CSCI 3104 Spring 2022
Instructor: Profs. Chen and Layer

Quiz 2 - Greedy Counterexamples

Due Date ..... February 4
Name ..... **Julia Troni**
Student ID **109280095**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Julia Troni. □

3 Standard 2- Greedy Counterexamples

3.1 Problem 2

Problem 2. Consider the Making Change problem where we have four coins: 1-cent pieces, 2-cent pieces, 8-cent pieces, and 10-cent pieces. We take as input an integer $n \geq 0$. The goal is to make change for n using the fewest number of coins. Consider a greedy algorithm which selects as many 10-cent pieces as possible, followed by as many 8-cent pieces, followed by as many 2-cent pieces, then lastly 1-cent pieces.

Give an integer $n \geq 0$ such that the greedy algorithm fails to give the optimal solution.

Proof. The greedy algorithm fails to give the optimal solution for $n > 8$ where $n\%8 = 0$ and $n\%8 \neq 0$. That is, for amounts greater than 8 cents, that are multiples of 8, but also NOT multiples of 10, the greedy algorithm is not optimal.
For example, if $n = 16$ the greedy algorithm will select 1 10-cent piece and 3 2-cent pieces, for a total for 4 pieces. However, it is optimal to select just 2 8-cent pieces because $2 < 4$.
Likewise, $n = 24$ the greedy algorithm will select 2 10-cent pieces and 2 2-cent pieces, for a total for 4 pieces. However, it is optimal to select just 3 8-cent pieces because $3 < 4$.

This is formalized as follows:
Suppose set I is an optimal solution produced from the greedy algorithm, meaning it is a set of coins that makes change for n using the fewest number of coins possible.
Now also suppose that I contains > 8 cents and it is a multiple of 8, but not a multiple of 10.
I will prove by counterexample that the greedy algorithm is not always optimal when this is the case. The key idea is that when $n\%8 = 0$ (and $n\%8 \neq 0$), we can reduce the number of total coins by choosing just 8 cent coins. The counterexample is to consider if $n = 16$. The greedy algorithm will select 1 10-cent piece and 3 2-cent pieces, for a total for 4 pieces. However, it is optimal to select just 2 8-cent pieces. And since $2 < 4$, $|I'| = |I| - 2 < |I|$. Thus, since we can produce a set I' that uses fewer coins that I, it follows that the greedy algorithm does not always give an optimal solution.

□