ICPY102: Physics II

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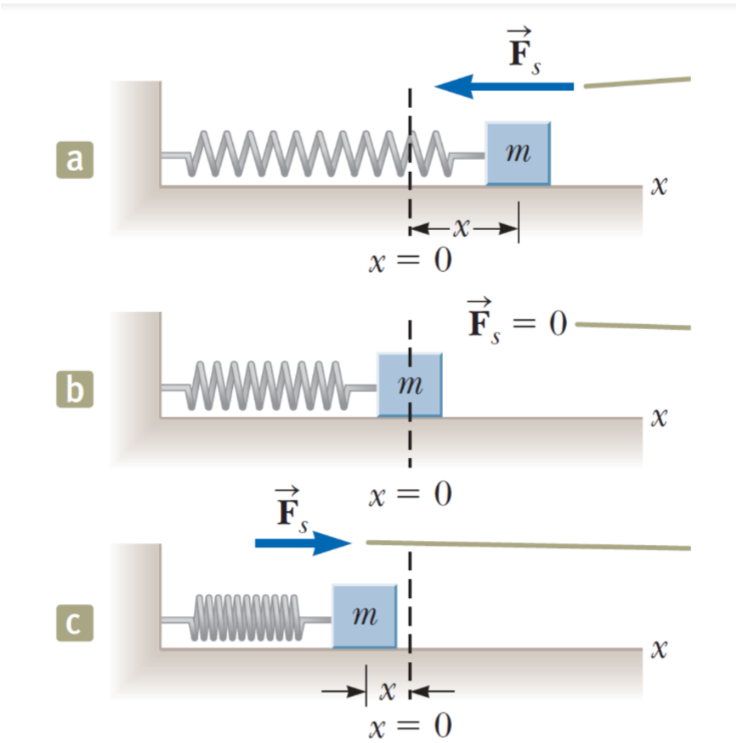
1 Oscillations and Waves

1.1 Simple Harmonic Motion (S.H.M.)

1.1.1 Equation of Motion

Consider the following systems.



From Physics I, we know that we can use Hooke's Law to calculate the spring force.

$$\vec{F_s} = -kx$$

When the spring is stretched, $\vec{F_s}$ is negative. When the spring is at equilibrium, $\vec{F_s} = 0$. If the spring is compressed, $\vec{F_s}$ is positive. There is only one force, everything is in 1D, and everything is equal to ma from Newton's $F = ma$. Thus, we can derive the **Equation of Motion**.

$$\sum F_x = -kx = ma_x$$

In our equation, we can turn a into d^2x/dt^2 since accerelation is just the second derivative of displacement. Then, through some manipulation, we can derive

$$-kx = ma_x = m\frac{d^2x}{dt^2} \implies \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

which is a differential equation we need to solve. Now, what sort of function $x(t)$ do we use such that its second derivative would still contain an x term and is negative?

We have a few choices: polynomic with negative power, exponential, logarithmic, and trigonometric functions. Our choice? *Trigonometric functions*, specifically sin and cos. Finally, we have found a suitable function x.

$$x(t) = A\sin(\omega t + \phi)$$

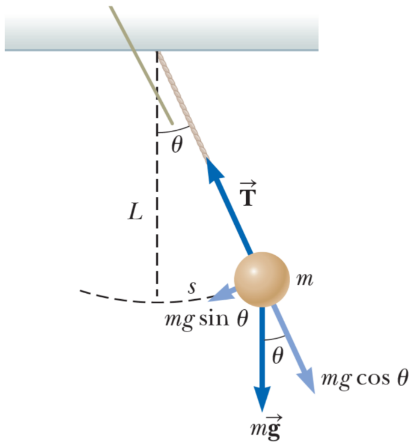
We can also choose to use the cos function as well as they give the same result. Essentially, we can convert between sin and cos since we have the ϕ term.

1.1.2 Interpretation

We can interpret ω as the angular frequency which is $2\pi f = 2\pi/T$. Knowing that $\omega = \sqrt{k/m}$, this means that for a given spring with a certain k and mass m, we cannot change the way that it oscillates. No matter how much we stretch, it's going to bounce with that frequency. This is called the **Natural Frequency**. The only way to change is to change the value of k and m.

Simple Harmonic Motion refers to a motion with a single frequency – ω. Obviously, there exists Harmonic Motion that are not simple. However, it is out of the scope of this course.

1.1.3 Equation of Motion on a Pendulum



In order to analyze the motion of the pendulum, we can break down said motion into the vertical and horizontal components. We can say that the tension $T = mg\cos\theta$ then completely ignore the motion's vertical component as they cancel out. Therefore, the only remaining forces will be

$$\sum F_t = ma_t \implies -mg\sin\theta = m\frac{d^2s}{dt^2}$$

where s is the tangential distance (displacement in circular motion). Once we cancel everything and rearrange it such that it looks like our previous differential equation, we will get

$$\frac{d^2s}{dt^2} = -g\sin\theta$$

This is where we need to face the harshness of reality. We cannot solve this equation, not without a computer. However, we can use some approximation tricks to help us.

If the angle θ of swing is small, $\sin\theta \approx \theta$. We can also change s into something easier to deal with. Since we know that $\theta = s/L$ where L is the radius, we can express s as θL. With some manipulation, our equation then becomes

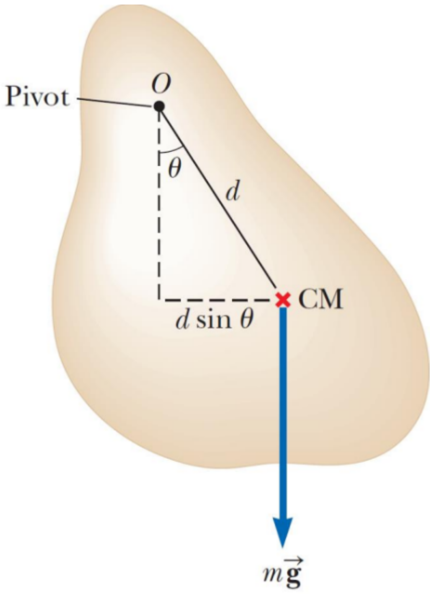
$$\frac{d^2(\theta)}{dt^2} \approx -\frac{g}{L}\theta$$

Therefore, the (approximate) solution to this equation of motion is

$$\theta(t) = A\sin(\omega t + \delta), \omega = \sqrt{\frac{g}{L}}$$

In this case, mass does not even matter to the oscillation. The only things that matter are the gravitational acceleration (g) and the length of the string attached to your pendulum (L). Interestingly, this was how people made clocks back in the day.

1.1.4 Equation of Motion on Rotation



Instead of Newton's equation, we now use $\vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F}$. From the diagram, we can see that the only thing generating torque is $m\vec{g}$. Let r be the distance from the pivot to the force (note that this distance is measured from O to the point perpendicular to the force), $r = d\sin\theta$. Now, torque will become $dmg\sin\theta$. Since the torque is going clock-wise, the sign is negative. With some manipulation, our final approximate solution becomes

$$\frac{d^2\theta}{dt^2} \approx -\frac{mgd}{I}\theta$$

Again, we can see that this is another simple harmonic motion where $\omega = \sqrt{(mgd)/I}$.

1.1.5 Energy of Simple Harmonic Oscillator

We can first find the kinetic and potential energy by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$
$$U = \frac{1}{2}kx^2 = \frac{1}{2}A^2\cos^2(\omega t + \phi)$$

Then, we can derive the total energy by

$$\begin{aligned} E &= K + U \\ &= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) + \frac{1}{2}A^2\cos^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2(\sin^2(\omega t + phi) + \cos^2(\omega t + phi)) \\ &= \frac{1}{2}kA^2 \end{aligned}$$

Note that since ω^2 k/m, $(1/2)m(k/m) = (1/2)k$.

What does this mean? At any moment in time, the sum of the energy of the system must remain the same. This aligns with the conservation of energy.