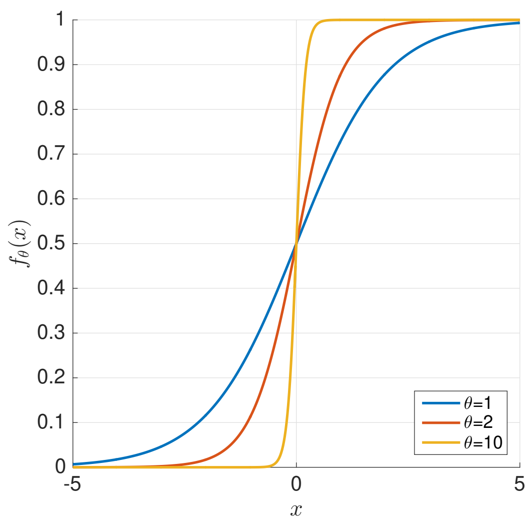
Numerical Computing :: Project Seven

Consider the parameterized family of functions,

$$f_\theta(x) \;=\; \frac{1}{1+\exp(-\theta x)}, \quad x \in [-5, 5].$$

The parameter θ controls how smooth f_θ is near $x = 0$, as shown:



To start this homework, let $\theta = 1$.

1. **Generate training data:** Create a vector with $n = 7$ evenly spaced points in the interval $[-5, 5]$. For each point x_i in this vector, compute $y_i = f_\theta(x_i)$. You should now have 7 pairs (x_i, y_i). Make a nice table with the seven input/output pairs.

2. **Train the model:** Construct the Vandermonde system and solve for the coefficients of the unique degree-6 interpolating polynomial $p_6(x)$. Make a nice table of the 7 coefficients. And make a plot showing both $f_\theta(x)$ and $p_6(x)$ over the domain $[-5, 5]$. Does this look like a good approximation? Explain your assessment.

3. **Generate testing data:** Create a new vector with 101 evenly spaced points in $[-5, 5]$. For each point x'_i, compute $y'_i = f_\theta(x'_i)$.

4. **Compute the testing error:** Compute and report the the absolute testing error:

$$\text{error} \;=\; \text{error}_{\theta=1,n=7} \;=\; \underset{i}{\text{maximum}} \;\frac{|\, y_i' - p_6(x_i')\,|}{|y_i'|}$$

   If you're wondering how to compute $p_6(x_i')$, look up `(np.polyval` and use the coefficients you computed in Step 2. You're evaluating the polynomial model's prediction of $f_\theta(x_i')$.

5. Repeat steps 1-4 with $\theta = 10$. How does the error change? What does that tell you about the quality of the polynomial approximation for the two functions?