Quiz 19 - DP: Identify subproblems

Due Date . . . . April 1
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.

- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.

- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).

- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 19 - Dynamic Programming: Identify Precise Subproblems

Problem 1. Given an undirected graph $G(V, E)$ with positive weight $c_e > 0$ for each edge $e \in E$, you are asked to find the shortest path from a source $s \in V$ to a destination $t \in V$. For each node $v \in V$, denote by d_v the cost of shortest path from v to the destination t. Clearly identify the precise sub-problems to consider. That is, what is the recursive structure to leverage when designing a dynamic programming algorithm?

Answer. To find the shortest path from a source $s \in V$ to a destination $t \in V$, we will need to know the shortest paths from the source node to each node on every possible path to the destination.

Let $D(i, v)$ be the cost of the shortest path from v to t that uses $\leq i$ edges.
We consider 2 cases:

1. The cheapest path $v \to t$ uses $\leq i - 1$ edges, in which cases $D(i, v) = D(i - 1, v)$

2. The cheapest path $v \to t$ uses exactly i edges.
   If (j, k) is the first edge, then D will use the edge (j, k) and then select the shortest path from $k \to t$ using $\leq i - 1$ edges, thus we have a smaller instance of the shortest path problem

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