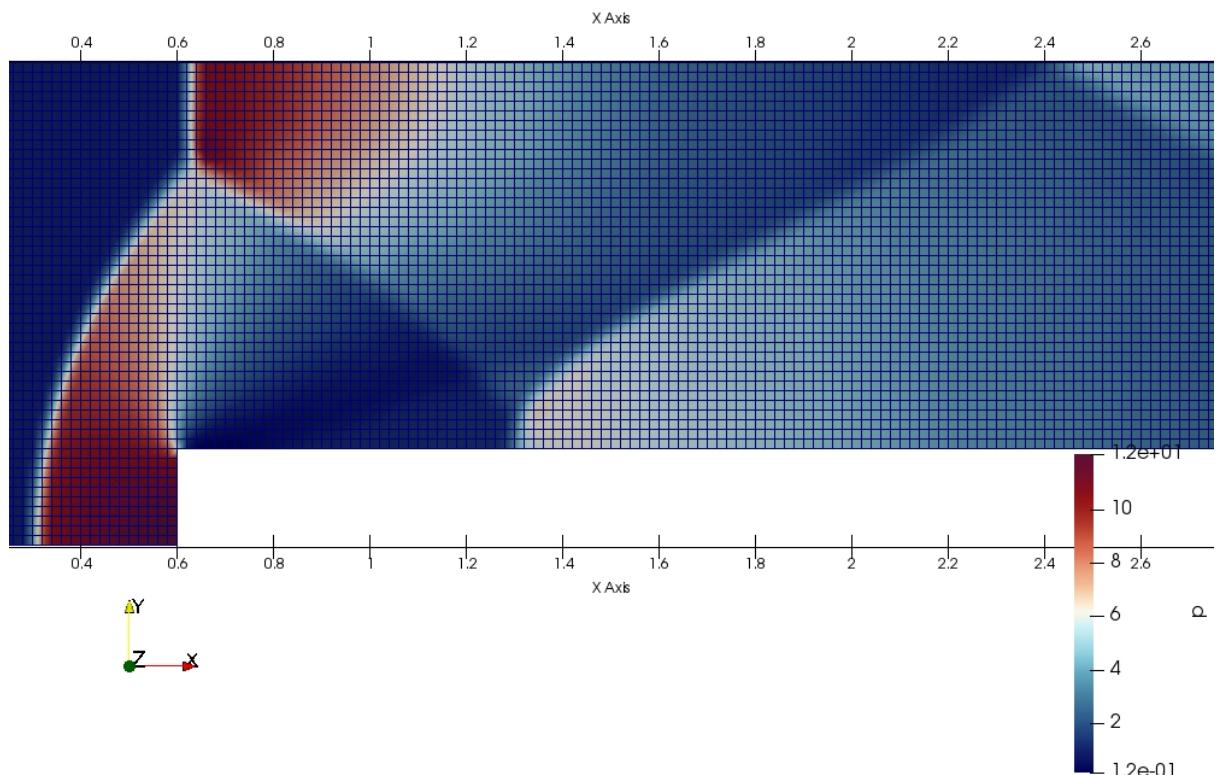


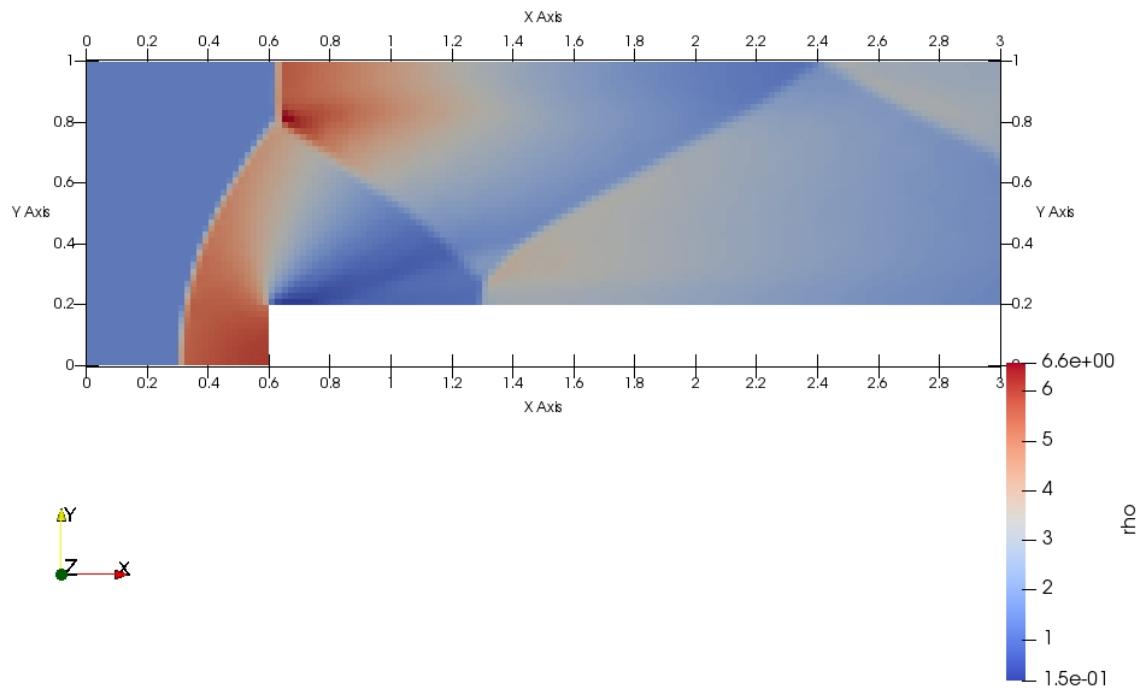
4. Mesh Assembly : Preliminaries

We assemble a preliminary solution at $T = 4s$ with 6300 cells, for the following inlet conditions: $M = 3, p = 1Pa, T = 1K, \rho = \gamma \frac{kg}{m^3}$ with $\gamma = 1.4$. All results have been non-dimensionalized: velocity, pressure, density and temperature are all measured with respect to inlet conditions.

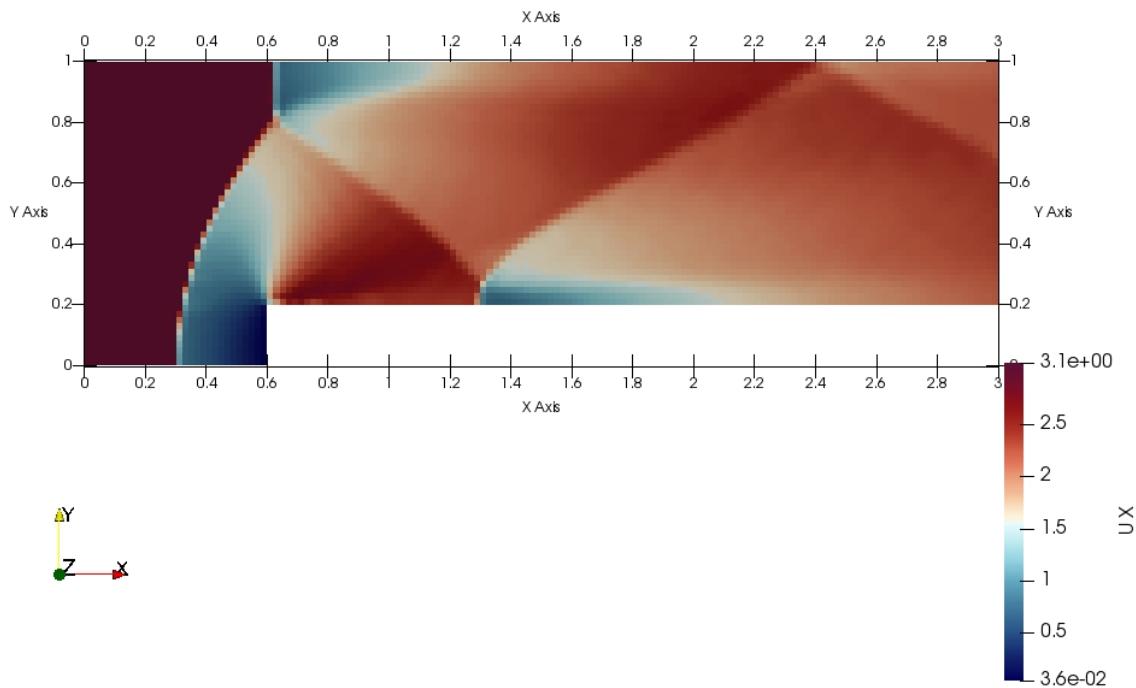
4.1. Mesh

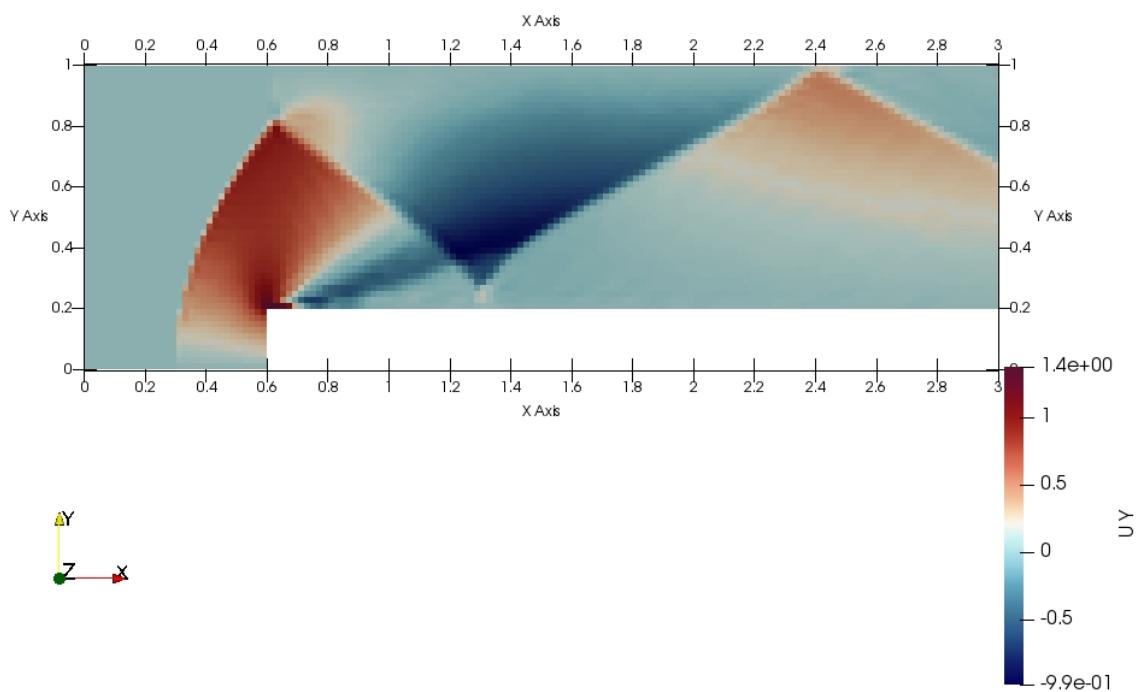


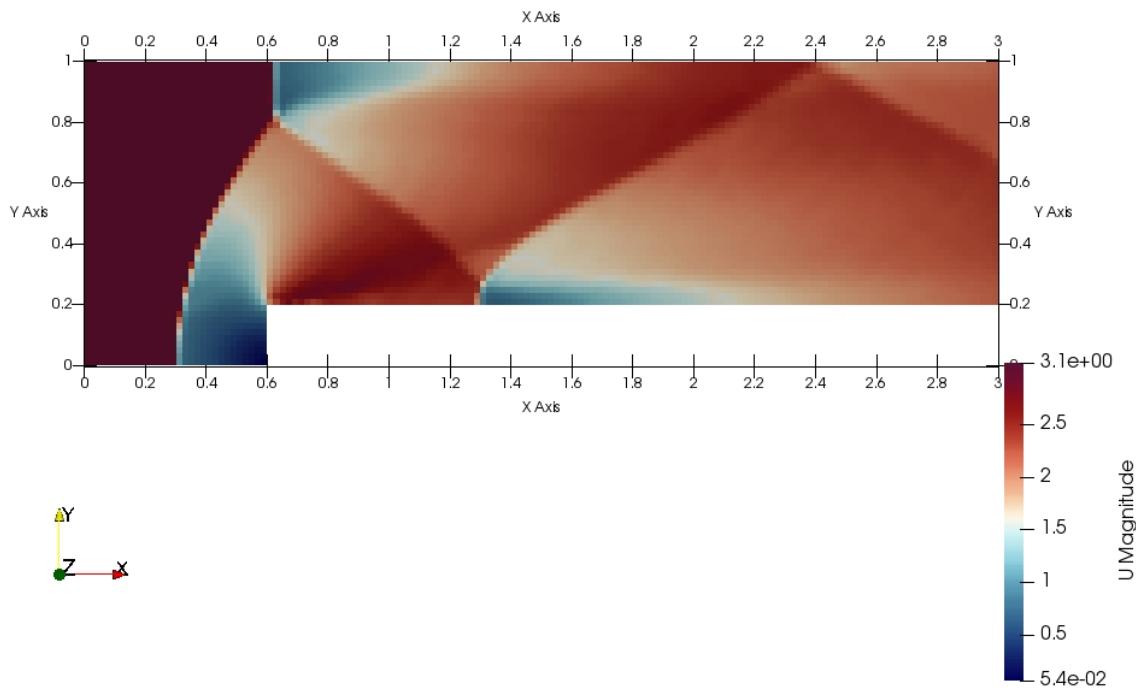
4.2. Density



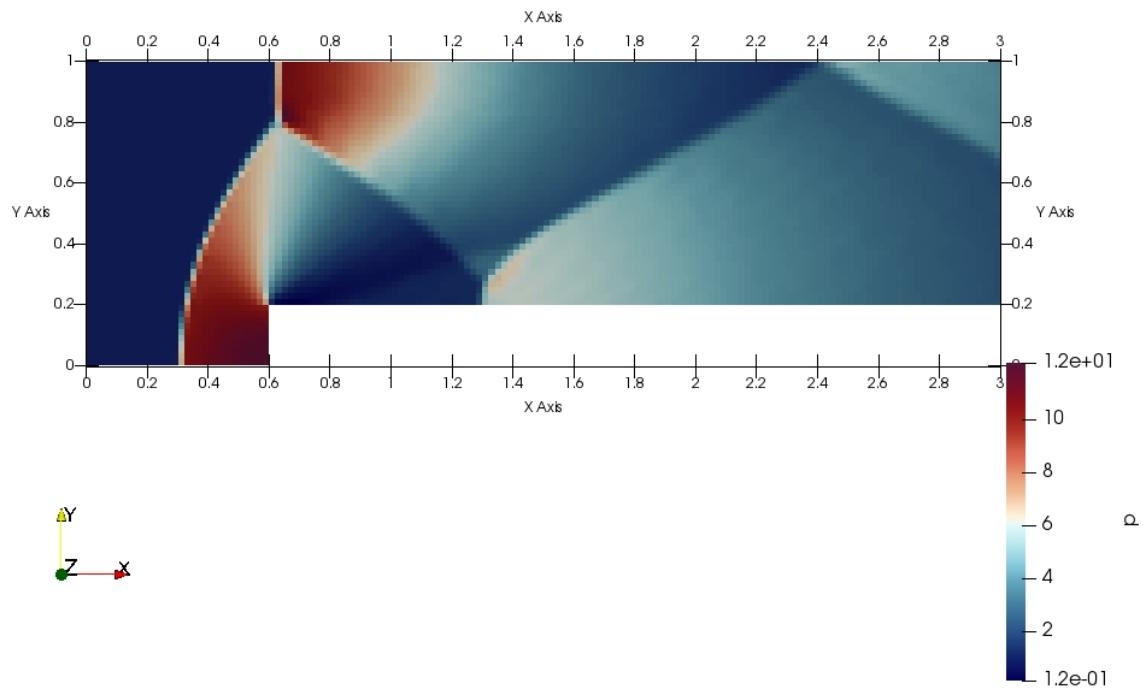
4.3. Velocity X, Y, Magnitude



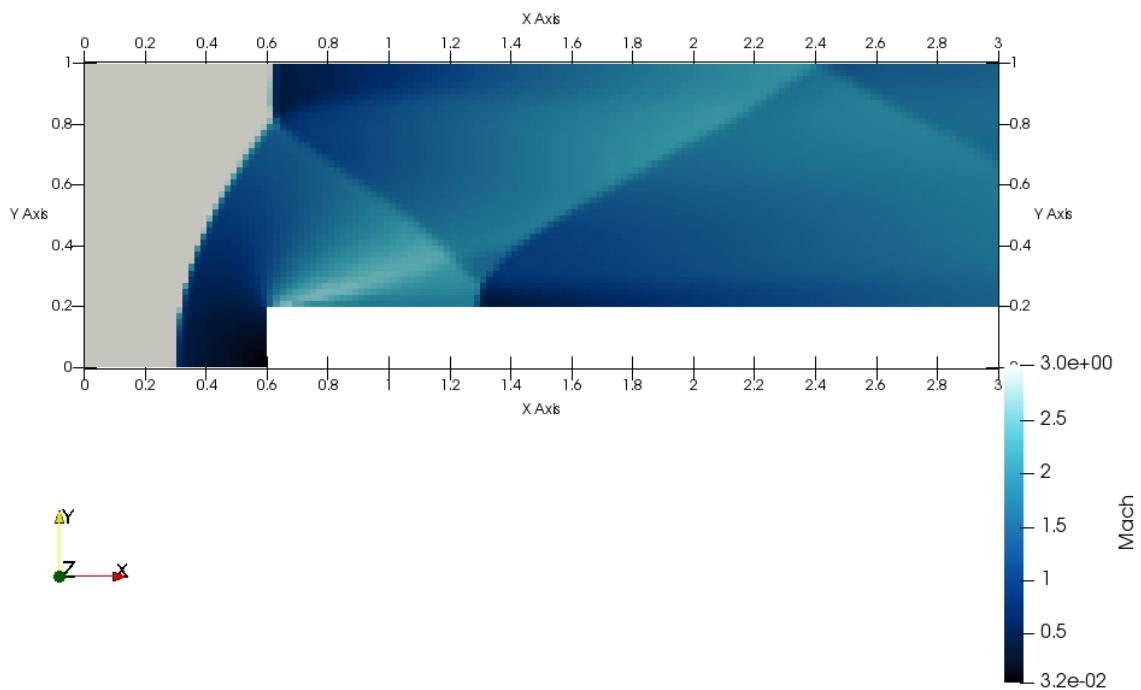




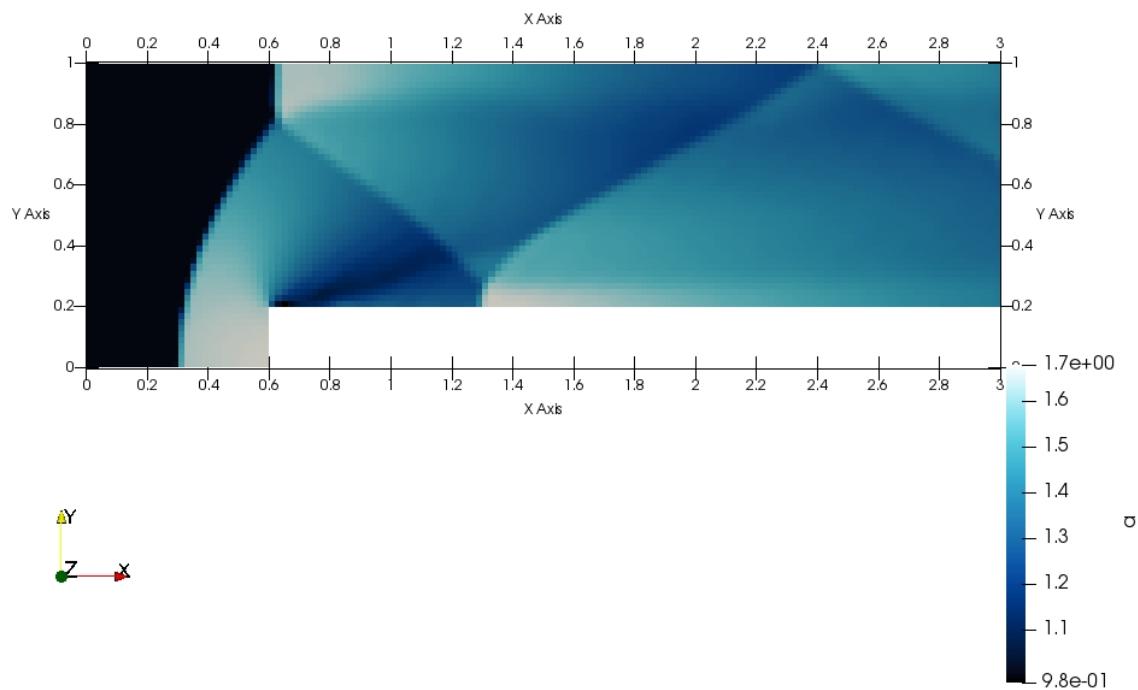
4.4. Pressure



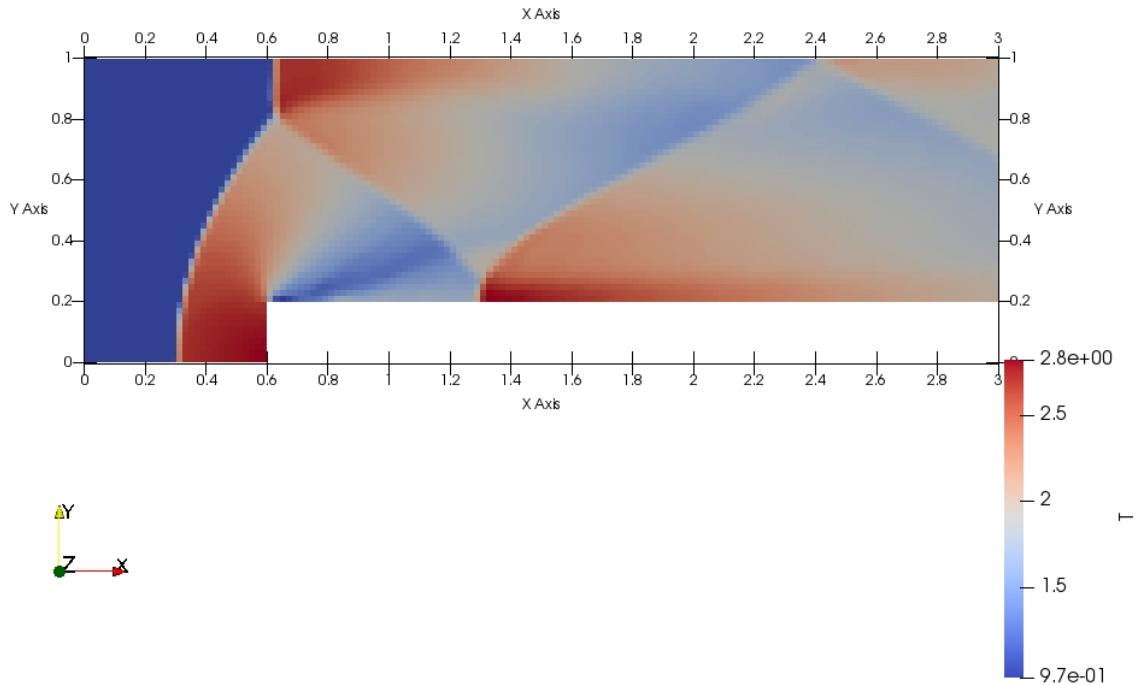
4.5. Mach Number



4.6. Speed of Sound



4.7. Temperature



5. Convergence

Three meshes were used, with increasing numbers of cells and smaller timesteps, as follows: (note the timesteps are approximate as the solver, RhoCentralFoam, adjusts timesteps as necessary to maintain a maximum Courant number of 0.2).

Run_10_64 : 6300 cells, $dx = 0.04000$, $dt = 0.00110$

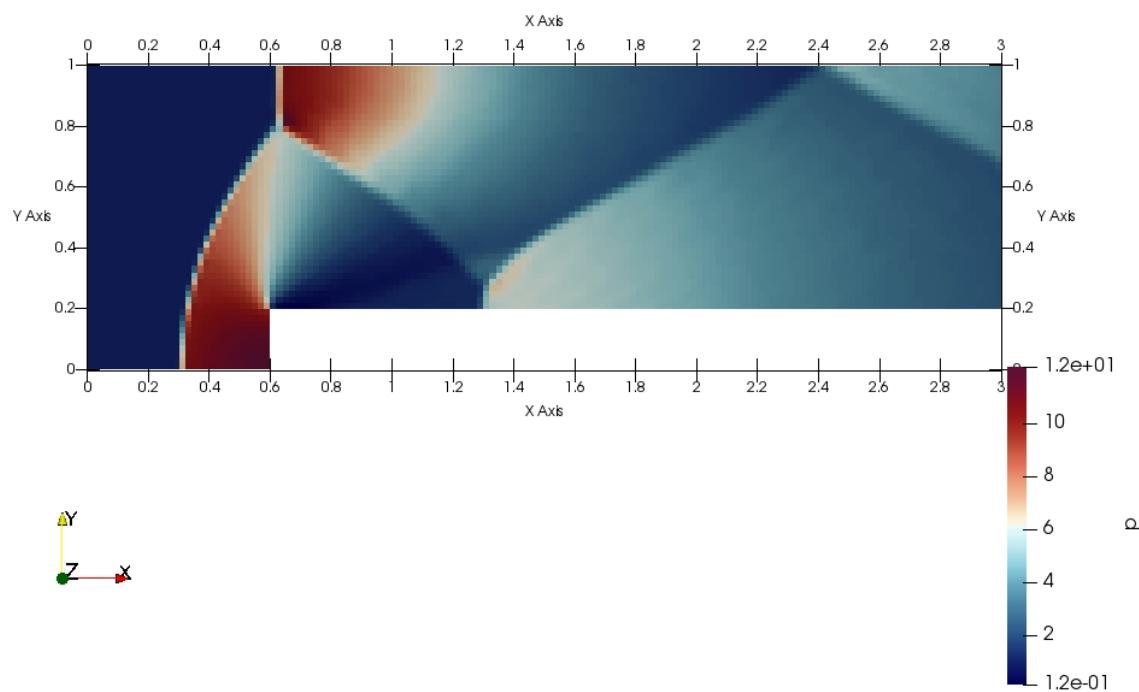
Run_20_64 : 25200 cells, $dx = 0.02000$, $dt = 0.00056$

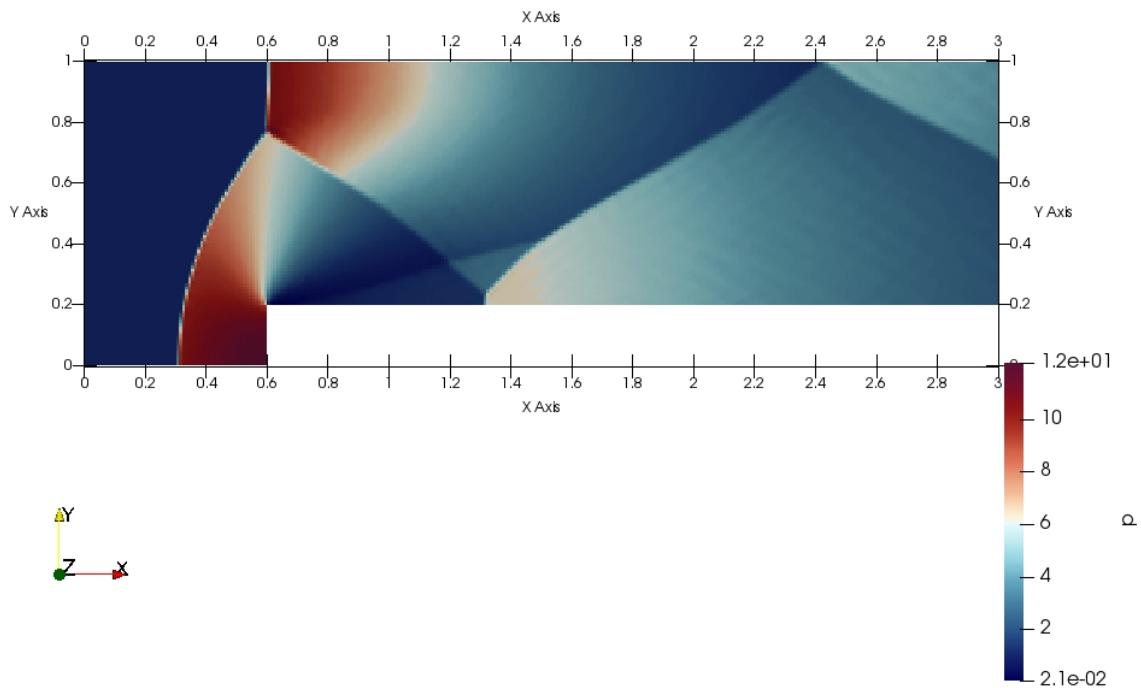
Run_32_64 : 64512 cells, $dx = 0.00625$, $dt = 0.00034$

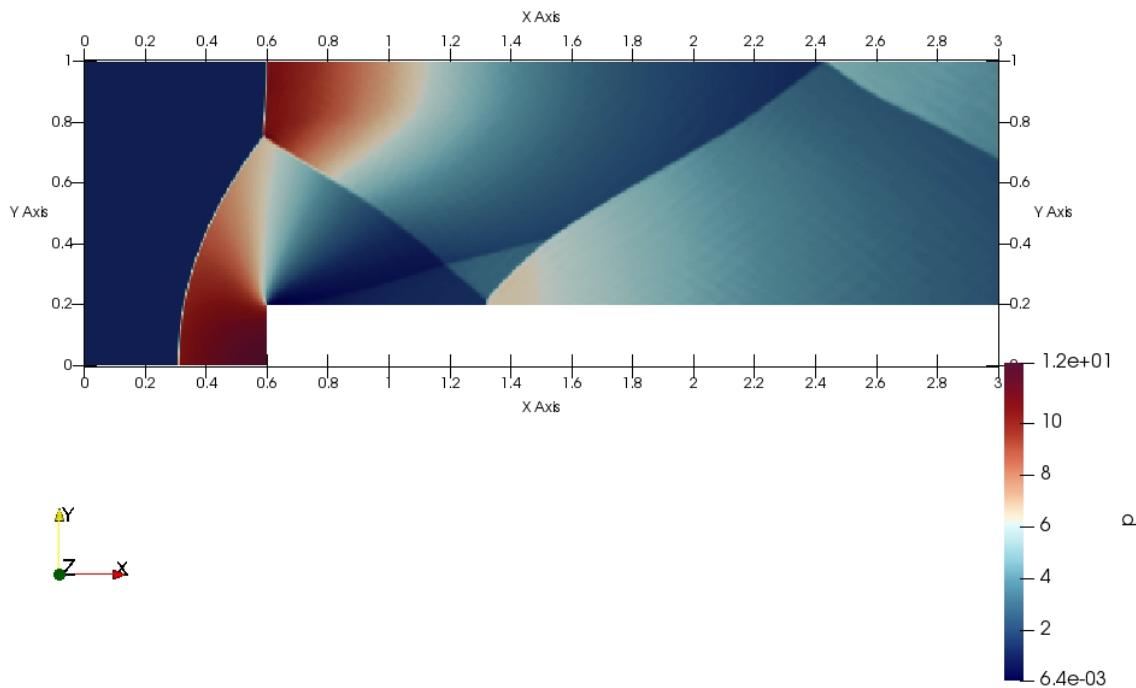
5.8. Improvements in Pressure, Density, and Y-Velocity with increasing resolution:

Note the upper shock turbulent features (the small waves), and how they resolve much more appropriately with higher resolution meshes.

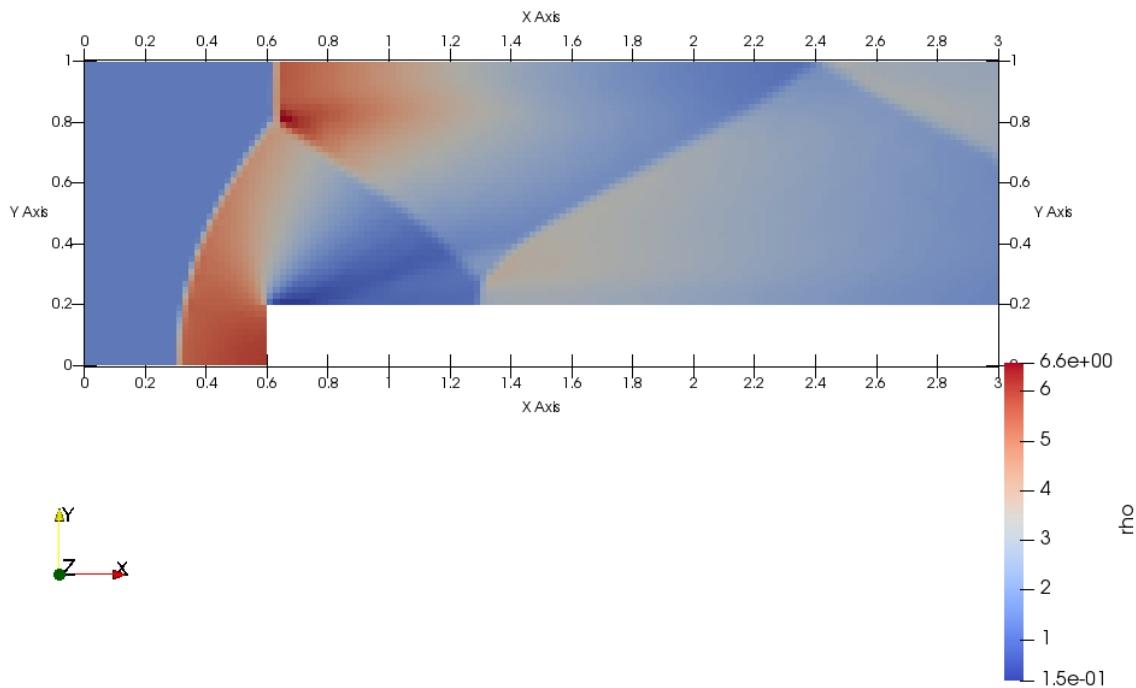
Pressure (from Run_10 to Run_32):

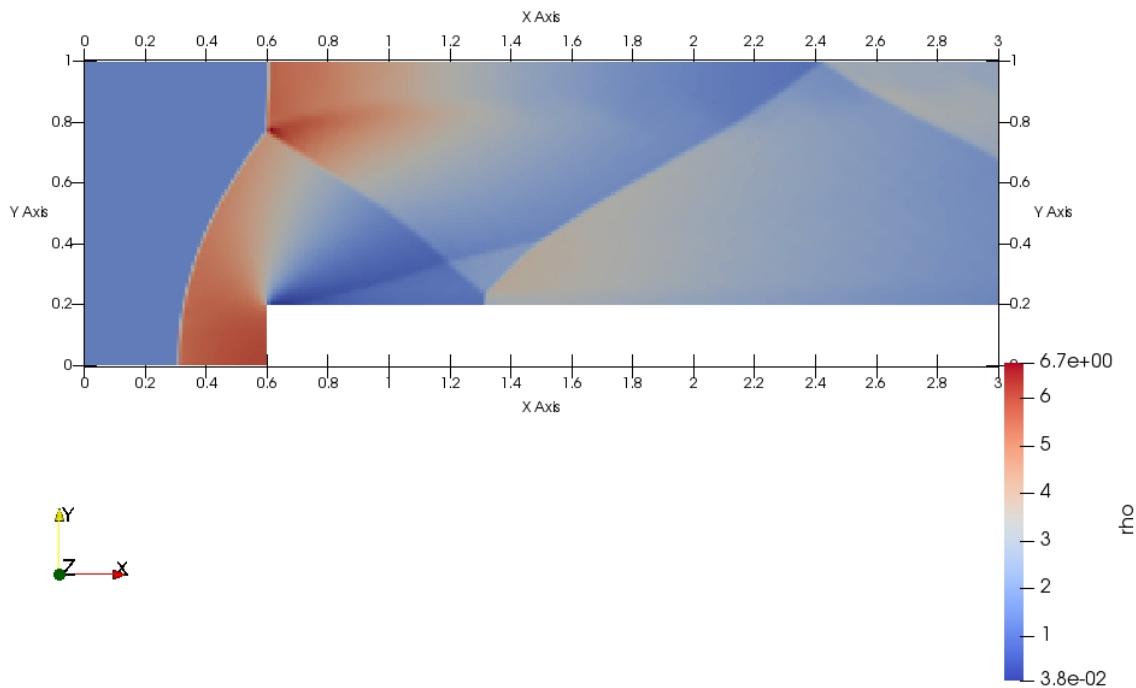


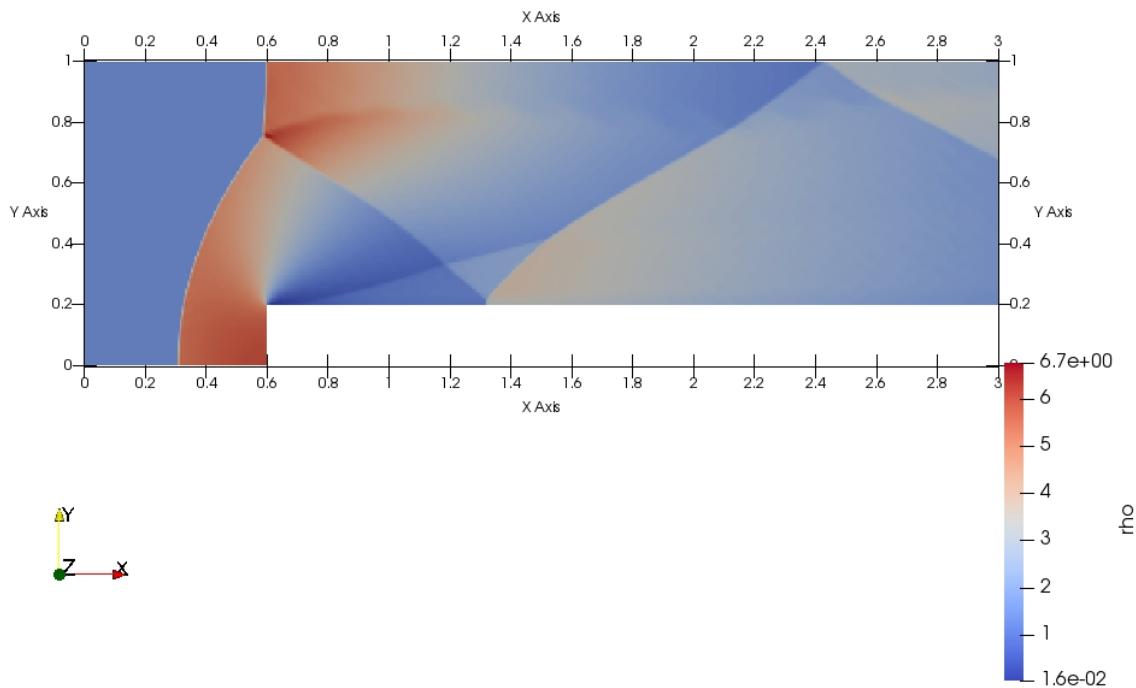




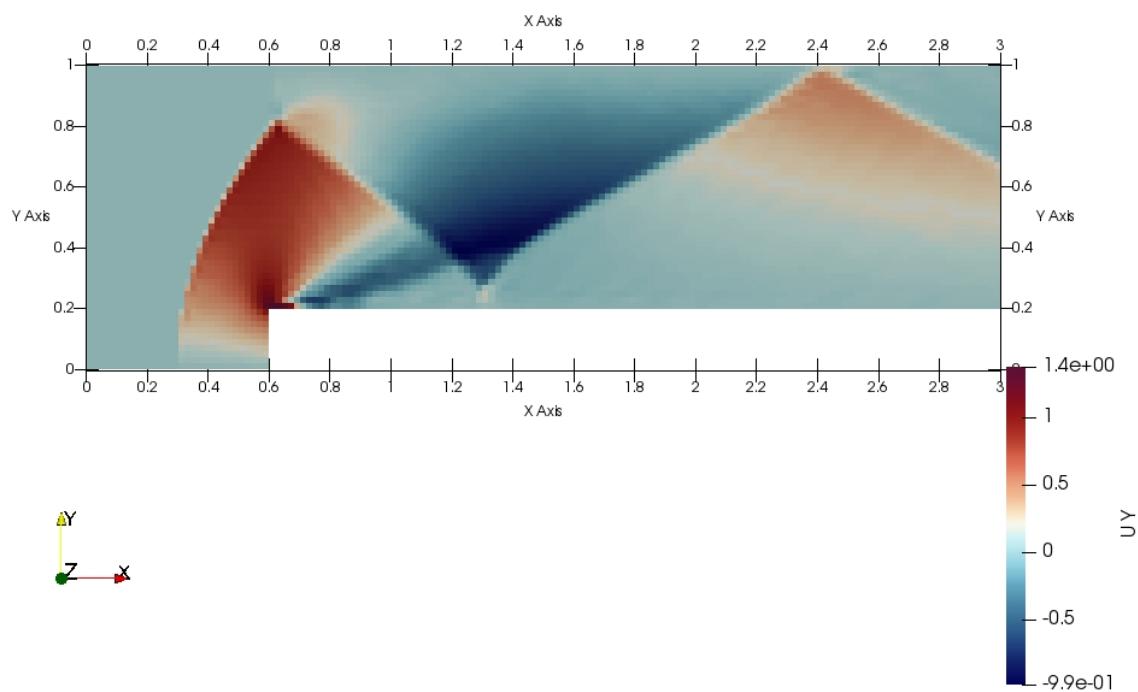
Density:

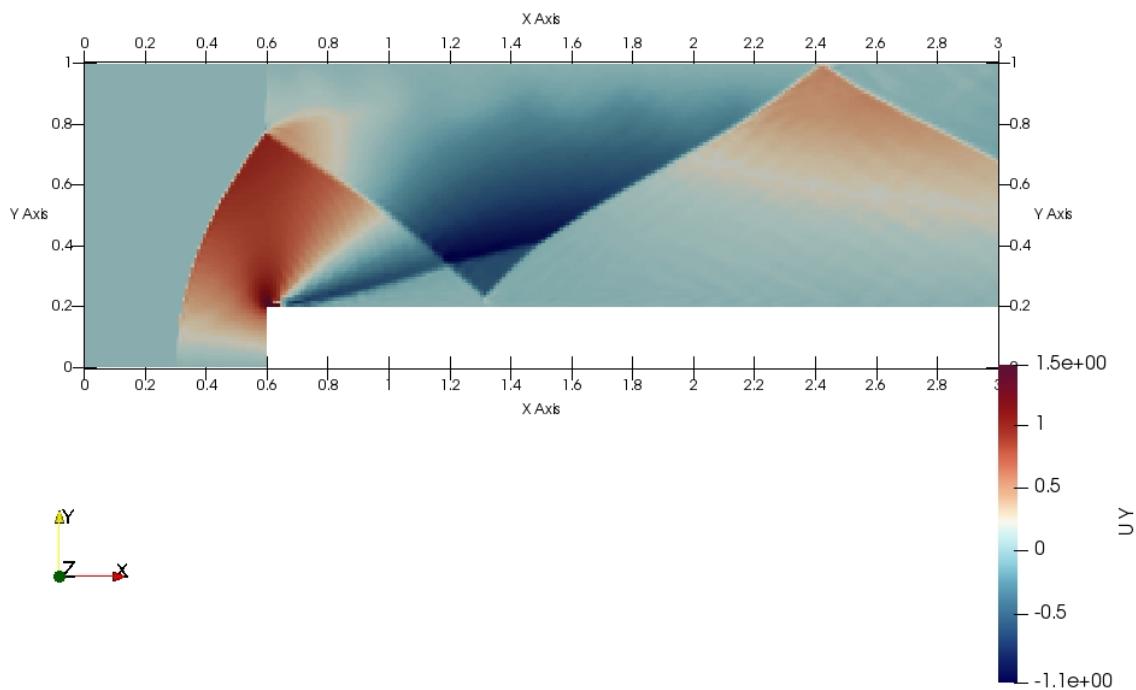


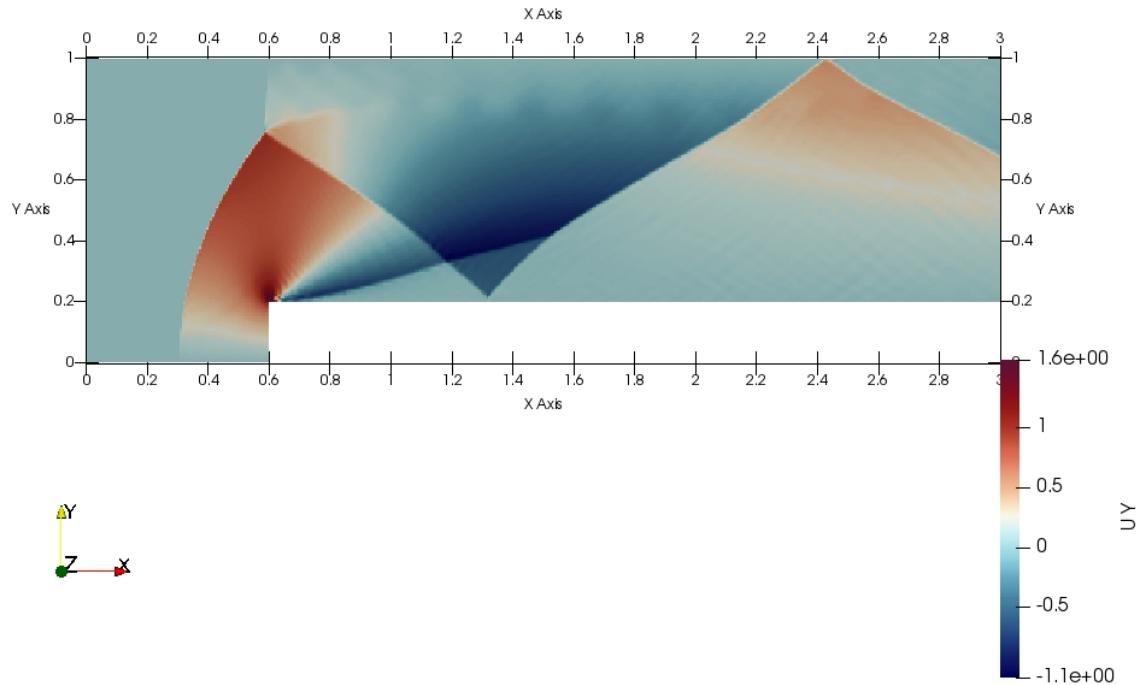




Y-velocity:





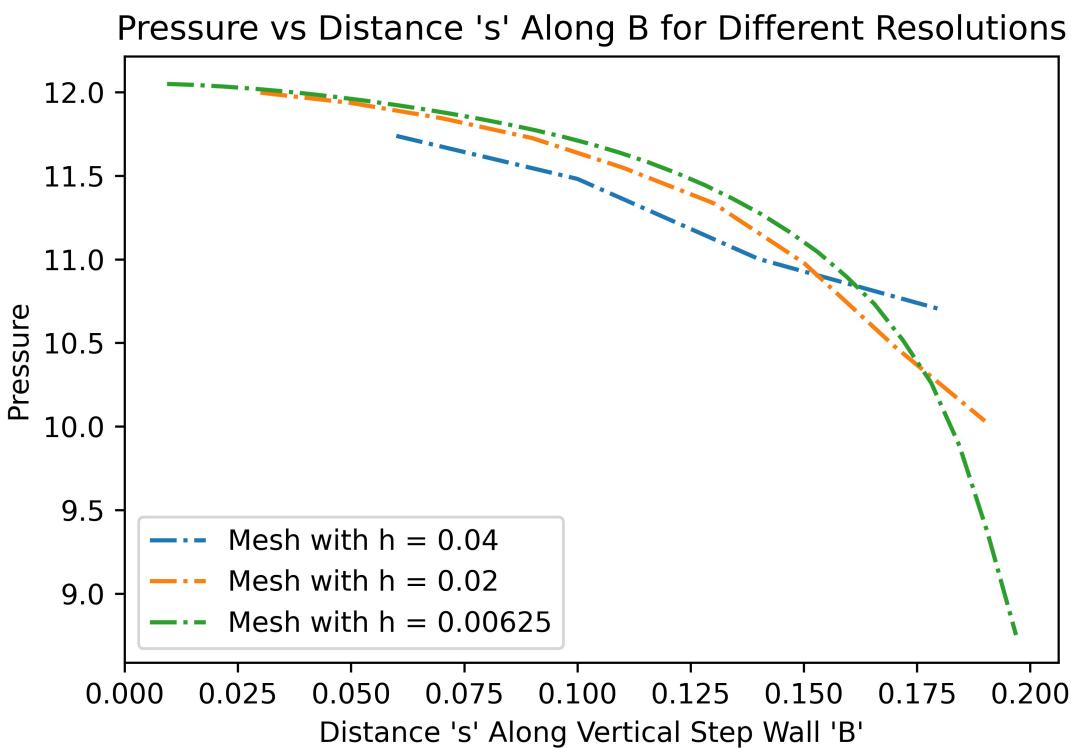
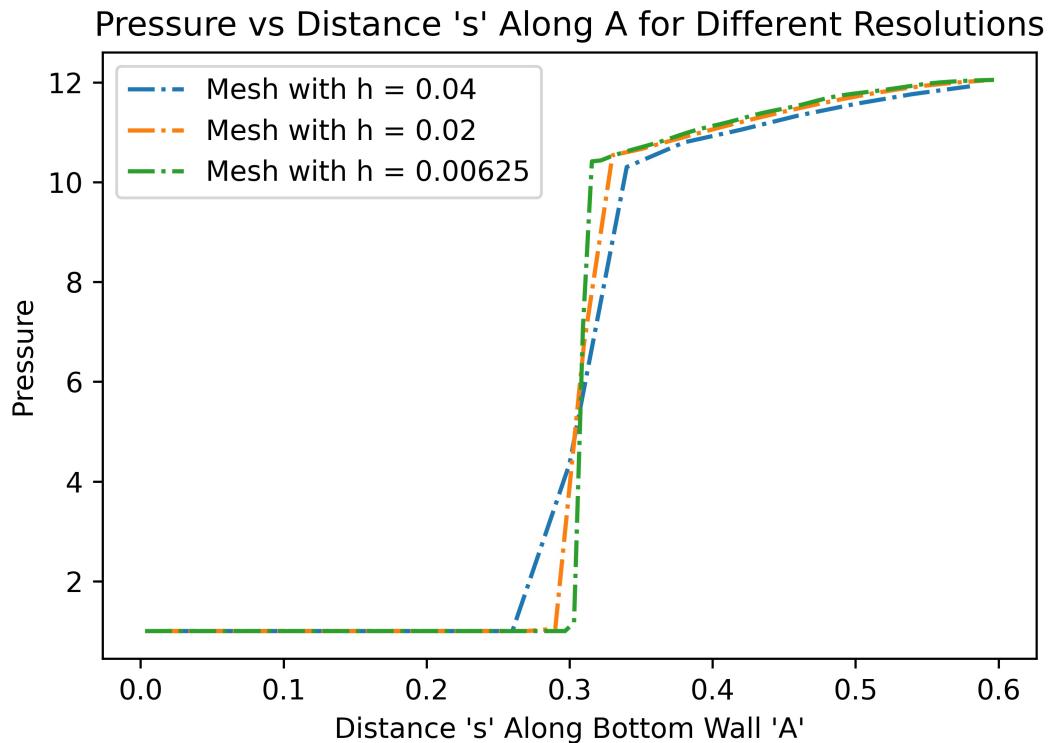


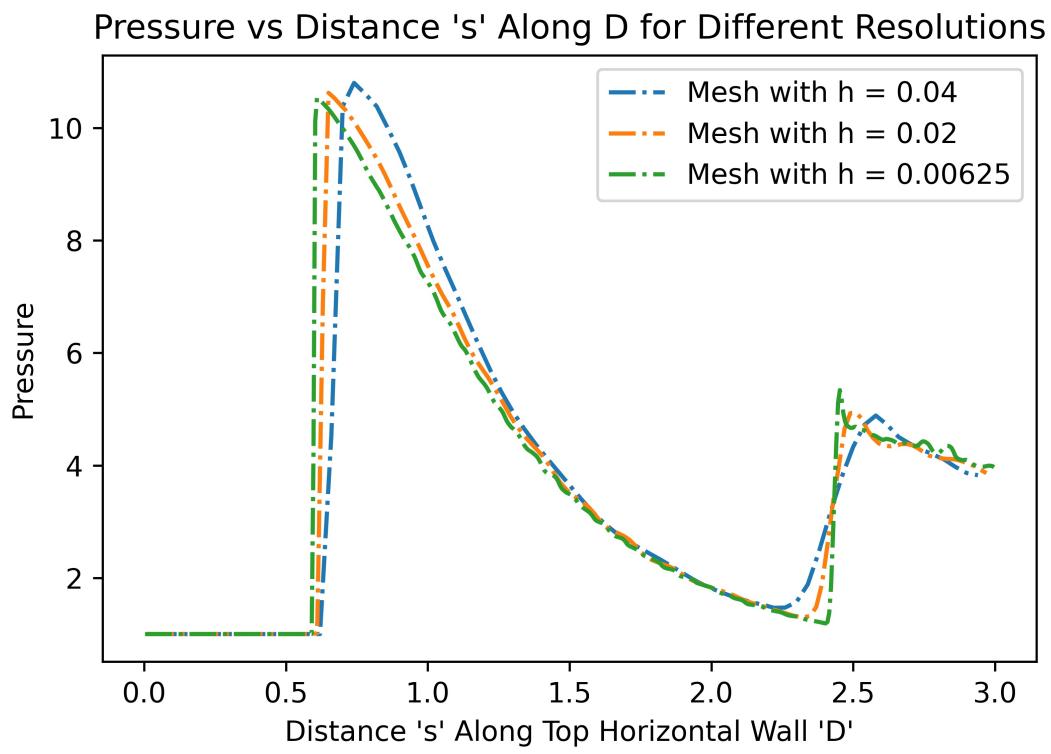
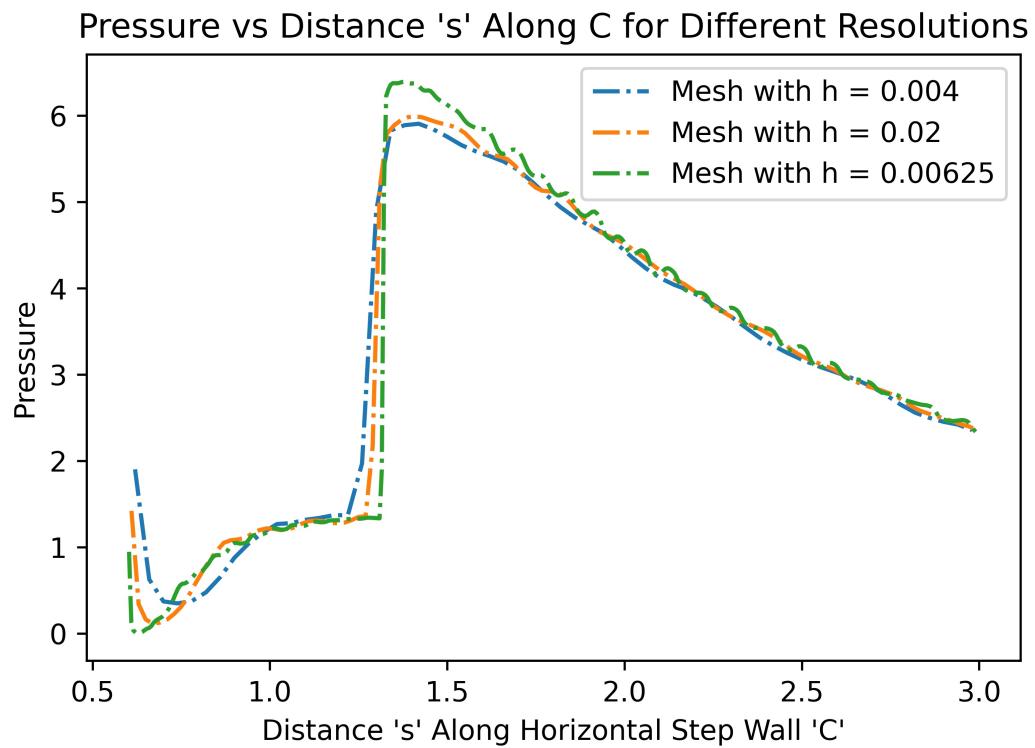
5.9. Wall Distributions w.r.t Resolution Levels:

Pressure, density, x-velocity, y-velocity, and temperature are reported in that order. Note that the walls are traversed from left to right, first the horizontal wall prior to the step, then the vertical step wall (traversing upward), then the horizontal wall (the step itself), and finally the upper horizontal wall.

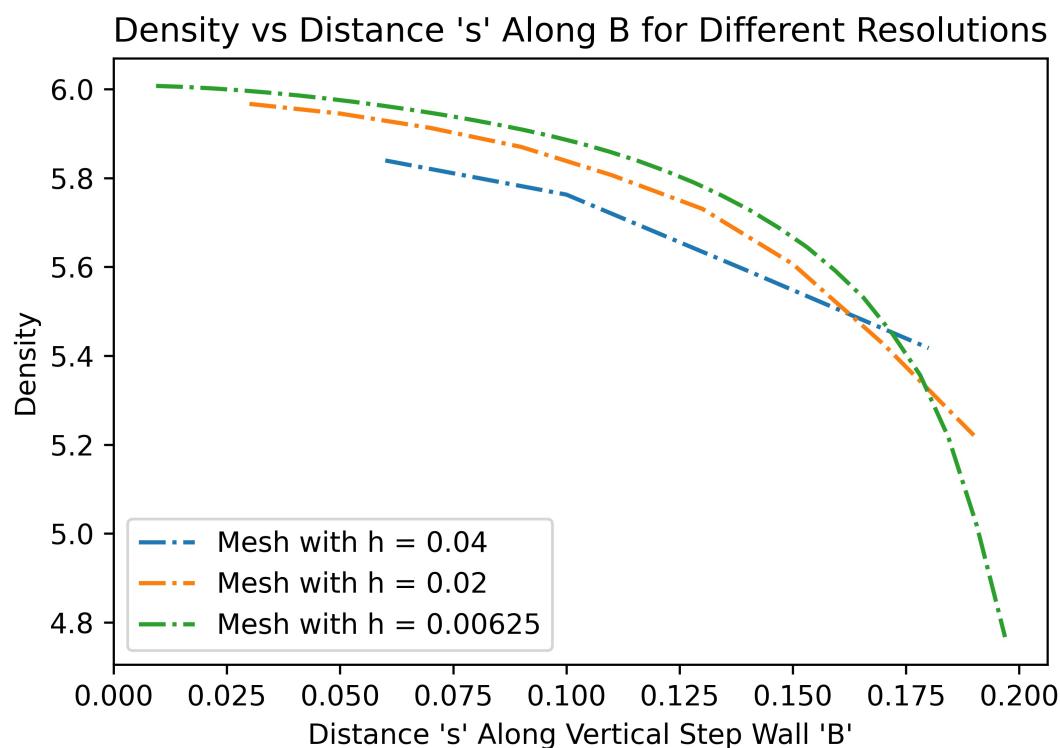
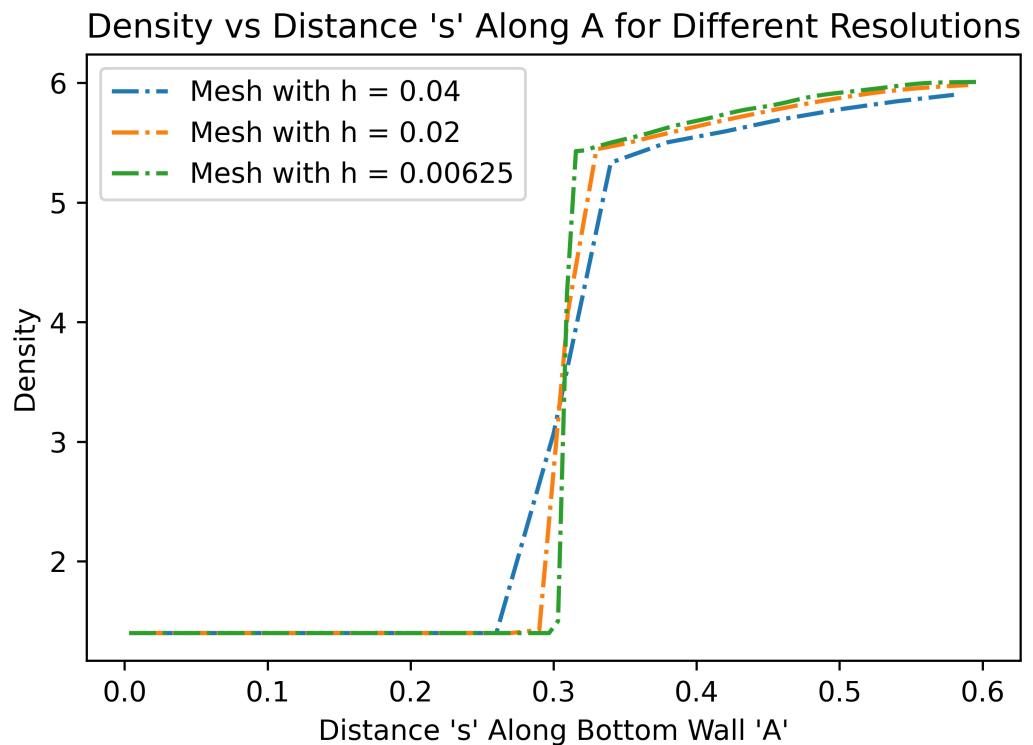
Convergence is shown through decreasing h (the spatial step-size).

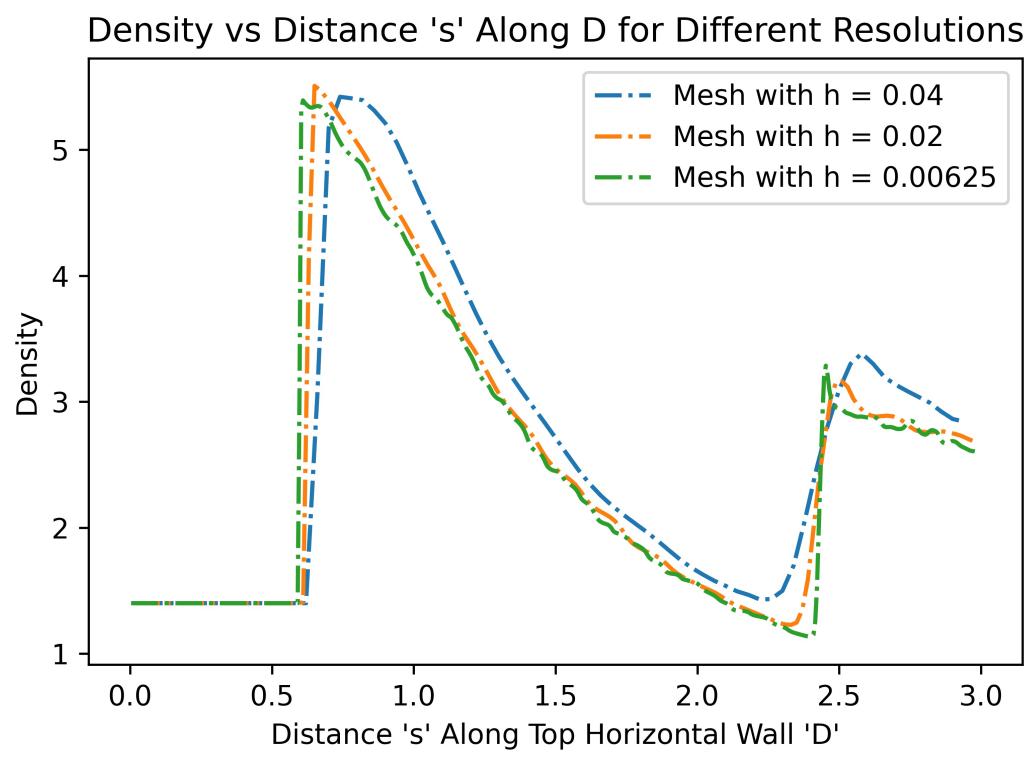
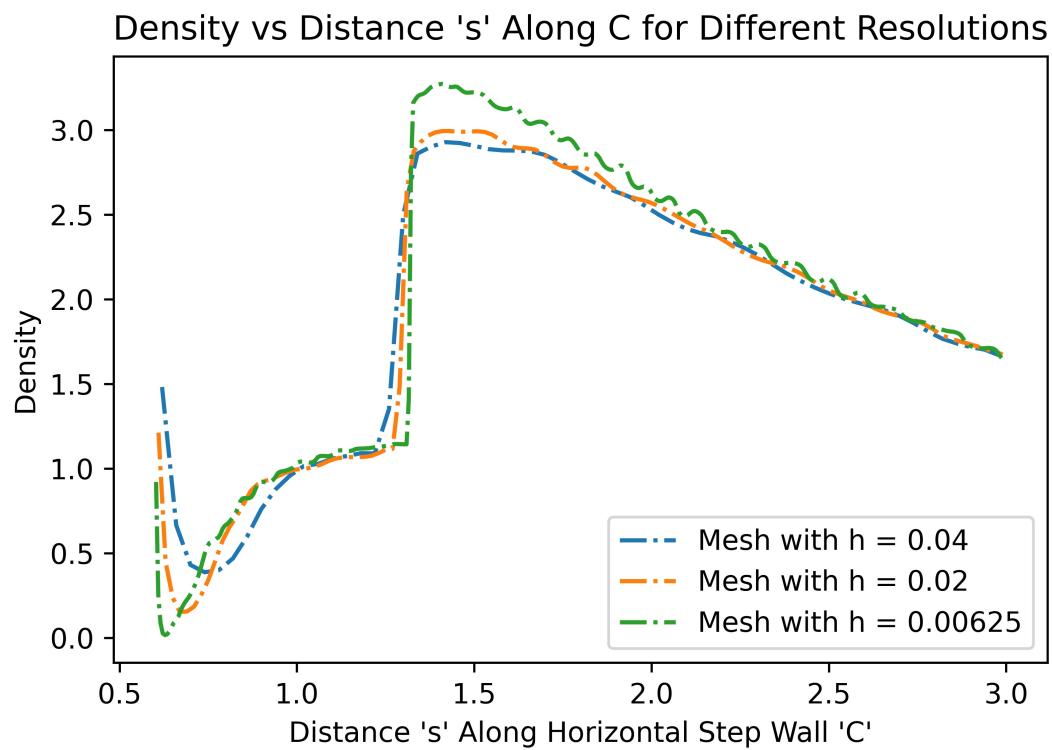
Pressure:





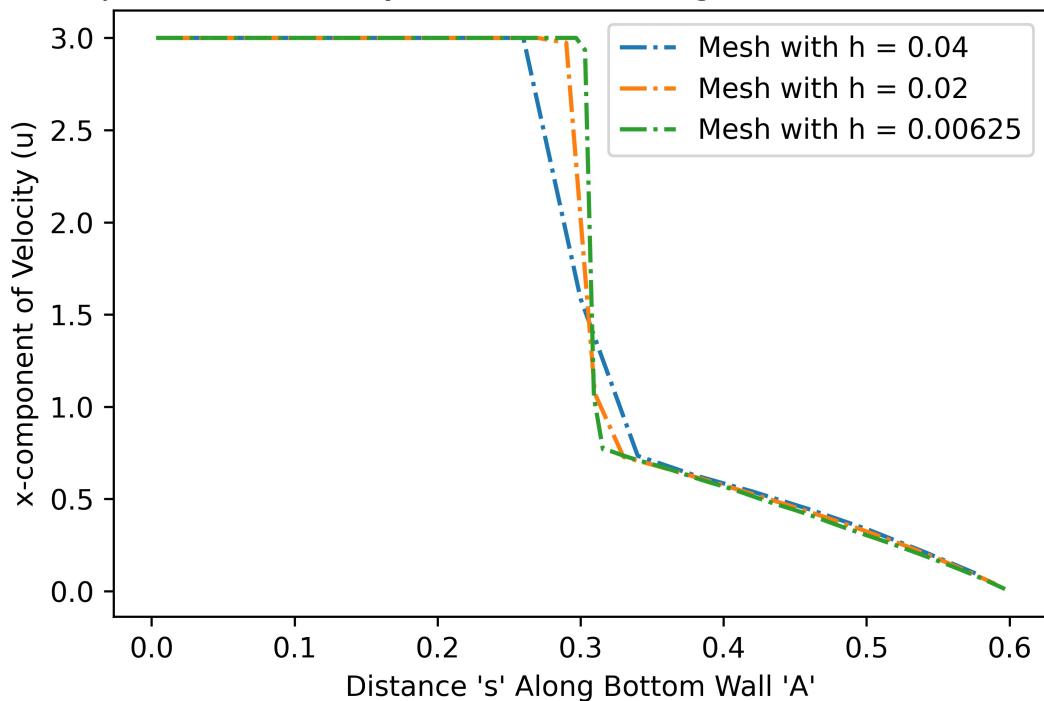
Density:



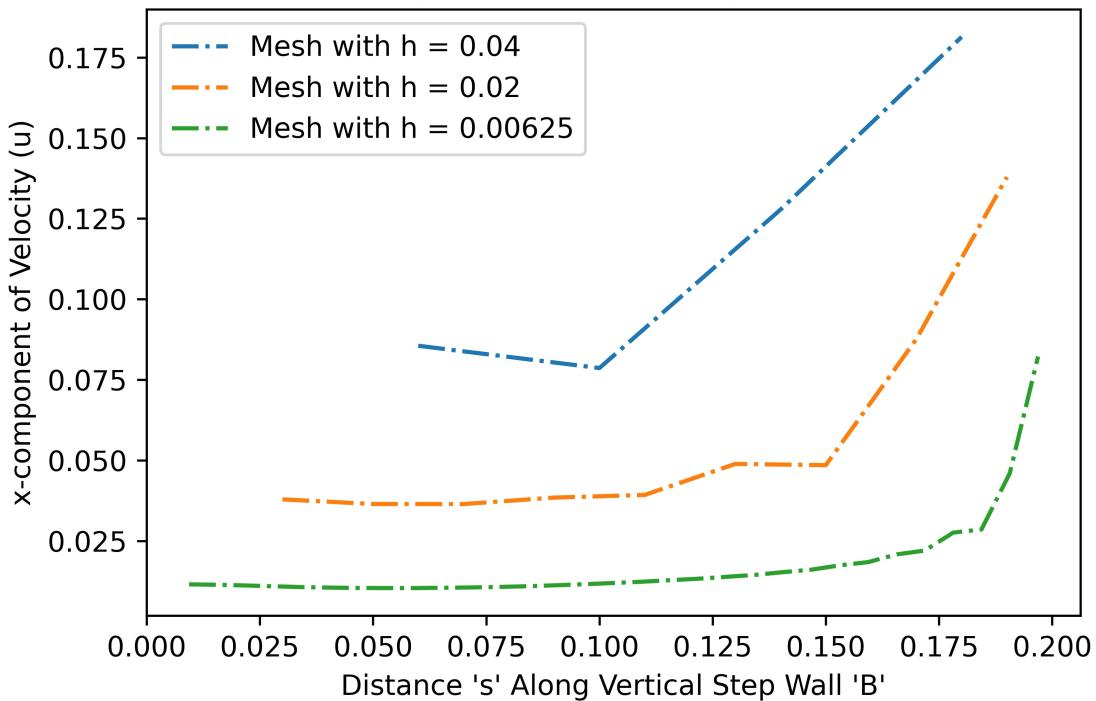


X-velocity:

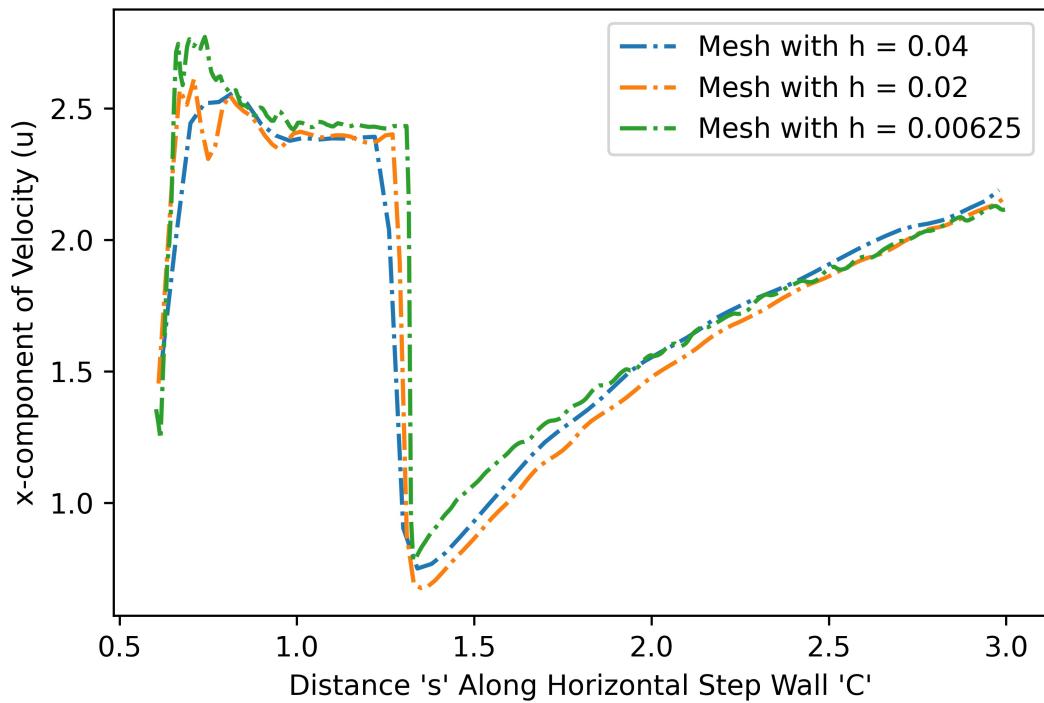
x-component of Velocity vs Distance Along A for Different Resolutions



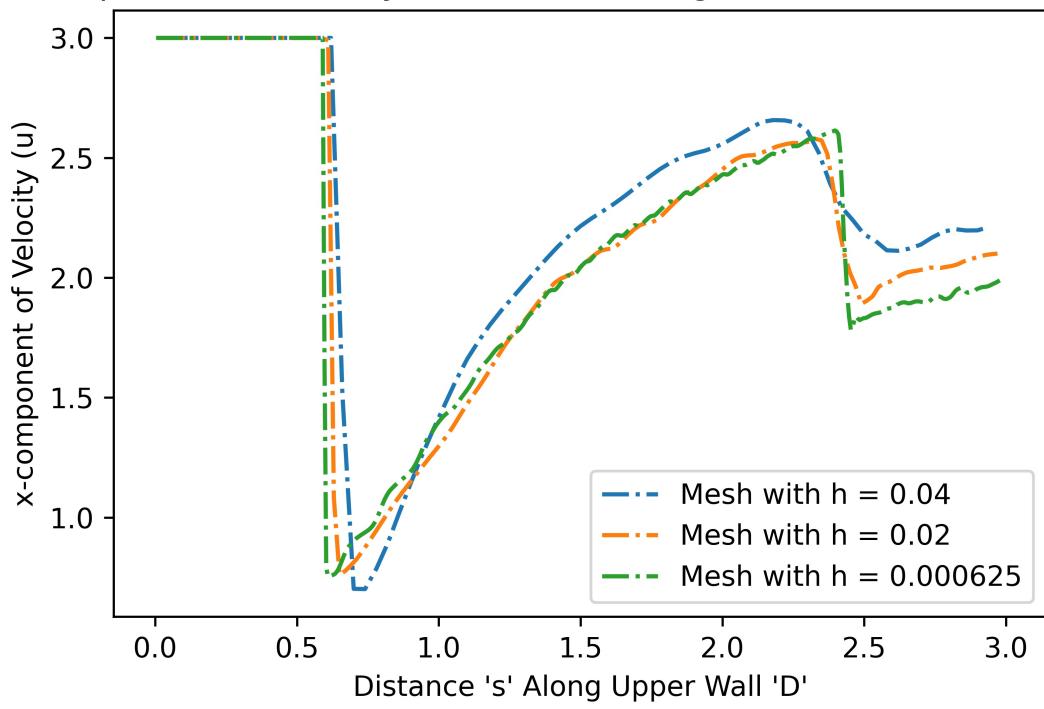
x-component of Velocity vs Distance Along B for Different Resolutions



x-component of Velocity vs Distance Along C for Different Resolutions

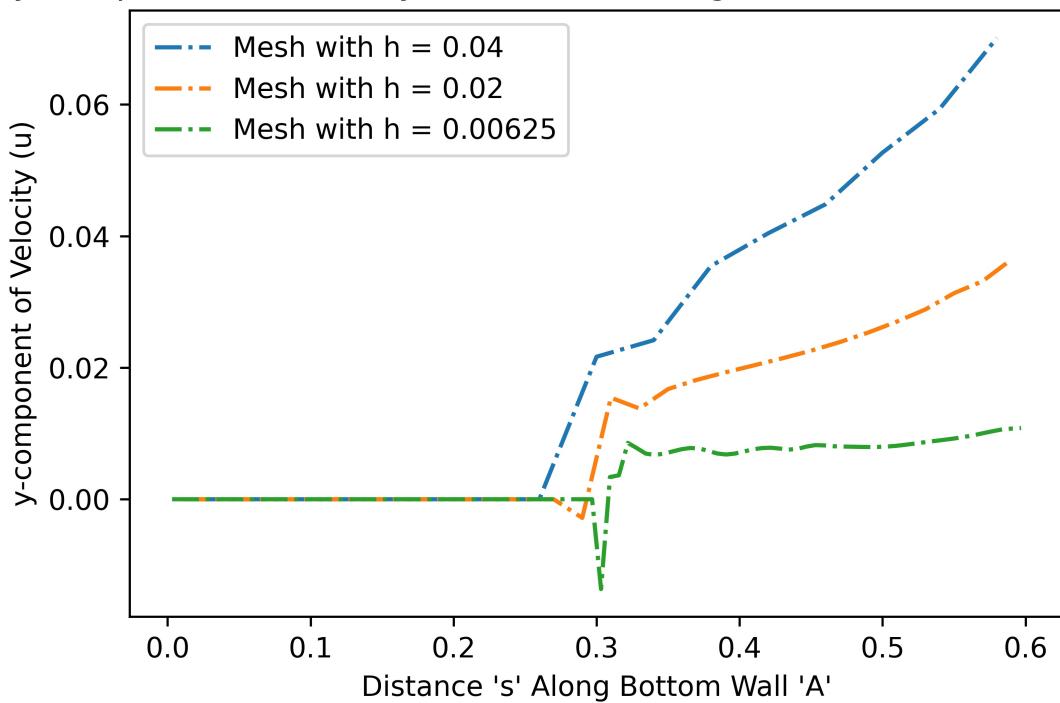


x-component of Velocity vs Distance Along D for Different Resolutions

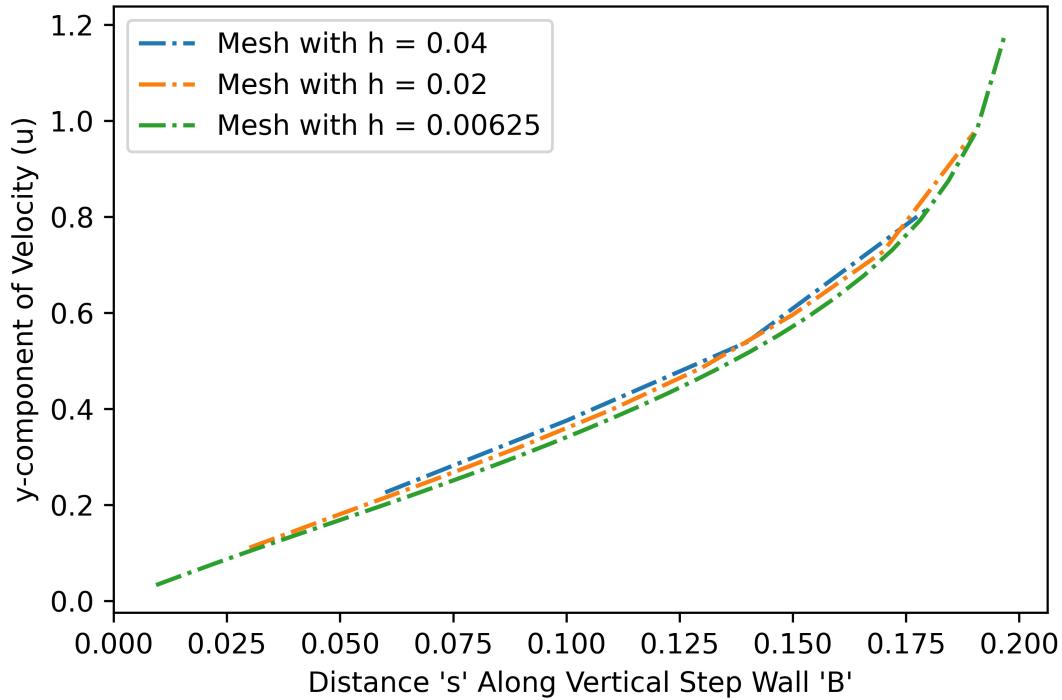


Y-velocity:

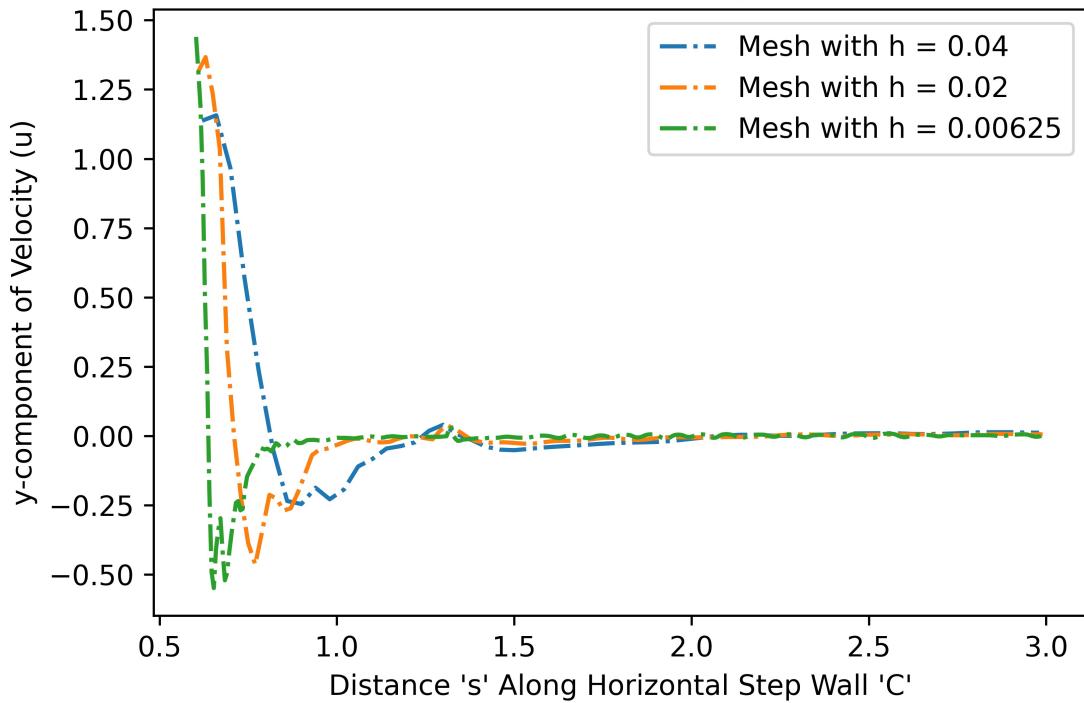
y-component of Velocity vs Distance Along A for Different Resolutions



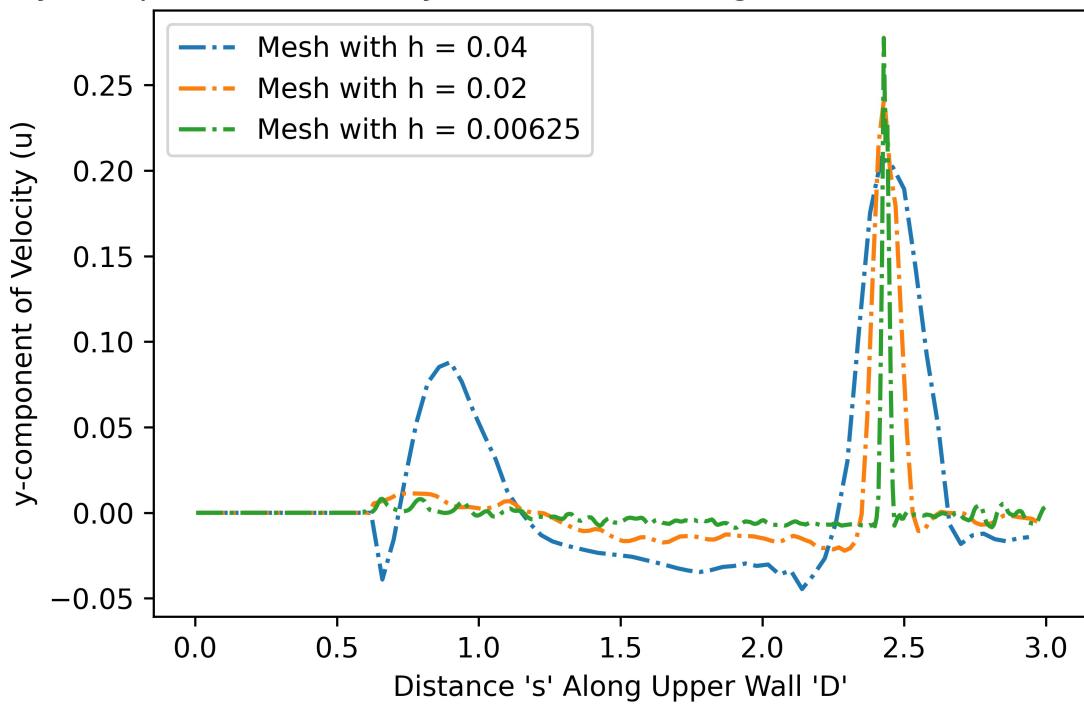
y-component of Velocity vs Distance Along B for Different Resolutions



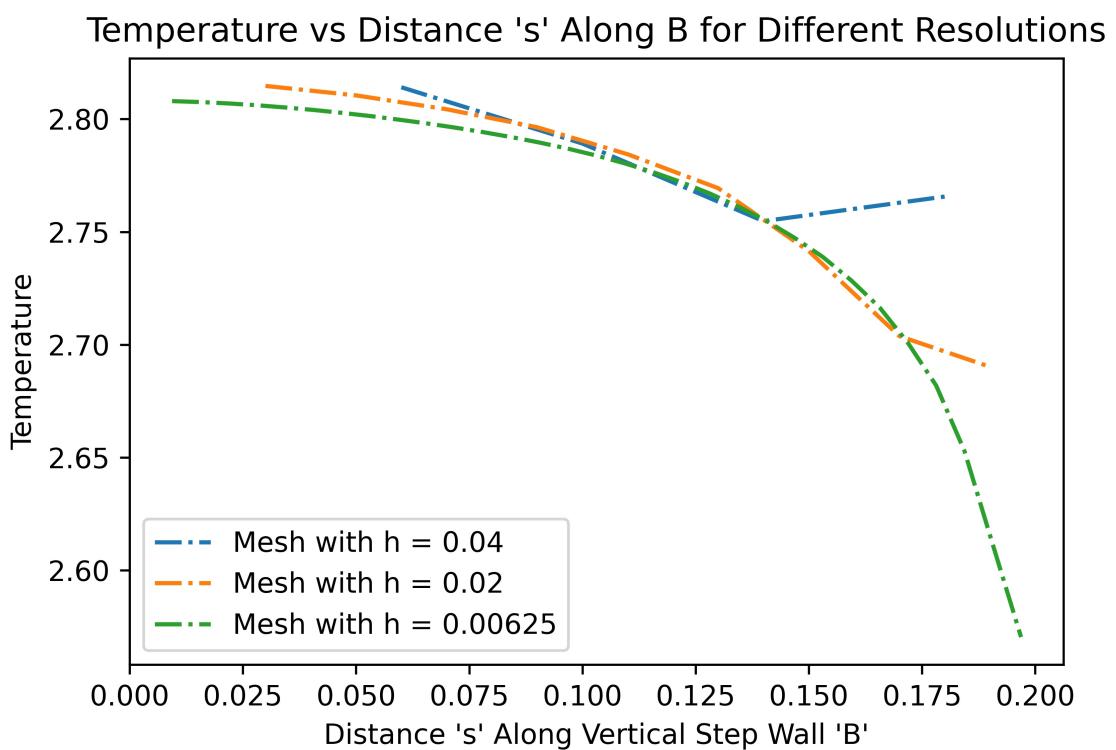
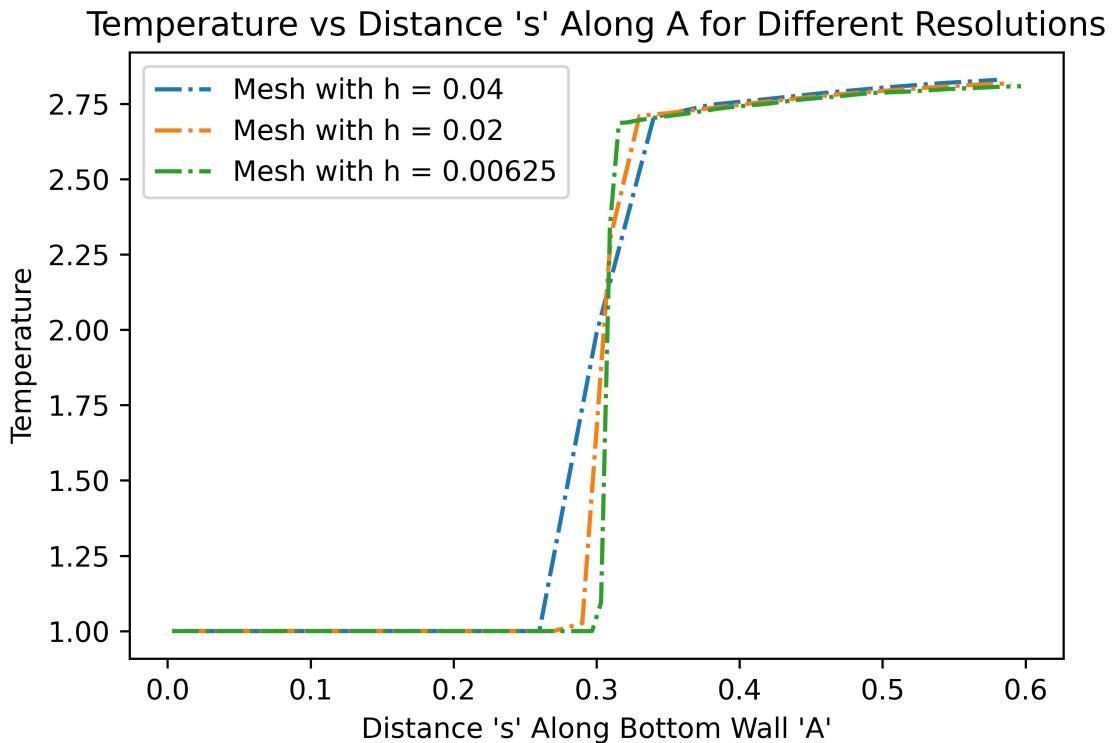
y-component of Velocity vs Distance Along C for Different Resolutions

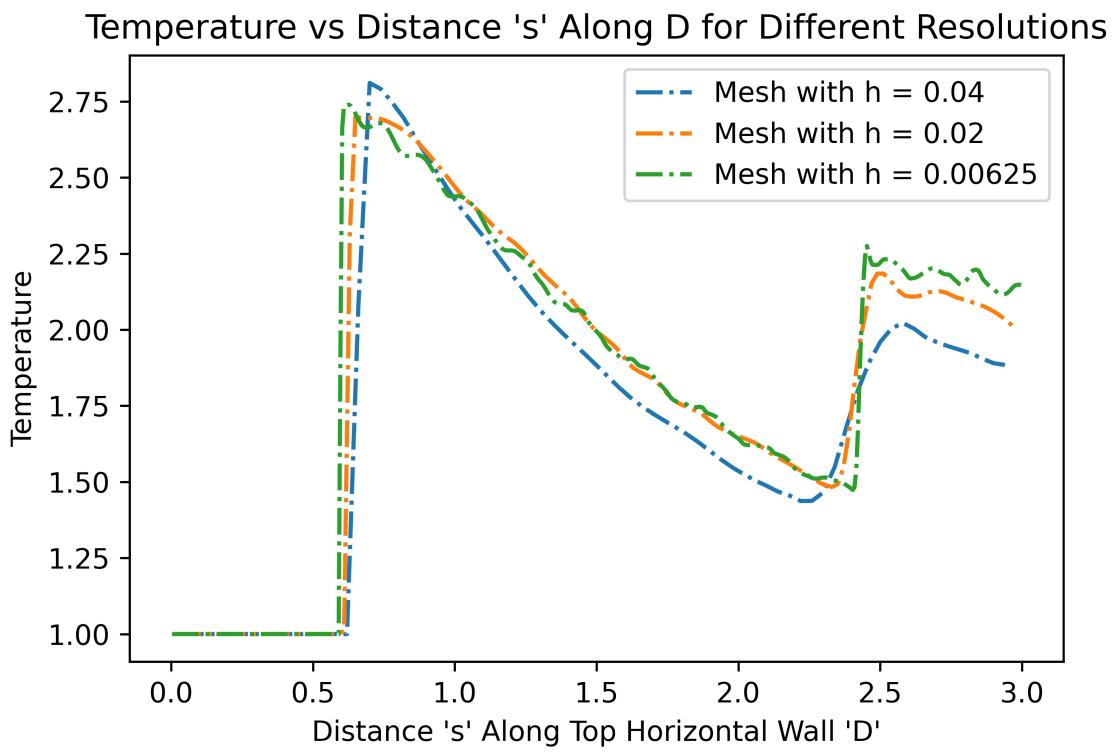
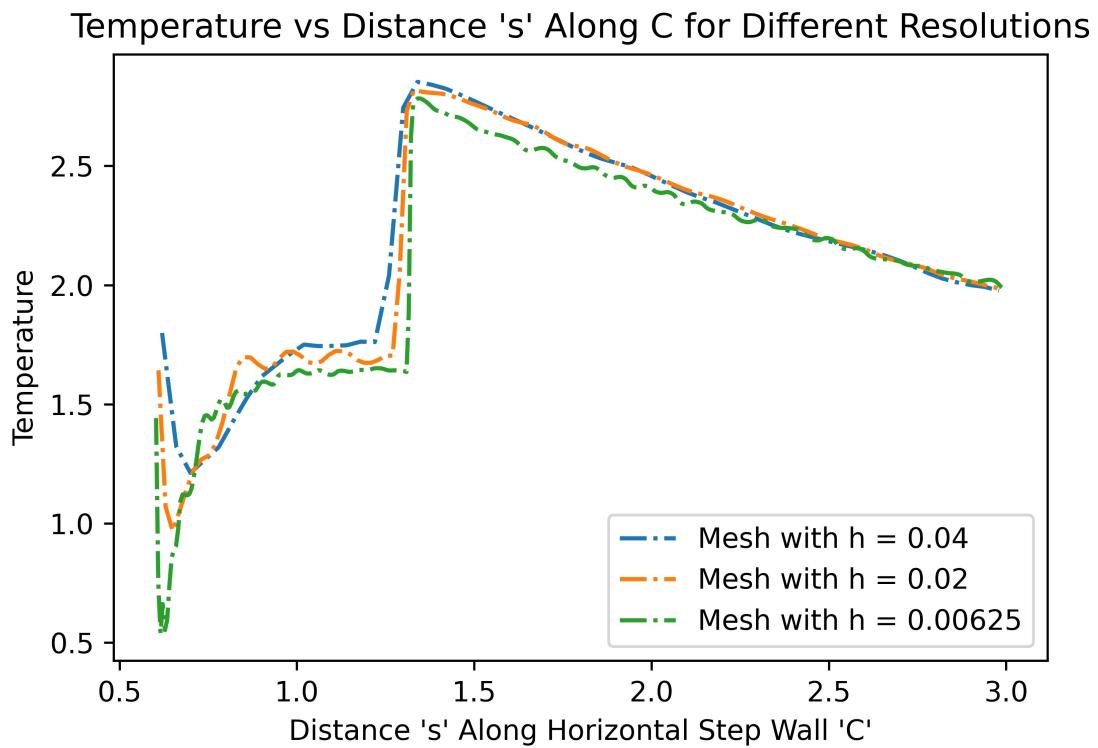


y-component of Velocity vs Distance Along D for Different Resolutions



Temperature:





5.10. Stagnation Pressure at Step

Note from the first two pressure plots, the pressure at the base of the step is slightly over 12Pa. The simulation measures a value of about 12.05Pa, which is in agreement with the theoretical calculation performed earlier.

6. Parallel Performance Improvements

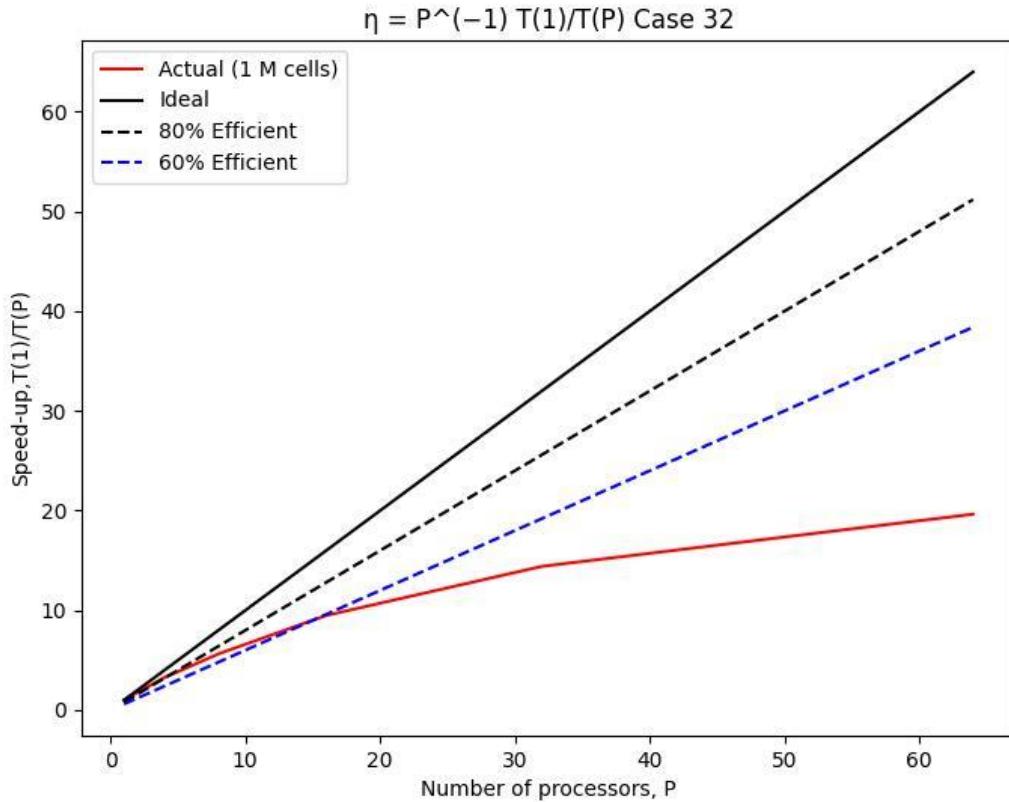
Define the speedup as the time taken for one processor ($T(1)$) divided by the time taken for P processors ($T(P)$). Similarly, let the parallel efficiency η be the speedup divided by P .

Note that the tables report the total $T(P)$, not that of single steps. The single step $T(P)$ is used instead for the plots.

On a mesh of 64512 cells, for P ranging from 1 to 64 in steps of 2, the following scaling is obtained.

$T(P)$ Speedup η

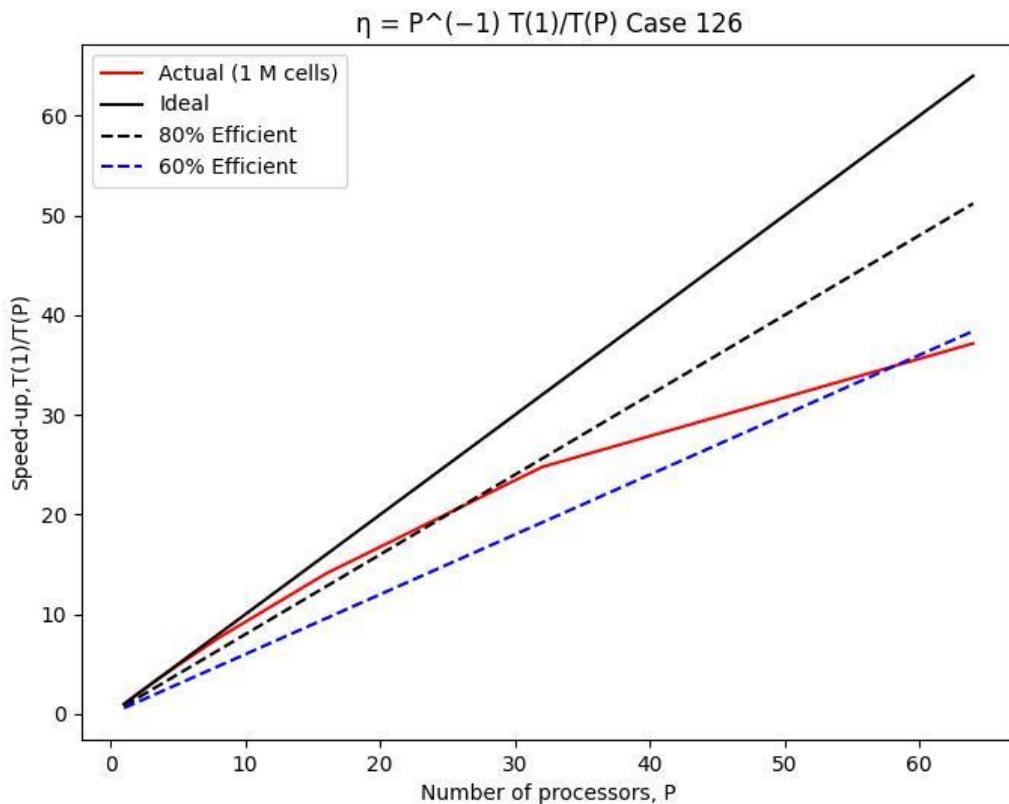
| | | |
|---------|--------|--------|
| 4870.28 | 1.0000 | 1.0000 |
| 2745.57 | 1.7738 | 0.8869 |
| 1493.30 | 3.2614 | 0.8154 |
| 0865.21 | 5.6290 | 0.7036 |
| 0513.54 | 9.4837 | 0.5927 |
| 0337.99 | 14.409 | 0.4503 |
| 0247.94 | 19.642 | 0.3069 |



A slightly improved scaling is obtained for a mesh of 1000188 cells.

$T(P)$ Speedup η

| | | |
|-----------|--------|--------|
| 270763.66 | 1.0000 | 1.0000 |
| 034192.10 | 2.0239 | 1.0119 |
| 034129.40 | 3.9783 | 0.9946 |
| 039850.40 | 7.6188 | 0.9524 |
| 021542.00 | 14.094 | 0.8808 |
| 012256.70 | 24.765 | 0.7739 |
| 008170.29 | 37.171 | 0.5808 |



Clearly the larger mesh case scales much more nicely, which can be expected due to the more even load distribution.

7. References

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8. Appendix

Thank you so much for reading this work!

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