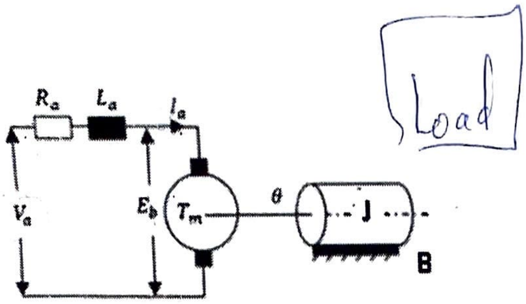

Figure 1. DC shunt motor model.

The armature voltage equation is given by:

$$V_a(t) = R_a\, I_a\,(t) + L_a.\frac{d\, I_a\,(t)}{dt} + E_B(t) \quad (1)$$

Equation for back emf of motor will be

$$E_B(t) = K_b\, \omega(t) \quad (2)$$

Now the torque balance equation will be given by:

$$T_m(t) = K_t.\, I_a\,(t) \quad (3)$$

$$T_m(t) = J\,\frac{d\omega(t)}{dt} + B\omega(t) \quad (4)$$

Where,
K_t = Torque constant (Nm/A)
K_b = back emf constant (Vs/rad)
Let us combine the upper equations together:

$$V_a(t) = R_a.I_a\,(t) + L_a.\frac{d\, I_a\,(t)}{dt} + K_b\, \omega(t) \quad (5)$$

$$K_t.\, I_a(t) = J\,\frac{d\omega(t)}{dt} + B\omega(t) \quad (6)$$

Taking Laplace Transform of (5) & (6), we get

$$V_a(s) = R_a.I_a(s) + L_a.I_a(s) + K_b\, \omega(s) \quad (7)$$

$$K_t.\, I_a(s) = J\omega(s) + B\,\omega(s) \quad (8)$$

If current is obtained from (8) and substituted in (7) we have...

$$V_a(s) = \omega(s)\frac{1}{K_t}[L_a.Js^2 + R_a.J + L_a.B(s) + K_b.K_t] \quad (9)$$

Then the relation between rotor shaft speed and applied armature voltage is represented by transfer function:

$$\frac{\omega(s)}{V_a(s)} = \frac{K_t}{(JL_a s^2 + (JR_a + BL_a)s + (K_t K_b + BR_a)} \quad (10)$$

This is the transfer function of the DC motor.
Consider the following values for the physical parameters [7,8]
Armature inductance (L_a) = 0.5 H
Armature resistance (R_a) = 1Ω
Armature voltage (V_a) = 200 V
Mechanical inertia (J) = 0.01 Kg.m2
Friction coefficient (B) = 0.1 N-m/rad/sec
Back emf constant K_b = 0.01 V/rad/sec
Motor torque constant K_t = 0.01N.m/A
Rated speed = 1450 rpm

Based on the data book, the transfer function is as

$$\frac{\omega(s)}{V_a(s)} = \frac{2}{s^2+12s+20.02} \quad (11)$$

3. Proportional-Integral-Derivative (PID) Controller

PID controllers are probably the most widely used industrial controller. In PID controller Proportional (P) control is not able to remove steady state error or offset error in step response. This offset can be eliminated by Integral (I) control action. Integral control removes offset, but may lead to oscillatory response of slowly decreasing amplitude or even increasing amplitude, both of which are error, initiates an early correction action and tends to increase stability of system.

Ideal PID controller in continuous time is given as

$$y(t) = K_p\left(e(t) + \frac{1}{T_i \int_0^t e(t)dt} + T_d\frac{de(t)}{dt}\right) \quad (12)$$

Laplace domain representation of ideal PID controller is

$$Gc(s) = \frac{Y(s)}{E(s)} = K_p(1 + \frac{1}{T_i s} + T_d s) \quad (13)$$

Ziegler and Nichols proposed rules for determining values of K_p, T_i *and* T_d based on the transient response characteristics of a given plant. Closed loop oscillation based PID tuning method is a popular method of tuning PID controller. In this kind of tuning method, a critical gain K_c is induced in the forward path of the control system. The high value of the gain takes the system to the verge of instability. It creates oscillation and from the oscillations, the value of frequency and time are calculated. Table 1 gives experimental tuning rules based on closed loop oscillation method [2,9].

Table 1. *Closed loop oscillation based tuning methods.*

Type of Controller	K_p	T_i	T_d
P	$0.5K_c$	∞	0
PI	$0.45K_c$	0.83T	0
PID	$0.6K_c$	0.5T	0.125T

From the Closed loop oscillation method, K_c = 13 and T = 2 sec, which implies K_p = 7.8 , T_i = 1 and T_d = 0.5

Usually, initial design values of PID controller obtained by all means needs to be adjusted repeatedly through computer simulations until the closed loop system performs or compromises as desired. These adjustments are done in MATLAB simulation.

4. Internal Model Control (IMC)

The theory of IMC states that "control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be