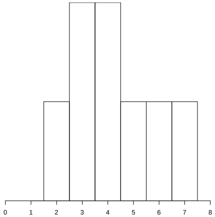


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It picks up a good chunk of the list, but not all.

No matter what the list, the vast majority of entries will be in the range **average ± a few SDs.**

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SD $=$

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SD = average

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SD = deviations from average

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The calculation is the same in both languages.

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But unless you're in those situations (and you won't be, in Stat 2.1X), there's no need to fuss about using $n - 1$.

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**0, 1**

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- Half the list is 480, and the other half is 500. So the **average is** 490.

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- So all the deviations are either $-10$ or 10. So the **SD is** 10.

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