1. $\frac{(2-2i)^{2020}}{(2+2i)^{2018}}$

$2-2i=z_1$

$|z_1|=\sqrt{4+4}=2\sqrt{2}$

$$\left.\begin{aligned}\cos\varphi_1&=\frac{1}{\sqrt{2}}\\ \sin\varphi_1&=-\frac{1}{\sqrt{2}}\end{aligned}\right\}\Rightarrow\varphi_1=-\frac{\pi}{4}$$

$z_1=2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$

$z_1^{2020}=8^{1010}(\cos(-505\pi)+i\sin(-505\pi))=8^{1010}(-1+i\cdot 0)=-8^{1010}$

$2+2i=z_2$

$|z_2|=\sqrt{4+4}=2\sqrt{2}$

$$\left.\begin{aligned}\cos\varphi_2&=\frac{1}{\sqrt{2}}\\ \sin\varphi_2&=\frac{1}{\sqrt{2}}\end{aligned}\right\}\Rightarrow\varphi_2=\frac{\pi}{4}$$

$z_2=2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$$z_2^{2018}=8^{1009}\left(\cos\left(\frac{1009\pi}{2}\right)+i\sin\left(\frac{1009\pi}{2}\right)\right)=8^{1009}\left(\cos\left(504\pi+\frac{\pi}{2}\right)+i\sin\left(504\pi+\frac{\pi}{2}\right)\right)=$$

$$=8^{1009}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=8^{1009}(0+i\cdot 1)=8^{1009}i$$

$$\frac{(2-2i)^{2020}}{(2+2i)^{2018}}=-\frac{8^{1010}}{8^{1009}i}=\frac{8^{1010}\cdot 8^{1009}i}{8^{2018}}=8i$$

2. $z^2=-\frac{8}{3}+2$

$z^2=w$

$|w|=\sqrt{\frac{64}{9}+4}=\sqrt{\frac{64+36}{9}}=\frac{10}{3}$

$$\left.\begin{aligned}\cos\varphi&=-\frac{8\cdot 3}{3\cdot 10}=-\frac{4}{5}\\ \sin\varphi&=\frac{2\cdot 3}{10}=\frac{3}{5}\end{aligned}\right\}\Rightarrow\varphi=\pi-arcsin\frac{3}{5}$$

$$w=\frac{10}{3}\left(cos\left(\pi-arcsin\frac{3}{5}\right)+i\,sin\left(\pi-arcsin\frac{3}{5}\right)\right)$$

$$z=\sqrt{w}=\sqrt{\frac{10}{3}}\left(\cos\left(\frac{\pi-arcsin\frac{3}{5}+2\pi k}{2}\right)+i\,sin\left(\frac{\pi-arcsin\frac{3}{5}+2\pi k}{2}\right)\right)$$

$$z_0=\sqrt{\frac{10}{3}}\left(\cos\left(\frac{\pi}{2}-\frac{arcsin\frac{3}{5}}{2}\right)+i\,sin\left(\frac{\pi}{2}-\frac{arcsin\frac{3}{5}}{2}\right)\right)=$$

$$=\sqrt{\frac{10}{3}}\left(\sin\left(\frac{arcsin\frac{3}{5}}{2}\right)+i\,cos\left(\frac{arcsin\frac{3}{5}}{2}\right)\right)$$

$$z_1=\sqrt{\frac{10}{3}}\left(\cos\left(\frac{3\pi}{2}-\frac{arcsin\frac{3}{5}}{2}\right)+i\,sin\left(\frac{3\pi}{2}-\frac{arcsin\frac{3}{5}}{2}\right)\right)=$$

$$=\sqrt{\frac{10}{3}}\left(-\sin\left(\frac{arcsin\frac{3}{5}}{2}\right)-i\,cos\left(\frac{arcsin\frac{3}{5}}{2}\right)\right)$$

3. $(1+i)z^2-2(1+2i)z+(1+7i)=0$

$$\frac{D}{4}=(1+2i)^2-(1+i)(1+7i)=(1+2i+2i-4)-(1+7i+i-7)=-3+4i+6-8i=$$
$$=3-4i$$

$$\sqrt{\frac{D}{4}}=\sqrt{3-4i}=\pm\left(\sqrt{\frac{3+5}{2}}-\frac{4}{\sqrt{2(3+5)}}i\right)=\pm(2-i)$$

$$z=\frac{(1+2i)\pm(2-i)}{1+i}$$

$$z_1=\frac{1+2i-2+i}{1+i}=\frac{(-1+3i)(1-i)}{(1+i)(1-i)}=\frac{-1+3i+i+3}{1+i-i+1}=\frac{2+4i}{2}=1+2i$$

$$z_2=\frac{1+2i+2-i}{1+i}=\frac{(3+i)(1-i)}{(1+i)(1-i)}=\frac{3+i-3i+1}{1+i-i+1}=\frac{4-2i}{2}=2-i$$

Ответ: $\{1+2i;2-i\}$

4. $z^4+5z^2+9=0$

$z^2=w$

$w^2+5w+9=0$

$D=25-36=-11$

$$w=\frac{-5\pm\sqrt{-11}}{2}=-\frac{5}{2}\pm i\frac{\sqrt{11}}{2}$$

$$z=\sqrt{w}=\pm\left(\sqrt{\frac{-\frac{5}{2}+3}{2}}+i\frac{\frac{\sqrt{11}}{2}}{\sqrt{2\left(-\frac{5}{2}+3\right)}}\right)=\pm\left(\frac{1}{2}\pm\frac{\sqrt{11}}{2}i\right)$$

Ответ: $\left\{\frac{1}{2}+\frac{\sqrt{11}}{2}i;\ \frac{1}{2}-\frac{\sqrt{11}}{2}i;\ -\frac{1}{2}+\frac{\sqrt{11}}{2}i;\ -\frac{1}{2}-\frac{\sqrt{11}}{2}i\right\}$

5. $z^6=\frac{\sqrt{6}+i\sqrt{2}}{1+i}=\frac{(\sqrt{6}+i\sqrt{2})(1-i)}{2}=\frac{\sqrt{6}+i\sqrt{2}-i\sqrt{6}+\sqrt{2}}{2}=\frac{\sqrt{2}+\sqrt{6}}{2}+i\frac{\sqrt{2}-\sqrt{6}}{2}$

$z^6=w$

$$|w|=\sqrt{\frac{\left(\sqrt{2}+\sqrt{6}\right)^2}{4}+\frac{\left(\sqrt{2}-\sqrt{6}\right)^2}{4}}=\sqrt{\frac{2+2\sqrt{12}+6+2-2\sqrt{12}+6}{4}}=\sqrt{\frac{16}{4}}=2$$

$$\left.\begin{aligned}\cos\varphi&=\frac{\sqrt{2}+\sqrt{6}}{4}\\ \sin\varphi&=\frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}\right\}\Rightarrow\varphi=-arccos\frac{\sqrt{2}+\sqrt{6}}{4}$$

$$w=2\left(\cos\left(-arccos\frac{\sqrt{2}+\sqrt{6}}{4}\right)+i\,sin\left(-arccos\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)$$

$$z=\sqrt[6]{w}=\sqrt[6]{2}\left(\cos\left(\frac{-arccos\frac{\sqrt{2}+\sqrt{6}}{4}+2\pi k}{6}\right)+i\,sin\left(\frac{-arccos\frac{\sqrt{2}+\sqrt{6}}{4}+2\pi k}{6}\right)\right)$$

$$z_0 = \sqrt[6]{2}\left(\cos\left(\frac{-arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\frac{-arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

$$z_1 = \sqrt[6]{2}\left(\cos\left(\frac{\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\frac{\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

$$z_2 = \sqrt[6]{2}\left(\cos\left(\frac{2\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\frac{2\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

$$z_3 = \sqrt[6]{2}\left(\cos\left(\pi - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\pi - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

$$z_4 = \sqrt[6]{2}\left(\cos\left(\frac{4\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\frac{4\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

$$z_5 = \sqrt[6]{2}\left(\cos\left(\frac{5\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right) + i\,sin\left(\frac{5\pi}{3} - \frac{arccos\frac{\sqrt{2}+\sqrt{6}}{4}}{6}\right)\right)$$

6. $z = \cos\varphi + i\sin\varphi$

$z^5 = \cos 5\varphi + i\sin 5\varphi$

$$\begin{aligned} z^5 &= (\cos\varphi + i\sin\varphi)^5 = \\ &= \cos^5\varphi + 5i\cos^4\varphi\sin\varphi + 10i^2\cos^3\varphi\sin^2\varphi + 10i^3\cos^2\varphi\sin^3\varphi + 5i^4\cos\varphi\sin^4\varphi \\ &\quad + i^5\sin^5\varphi = \\ &= (\cos^5\varphi - 10\cos^3\varphi\sin^2\varphi + 5\cos\varphi\sin^4\varphi) \\ &\quad + i(\sin^5\varphi - 10\sin^3\varphi\cos^2\varphi + 5\sin\varphi\cos^4\varphi) \end{aligned}$$

$\sin 5\varphi = \sin\varphi\,(\sin^4\varphi - 10\sin^2\varphi\cos^2\varphi + 5\cos^4\varphi)$

7. $\cos\varphi = \frac{e^{i\varphi}+e^{-i\varphi}}{2}$

$$\begin{aligned} \cos^5\varphi &= \left(\frac{e^{i\varphi}+e^{-i\varphi}}{2}\right)^5 = \\ &= \frac{e^{i\cdot 5\varphi} + 5e^{i\cdot 4\varphi}e^{-i\varphi} + 10e^{i\cdot 3\varphi}e^{-i\cdot 2\varphi} + 10e^{i\cdot 2\varphi}e^{-i\cdot 3\varphi} + 5e^{i\varphi}e^{-i\cdot 4\varphi} + e^{-i\cdot 5\varphi}}{32} = \\ &= \frac{e^{i\cdot 5\varphi} + e^{-i\cdot 5\varphi} + 5\left(e^{i\cdot 3\varphi} + e^{-i\cdot 3\varphi}\right) + 10\left(e^{i\varphi} + e^{-i\varphi}\right)}{32} = \frac{\cos 5\varphi}{16} + \frac{5\cos 3\varphi}{16} + \frac{5\cos\varphi}{8} \end{aligned}$$

8. $f(x) = (x^2-4)(x+4)Q(x) + R(x)$

$f(x) = (x^3+4x^2-4x-16)Q(x) + R(x)$

$R(x) = ax^2 + bx + c$

$f(-2) = 4a - 2b + c = 7$

$f(2) = 4a + 2b + c = 3$

$f(-4) = 16a - 4b + c = 21$

$$\begin{cases} 4a-2b+c=7 \\ 4a+2b+c=3 \\ 16a-4b+c=21 \end{cases}$$

$$\left(\begin{array}{ccc|c} 4 & -2 & 1 & 7 \\ 4 & 2 & 1 & 3 \\ 16 & -4 & 1 & 21 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & -2 & 1 & 7 \\ 0 & 4 & 0 & -4 \\ 0 & 4 & -3 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 4 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\begin{cases} a=1 \\ b=-1 \\ c=1 \end{cases}$$

$R(x) = x^2 - x + 1$

9. $2x^5 + 18x^4 + 55x^3 + 63x^2 + 27x + 27$

$x_1 = -3$

	|	2	18	55	63	27	27
−	+	−	−	−	−	−	−
−3	|	2	12	19	6	9	0
−3	|	2	6	1	3	0	
−3	|	2	0	1	0		

$2x^5 + 18x^4 + 55x^3 + 63x^2 + 27x + 27 = (x+3)^3(2x^2+1)$

$$2x^2 + 1 = 0 \Leftrightarrow x^2 = -\frac{1}{2} \Leftrightarrow x = \pm i\sqrt{\frac{1}{2}}$$

$$2x^5 + 18x^4 + 55x^3 + 63x^2 + 27x + 27 = 2(x+3)^3\left(x - i\sqrt{\frac{1}{2}}\right)\left(x + i\sqrt{\frac{1}{2}}\right)$$

10. $x^8 + 4x^4 + 4 = (x^4+2)^2$

11. $\begin{cases} 2(1+i)x + iy = -1+i \\ 2(1+2i)x + (1-i)y = -3-3i \end{cases}$

$\Delta = \begin{vmatrix} 2+2i & i \\ 2+4i & 1-i \end{vmatrix} = (2+2i)(1-i) - i(2+4i) = 2 - 2i + 2i + 2 - 2i + 4 = 8 - 2i$

$\Delta_x = \begin{vmatrix} -1+i & i \\ -3-3i & 1-i \end{vmatrix} = (-1+i)(1-i) - i(-3-3i) = -1 + i + i + 1 + 3i - 3 = -3 + 5i$

$\Delta_y = \begin{vmatrix} 2+2i & -1+i \\ 2+4i & -3-3i \end{vmatrix} = (2+2i)(-3-3i) - (2+4i)(-1+i) =$

$= -6 - 6i - 6i + 6 + 2 - 2i + 4i + 4 = 6 - 10i$

$$x = \frac{\Delta_x}{\Delta} = \frac{-3+5i}{8-2i} = \frac{(-3+5i)(8+2i)}{(8-2i)(8+2i)} = \frac{-24-6i+40i-10}{64+4} = \frac{-34+34i}{68} = -\frac{1}{2} + \frac{1}{2}i$$

$$y = \frac{\Delta_y}{\Delta} = \frac{6-10i}{8-2i} = \frac{(6-10i)(8+2i)}{(8-2i)(8+2i)} = \frac{48+12i-80i+20}{64+4} = \frac{68-68i}{68} = 1 - i$$

Ответ: $\left(-\frac{1}{2} + \frac{1}{2}i; 1-i\right)$

12. $\begin{cases} 5(m-1)x - (2m^2+m-3)y = 5(m-1) \\ (2m-1)x - my = 3m-2 \end{cases}$

$\Delta = \begin{vmatrix} 5m-5 & c \\ 2m-1 & -m \end{vmatrix} = -m(5m-5) + (2m^2+m-3)(2m-1) =$

$= 5m - 5m^2 + 4m^3 + 2m^2 - 6m - 2m^2 - m + 3 = 4m^3 - 5m^2 - 2m + 3$

$\Delta = 0$:

$4m^3 - 5m^2 - 2m + 3 = 0$

$m_1 = 1$

$$\begin{array}{cccccc} & | & 4 & -5 & -2 & 3 \\ - & + & - & - & - & - \\ 1 & | & 4 & -1 & -3 & 0 \\ 1 & | & 4 & 3 & 0 & \end{array}$$

$4m^3-5m^2-2m+3=(m-1)^2(4m+3)$

$m_2=-\dfrac{3}{4}$

$m=1\Rightarrow\begin{cases}0x-0y=0\\ x-y=1\end{cases}\Rightarrow$ бесконечно много решений

$$\mathrm{m}=-\frac{3}{4}\Rightarrow\begin{cases}-\dfrac{35}{4}x+\dfrac{21}{8}y=-\dfrac{35}{4}\\ -\dfrac{5}{2}x+\dfrac{3}{4}y=-\dfrac{17}{4}\end{cases}\Leftrightarrow\begin{cases}-10x+3y=-10\\ -10x+3y=-17\end{cases}\Rightarrow \text{нет решений}$$

$\Delta\neq 0$:

$$\Delta_x=\begin{vmatrix}5m-5 & -2m^2-m+3\\ 3m-2 & -m\end{vmatrix}=-m(5m-5)+(2m^2+m-3)(3m-2)=$$
$$=5m-5m^2+6m^3+3m^2-9m-4m^2-2m+6=6m^3-6m^2-6m+6$$

$$\Delta_y=\begin{vmatrix}5m-5 & 5m-5\\ 2m-1 & 3m-2\end{vmatrix}=(5m-5)\begin{vmatrix}1 & 1\\ 2m-1 & 3m-2\end{vmatrix}=(5m-5)(3m-2-2m+1)=$$
$$=(5m-5)(m-1)=5m^2-5m-5m+5=5m^2-10m+5$$

$$x=\frac{\Delta_x}{\Delta}=\frac{6(m^3-m^2-m+1)}{4m^3-5m^2-2m+3}=\frac{6(m-1)^2(m+1)}{(m-1)^2(4m+3)}=\frac{6(m+1)}{4m+3}$$

$$y=\frac{\Delta_y}{\Delta}=\frac{5(m^2-2m+1)}{4m^3-5m^2-2m+3}=\frac{5(m-1)^2}{(m-1)^2(4m+3)}=\frac{5}{4m+3}$$

$$\text{Ответ: }\left[\begin{array}{c}\left(\dfrac{6(m+1)}{4m+3};\dfrac{5}{4m+3}\right), m\in\left(-\infty;-\dfrac{3}{4}\right)\cup\left(-\dfrac{3}{4};1\right)\cup(1;+\infty)\\ \text{бесконечно много решений}, m=1\\ \text{нет решений}, m=-\dfrac{3}{4}\end{array}\right.$$

13. $$\begin{vmatrix}1 & 1 & 1 & \dots & 1\\ 1 & 2 & 1 & \dots & 1\\ 1 & 1 & 3 & \dots & 1\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & 1 & 1 & \dots & n\end{vmatrix}=\begin{vmatrix}1 & 1 & 1 & \dots & 1\\ 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 2 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & n-1\end{vmatrix}=1\cdot1\cdot2\cdot\ldots\cdot(n-1)=(n-1)!$$

14. $A=\begin{pmatrix}2 & 3 & 3\\ 3 & 3 & 3\\ 3 & 3 & 4\end{pmatrix}, B=\begin{pmatrix}5 & 2 & 0\\ 2 & 1 & 3\\ 0 & 3 & 5\end{pmatrix}$

$$C=-5AB=-5\begin{pmatrix}2 & 3 & 3\\ 3 & 3 & 3\\ 3 & 3 & 4\end{pmatrix}\begin{pmatrix}5 & 2 & 0\\ 2 & 1 & 3\\ 0 & 3 & 5\end{pmatrix}=\begin{pmatrix}-10 & -15 & -15\\ -15 & -15 & -15\\ -15 & -15 & -20\end{pmatrix}\begin{pmatrix}5 & 2 & 0\\ 2 & 1 & 3\\ 0 & 3 & 5\end{pmatrix}=$$
$$=\begin{pmatrix}-80 & -80 & -120\\ -105 & -90 & -120\\ -105 & -105 & -145\end{pmatrix}$$

$$|\mathrm{C}|=\begin{vmatrix}-80 & -80 & -120\\ -105 & -90 & -120\\ -105 & -105 & -145\end{vmatrix}=(-40)(-15)(-5)\begin{vmatrix}2 & 2 & 3\\ 7 & 6 & 8\\ 21 & 21 & 29\end{vmatrix}=-3000\begin{vmatrix}2 & 2 & 3\\ 7 & 6 & 8\\ 0 & 3 & 5\end{vmatrix}=$$
$$=-3000(60+63-48-70)=-3000\cdot5=-15000$$

15. $$A=\begin{pmatrix}2 & -4 & 3 & -3 & 5\\ 1 & -2 & 1 & 5 & 3\\ 1 & -2 & 4 & -34 & 0\end{pmatrix}\sim\begin{pmatrix}1 & -2 & 1 & 5 & 3\\ 0 & 0 & 1 & -13 & -1\\ 0 & 0 & 3 & -39 & -3\end{pmatrix}\sim\begin{pmatrix}1 & -2 & 1 & 5 & 3\\ 0 & 0 & 1 & -13 & -1\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}\Rightarrow rg(A)=2$$

16. $\begin{pmatrix}n & 1\\ 0 & n\end{pmatrix}^{2019}$

$$\begin{pmatrix}n & 1\\ 0 & n\end{pmatrix}^2=\begin{pmatrix}n & 1\\ 0 & n\end{pmatrix}\begin{pmatrix}n & 1\\ 0 & n\end{pmatrix}=\begin{pmatrix}n^2 & 2n\\ 0 & n^2\end{pmatrix}$$

$\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^3 = \begin{pmatrix} n^2 & 2n \\ 0 & n^2 \end{pmatrix}\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} n^3 & 3n^2 \\ 0 & n^3 \end{pmatrix}$

$\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^4 = \begin{pmatrix} n^3 & 3n^2 \\ 0 & n^3 \end{pmatrix}\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} n^4 & 4n^3 \\ 0 & n^4 \end{pmatrix}$

$] \begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^x = \begin{pmatrix} n^x & xn^{x-1} \\ 0 & n^x \end{pmatrix}$

ММИ:

1. База: $x = 1$: $\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}$ – верно

2. Предположение:] при $x = y$: $\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^y = \begin{pmatrix} n^y & yn^{y-1} \\ 0 & n^y \end{pmatrix}$ – верно

3. Шаг:] $x = y + 1$: $\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^{y+1} = \begin{pmatrix} n^y & yn^{y-1} \\ 0 & n^y \end{pmatrix}\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} n^{y+1} & (y+1)n^y \\ 0 & n^{y+1} \end{pmatrix}$ – верно

Согласно ММИ $\forall x \in \mathbb{N}$: $\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^x = \begin{pmatrix} n^x & xn^{x-1} \\ 0 & n^x \end{pmatrix}$

$\begin{pmatrix} n & 1 \\ 0 & n \end{pmatrix}^{2019} = \begin{pmatrix} n^{2019} & 2019n^{2018} \\ 0 & n^{2019} \end{pmatrix}$

17. $A = \begin{pmatrix} 8 & 8 & 11 \\ 6 & 3 & 4 \\ -5 & 2 & 3 \end{pmatrix}$

$\left(\begin{array}{ccc|ccc} 8 & 8 & 11 & 1 & 0 & 0 \\ 6 & 3 & 4 & 0 & 1 & 0 \\ -5 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 8 & 8 & 11 & 1 & 0 & 0 \\ 1 & 5 & 7 & 0 & 1 & 1 \\ -5 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 7 & 0 & 1 & 1 \\ 0 & -32 & -45 & 1 & -8 & -8 \\ 0 & 27 & 38 & 0 & 5 & 6 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 7 & 0 & 1 & 1 \\ 0 & -5 & -7 & 1 & -3 & -2 \\ 0 & 27 & 38 & 0 & 5 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & -5 & -7 & 1 & -3 & -2 \\ 0 & 2 & 3 & 5 & -10 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & -1 & -1 & 11 & -23 & -10 \\ 0 & 2 & 3 & 5 & -10 & -4 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & 1 & 1 & -11 & 23 & 10 \\ 0 & 0 & 1 & 27 & -56 & -24 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & 1 & 0 & -38 & 79 & 34 \\ 0 & 0 & 1 & 27 & -56 & -24 \end{array}\right)$

$\mathrm{A}^{-1} = \begin{pmatrix} 1 & -2 & -1 \\ -38 & 79 & 34 \\ 27 & -56 & -24 \end{pmatrix}$

18. $A = \begin{pmatrix} 0 & 3 & 3 \\ 2 & -1 & -1 \\ 1 & -2 & -1 \end{pmatrix}, B = \begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}$

$AXB = C$

$A^{-1}AXBB^{-1} = A^{-1}CB^{-1}$

$EXE = A^{-1}CB^{-1}$

$X = A^{-1}CB^{-1}$

$\left(\begin{array}{ccc|ccc} 0 & 3 & 3 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 0 & 1 \\ 0 & 3 & 1 & 0 & 1 & -2 \\ 0 & 3 & 3 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 0 & 1 \\ 0 & 3 & 1 & 0 & 1 & -2 \\ 0 & 0 & 2 & 1 & -1 & 2 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1/3 & 0 & 1/3 & -2/3 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1/3 & 0 & 2/3 & -1/3 \\ 0 & 1 & 1/3 & 0 & 1/3 & -2/3 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & 1/2 & 0 \\ 0 & 1 & 0 & -1/6 & 1/2 & -1 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1 \end{array}\right)$

$A^{-1} = \begin{pmatrix} 1/6 & 1/2 & 0 \\ -1/6 & 1/2 & -1 \\ 1/2 & -1/2 & 1 \end{pmatrix}$

$$\left(\begin{array}{cc|cc}5 & 2 & 1 & 0\\ 1 & 3 & 0 & 1\end{array}\right)\sim\left(\begin{array}{cc|cc}1 & 3 & 0 & 1\\ 0 & -13 & 1 & -5\end{array}\right)\sim\left(\begin{array}{cc|cc}1 & 0 & {}^{3}/_{13} & -{}^{2}/_{13}\\ 0 & 1 & -{}^{1}/_{13} & {}^{5}/_{13}\end{array}\right)$$

$$B^{-1}=\begin{pmatrix}{}^{3}/_{13} & -{}^{2}/_{13}\\ -{}^{1}/_{13} & {}^{5}/_{13}\end{pmatrix}$$

$$X=\begin{pmatrix}{}^{1}/_{6} & {}^{1}/_{2} & 0\\ -{}^{1}/_{6} & {}^{1}/_{2} & -1\\ {}^{1}/_{2} & -{}^{1}/_{2} & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ 2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}{}^{3}/_{13} & -{}^{2}/_{13}\\ -{}^{1}/_{13} & {}^{5}/_{13}\end{pmatrix}=\begin{pmatrix}{}^{4}/_{3} & {}^{2}/_{3}\\ -{}^{1}/_{3} & -{}^{5}/_{3}\\ 1 & 2\end{pmatrix}\begin{pmatrix}{}^{3}/_{13} & -{}^{2}/_{13}\\ -{}^{1}/_{13} & {}^{5}/_{13}\end{pmatrix}=$$

$$=\begin{pmatrix}{}^{4}/_{39} & {}^{2}/_{39}\\ -{}^{1}/_{39} & -{}^{5}/_{39}\\ {}^{1}/_{13} & {}^{8}/_{13}\end{pmatrix}$$

19. $a_1=(1\quad 4\quad 5\quad 6)$

$a_2=(-1\quad 2\quad 5\quad 8)$

$a_3=(2\quad 0\quad -2\quad -4)$

$a_4=(2\quad 3\quad 4\quad -9)$

$$A=\begin{pmatrix}1 & 4 & 5 & 6\\ -1 & 2 & 5 & 8\\ 2 & 0 & -2 & -4\\ 2 & 3 & 4 & -9\end{pmatrix}\sim\begin{pmatrix}1 & 4 & 5 & 6\\ 0 & 6 & 10 & 14\\ 0 & -8 & -12 & -16\\ 0 & -5 & -6 & -21\end{pmatrix}\sim\begin{pmatrix}1 & 4 & 5 & 6\\ 0 & 1 & 4 & -7\\ 0 & -2 & -3 & -4\\ 0 & 3 & 5 & 7\end{pmatrix}\sim$$

$$\sim\begin{pmatrix}1 & 4 & 5 & 6\\ 0 & 1 & 4 & -7\\ 0 & 0 & 5 & -18\\ 0 & 0 & -7 & -14\end{pmatrix}\sim\begin{pmatrix}1 & 4 & 5 & 6\\ 0 & 1 & 4 & -7\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & -28\end{pmatrix}\Rightarrow rg(A)=4\Rightarrow$$

$\Rightarrow$ Система векторов линейно независима

Максимальная линейно независимая система: a_1, a_2, a_3, a_4

20. ΔABC

$A(8;-1;2), B(9;-3;4), C(4;3;-4)$

$\overrightarrow{BA}=\{1;-2;2\}$

$\overrightarrow{BC}=\{-5;6;-8\}$

$$S_{ABC}=\frac{1}{2}\left|\overrightarrow{BA}\times\overrightarrow{BC}\right|=\frac{1}{2}\left\|\begin{matrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 1 & -2 & 2\\ -5 & 6 & -8\end{matrix}\right\|=\frac{1}{2}\left|\vec{\imath}\begin{vmatrix}-2 & 2\\ 6 & -8\end{vmatrix}-\vec{\jmath}\begin{vmatrix}1 & 2\\ -5 & -8\end{vmatrix}+\vec{k}\begin{vmatrix}1 & -2\\ -5 & 6\end{vmatrix}\right|=$$

$$=\frac{1}{2}\left|4\vec{\imath}-2\vec{\jmath}-4\vec{k}\right|=\frac{1}{2}\sqrt{16+4+16}=3$$

$$S_{ABC}=\frac{1}{2}h\left|\overrightarrow{BC}\right|\Leftrightarrow h=\frac{2S_{ABC}}{\left|\overrightarrow{BC}\right|}=\frac{6}{\sqrt{25+36+64}}=\frac{6}{\sqrt{125}}=\frac{6\sqrt{5}}{25}$$

21. ΔABC

$A(-1;-4), B(4;1), C(-2;4)$

$BC: y=kx+b$

$$\begin{cases}1=4k+b\\ 4=-2k+b\end{cases}\Leftrightarrow\left(\begin{array}{cc|c}4 & 1 & 1\\ -2 & 1 & 4\end{array}\right)\sim\left(\begin{array}{cc|c}6 & 0 & -3\\ -2 & 1 & 4\end{array}\right)\sim\left(\begin{array}{cc|c}1 & 0 & -{}^{1}/_{2}\\ 0 & 1 & 3\end{array}\right)\Rightarrow BC: y=-\frac{1}{2}x+3$$

$$h: y=-\frac{1}{k}x+b_h$$

$$h: y=2x+b_h$$

$$-4=-2+b_h\Leftrightarrow b_h=-2\Rightarrow h: y=2x-2\Leftrightarrow\frac{x}{1}=\frac{y+2}{2}$$

22. $ABCD$ − тэтраэдр

$A(2;-1;1), B(3;2;-1), C(5;5;4), D(4;1;3)$

$\overrightarrow{AB} = \{1; 3; -2\}$
$\overrightarrow{AC} = \{3; 6; 3\}$
$\overrightarrow{AD} = \{2; 2; 2\}$

$$V_{ABCD} = \frac{1}{6}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right) \cdot \overrightarrow{AD}\right| = \frac{1}{6}\left\|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ 3 & 6 & 3 \end{matrix}\right|\overrightarrow{AD}\right| =$$

$$= \frac{1}{6}\left|\left(3\vec{i}\begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} - 3\vec{j}\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + 3\vec{k}\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}\right)\overrightarrow{AD}\right| =$$

$$= \frac{1}{6}\left|\left(21\vec{i} - 9\vec{j} - 3\vec{k}\right)\left(2\vec{i} + 2\vec{j} + 2\vec{k}\right)\right| = \frac{|42 - 18 - 6|}{6} = 3$$

23. $M(3; 2; -1), N(2; -3; 4), K(4; -2; 2)$

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ -1 & -5 & 5 \\ 1 & -4 & 3 \end{vmatrix} = 5(x-3)\begin{vmatrix} -1 & 1 \\ -4 & 3 \end{vmatrix} - (y-2)\begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix} + (z+1)\begin{vmatrix} -1 & -5 \\ 1 & -4 \end{vmatrix} =$$

$$= 5x - 15 + 8y - 16 + 9z + 9 = 5x + 8y + 9z - 22$$

$P: 5x + 8y + 9z - 22 = 0$

24.

$$L_1: \frac{x+2}{-5} = \frac{y+4}{10} = \frac{z-3}{5}$$

$$L_2: \frac{x+4}{4} = \frac{y+5}{-8} = \frac{z-7}{-4}$$

$$L_1: \begin{cases} x = -5t - 2 \\ y = 10t - 4 \\ z = 5t + 3 \end{cases}$$

$$L_2: \begin{cases} x = 4t - 4 \\ y = -8t - 5 \\ z = -4t + 7 \end{cases}$$

$M_0(-2; -4; 3), M_1(-4; -5; 7), M_2(0; -13; 3)$

$$\begin{vmatrix} x+2 & y+4 & z-3 \\ -2 & -1 & 4 \\ 2 & -9 & 0 \end{vmatrix} = (x+2)\begin{vmatrix} -1 & 4 \\ -9 & 0 \end{vmatrix} - (y+4)\begin{vmatrix} -2 & 4 \\ 2 & 0 \end{vmatrix} + (z-3)\begin{vmatrix} -2 & -1 \\ 2 & -9 \end{vmatrix} =$$

$$= 36(x+2) + 8(y+4) + 20(z-3) = 36x + 72 + 8y + 32 + 20z - 60 =$$

$$= 36x + 8y + 20z + 44$$

$P: 9x + 2y + 5z + 11 = 0$

25. $M(3; 14; -6)$

$P: -3x - 3y + 2z - 7 = 0$

$\vec{n} = \{-3; -3; 2\}$

$M \in l, l \perp P$

$$l: \frac{x-3}{-3} = \frac{y-14}{-3} = \frac{z+6}{2} \Leftrightarrow \begin{cases} -3(x-3) = -3(y-14) \\ 2(x-3) = -3(z+6) \\ 2(y-14) = -3(z+6) \end{cases} \Leftrightarrow \begin{cases} x - 3 = y - 14 \\ 2x - 6 = -3z - 18 \\ 2y - 28 = -3z - 18 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x - y + 11 = 0 \\ 2x + 3z + 12 = 0 \\ 2y + 3z - 10 = 0 \end{cases}$$

$$\begin{cases} x - y + 11 = 0 \\ 2x + 3z + 12 = 0 \\ -3x - 3y + 2z - 7 = 0 \end{cases} \Leftrightarrow \begin{cases} x - y = -11 \\ 2x + 3z = -12 \\ 3x + 3y - 2z = -7 \end{cases} \Leftrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & -11 \\ 2 & 0 & 3 & -12 \\ 3 & 3 & -2 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -11 \\ 0 & 2 & 3 & 10 \\ 0 & 6 & -2 & 26 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -11 \\ 0 & 2 & 3 & 10 \\ 0 & 3 & -1 & 13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -11 \\ 0 & 1 & -4 & 3 \\ 0 & 2 & 3 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -4 & -8 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 11 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -{}^{72}/_{11} \\ 0 & 1 & 0 & {}^{49}/_{11} \\ 0 & 0 & 1 & {}^{4}/_{11} \end{array}\right)$$

$$H \in P, H \in l, H\left(-\frac{72}{11}; \frac{49}{11}; \frac{4}{11}\right)$$

$$\overrightarrow{MH}=\left\{-\frac{105}{11};-\frac{105}{11};\frac{70}{11}\right\}$$

$$\left|\overrightarrow{MH}\right|=\sqrt{\frac{11025}{121}+\frac{11025}{121}+\frac{4900}{121}}=\frac{\sqrt{26950}}{11}=\frac{35\sqrt{22}}{11}$$

26. $M(6;-3;7)$

$P: 16x-3y=10$

$\vec{n}=\{16;-3;0\}$

$M\in l, l\perp P$

$$l: \frac{x-6}{16}=\frac{y+3}{-3}=\frac{z-7}{0}\Leftrightarrow\begin{cases}-3(x-6)=16(y+3)\\ z-7=0\end{cases}\Leftrightarrow\begin{cases}-3x-16y=30\\ z=7\end{cases}$$

$$\begin{cases}-3x-16y=30\\ 16x-3y=10\\ z=7\end{cases}\Leftrightarrow\left(\begin{array}{ccc|c}-3&-16&0&30\\16&-3&0&10\\0&0&1&7\end{array}\right)\sim\left(\begin{array}{ccc|c}1&-83&0&160\\-3&-16&0&30\\0&0&1&7\end{array}\right)\sim\left(\begin{array}{ccc|c}1&-83&0&160\\0&-265&0&510\\0&0&1&7\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c}1&0&0&\frac{14}{53}\\0&1&0&-\frac{102}{53}\\0&0&1&7\end{array}\right)\Rightarrow M'\left(\frac{14}{53};-\frac{102}{53};7\right)$$

27. $L: \begin{cases}3x-2y+3z=10\\ -x+2y-4z=6\end{cases}$

$$z=0\Rightarrow\begin{cases}3x-2y=10\\ -x+2y=6\end{cases}\Leftrightarrow\left(\begin{array}{cc|c}3&-2&10\\-1&2&6\end{array}\right)\sim\left(\begin{array}{cc|c}0&4&28\\1&-2&-6\end{array}\right)\sim\left(\begin{array}{cc|c}0&1&7\\1&0&8\end{array}\right)\Rightarrow M(8;7;0)$$

$\overrightarrow{n_1}=\{3;-2;3\}$

$\overrightarrow{n_2}=\{-1;2;-4\}$

$$\vec{l}=\overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\3&-2&3\\-1&2&-4\end{vmatrix}=\vec{i}\begin{vmatrix}-2&3\\2&-4\end{vmatrix}-\vec{j}\begin{vmatrix}3&3\\-1&-4\end{vmatrix}+\vec{k}\begin{vmatrix}3&-2\\-1&2\end{vmatrix}=2\vec{i}+9\vec{j}+4\vec{k}=$$

$$=\{2;9;4\}$$

$$L: \frac{x-8}{2}=\frac{y-7}{9}=\frac{z}{4}$$

28. $L: \frac{x+3}{0}=\frac{y}{1}=\frac{z+5}{1}$

$\vec{s}=\{0;1;1\}\Rightarrow|\vec{s}|=\sqrt{2}$

$P: y-5z-6=0$

$\vec{n}=\{0;1;-5\}\Rightarrow|\vec{n}|=\sqrt{26}$

$\vec{s}\vec{n}=0+1-5=-4$

$$\vec{s}\vec{n}=\sqrt{52}\cos(\widehat{\vec{s},\vec{n}})\Leftrightarrow\cos(\widehat{\vec{s},\vec{n}})=-\frac{4}{\sqrt{52}}=-\frac{2\sqrt{13}}{13}\Rightarrow(\widehat{\vec{s},\vec{n}})>\frac{\pi}{2}$$

$$(\widehat{L,P})=(\widehat{\vec{s},\vec{n}})-\frac{\pi}{2}$$

$$\sin(\widehat{L,P})=\sin\left((\widehat{\vec{s},\vec{n}})-\frac{\pi}{2}\right)=-\cos(\widehat{\vec{s},\vec{n}})=\frac{2\sqrt{13}}{13}\Rightarrow(\widehat{L,P})=\arcsin\frac{2\sqrt{13}}{13}$$

29. $M(2;3;-1)$

$$L: \frac{x-5}{-3}=\frac{y}{-2}=\frac{z+25}{2}\Leftrightarrow\begin{cases}x=-3t+5\\ y=-2t\\ z=2t-25\end{cases}$$

$\vec{s}=\{-3;-2;2\}\Rightarrow|\vec{s}|=\sqrt{9+4+4}=\sqrt{17}$

$M_0(5;0;-25)$

$\overrightarrow{MM_0}=\{3;-3;-24\}$

$H\in L, MH\perp L$

$$MH=\frac{\left|\overrightarrow{MM_0}\times\vec{s}\right|}{|\vec{s}|}=\frac{\left\|\begin{matrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 3 & -3 & -24\\ -3 & -2 & 2\end{matrix}\right\|}{\sqrt{17}}=\frac{1}{\sqrt{17}}\left|6\vec{\imath}\begin{vmatrix}-1 & -8\\ -1 & 1\end{vmatrix}-3\vec{\jmath}\begin{vmatrix}1 & -8\\ -3 & 2\end{vmatrix}+3\vec{k}\begin{vmatrix}1 & -1\\ -3 & -2\end{vmatrix}\right|=$$

$$=\frac{1}{\sqrt{17}}\left|-54\vec{\imath}+66\vec{\jmath}-15\vec{k}\right|=\frac{\sqrt{2916+4356+225}}{\sqrt{17}}=\sqrt{\frac{7497}{17}}=\sqrt{441}=21$$

30. $Ax+By+3z=5$

$L:\begin{cases}x=2t+3\\ y=-3t+5\\ z=-2t-2\end{cases}$

$\vec{n}=\{A;B;3\}$

$\vec{s}=\{2;-3;-2\}$

$\vec{s}\times\vec{n}=\vec{0}$

$$\vec{s}\times\vec{n}=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 2 & -3 & -2\\ A & B & 3\end{vmatrix}=\vec{\imath}\begin{vmatrix}-3 & -2\\ B & 3\end{vmatrix}-2\vec{\jmath}\begin{vmatrix}1 & -1\\ A & 3\end{vmatrix}+\vec{k}\begin{vmatrix}2 & -3\\ A & B\end{vmatrix}=$$

$$=(2B-9)\vec{\imath}+(-2A-6)\vec{\jmath}+(3A+2B)\vec{k}=\vec{0}\Leftrightarrow\begin{cases}2B-9=0\\ -2A-6=0\\ 3A+2B=0\end{cases}\Leftrightarrow\begin{cases}B=\dfrac{9}{2}\\ A=-3\end{cases}$$