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- **Multiplying by a constant, then adding a constant**

- **Can also first add a constant, then multiply by a constant:**

new variable $=$ (old variable $+ \ b) \times a$

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- **Multiplying by a constant**

$^\circ$F $= (9/5)\ ^\circ$C $+ 32$

- **Multiplying by a constant, then adding a constant**

- **Can also first add a constant, then multiply by a constant:**

new variable $=$ (old variable $+ b) \times a$

- **Hence can also subtract a constant and then divide by a constant:**

$$\text{new variable} = \frac{\text{old variable} - b}{a}$$

Adding a constant

Original list: 1, 2, 2, 3
Add 4 to each entry
New list: 5, 6, 6, 7

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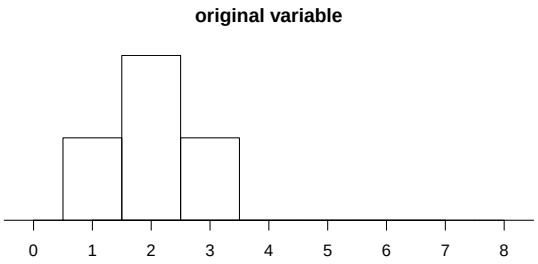

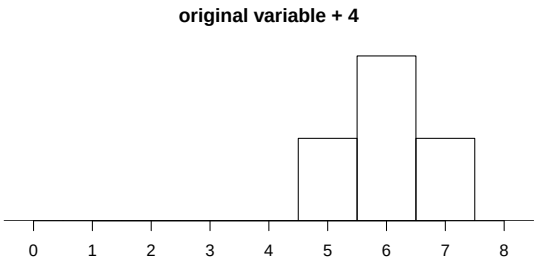

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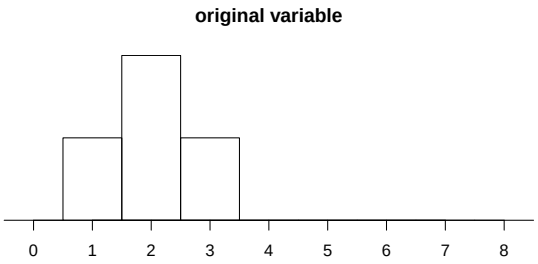

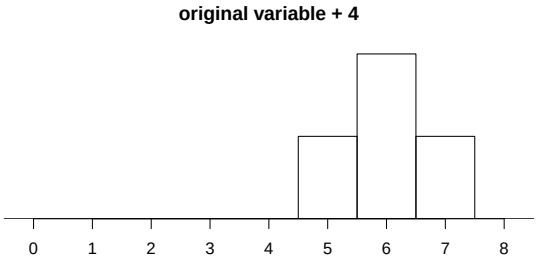

- **histogram slides 4 units to the right**

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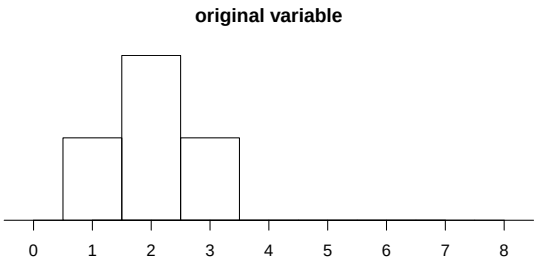

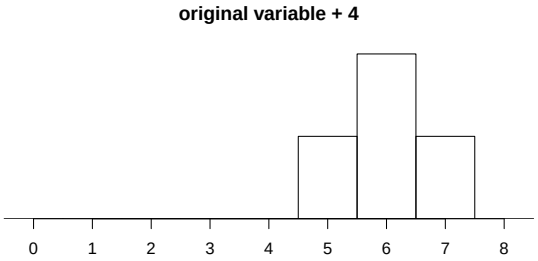

- **histogram slides 4 units to the right**
- **new average = old average + 4**

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Original list: 1, 2, 2, 3
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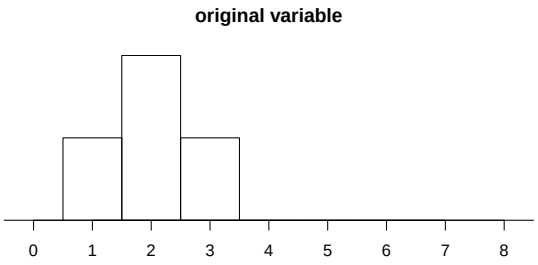

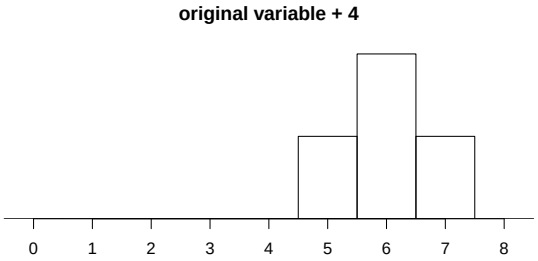

- **histogram slides 4 units to the right**
- **new average = old average + 4**
- **new SD = old SD**

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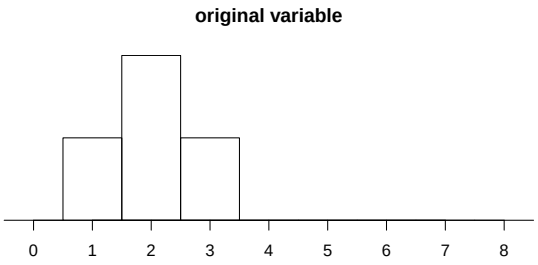

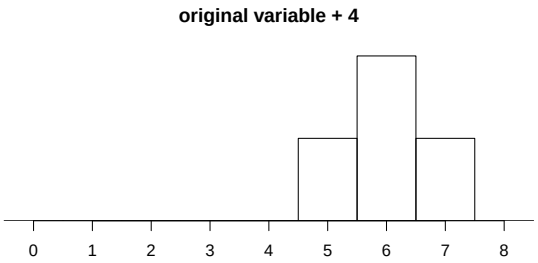

- **histogram slides 4 units to the right**
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Adding a constant doesn't change the SD.

Multiplying by a constant

Original list: 1, 2, 2, 3
Multiply each entry **by 2**
New list: 2, 4, 4, 6
Multiply each entry by -2
New new list: -2, -4, -4, -6

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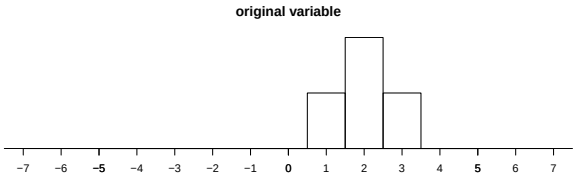

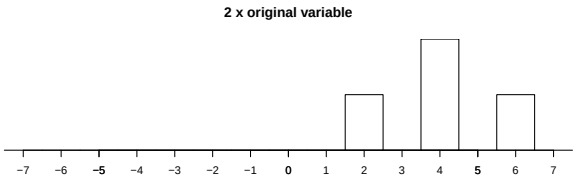

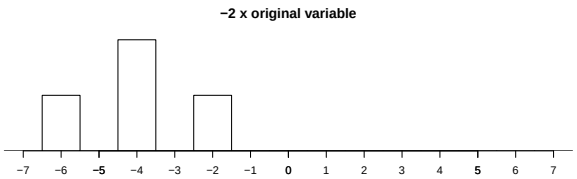

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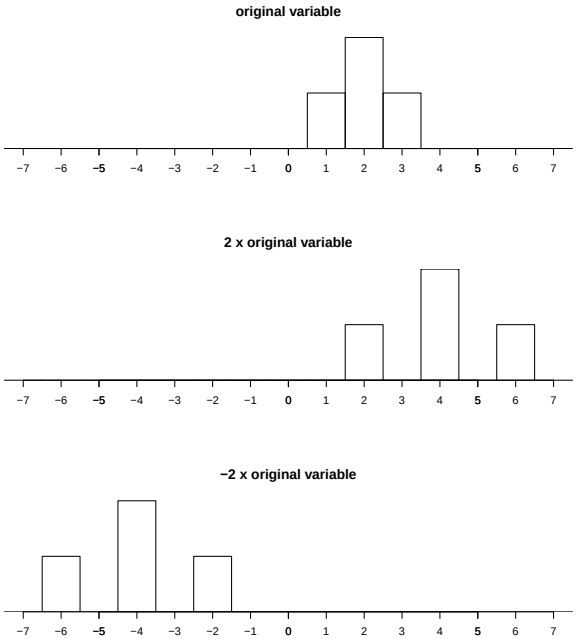

Multiplying by 2:

- **histogram gets stretched out by a factor of 2**
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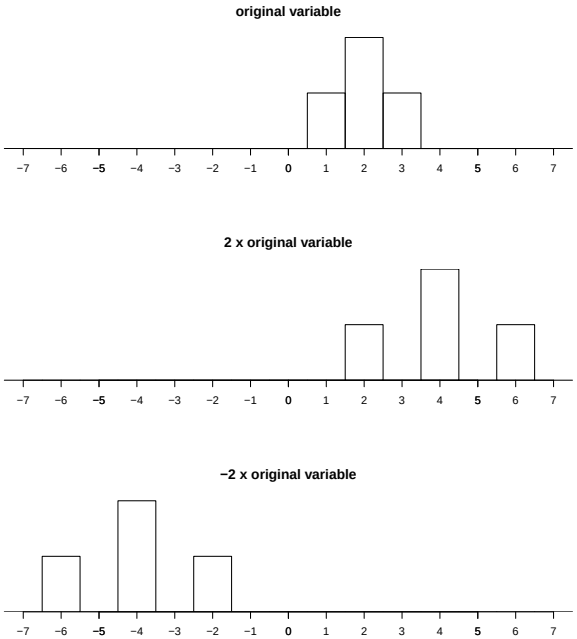

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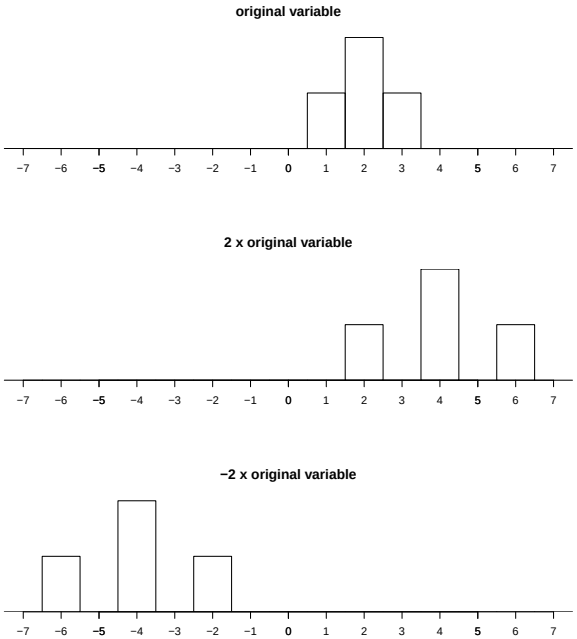

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No negative SDs!

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- **new list** $= a \times$ **(old list)** $+ b$

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average, when measured in °F $= (9/5)\times 20 + 32 =$ **68°F**

SD, when measured in °F $= (9/5)\times 5 =$ **9°F**

The most important linear transformation

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Heights: average 67 inches, SD 3 inches

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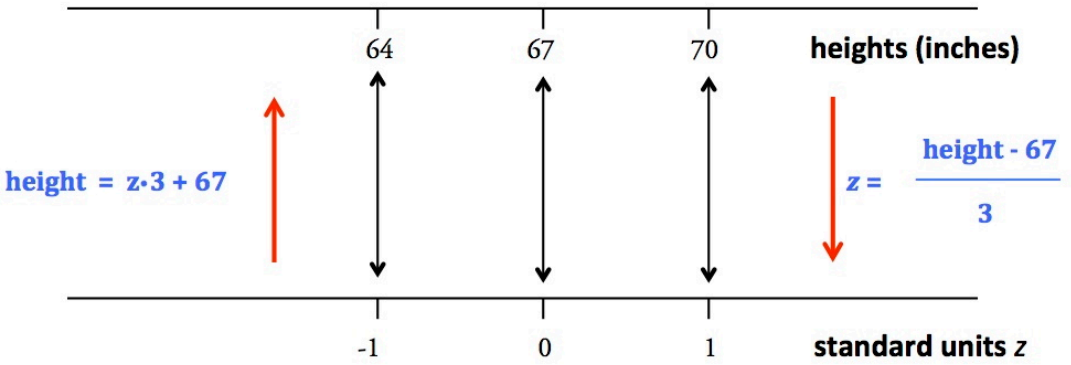

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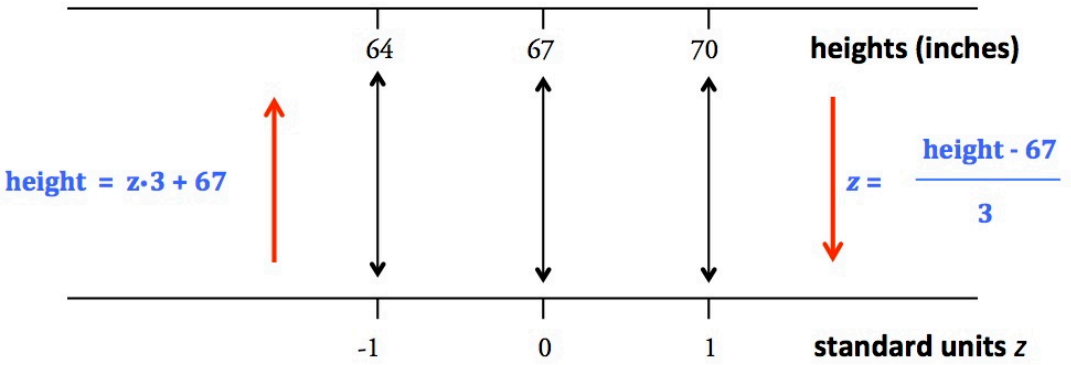

Standard units measure "how many SDs above average"

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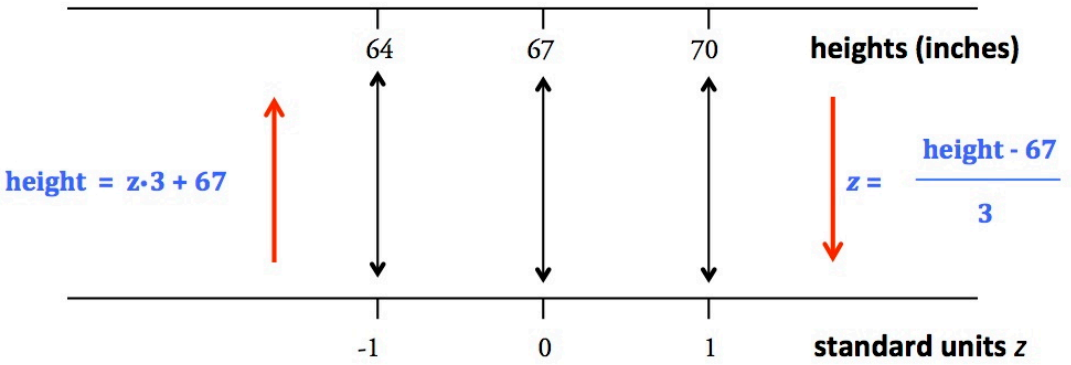

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Examples:

$$\text{73 inches, in standard units} = \frac{\text{73 inches - 67 inches}}{\text{3 inches}} = \frac{\text{6 inches}}{\text{3 inches}} = 2$$

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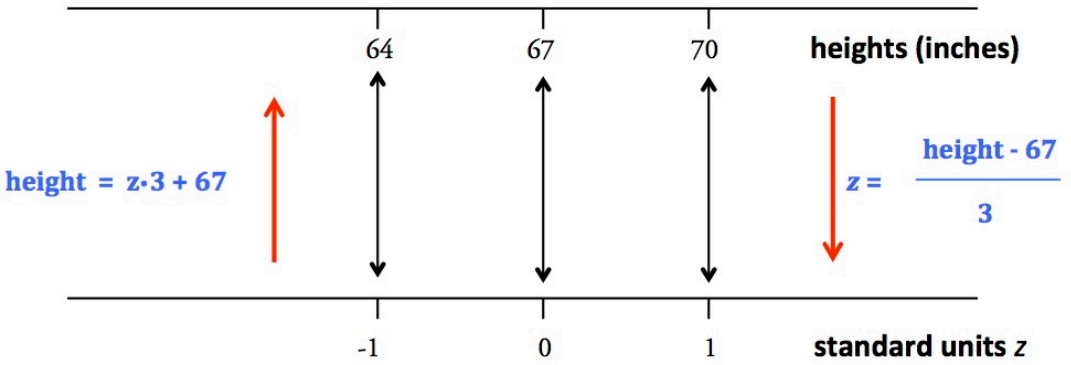


Standard units measure "how many SDs above average"

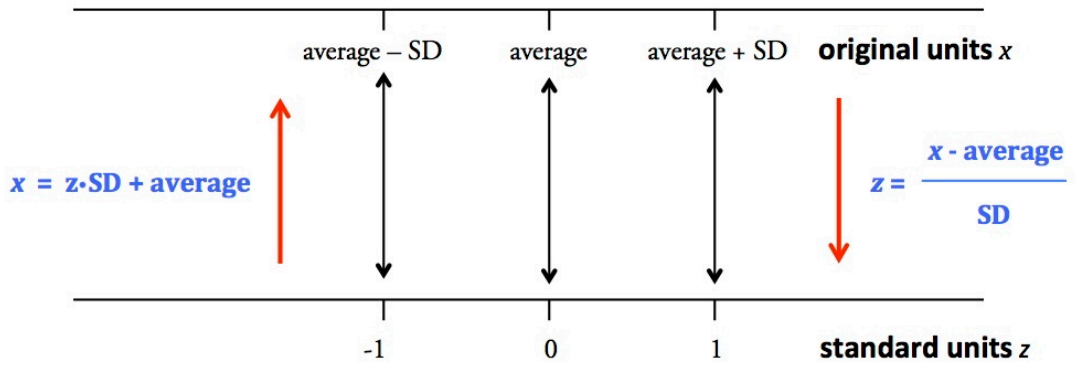
Examples:

$$\text{73 inches, in standard units} = \frac{\text{73 inches - 67 inches}}{\text{3 inches}} = \frac{\text{6 inches}}{\text{3 inches}} = 2$$

$$\text{-0.5 standard units, in inches} = -0.5 \times \text{3 inches} + \text{67 inches} = \text{65.5 inches}$$

Standard units: the *z-score*

Converting to and from standard units:



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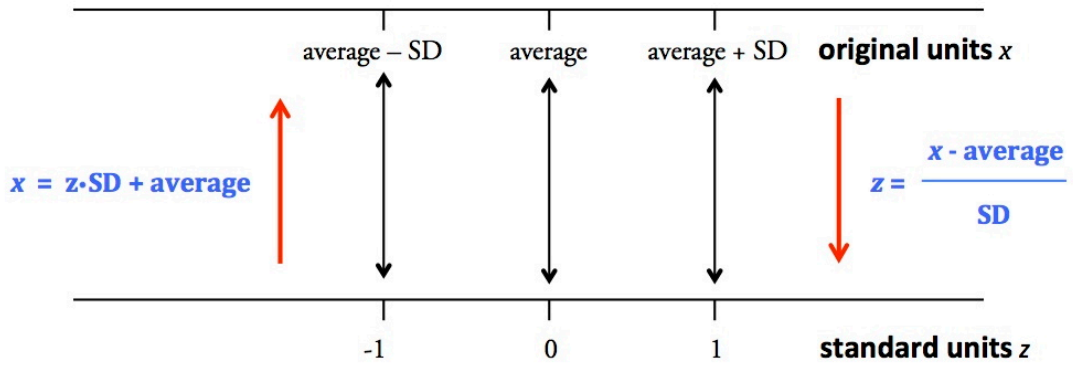

- z measures "how many SDs above average"

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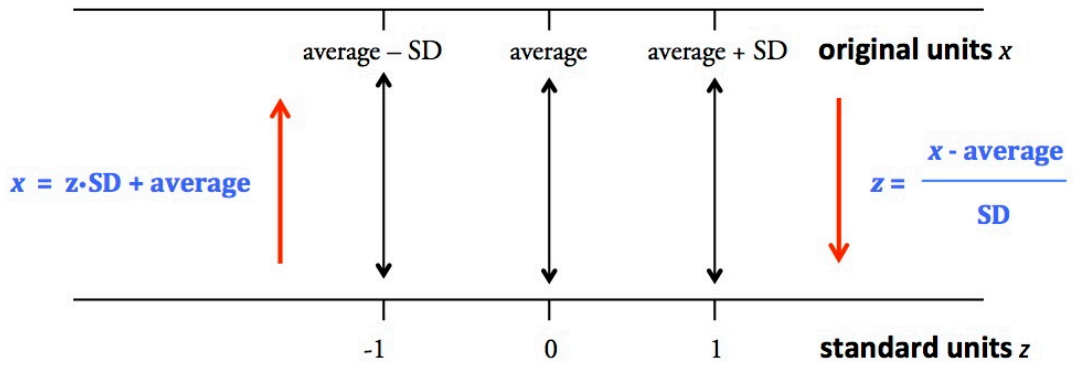
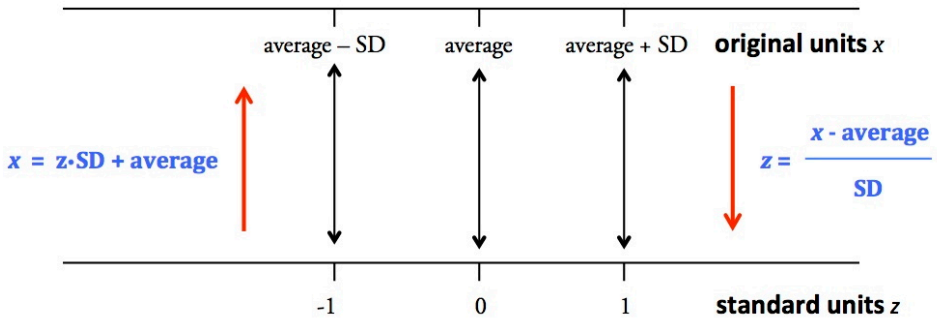


- z measures "how many SDs above average"
- positive z: bigger than average
- negative z: less than average
- $z = 0$: equal to average

The average and SD of a list in standard units



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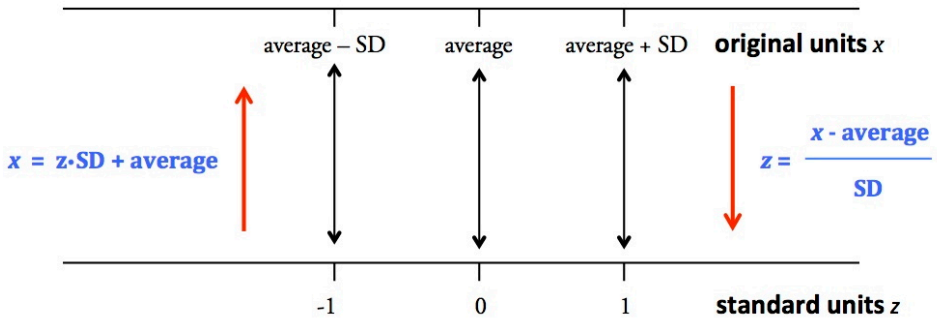

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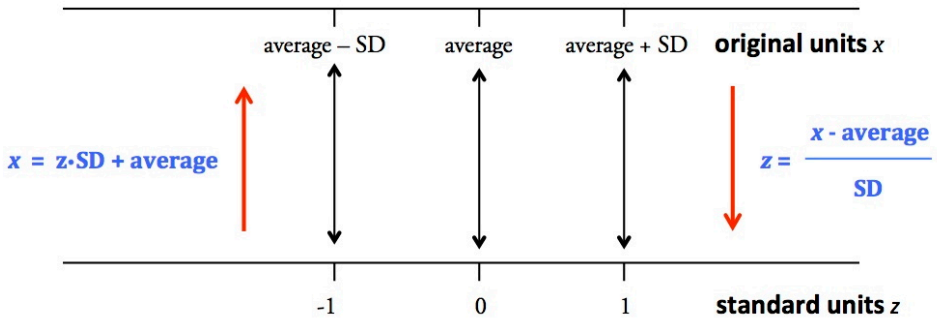
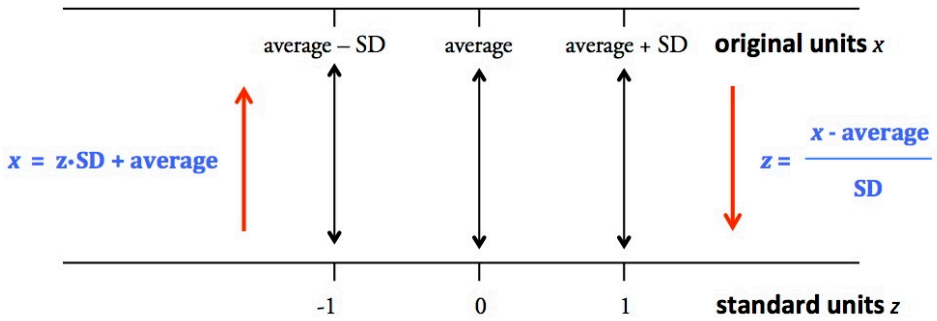


- z is a linear transformation of x
- the average of any list in standard units is 0

The average and SD of a list in standard units



- z is a linear transformation of x
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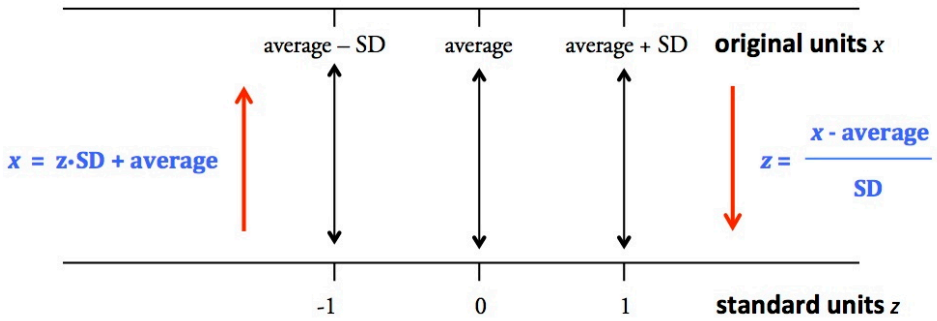
The average and SD of a list in standard units



- z is a linear transformation of x
- the average of any list in standard units is 0
- the SD of any list in standard units is 1
- the vast majority (at least 8/9) of any list in standard units will be in the range -3 to 3