CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 21 - DP: Use recurrence to solve

Due Date .... April 1
Name .... **Julia Troni**
Student ID **109280095**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 21 - DP: Use Recurrence to Solve

Problem 1. Consider the following modified Rod Cutting Problem:

- Instance: Let $n \geq 0$ be an integer where n is the length of the rod, and let $p_1, p_2, \ldots, p_n$ be non-negative real numbers. Here, p_i is the price of selling a rod of length i.

- Solution: The maximum revenue, which we denote r_n, obtained by cutting the rod into pieces of integer lengths and selling the pieces.

Now suppose that our cutting tool is **malfunctioning** and **we are limited in the cuts we can make**. We can always cut off a piece of length 1, and additionally we can cut rods into halves (even n) or nearly halves (odd n): $\lfloor \frac{n}{2} \rfloor$ and $\lceil \frac{n}{2} \rceil$. A recurrence relation describing r_n is:

$$r_n = \begin{cases} 0 & \text{for } n = 0, \\ \max(p_n, r_1 + r_{n-1}, 2r_{n/2}) & \text{for even } n > 0, \\ \max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2}) & \text{for odd } n > 0. \end{cases}$$

Suppose we have a rod of length 6, with prices given as follows:

$$p_1 = 1, \quad p_2 = 5, \quad p_3 = 8, \quad p_4 = 10, \quad p_5 = 12, \quad p_6 = 15.$$

Using a bottom-up dynamic programming approach, determine the maximum revenue r_6 obtained by cutting up this rod **under the limited cut assumption described above**. For your convenience, we have provided the lookup table for you to use. You may also hand-draw your lookup tables.
Clearly show all work for how you filled in each cell of the lookup table.

Answer.

r_0	r_1	r_2	r_3	r_4	r_5	r_6
0	1	5	8	10	13	16

$r_0 = 0$ by the base case

r_1: since n is odd apply $\max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2})$

$r_1 = \max(p_1, r_1 + r_0, r_{0/2} + r_{2/2})$

$= \max(1, r_1 + 0, r_0 + r_1)$

$= 1$

$\therefore r_1 = 1$

r_2: apply $\max(p_n, r_1 + r_{n-1}, 2r_{n/2})$ because n is even

$r_2 = \max(p_2, r_1 + r_1, 2r_{2/2})$

$= \max(5, 1+1, 2r_1)$

$= \max(5, 2, 2)$

$= 5$

$\therefore r_2 = 5$

2

r_3 : n is odd $\Rightarrow$ apply $\max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2})$

$$r_3 = \max(p_3,\ r_1 + r_2,\ r_{2/2} + r_{4/2})$$
$$= \max(8,\ 1+5,\ r_1 + r_2)$$
$$= \max(8,\ 6,\ 1+5)$$
$$= 8$$
$$\therefore r_3 = 8$$

r_4 : n is even $\Rightarrow$ $\max(p_n, r_1 + r_{n-1}, 2r_{n/2})$

$$r_4 = \max(p_4,\ r_1 + r_3,\ 2r_{4/2})$$
$$= \max(10,\ 1+8,\ 2 \cdot 5)$$
$$= 10$$
$$\therefore r_4 = 10$$

r_5 : n is odd $\Rightarrow$ $\max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2})$

$$r_5 = \max(p_5,\ r_1 + r_{5-1},\ r_{4/2} + r_{6/2})$$
$$= \max(12,\ 1+10,\ 5+8)$$
$$= \max(12,\ 11,\ 13)$$
$$= 13$$
$$\therefore r_5 = 13$$

r_6 is even $\Rightarrow$ $\max(p_n, r_1 + r_{n-1}, 2r_{n/2})$

$$r_6 = \max(p_6,\ r_1 + r_5,\ 2r_{6/2})$$
$$= \max(15,\ 1+13,\ 2 \cdot 8)$$
$$= \max(15,\ 14,\ 16)$$
$$= 16$$
$$\therefore r_6 = 16$$

$\square$