CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 20 - DP: Write down recurrence

Due Date .... April 1
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 20 - DP: Write down Recurrence

Problem 1. Suppose we have an m-letter alphabet, $\Sigma = \{0, 1, \ldots, m-1\}$. Determine a recurrence for the number of strings ω of length n, such that no two consecutive characters in ω are the same. Clearly justify your recurrence.

Answer. Let $f_n := |W_n|$ where and define w_i as the character at the position i.

If n = 1 then every string does not contain two consecutive characters and there are m strings.

If n = 2 and if $w_0 \neq w_1$ then there are $m \cdot (m-1)$ ways to pick the string.
If $n \geq 3$:
if $w_0 \neq w_1$ then there are f_{n-1} ways to pick $w_2 = w_n$
if $w_0 = w_1$ then there are m ways to pick w_0 and 1 way for w_1 and f_{n-2} ways to pick $w_2 = w_n$
Because each of these cases are disjoint (no overlap), we can sum the contribution of each case. The final recurrence is then

$$f_n = \begin{cases} \text{m} & : \text{n=1}, \\ m^2 - m & : \text{n=2} \\ m(m-1)f_{n-1} + m \cdot f_{n-2} & : \text{otherwise.} \end{cases}$$

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