Question on NN

Build a methodology of neural network

Given inputs x_1 and x_2
Build a MLP with one hidden layer and the output layer
there is only one neuron to solve a binary classification
problem, predict 0 or 1.

1) Forward Propagation (5 1-5)

1) Take the input as a matrix

$$
\begin{array}{ccc}
\text{input} & x_1 & x_2 \\
 & \left[\begin{matrix} 0 \\ 0 \\ 1 \\ 1 \end{matrix}\right. & \left.\begin{matrix} 0 \\ 1 \\ 0 \\ 1 \end{matrix}\right]
\end{array}
$$

2) Align the x's $\quad x_0 = [\quad]$ always $\quad [x_0, x_1, x_2]$

$$
\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}
$$

3) Initialize the weights and biases (random)

$$[w_0, w_1, w_2] = [1, 1, 1]$$

4) Take the dot product (only for 1)

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = a_1 b_1 + a_2 b_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot [1, 1, 1] = (W_0 \cdot x_0) + (W_1 \times x_1) + (W_2 \times x_2)$$

$$= (1 \times 1) + (1 \times 0) + (1 \times 0) = 1 = \text{weighted sum}$$

5) Perform non-linear transformation using an activation function like sigmoid.

Sigmoid says if weighted sum is bigger than $\emptyset = 1$
Else $= 0$

- $Y = 1$ ⟹ this is now a single neuron

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2) Backward Propogation (Here we must know the output!

1) compare input with output → let's say XOR gate

$$\underset{x}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ ← output y

learning rate 0,5

2) Compare the actual output and calculate the gradient of error (actual - predicted) → $E = y - \text{Output}$

Compute slope W_0 = (learning coeff) × (input x) × (actual - predicted)

$W_0 = 0{,}5 \times 1 \times (0 - 1) = -0{,}5$

$W_1 = 0{,}5 \times 0 \times (0 - 1) = 0$

$W_2 = 0{,}5 \times 0 \times (0 - 1)$ 0

3) Update the weights for the next values + make dot product

Old weight + (new weight) = $1 + (-0,5)$

$$\begin{array}{c} x_0 \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{ccc} w_0 & w_1 & w_2 \end{array} \begin{bmatrix} 1 & 1 & 1 \\ 0,5 & 1 & 1 \end{bmatrix} = (1 \times 0,5) + (0 \times 1) + (1 \times 1) = 1,5$$

4) sigmoid = $\sigma(1,5) = 1$

5) Make gradient 0,5

$0,5 \times 1 \times (0-1) = -0,5 = (0,5) + (-0,5) = 0$

$0,5 \times 0 \times (0-1) = 0 = (1) + 0 = 1$

$0,5 \times 1 \times (0-1) = -0,5 = -0,5 + 1 = 0,5$

6)

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0,5 & 1 & 1 \\ 0 & 1 & 0,5 \end{bmatrix} = (1 \times 0) + (1 \times 1) + (0 \times 0,5) = 1$$

7) sigmoid(1) = 1

8) Gradient

$0,5 \times 1 \times (0-1) = -0,5 = 0 + (-0,5) = -0,5$

$0,5 \times 1 \times (0-1) = -0,5 = 1 + (-0,5) = 0,5$

$0,5 \times 0 \times (0-1) = 0 = 0,5 + 0 = 0,5$

9)

$$\times \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{array}{c} w \\ \begin{bmatrix} 1 & 1 & 1 \\ 0,5 & 1 & 1 \\ 0 & 1 & 0,5 \\ -0,5 & 0,5 & 0,5 \end{bmatrix} \end{array} \times \begin{array}{c} Y \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{array} = (1 \times -0,5) + (1 \times 0,5) + (1 \times 0,5) = 0,5$$

=

10) sigmoid(0,5) = 1

11) Gradient

$0,5 \times 1$

$0,5 \times 1$

$0,5 \times 1$

m) Gradient

$$0{,}5 \times 1 \times (1 - 1) = 0$$
$$0{,}5 \times 1 \times (1 - 1) = 0$$
$$0{,}5 \times 1 \times (1 - 1) = 0$$

new first weights