Übung 4

Tuesday, May 11, 2021 2:48 PM

Aufg. 12

$$\frac{dn}{dt} = r_0 \left(1 - K \cdot n(t)\right) \cdot n(t)$$

a)

$$n(t) = \frac{n_0 e^{r_0 t}}{1 + K n_0 (e^{r_0 t} - 1)}$$

$$\frac{dn}{dt} = \frac{r_0 n_0 e^{r_0 t} \left[1 + K n_0 (e^{r_0 t} - 1)\right] - K n_0 r_0 e^{r_0 t} n_0 e^{r_0 t}}{\left(1 + K n_0 (e^{r_0 t} - 1)\right)^2}$$

$$= r_0 \cdot n(t) - \frac{K n_0^2 r_0 e^{r_0 t} \cdot e^{r_0 t}}{\left(1 + K n_0 (e^{r_0 t} - 1)\right)^2}$$

$$= r_0 \cdot n(t) - K \cdot r_0 \cdot n^2(t)$$

$$= r_0 \left(1 - K \cdot n(t)\right) \cdot n(t)$$

$$\lim_{t \to \infty} n = \lim_{t \to \infty} \frac{n_0}{e^{-r_0 t} + K n_0 (1 - e^{-r_0 t})} = \frac{1}{K}$$

c)

$$X_{i+1} = 4 \mu x_i (1 - x_i)$$

Definiere

$$T(x_i) = 4 \mu x_i (1 - x_i)$$

$$T'(x_i) = 4 \mu (1 - x_i) - 4 \mu x_i = 4 \mu (1 - 2 x_i)$$

$$T'(x_i) = 4\mu(1-x_i) - 4\mu x_i = 4\mu(1-2x_i)$$

$$x_i = \frac{\Delta t r_0 k}{1+\Delta t r_0} n_i$$

$$4\mu = 1+\Delta t r_0$$

$$\Rightarrow T' = (1+\Delta t r_0)\left(1 - 2\,\frac{\Delta t r_0 k}{1+\Delta t r_0} n_i\right)$$

$$= 1+\Delta t r_0 - 2\Delta t r_0 k n_i$$

$$= 1+\Delta t r_0(1-2kn_i)$$

Stabilitätskriterium:

$$|1+\Delta t r_0(1-2kn_i)|$$

$$= |1+(4\mu-1)(1-2kn_i)|$$

$$= |4\mu(1-2kn_i)+2kn_i| \overset{!}{\leq} 1$$

$$\leq |-4\mu+2| \leq 1$$

$$\Rightarrow \frac{1}{4} \leq \mu \leq \frac{3}{4}$$