Quiz 7 – Safe and useless edges

Due Date ....February 18
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.

- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.

- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).

- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 6- Safe and Useless Edges

2.1 Problem 1

Problem 1. Consider the following graph $G(V, E, w)$. Suppose we have the intermediate spanning forest $\mathcal{F}$ (indicated using thick edges) consisting of the edges $\{A, B\}$, $\{A, C\}$, and $\{D, E\}$. Clearly identify the safe, useless, and undecided edges. Justify your reasoning.
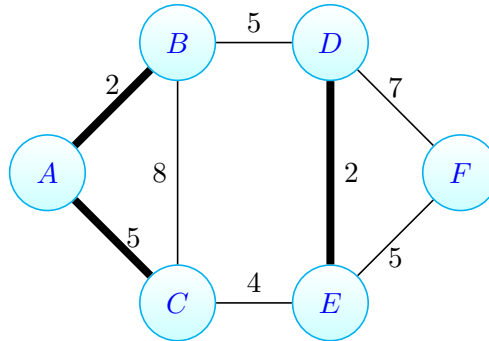

Answer. $\{C, E\}$, and $\{E, F\}$ are **safe** edges with respect to $\mathcal{F}$ because they are each the minimum weight edges that crosses a partition of the vertices (a cut), so they are each light edges. Since any light edge that connects two components in an intermediate spanning forest is safe, they are safe.

$\{B, C\}$ is a **useless** edge with respect to $\mathcal{F}$ because it would create a cycle

$\{B, D\}$, $\{D, F\}$ are **undecided** edges with respect to $\mathcal{F}$ because they are neither safe, nor useless

1. $\{C, E\}$ is the minimum weight edge with exactly one endpoint in the the component $\{A, B, C\}$ and the minimum weight edge with exactly one endpoint in the component $\{D, F, E\}$. Therefore $\{C, E\}$ is a light edge and hence a **safe** edge with respect to $\mathcal{F}$

2. $\{E, F\}$ is the minimum weight edge with exactly one endpoint in the the component $\{A, B, D, C, E\}$ and the minimum weight edge with exactly one endpoint in the component $\{F\}$. Therefore $\{E, F\}$ is a light edge and hence a **safe** edge with respect to $\mathcal{F}$

   See below for a visual of the safe edges
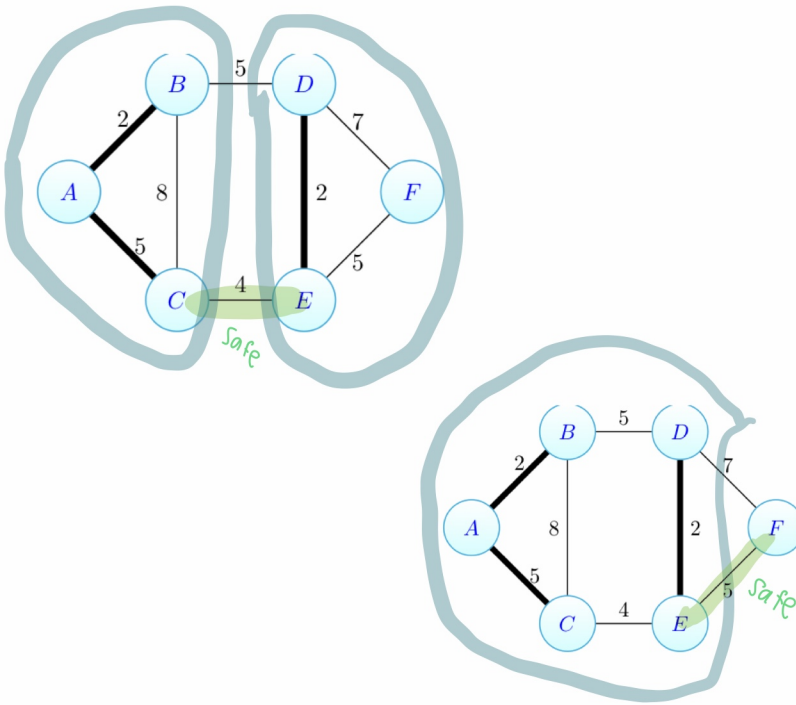
3. While the edge $\{B, D\}$ hasexactly one endpoint in the the component $\{A, B, C, E, F\}$ and one endpoint in the component $\{D\}$, it is not the minimum weight edge in doing so. Therefore $\{B, D\}$ is an **undecided** edge with respect to $\mathcal{F}$

4. While the edge $\{D, F\}$ has exactly one endpoint in the component $\{A, B, C, E, F\}$ and one endpoint in the component $\{D\}$, it is not a minimum weight edge in doing so. Therefore, $\{D, F\}$ is an **undecided** edge with respect to $\mathcal{F}$

5. Lastly, the edge $\{B, C\}$ has both endpoints in the component $\{A, B, C\}$, thus creating a cycle. So $\{B, C\}$ is a **useless** edge with respect to $\mathcal{F}$