Numerical Computing :: Project Six

Consider the following nonlinear system of equations with two equations and two unknowns. The math problem can be stated as follows. Given $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ defined as

$$\begin{aligned} f_1(x_1, x_2) &= x_1^3 - x_2^3 + x_1, \\ f_2(x_1, x_2) &= x_1^2 + x_2^2 - 1. \end{aligned} \tag{1}$$

Find r_1 and r_2 such that $f_1(r_1, r_2) = 0$ and $f_2(r_1, r_2) = 0$.

1. Note that all the points such that $f_2 = 0$ define a circle of radius 1 centered at the origin. Make a plot that shows (i) all the points that satisfy $f_1 = 0$ and (ii) all the points that satisfy $f_2 = 0$. Identify the points on the plot that satisfy both $f_1 = 0$ and $f_2 = 0$.

2. By hand, calculate the 2×2 Jacobian matrix of the system (f_1, f_2).

3. Use Newton's method for systems to find the two solutions to the system of equations $(f_1 = 0, f_2 = 0)$. Try several (10 or so) different initial guesses. Make a table of the answer that Newton's method gives—something like:

Initial guess $(x_1^{(0)}, x_2^{(0)})$	Newton's answer (r_1, r_2)
####, ####	####, ####

   The superscript in the column heading indicates the iteration number, i.e., 0 means the initial guess. Check the plot you made in problem 1 to see whether the answers you're getting make sense.

4. Find a starting point where Newton's method fails. Why did it fail?