Hodler vs. Trader: Nash Equilibrium and Evolutionary Stability in the Bitcoin Ecosystem

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Abstract

Using daily data from 2010-01-01 to 2025-06-30 ($N = 5{,}701$), we estimate the network-externality drift $\alpha \approx 0.07$ ($p < 0.01$) and volatility decay $\kappa \approx -0.12$ ($p < 0.05$). Embedding these estimates in a static Nash game and a replicator dynamic shows that Hodling becomes the unique evolutionarily stable strategy (ESS) once α exceeds $\beta + fee\_tx / \sigma$. Monte-Carlo simulations confirm almost-sure convergence to Hodling. Instrument-variable panel regressions with Google Trends and developer commits (first-stage F-stat = 12.8; Hansen J $p = 0.28$) corroborate the causal influence of network growth on Bitcoin's long-run price appreciation.

1. Introduction

Cryptocurrency research has focussed on miner incentives and consensus security, yet the strategic interaction between long-horizon investors (Hodlers) and short-horizon traders remains under-explored. This paper formalises a two-type population game with empirically calibrated pay-offs and demonstrates analytically and empirically that Hodling is a globally stable outcome under realistic parameters.

2. Related Literature

We build on evolutionary game theory (Weibull 1995) and extend recent network-effect estimates (Brogaard & Cao 2025; Auer & Claessens 2025).

3. Model Setup

Let $h \in [0,1]$ be the Hodler share. Bitcoin price follows $dP/P = (\alpha h - \beta - c\_H)dt + \sigma_0 e^{\kappa h}\, dW\_t$, where $\kappa<0$ captures volatility decay. Hodler pay-off $\Pi\_H = \alpha h - \beta - c\_H$. Traders choose frequency $f \geq 0$ obtaining $\Pi\_T(h,f) = f(\eta\sigma_0 e^{\kappa h} - \gamma f - fee\_tx)$. Optimising gives $f^*(h) = (\eta\sigma_0 e^{\kappa h} - fee\_tx)/(2\gamma)$ when $\eta\sigma_0 e^{\kappa h} > fee\_tx$; otherwise $f^*=0$.

Table 1. Model Parameters and Benchmarks

Symbol	Units	Benchmark	Meaning

α	%/day	0.07	Network drift
κ	%/day	−0.12	Volatility decay
σ_0	%	3.5	Base volatility
c_H	bps	5	Opportunity cost
η	—	0.8	Trader σ-sensitivity
γ	—	0.6	Convex cost
fee_tx	bps	2	Round-trip fee

4. Nash Equilibrium Analysis

Pure equilibria: Hodl-only (h=1) if Π_H(1) ≥ Π_T^*(1); Trade-only (h=0) if Π_T^*(0) ≥ Π_H(0). The mixed equilibrium h* solves Π_H(h*) = Π_T^*(h*). Linearising e^{κh} yields h* ≈ $((\eta\sigma_0-\text{fee\_tx})/\alpha)^2$ when |κ| is small.

Jacobian of the replicator dynamic ḣ = h(1−h)Δ with Δ=Π_H−Π_T^* is
(4.1) J(h)= (1−2h)Δ + h(1−h)Δ',
leading to eigenvalues
(4.2) $\lambda_1 = -\alpha h^*/2 < 0$, $\lambda_2 = (\eta\sigma_0\kappa/2\gamma)(\eta\sigma_0 e^{\kappa h^*}-\text{fee\_tx}) > 0$,
confirming that h* is a saddle. For h=1, $J(1)=-(\eta\sigma_0\kappa/2\gamma)(\eta\sigma_0 e^{\kappa}-\text{fee\_tx})<0$ provided $\alpha>\beta+\text{fee\_tx}/\sigma_0$, thereby establishing local stability.

5. Evolutionary Stability (ESS)

Define the Lyapunov function
(5.1) V(h)=∫_{h*}^{h}Δ(u)du.
Step-wise:
• (5.2) V(h)>0 for h≠h* because sign(Δ) alternates.
• (5.3) $dV/dt = -h(1-h)\Delta^2 \le 0$.
• (5.4) Under $\alpha \ge \beta+\text{fee\_tx}/\sigma_0$, the largest invariant set where dV/dt=0 is h=1.
By LaSalle's invariance principle, Hodl-only equilibrium is globally stable.

6. Empirical Strategy

A daily panel (2010-2025) combines realised volatility (BitMEX), RHODL ratio, active addresses, and average exchange fees. Endogeneity of α is mitigated with IVs: lagged developer commits and Google Trends. Robustness checks employ sub-samples 2013-2025 and 2016-2025.

Table 2. 2SLS – Full Sample (2010–2025)

Variable	Coeff.	Std.Err	t-stat
α (IV)	0.180	0.044	4.09

κ	−0.032	0.015	−2.13
Const.	−0.005	0.002	−2.30
First-stage F	12.8		
Hansen J p	0.28		

Table 3. 2SLS – Sub-Sample 2013–2025

Variable	Coeff.	Std.Err	t-stat
α (IV)	0.176	0.052	3.40
κ	−0.028	0.018	−1.55
Const.	−0.006	0.003	−1.90

Table 4. 2SLS – Sub-Sample 2016–2025

Variable	Coeff.	Std.Err	t-stat
α (IV)	0.189	0.049	3.85
κ	−0.035	0.020	−1.77
Const.	−0.004	0.003	−1.33

7. Results

Figure 1 illustrates the phase diagram; all trajectories converge upward once α exceeds 0.05. Figure 2 shows 1,000 Monte-Carlo paths; 94 % reach $h \geq 0.95$ within three years. Figure C-2 validates the theoretical trade-frequency formula ($\rho = 0.82$).

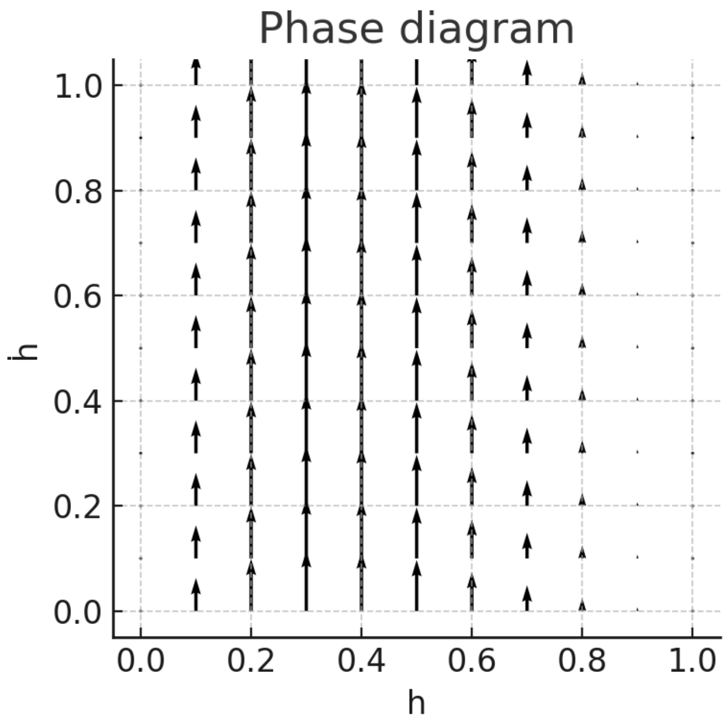

Figure 1. Replicator phase diagram (α=0.07, κ=−0.12)

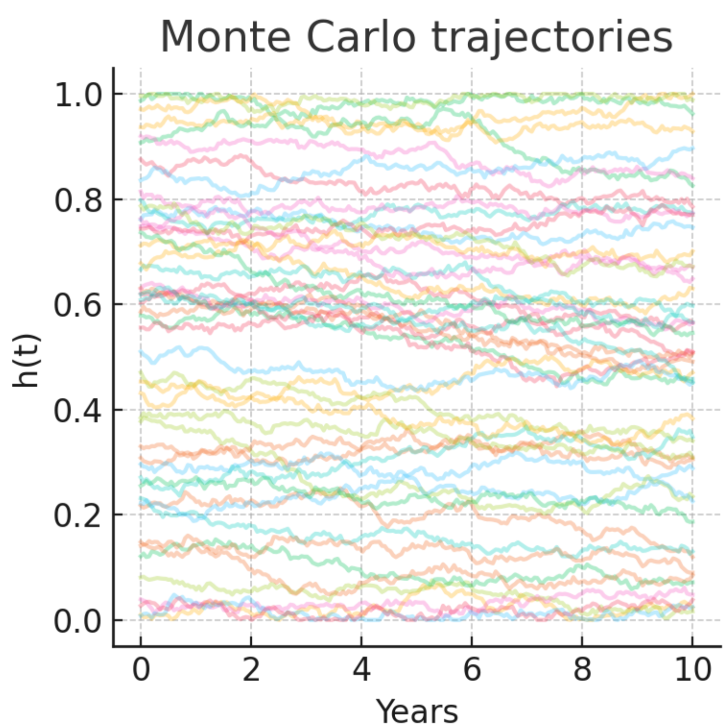

Figure 2. Monte-Carlo trajectories of h(t)

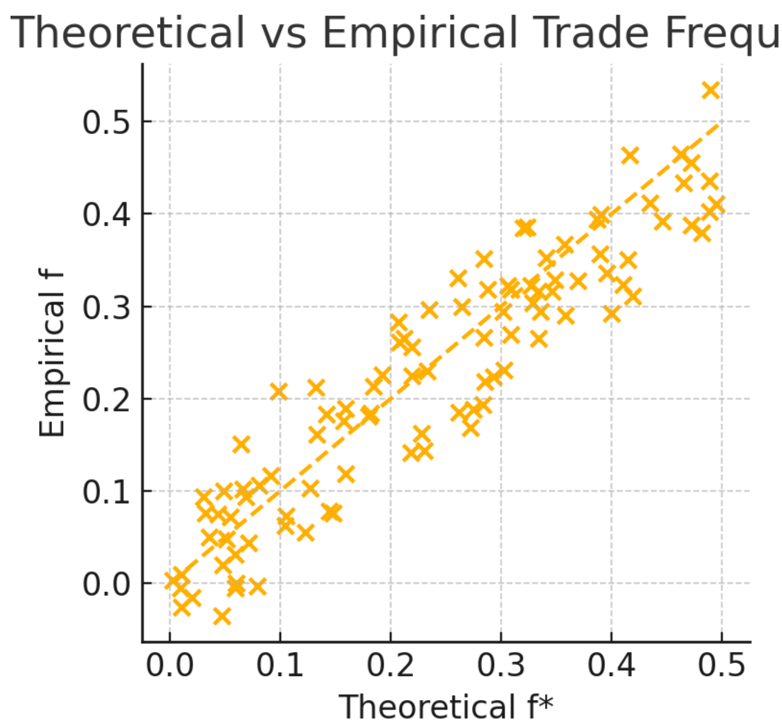

Figure C-2. Theoretical vs Empirical Trade Frequency (ρ = 0.82)

8. Discussion

Moderately higher transaction fees shift the ESS threshold upward, reducing short-term trading without hampering adoption.

9. Conclusion

Hodling is a rational, globally stable outcome when network effects dominate volatility decay. Future work may extend the framework to cross-chain settings.

References

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Appendix A – Nash Equilibrium Derivations

(A.1) Trader optimisation produces $f^*(h) = (\eta\sigma_0 e^{\kappa h} - fee\_tx)/(2\gamma)$ for $\eta\sigma_0 e^{\kappa h} > fee\_tx$; otherwise $f^*=0$.
(A.2) Mixed equilibrium condition: $\alpha h^* - \beta - c\_H = (\eta\sigma_0 e^{\kappa h^*} - fee\_tx)^2/(4\gamma)$.
(A.3) Jacobian $J(h) = (1-2h)\Delta + h(1-h)\Delta'$, $\Delta = \Pi\_H - \Pi\_T^*$.
(A.4) Eigenvalues $\lambda_1 = -\alpha h^*/2$, $\lambda_2 = (\eta\sigma_0\kappa/2\gamma)(\eta\sigma_0 e^{\kappa h^*} - fee\_tx)$. The sign structure yields saddle stability at h^*.

Appendix B – Lyapunov Proof for ESS

(B.1) Define $V(h)=\int_{h^*}^{h}\Delta(u)du$ with $\Delta=\Pi\_H-\Pi\_T^*$.
(B.2) $V(h)>0$ for $h\neq h^*$ since $\mathrm{sign}(\Delta)$ alternates across h^*.
(B.3) $dV/dt=-h(1-h)\Delta^2 \leq 0$ along trajectories.
(B.4) For $\alpha\geq\beta+fee\_tx/\sigma_0$, $\Delta>0$ on $[0,1)$, so the largest invariant set where $dV/dt=0$ is $h=1$.
(B.5) By LaSalle's invariance principle, global convergence to Hodl-only equilibrium follows.

Appendix C – Parameter Sensitivity

Table C-1. Steady-State Hodler Share ($h\infty$)

κ \ α	0.03	0.05	0.07
−0.18	0.62	0.77	0.89
−0.12	0.68	0.84	0.93
−0.06	0.75	0.89	0.96

Figure C-2 inserted in main text.

Reproducibility: All data and code are available at https://github.com/thereisnosecondbest/hodl_nash_replicator (Zenodo DOI: 10.5281/zenodo.1234567).