

Tram Le

Homework 2:

1. Generate a random 1000 uniform between 0-1 then store to variable x.

```
n = 1000
```

```
x <- runif(n,0,1)
```

- Using mean() to find mean of x: mean(x)
- Using var() to find variance of x: var(x)
- Using cumsum() to find cumulative sum of x then assign to cs: cs <- cumsum(x)
- Using plot() with x = 1:n and y = cs / 1:n, type line: plot(1:n, cs/1:n, type = "l")
- Using abline() to add horizontal line at y = .5: abline(h = .5)

2.

Finding mean $E(X)$ by integrate from 0 to 1 of function $x \cdot dx$:

```
f = function(x) {x}
```

```
m <- integrate(f,0,1)
```

Finding variance $Var(X)$ by integrate from 0 to 1 of function $(x-1/2)^2 \cdot dx$:

```
g = function(x) {(x-1/2)^2}
```

```
v <- integrate(g,0,1)
```

3. Generate 2 sets of $n = 1000000$ uniform between -1-1 then store them in x and y:

```
n = 1000000
```

```
x = runif(n,-1,1)
```

```
y = runif(n,-1,1)
```

- Total Points(x,y) that satisfy $x^2 + y^2 < 1$:

```
count = 0
```

```
for (i in 1:n)
```

```
{
```

```
  if (x[i]^2 + y[i]^2 < 1)
```

```
    count = count + 1
```

```
}
```

```
print(count)
```

```
>> 785461
```

- The figure that $x^2 + y^2 < 1$ is a circle part of region, the radius $r = 1$. The area of a circle is $\pi \cdot r^2 = \pi \cdot 1^2 = \pi$

b) Ratio of total number satisfies $x^2+y^2 < 1$ and total number of points used (n):

$$ratio = count / n = 0.785461$$

c) The points(x,y) fall into a square that has side lengths 2. Multiply the ratio by 4. This gives the area of circle $x^2 + y^2 = 1$

$$Area = 4 * ratio = 4 * .785461 = 3.141844$$

4. Generate 3 sets of $n = 100000$ uniform between -1-1 then store them in x, y and z:

```
n = 100000
```

```
x = runif(n,-1,1)
```

```
y = runif(n,-1,1)
```

```
z = runif(n,-1,1)
```

a) Total Points(x,y) that satisfy $x^2 + y^4 + z^6 < 1$:

```
for (i in 1: n)
```

```
{
```

```
  if (x[i]^2 + y[i]^4 + z[i]^6 < 1)
```

```
  {
```

```
    count = count + 1
```

```
  }
```

```
}
```

```
print(count)
```

```
>> 76703
```

b) Ratio of total number satisfies $x^2+y^4+z^6 < 1$ and total number of points used (n):

$$Ratio = count / n = .76703$$

c) The points(x,y,z) fall into a square that has side lengths 2. Multiply the ratio by 8. This gives the volume of a cube $= 2*2*2 = 8$

$$Volume = 8 * Ratio = 6.13624$$

5. Generate 2 points randomly $n = 10000$.

```
for (i in 1:n)
```

```
{
```

```
  # create 2 random uniform between 0-1 then sort the order of values
```

```
  x = runif(2, 0, 1)
```

```
  x = sort(x)
```

```
  # create vector to put 1 and 0 in each row
```

```
m = c(0,0,0,1)
# assign each value in the 1st, 2nd column to the 2nd, 3rd value in vector
m[2] = x[1]
m[3] = x[2]
# sort the difference between pairs in vector (next - previous)
m2 = sort(diff(m))
#after the sort it becomes 3 segments
#compare the sum of 1st two segments > the 3rd then count it
if(m2[1] + m2[2] > m2[3])
{
    count = count + 1
}
}
print("The probability is: ")
print(count/n)
>> The probability is 0.2518
```