CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 12 - Nested Independent Loops

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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 12 - Nested Independent Loops

Problem 1. Analyze the runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops. [**Note:** $A[n, n]$ denotes an n-by-n matrix with entries $a(i, j)$, $1 \leq i \leq n$, $1 \leq j \leq n$.]

```
1: column_power_sum(A[n, n])
2:   sum = 0
3:   for i = 1, i <= n
4:       i = i + 1
5:       for j = 1, j <= n
6:           j = 2 * j
7:           sum = sum + a(i, j)
8:   return sum
```

Answer.

- Line 2 takes 1 step to initialize sum=0
- The return statement takes 1 step at line 8 I begin by analyzing the inner j loop:
- At line 5, the j loop takes 1 step to initialize j=1
- The loop terminates when $2^k > n$. Solving for k, we have that: $k > log_2 n$. The loop takes at least one iteration to compare j to n, so the loop takes $\lfloor log_2 n \rfloor + 1$ iterations
- At each iteration, the loop does the following:
  - The comparison $j <= n$ takes 1 step
  - The update $j = 2 * j$ takes 2 steps: 1 step to evaluate $2 * j$ and 1 step for the assignment
  - The body of the loop consists of $sum = sum + a(i, j)$ which takes 3 steps: 1 step to access an array element, 1 step to calculate $sum + a(i, j)$, and 1 step to assign $sum = sum + a(i, j)$.

  So the run time complexity of the j loop is :

$$
\begin{aligned}
1 + \sum_{j=1}^{\lfloor log_2 n \rfloor + 1} (1 + 2 + 3) &= 1 + \sum_{j=1}^{\lfloor log_2 n \rfloor + 1} (6) \\
&= 1 + 6(\lfloor log_2 n \rfloor + 1) \\
&= 7 + 6\lfloor log_2 n \rfloor
\end{aligned}
$$

Now analyze the outer loop:

- Initializing the loop at line 3 takes 1 step
- The i loop takes n iterations
- At each iteration, the loop does the following:
  - The comparison $i <= n$ takes 1 step
  - The update $i = i + 1$ takes 2 steps: 1 step to evaluate $i + 1$ and 1 step for the assignment
  - The body of the loop consists just inner j loop.

So the run time complexity function is:

$$
\begin{aligned}
T(n) = 1 + 1 + 1 + \sum_{i=1}^{n}((1+2) + (7 + 6\lfloor log_2 n \rfloor)) \\
T(n) = 3 + \sum_{i=1}^{n}(10 + 6\lfloor log_2 n \rfloor) \\
T(n) = 3 + \sum_{i=1}^{n}(10) + \sum_{i=1}^{n}(6\lfloor log_2 n \rfloor) \\
T(n) = 3 + 10n + 6n\lfloor log_2 n \rfloor
\end{aligned}
$$

So the run time complexity is $T(n) = \Theta(n \log_2 n)$

$\square$