

# **Sistem Kontrol Optimal**

## **Modul 1 & 2**

Kelompok 12 :

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Operasi Matematika sederhana pada matriks

# Penjumlahan

```
1 static void add_matrix(int row, int col, int matrix_result[row][col],int matrix_input_1[row][col],int
  matrix_input_2[row][col]){
2
3     for (int x = 0; x < row; x++){
4         for (int y = 0; y < col; y++)
5             matrix_result[x][y] = matrix_input_1[x][y] + matrix_input_2[x][y];
6     }
7 }
```

# Pengurangan

```
1 static void subtract_matrix(int row, int col, int matrix_result[row][col],int matrix_input_1[row][col],int  
  matrix_input_2[row][col]){  
2  
3     for (int x = 0; x < row; x++){  
4         for (int y = 0; y < col; y++)  
5             matrix_result[x][y] = matrix_input_1[x][y] - matrix_input_2[x][y];  
6     }  
7 }
```

# Perkalian

```
1 static void multiply_matrix(int row, int col, int matrix_result[row][col],int matrix_input_1[row][col],int
  matrix_input_2[row][col]){
2
3     for (int x = 0; x < row; x++){
4         for (int y = 0; y < col; y++){
5             matrix_result[x][y] = 0;
6             for (int z = 0; z < col; z++)
7                 matrix_result[x][y] += matrix_input_1[x][z]*matrix_input_2[z][y];
8         }
9     }
10 }
```

# Transpose

```
1 static void transpose_matrix(int row, int col, int matrix_result[row][col],int matrix_input[row][col]){  
2  
3     int matrix_tmp[row][col];  
4  
5     for (int x = 0; x < row; x++){  
6         for (int y = 0; y < col; y++){  
7             matrix_tmp[y][x] = matrix_input[x][y];  
8         }  
9  
10    for (int x = 0; x < row; x++){  
11        for (int y = 0; y < col; y++){  
12            matrix_result[x][y] = matrix_tmp[x][y];  
13        }  
14 }
```

# Program 1 Modul 1

```
1 #define ROW 2
2 #define COL 2
3
4 #include "MatrixLib.h"
5
6 struct matrix_t{
7     int data[2][2];
8 };
9
10 struct matrix_t A,B,C;
11
12 int main(void){
13
14     // INPUT MATRIX
15     input_matrix(ROW,COL,A.data);
16     printf("Print Matrix A : \r\n");
17     print_matrix(ROW,COL,A.data);
18
19     input_matrix(ROW,COL,B.data);
20     printf("Print Matrix B : \r\n");
21     print_matrix(ROW,COL,B.data);
22
```

```
23     // ADD MATRIX
24     add_matrix(ROW,COL,C.data,A.data,B.data);
25     printf("Add Matrix : \r\n");
26     print_matrix(ROW,COL,C.data);
27
28     // SUBTRACT MATRIX
29     subtract_matrix(ROW,COL,C.data,A.data,B.data);
30     printf("Subtract Matrix : \r\n");
31     print_matrix(ROW,COL,C.data);
32
33     // MULTIPLY MATRIX
34     multiply_matrix(ROW,COL,C.data,A.data,B.data);
35     printf("Multiply Matrix : \r\n");
36     print_matrix(ROW,COL,C.data);
37
38     //TRANSPOSE MATRIX
39     transpose_matrix(ROW,COL,A.data,A.data);
40     printf("Transpose Matrix A : \r\n");
41     print_matrix(ROW,COL,A.data);
42
43     transpose_matrix(ROW,COL,B.data,B.data);
44     printf("Transpose Matrix B : \r\n");
45     print_matrix(ROW,COL,B.data);
46
```

# Hasil Program

```
matrix[0][0] = 1  
matrix[0][1] = 2  
matrix[1][0] = 3  
matrix[1][1] = 4
```

Print Matrix A :

|   |   |
|---|---|
| 1 | 2 |
| 3 | 4 |

```
matrix[0][0] = 5  
matrix[0][1] = 6  
matrix[1][0] = 7  
matrix[1][1] = 8
```

Print Matrix B :

|   |   |
|---|---|
| 5 | 6 |
| 7 | 8 |

Add Matrix :

|    |    |
|----|----|
| 6  | 8  |
| 10 | 12 |

Subtract Matrix :

|    |    |
|----|----|
| -4 | -4 |
| -4 | -4 |

Multiply Matrix :

|    |    |
|----|----|
| 19 | 22 |
| 43 | 50 |

Transpose Matrix A :

|   |   |
|---|---|
| 1 | 3 |
| 2 | 4 |

Transpose Matrix B :

|   |   |
|---|---|
| 5 | 7 |
| 6 | 8 |



# Program 2 Modul 1

```
1 #define ROW 2
2 #define COL 2
3
4 #include "MatrixLib.h"
5
6 struct matrix_t{
7     int data[2][2];
8 };
9
10 struct matrix_t A,B,P, C,D,E, F;
11
12 int main (){
13
14     // Berapakah hasil dari persamaan berikut:
15     //  $A^T + P \cdot A - B = C + P \cdot A - B$ 
16     // Dengan nilai matriks A, P dan B ditentukan oleh kalian sendiri
17
18     printf("Masukkan nilai untuk matriks A : \n");
19     input_matrix(ROW,COL,A.data);
20     printf("Masukkan nilai untuk matriks B : \n");
21     input_matrix(ROW,COL,B.data);
22     printf("Masukkan nilai untuk matriks P : \n");
23     input_matrix(ROW,COL,P.data);
24 }
```

```
25     printf("Matrix A : \n");
26     print_matrix(ROW,COL,A.data);
27     printf("Matrix B : \n");
28     print_matrix(ROW,COL,B.data);
29     printf("Matrix P : \n");
30     print_matrix(ROW,COL,P.data);
31
32     multiply_matrix(ROW,COL,C.data,P.data,A.data);
33     printf("P * A : \n");
34     print_matrix(ROW,COL,C.data);
35
36     transpose_matrix(ROW,COL,A.data,A.data);
37     printf("Transpose matrix A : \n");
38     print_matrix(ROW,COL,A.data);
39
40     add_matrix(ROW,COL,D.data,A.data,C.data);
41     printf("A^T + [P * A] : \n");
42     print_matrix(ROW,COL,D.data);
43
44     subtract_matrix(ROW,COL,E.data,D.data,B.data);
45     printf("[A^T + [P * A]] - B : \n");
46     print_matrix(ROW,COL,E.data);
47
48     return 0;
49 }
```

# Hasil Program

```
Masukkan nilai untuk matriks A :  
matrix[0][0] = 1  
matrix[0][1] = 2  
matrix[1][0] = 3  
matrix[1][1] = 4
```

```
Masukkan nilai untuk matriks B :  
matrix[0][0] = 5  
matrix[0][1] = 6  
matrix[1][0] = 7  
matrix[1][1] = 8
```

```
Masukkan nilai untuk matriks P :  
matrix[0][0] = 9  
matrix[0][1] = 10  
matrix[1][0] = 11  
matrix[1][1] = 12
```

Matrix A :

|   |   |
|---|---|
| 1 | 2 |
| 3 | 4 |

Matrix B :

|   |   |
|---|---|
| 5 | 6 |
| 7 | 8 |

Matrix P :

|    |    |
|----|----|
| 9  | 10 |
| 11 | 12 |

P \* A :

|    |    |
|----|----|
| 39 | 58 |
| 47 | 70 |

Transpose matrix A :

|   |   |
|---|---|
| 1 | 3 |
| 2 | 4 |

$A^T + [P * A]$  :

|    |    |
|----|----|
| 40 | 61 |
| 49 | 74 |

$[A^T + [P * A]] - B$  :

|    |    |
|----|----|
| 35 | 55 |
| 42 | 66 |

determinant ,minor, cofactor, adjoint dan  
invers

# Determinant

```
1 static float determinant_matrix(int row, int col, int matrix[row][col]){
2
3     float a = 0, b = 0, c = 0;
4
5     if (row == 3 && col == 3) {
6
7         a = matrix[0][0]*(matrix[1][1]*matrix[2][2] - matrix[1][2]*matrix[2][1]);
8         b = matrix[0][1]*(matrix[1][0]*matrix[2][2] - matrix[1][2]*matrix[2][0]);
9         c = matrix[0][2]*(matrix[1][0]*matrix[2][1] - matrix[1][1]*matrix[2][0]);
10
11         return a-b+c;
12     }
13
14     else if (row == 2 && col == 2) {
15
16         a = matrix[0][0]*matrix[1][1];
17         b = matrix[0][1]*matrix[1][0];
18
19         return a-b;
20
21     }
22 }
```

# Minor

```
1 static void minor_matrix(int row, int col, int matrix_result[row][col][row - 1][col - 1],int matrix_input[row]
   [col],int dimension){
2
3     for(int target_row = 0; target_row < row; target_row++){
4         for(int target_col = 0; target_col < col; target_col++){
5
6             int minor_row = 0,minor_col = 0;
7
8             for (int x = 0; x < row; x++){           // row matrix
9                 for (int y = 0; y < col; y++){       // col matrix
10                     if(x != target_row && y != target_col)
11                         matrix_result[target_row][target_col][minor_row][minor_col++] = matrix_input[x][y];
12
13                     if(minor_col == dimension - 1){ // dimension of minor matrix
14                         minor_col = 0;             // reset minor col
15                         minor_row++;               // add new minor row
16                     }
17                 }
18             }
19         }
20     }
21 }
```

# Cofactor

```
1 static void get_cofactor(int row, int col, int matrix_result[row][col],int matrix_input[row][col][row - 1][col- 1]){  
2  
3     for (int x = 0; x < row; x++){  
4         for (int y = 0; y < col; y++){  
5             matrix_result[x][y] = pow(-1,x+y) * (matrix_input[x][y][0][0]*matrix_input[x][y][1][1]) -  
6             (matrix_input[x][y][0][1]*matrix_input[x][y][1][0]);  
7         }  
8     }
```

# Adjoint = Transpose cofactor

```
1 static void transpose_matrix(int row, int col, int matrix_result[row][col],int matrix_input[row][col]){  
2  
3     int matrix_tmp[row][col];  
4  
5     for (int x = 0; x < row; x++){  
6         for (int y = 0; y < col; y++){  
7             matrix_tmp[y][x] = matrix_input[x][y];  
8         }  
9  
10    for (int x = 0; x < row; x++){  
11        for (int y = 0; y < col; y++){  
12            matrix_result[x][y] = matrix_tmp[x][y];  
13        }  
14 }
```

# Invers

```
1 static void invers_matrix(int row, int col, float matrix_result[row][col],int matrix_input[row][col],float
   determinant){
2
3     for (int x = 0; x < row; x++){
4         for (int y = 0; y < col; y++){
5             matrix_result[x][y] = matrix_input[x][y] / fabs(determinant);
6         }
7     }
8 }
```



# Program 3 Modul 1

```
1 #define ROW 3
2 #define COL 3
3
4 #define MINOR_ROW ROW-1
5 #define MINOR_COL COL-1
6
7 #define DIMENSION 3
8
9 #include "MatrixLib.h"
10
11 int A[ROW][COL] = {1,2,3,
12                   0,1,4,
13                   5,6,0};
14
15 int B[ROW][COL][MINOR_ROW][MINOR_COL];
16 int C[ROW][COL];
17 float D[ROW][COL];
18
```

```
18
19 int main(){
20
21     // input_matrix(A);
22     printf("Elemen Matrix A : \n");
23     print_matrix(ROW,COL,A);
24     printf("determinan Matrix A : %0.2f\n\n",determinant_matrix(ROW,COL,A));
25     minor_matrix(ROW,COL,B,A,DIMENSION);
26     print_minor_matrix(ROW,COL,B);
27     get_cofactor(ROW,COL,C,B);
28     printf("cofactor matrix : \n");
29     print_matrix(ROW,COL,C);
30     printf("adjoint matrix : \n"); //adjoint is transpose of cofactor matrix
31     transpose_matrix(ROW,COL,C,C);
32     print_matrix(ROW,COL,C);
33     printf("Invers matrix : \n");
34     invers_matrix(ROW,COL,D,C,determinant_matrix(ROW,COL,A));
35     print_float_matrix(ROW,COL,D);
36
37     return 0;
38 }
```

# Hasil Program

Elemen Matrix A :

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 5 | 6 | 0 |

determinan Matrix A : 1.00

```
Minor matrix [0][0] :  
1 4  
6 0  
Minor matrix [0][1] :  
0 4  
5 0  
Minor matrix [0][2] :  
0 1  
5 6  
Minor matrix [1][0] :  
2 3  
6 0  
Minor matrix [1][1] :  
1 3  
5 0
```

```
Minor matrix [1][2] :  
1 2  
5 6  
Minor matrix [2][0] :  
2 3  
1 4  
Minor matrix [2][1] :  
1 3  
0 4  
Minor matrix [2][2] :  
1 2  
0 1
```

cofactor matrix :

|     |     |    |
|-----|-----|----|
| -24 | -20 | -5 |
| -18 | -15 | -4 |
| 5   | 4   | 1  |

adjoint matrix :

|     |     |   |
|-----|-----|---|
| -24 | -18 | 5 |
| -20 | -15 | 4 |
| -5  | -4  | 1 |

Invers matrix :

|        |        |      |
|--------|--------|------|
| -24.00 | -18.00 | 5.00 |
| -20.00 | -15.00 | 4.00 |
| -5.00  | -4.00  | 1.00 |

Eigen Value

# Eigen value

```
1 static void eigen_val(int row, int col, int matrix[row][col]){
2
3     float lamda[2];
4     int matrix_temp[2][2];
5
6     // solve using quadratic formula
7
8     int a = 1;
9     int b = -1 * (matrix[0][0] + matrix[1][1]);
10    int c = (-1 * matrix[0][0] * -1 * matrix[1][1]) - (-1 * matrix[0][1] * -1 * matrix[1][0]);
11
12    printf("%d\n",b);
13    printf("%d\n",c);
14
15    print_matrix(2,2,matrix);
16
17    for(int x = 1; x <= 2; x++){
18        lamda[x] = (-b + pow(-1,x) * sqrt(pow(b,2) - (4 * a * c)))/ (2 * a);
19        printf("lamda [%d] : %0.2f\n",x,lamda[x]);
20    }
21 }
```

# Program 4 Modul 1

```
1 #include "MatrixLib.h"
2
3 static int A[2][2] = {3,0,
4                       8,-1};
5
6 int main(){
7
8     eigen_val(2,2,A);
9
10    return 0;
11 }
```

# Hasil Program

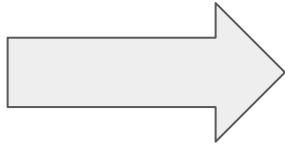
```
3      0
8      -1

lamda [1] : -1.00
lamda [2] : 3.00
```

Mencari matriks hessian

# Representasi Fungsi ke matriks

$$f(x, y) = x^3 y^3 + 3x^2 + y^2$$



|       | $x^0$ | $x^1$ | $x^2$ | $x^3$ |
|-------|-------|-------|-------|-------|
| $y^0$ | 0     | 0     | 3     | 0     |
| $y^1$ | 0     | 0     | 0     | 0     |
| $y^2$ | 1     | 0     | 0     | 0     |
| $y^3$ | 0     | 0     | 0     | 1     |

# Fungsi turunan terhadap x y

```
1 void get_differential_multivar(int row,int col, int diff_x, int diff_y, int function_input[row][col], int
  function_output[row][col]){
2
3   int function_temp[row][col];
4
5   if(diff_x != 0){
6     for (int i = 0; i < row; i++){
7       for (int j = 0; j < col; j++){
8         int x = (diff_x != diff_y) ? (get_factorial(j)) : (j * diff_x);
9         function_temp[i][j - diff_x] = function_input[i][j] * x;
10      }
11    }
12  }
13  else
14    transfer_matrix(row,col,function_temp,function_input);
15
16  if(diff_y != 0){
17    for (int i = 0; i < row; i++){
18      for (int j = 0; j < col; j++){
19        int x = (diff_x != diff_y) ? (get_factorial(i)) : (i * diff_y);
20        function_output[i - diff_y][j] = function_temp[i][j] * x;
21      }
22    }
23  }
24  else
25    transfer_matrix(row,col,function_output,function_temp);
26
27  for(int i = 0; i < row; i++){ // clean error value
28    for(int j = 0; j < col; j++)
29      if(i >= row - diff_y || j >= col - diff_x)
30        function_output[i][j] = 0;
31  }
32 }
```



# Menghitung matriks hessian

```
1 void hessian_matrix(int row, int col,int function_input[row][col]){
2
3     int x = 0;
4
5     print("\n");
6
7     for (int i = 0; i < 2; i++){
8         for (int j = 0; j < 2; j++){
9             printf("Matrix hessian [%d][%d] : \n",i,j);
10            get_differential_multivar(row,col,step[x][0],step[x][1],function_input,hessian_output[i][j].data);
11            print_function_matrix(row,col,hessian_output[i][j].data);
12            x++;
13        }
14    }
15 }
```

# Program 1 Modul 2

```
1 #define poly_x 3
2 #define poly_y 3
3
4 #define x_index poly_x + 1 //COL
5 #define y_index poly_y + 1 //ROW
6
7 #include "CalculusLib.h"
8
9 static int function_input[4][4] = {0, 0, 3, 0, // f(x,y) = x^3*y^3 + 3x^2 + y^2
10                                     0, 0, 0, 0,
11                                     1, 0, 0, 0,
12                                     0, 0, 0, 1};
13
14 int function_output[4][4];
15
16 int main(){
17
18     printf("Representasi fungsi polynomial dengan matrix :\n");
19     print_function_matrix(y_index,x_index,function_input);
20     printf("Hessian matrix : \n");
21     hessian_matrix(y_index,x_index,function_input,2,-1);
22
23     return 0;
24
25 }
```

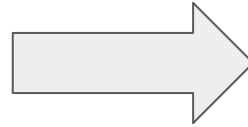
# Hasil program

Matrix hessian [0][0] :

|   |   |   |   |
|---|---|---|---|
| 6 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 6 | 0 | 0 |

Matrix hessian [0][1] :

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 9 | 0 |
| 0 | 0 | 0 | 0 |



$$\frac{\partial^2}{\partial x^2} \Rightarrow 6xy^3 + 6$$



$$\frac{\partial^2}{\partial x \partial y} \Rightarrow 9x^2y^2$$

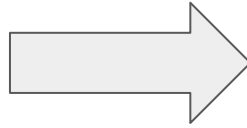
# Hasil program

Matrix hessian [1][0] :

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 9 | 0 |
| 0 | 0 | 0 | 0 |

Matrix hessian [1][1] :

|   |   |   |   |
|---|---|---|---|
| 2 | 0 | 0 | 0 |
| 0 | 0 | 0 | 6 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |



$$\frac{\partial^2}{\partial y \partial x} \Rightarrow 9x^2y^2$$



$$\frac{\partial^2}{\partial y^2} \Rightarrow 6x^3y + 2$$

Terimakasih