CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 14 - Write Recurrence

Due Date ..... March 11
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 14 - Writing Recurrences

Problem 1. Write down a recurrence for the runtime complexity of this algorithm. Clearly justify your answer. You are **not** being asked to solve the recurrence.

Algorithm 1 Recurrences

```
1: procedure Foo1(Integer n)
2:     if n < 4 then return 0
3:     Foo1(n/4)
4:     Foo1(n/4)
5:
6:     for i ← 1; i ≤ 3 * n; i ← i * 2 do
7:         print i
```

Answer.

$$T(n) = \begin{cases} 1 & : n < 4, \\ 2T(n/4) + \Theta(log_2(3n)) & : n \geq 4. \end{cases}$$

The base case is constant time when $n < 4$.
Then when $n \geq 4$, there are two recursive calls to Foo1 with imput size $n/4$, hence $2T(n/4)$
Then analyze the non recursive part:

- 1 step to initialize the loop
- let k denote the number of iterations of the i loop. The i loop terminates when$2^k > 3n$. So $k > log_2(3n)$ Thus the loop takes $k = \lfloor log_2(3n) \rfloor + 1$ iterations
- each iteration, there is 1 step for comparison, 2 steps for the update, and 1 step for the print statement

Thus the non recursive work is

$$1 + \sum_{i=1}^{\lfloor log_2(3n) \rfloor + 1} (4)$$
$$1 + 4 \cdot (\lfloor log_2(3n) \rfloor + 1)$$

Which is $\Theta(log_2(3n))$

□