POLITECNICO DI MILANO
DEPARTMENT OF AEROSPACE ENGINEERING

ORBITAL MECHANICS

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Professor: Colombo Camilla

Person Code	**Surname**	**Name**	**Bachelor (type and University)**
10717230	Beretta	Beatrice	Management Engineering - Politecnico di Milano
10726898	Juvara	Matteo Giovanni	Aerospace Engineering - Politecnico Di Milano
10782473	Miccoli	Francesco Antonio	Aerospace Engineering - Politecnico Di Milano
10973642	Separovic	Tomislav Marko	Mechanical Engineering - Instituto Tecnologico de Buenos Aires

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1 Assignment 1

1.1 Introduction

The PoliMi Space Agency is carrying out a feasibility study for a potential Interplanetary Explorer Mission departing from Venus and visiting the asteroid VF32 Aten, with an intermediate flyby on Earth. A window has been provided with the earliest departure from Venus and latest arrival to the Asteroid which is shown in Table 1.

The method of patched conics will be used to find a solution that minimizes ΔV. The initial heliocentric orbit is considered as the orbit of Venus and the final heliocentric orbit is considered as the orbit of the asteroid, without taking into account planetary departure and insertion.

The mission is characterized by three orbital trajectories. The 1st leg: a heliocentric trajectory from Venus to Earth. The flyby: an incoming and an outcoming hyperbola in Earth centered frame. The 2nd leg: a heliocentric trajectory from Earth to the asteroid VF32 Aten.

Earliest Departure	00:00:00 01/01/2028
Latest Arrival	00:00:00 01/01/2058

Table 1: Boundary mission dates

1.2 Design process

1.2.1 Critical Mission Dates

There are 3 dates that define the mission: departure date, flyby date and arrival date.

For the departure date window the boundary mission dates from Table 1 are used.

For the flyby dates they are defined as a launch date plus the flyby transfer (1st leg) time. For the arrival dates they are defined as a flyby date plus the arrival transfer (2nd leg) time.

As both transfers are direct and the body with the biggest period is Earth, with a period of 365.25 days, it is assumed that the time of flight of all transfers will be shorter than Earth's period.

The transfer windows are detailed in Table 2.

Flyby Transfer Window	1-365 days
Arrival Transfer Window	1-365 days

Table 2: Transfer windows

1.2.2 Flyby Parameters

For this analysis the flyby is treated as an impulsive maneuver in the heliocentric frame and analyzed separately in the Earth centered frame with the method of patched conics.

The main consideration for the flyby is that the periapsis is at a high enough altitude such that the spacecraft doesn't crash into the Earth or it's atmosphere. Based on previous missions that did flybys on Earth [1] the minimum periapsis is set as the radius of the Earth plus $300km$.

1.3 Matlab Method

1.3.1 Structure of the program

The search method used consists on iterating every possible launch date, flyby transfer time and arrival transfer time respecting their defined windows. The time step defines the discretization of the time windows. The code takes the form of 3 nested for loops.

The first value to be iterated is the launch date, where the initial position and velocity of the spacecraft are defined as the position and velocity of Venus at the launch date.

Then the flyby transfer time is iterated. The position and velocity of Earth are calculated at the flyby date. With this information the 1st leg of the mission can be obtained by solving the Lambert problem. The departure ΔV_{dep} can be calculated as:

$$\Delta V_{dep} = ||V_{1,dep} - V_V||$$

with $V_{1,dep}$ being the velocity of the spacecraft at the start of the 1st leg and V_V the velocity of Venus.

Then the arrival transfer time is iterated. The position and velocity of the asteroid are calculated at the arrival date. With this information the second leg of the mission can be obtained by solving the Lambert problem. The arrival ΔV_{arr} can be calculated as:

$$\Delta V_{arr} = ||V_{2,arr} - V_{as}||$$

with $V_{2,arr}$ being the velocity of the spacecraft at the end of the 2nd leg and V_{as} the velocity of the asteroid.

Knowing the velocity at the end of the first leg and at the beginning of the second the parameters of the flyby can be calculated. The entry and exit velocities, relative to Earth are computed as:

$$V_\infty^- = V_{1,arr} - V_E$$
$$V_\infty^+ = V_{2,dep} - V_E$$

with $V_{1,arr}$ being the velocity of the spacecraft at the end of the 1st leg, $V_{2,dep}$ the velocity of the spacecraft at the start of the second leg and V_E the velocity of Earth.
Then the turn angle can be calculated as:

$$\delta = cos^{-1}\left(\frac{V_\infty^- \cdot V_\infty^+}{||V_\infty^- \cdot V_\infty^+||}\right)$$

The the periapsis can be found by solving:

$$\begin{cases} f(r) = \delta - sin^{-1}\left(\frac{1}{1+\frac{r||V_\infty^-||^2}{\mu_E}}\right) - sin^{-1}\left(\frac{1}{1+\frac{r||V_\infty^+||^2}{\mu_E}}\right) \\ f(r_p) = 0 \end{cases} \tag{1}$$

The semi-major axis of each hyperbola can be calculated as:

$$a^- = -\frac{\mu_E}{||V_\infty^-||^2}$$
$$a^+ = -\frac{\mu_E}{||V_\infty^+||^2}$$

The the velocities at periapsis can be calculated as:

$$V_p^- = \sqrt{\mu_E\left(\frac{2}{r_p} - \frac{1}{a^-}\right)}$$
$$V_p^+ = \sqrt{\mu_E\left(\frac{2}{r_p} - \frac{1}{a^+}\right)}$$

The gravity assist ΔV and flyby ΔV can be calculated as:

$$\Delta V_{ga} = |V_p^+ - V_p^-|$$
$$\Delta V_{fb} = |V_\infty^+ - V_\infty^-|$$

Finally the total delta V can be calculated as:

$$\Delta V_{tot} = \Delta V_{dep} + \Delta V_{ga} + \Delta V_{arr}$$

Finding the optimal launch, flyby and arrival dates is just a matter of analysing all possible cases and choosing the one with the least ΔV_{tot}.

1.3.2 Optimization

Although implementing the method described in the above section would work, analyzing the full departure window with a time step of 2 days would mean analyzing more than 100 million cases, which could take hours or days to run on a normal computer.

A series of optimizations are implemented in order to be able to run the program in a reasonable time frame.

Some conditions are defined to avoid wasting processing time on unviable transfers, a maximum delta V ΔV_{max} is defined.

On the flyby date iteration if ΔV_{dep} is larger than ΔV_{max} then the program moves on to the next flyby date.

On the arrival date iteration if $\Delta V_{dep} + \Delta V_{arr}$ is larger than ΔV_{max} then the program moves on to the next arrival date. Then before solving the flyby periapsis it checks if the function from Equation 1 evaluated in $r = R_E + 300\text{km}$ and $r = r_{SOI}$ have the same sign, if that's the case then the imposed solution of r_p is outside the boundaries and the program moves on to the next arrival date.

By choosing the right ΔV_{max} this optimizations reduce the number of cases analyzed by an order of magnitude. However the time required to run the program would still be high, so two further optimizations are implemented.

First the launch date iteration is done with a parallel for, as the analysis of each launch date is independent. This reduces the run time by a factor of 4 to 32 depending on the computer.

Second the code is compiled using the native MatLab compiler, which further decreases the run time by an order of magnitude.

With all this optimizations implemented the code runs in a matter of seconds for the full departure window with a time step of 2 days, allowing a precise analysis of all transfer options.

1.4 Solution

During development and testing of the program it was found that there are solutions with less than $12\frac{km}{s}$ of ΔV, as such ΔV_{max} is set at $12\frac{km}{s}$.

The program is run with the whole departure window and a time step of 2 days. The minimum ΔV required for each departure date is shown in Figure 1. The solution with minimum ΔV found has a ΔV_{tot} of $11.83\,\text{km}\,\text{s}^{-1}$ and a departure date in 2049. However there is a solution that is almost as good with a launch date on 2030. This departure date is selected as we consider that the scientific cost of delaying the mission for 20 years is higher than the cost of adding $0.2\,\text{km}\,\text{s}^{-1}$ of ΔV to the spacecraft.

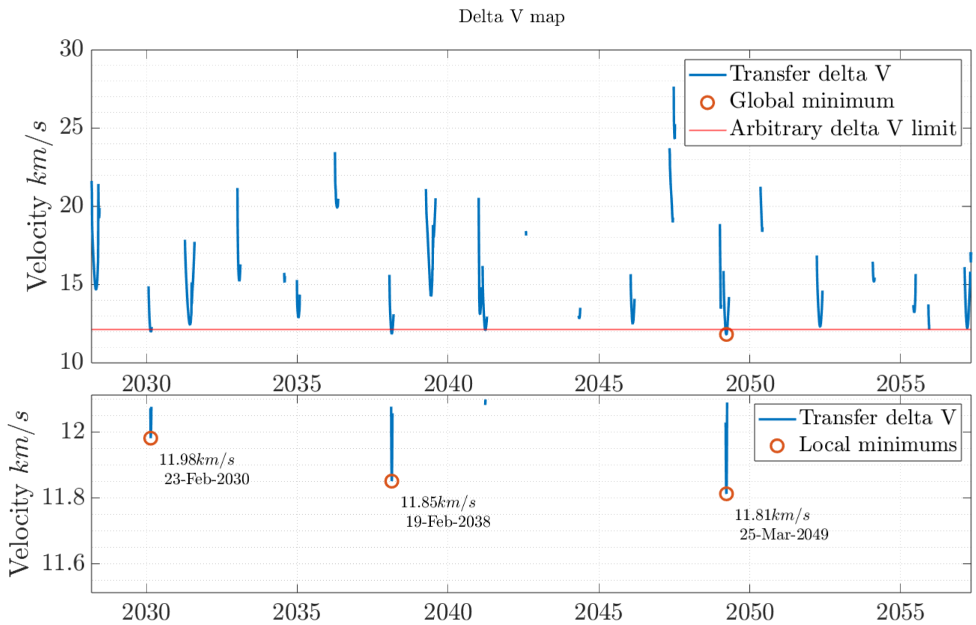


Fig. 1: Minimum delta V required for each launch date

The program is run again with the departure window in Table 3 and a time step of 0.25 days. The results are plotted in Figure 2. The mission dates of the final solution are in Table 4.

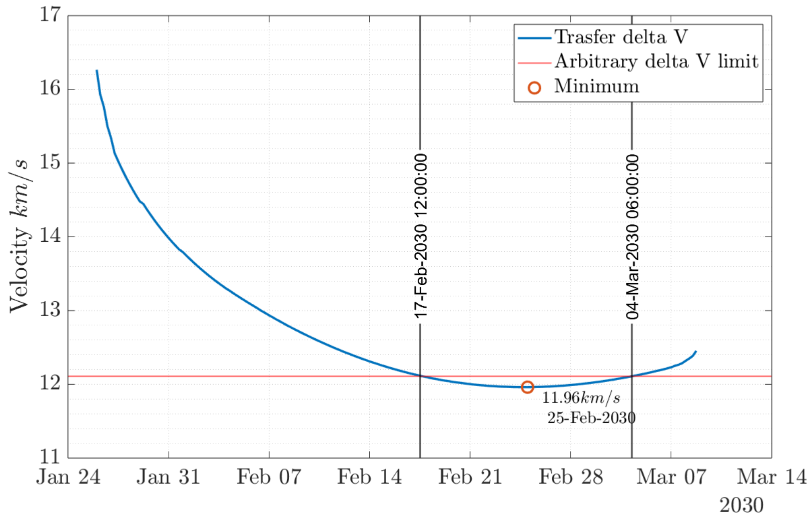


Fig. 2: Refined minimum delta V required for each launch date

Earliest departure	00:00:00 20/02/2030
Latest departure	00:00:00 01/03/2030

Table 3: Departure window for final analysis

Departure Date	00:00:00 25/02/2030
Flyby Date	12:00:00 13/12/2030
Arrival Date	18:00:00 29/10/2031

Table 4: Mission dates of chosen solution

1.5 Mission analysis outputs

1.5.1 Heliocentric trajectory

The orbital elements of the 1st and 2nd leg of the heliocentric trajectory are described in Table 5 and are plotted in Figure 3.

	a	e	i	Ω	ω	θ_{dep}	θ_{arr}
Leg 1	149372383.24 km	0.278504	3.311924 °	81.516470 °	103.376511 °	1.605808 °	256.623489 °
Leg 2	152110170.64 km	0.244247	11.637907 °	261.516470 °	276.768937 °	263.231063 °	218.851190 °

Table 5: Orbital parameters of the transfer arcs

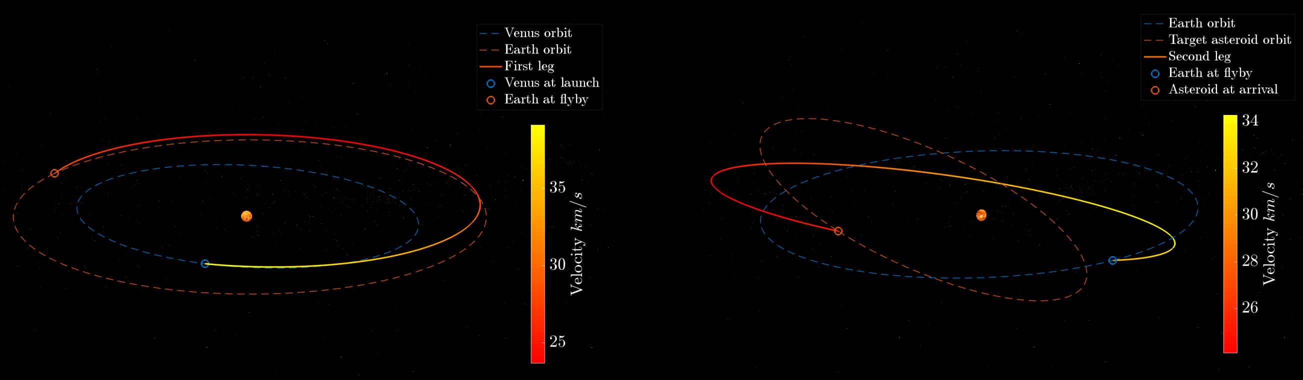


Fig. 3: First and second respectively

1.5.2 Flyby

The flyby has a periapsis of $r_p = 6671.59km$ with an altitude above Earth of $h_p = 300.59km$.

The duration of the flyby considering the SOI of Earth is 56.1975 hours.

The flyby provides a total $\Delta V_{fb} = 7.7130\,\mathrm{km\,s^{-1}}$ out of which $\Delta V_{ga} = 0.6040\frac{km}{s}$ comes from the powered maneuver. The proportion of gained to consumed ΔV is $\frac{\Delta V_{fb}}{\Delta V_{ga}} = 12.7701$.

The trajectory of the flyby is plotted in Figure 4.

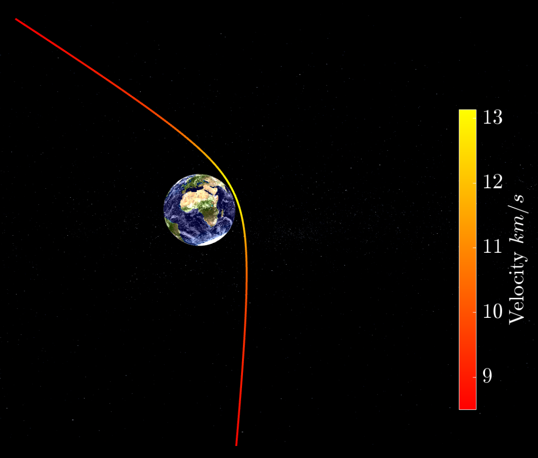


Fig. 4: Flyby trajectory

1.5.3 Cost of the mission

The mission has a total cost of $\Delta V_{tot} = 11.9612\frac{km}{s}$. With a departure $\Delta V_{dep} = 4.5201\frac{km}{s}$, gravity assist $\Delta V_{ga} = 0.6040\frac{km}{s}$ and arrival $\Delta V_{arr} = 6.8371\frac{km}{s}$.

2 Assignment 2

2.1 Introduction

The PoliMi Space Agency wants to perform a preliminary study of a Planetary Explorer Mission with the aim of Earth's observation. In this report, ground track estimation, orbit propagation, and analysis of perturbations are carried out.
The following research takes into account the effects of air drag and J2 on the orbit and defines the ground track of both the unperturbed and perturbed orbits. Any other kind of perturbation is assumed to be null.
The propagation of the perturbed orbit is obtained through two distinct methods:

- Direct time integration of the Cartesian equations of motion in the ECI frame.
- Integration of the Gauss equations for the Keplerian elements in the TNH frame.

2.1.1 Initial Data

The initial parameters used to characterize the Earth centered orbit are indicated in Table 6, along with the additional data in Table 7.

a $[km]$	e $[-]$	i $[deg]$	Ω $[deg]$	ω $[deg]$	θ $[deg]$
16 882	0.6026	19.3243	277.5732	162.3587	270

Table 6: Orbital parameters

$k:m$ $[-]$	cD $[-]$	A/M $[m^2/kg]$	μ_E $[km^3/s^2]$	R_E $[km]$	$J2$ $[-]$	ω_E $[rad/s]$
13/3	2.1	0.0569	398600	6371.01	$1.082627 \cdot 10^{-3}$	$7.2916 \cdot 10^{-5}$

Table 7: Useful data

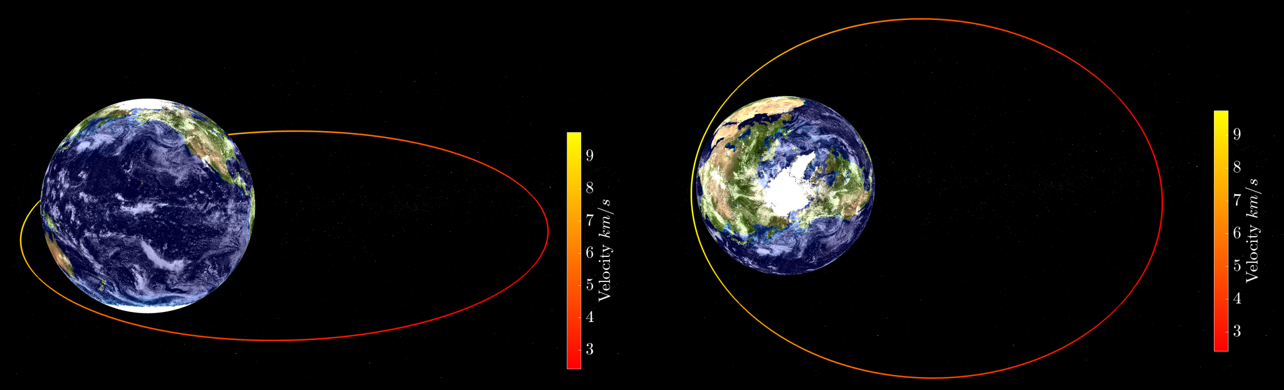
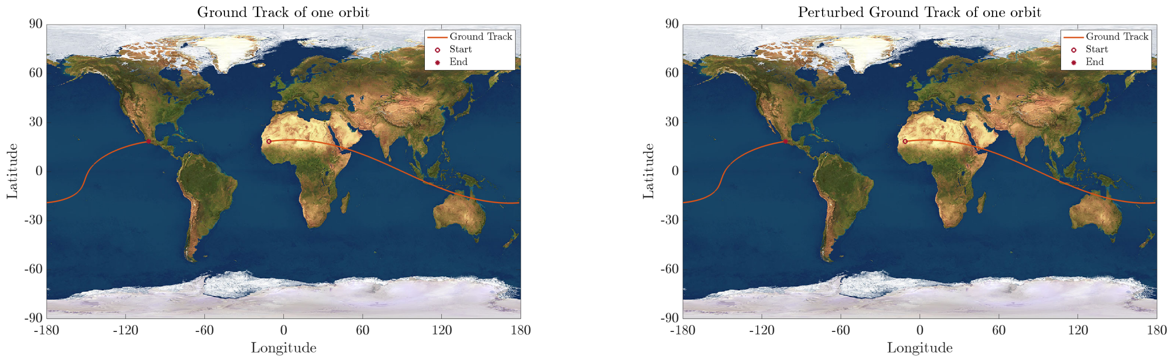


Fig. 5: Initial Unperturbed Orbit

2.2 Ground Track

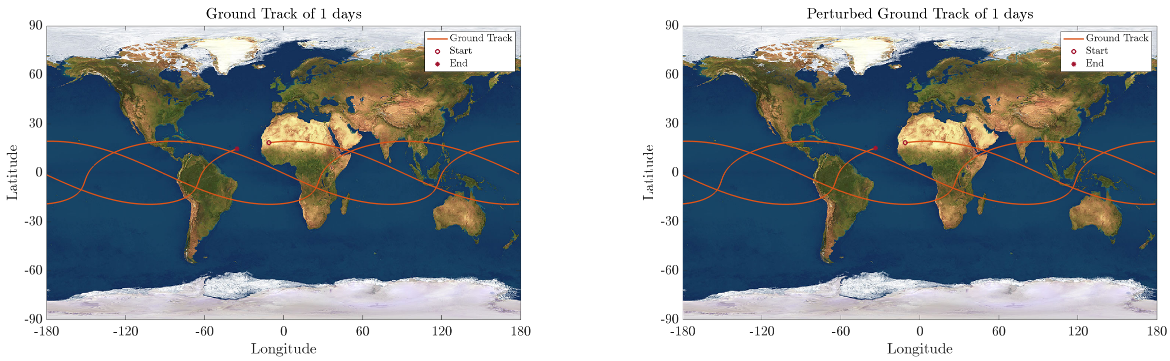
In the following section, the ground track for both the unperturbed and perturbed orbits is shown during one period, one day, and ten days.
The effect of the perturbations on the orbit in this time frame is barely visible and has an effect only in longer time windows.



(a) Unperturbed orbit

(b) Perturbed orbit

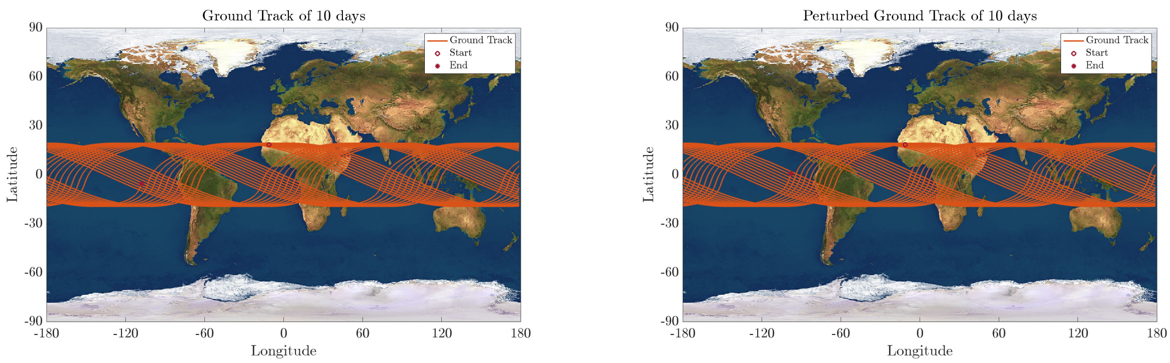
Fig. 6: Ground track of one period



(a) Unperturbed orbit

(b) Perturbed orbit

Fig. 7: Ground track of one day



(a) Unperturbed orbit

(b) Perturbed orbit

Fig. 8: Ground track of ten days

2.2.1 Repeating Ground Track

To achieve a repeating ground track in the unperturbed case, it is sufficient to change the semi-major axis of the initial orbit through the equation:

$$a_{rep} = \sqrt[3]{\mu_E \left(\frac{m}{k\omega_E}\right)^2} \tag{2}$$

where:

- m is the number of Earth's rotations for each repetition of the ground track;

- k is the number of satellite's revolutions for each repetition of the ground track;

The new semi-major axis depends only on the ratio $k : m$, as the Earth's rotation rate ω_E and the gravitational parameter μ_E are considered constant.
Using the new semi-major axis, it can be seen that, after 3 days, the ground track starts repeating itself.

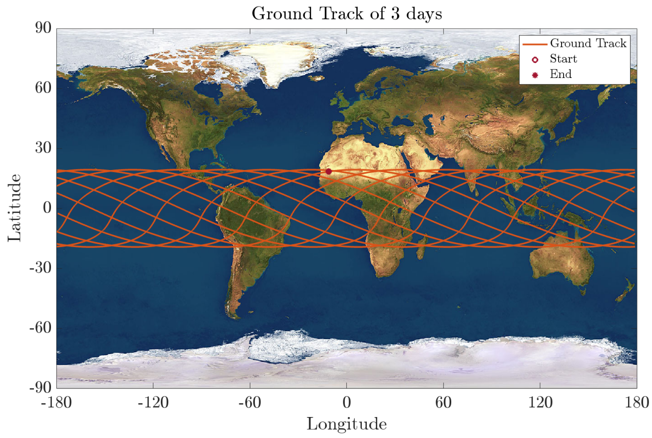


Fig. 9: Unperturbed repeating ground track for 3 days

2.2.2 Perturbed Repeating Ground Track

In the perturbed case, it is possible to calculate a new semi-major axis, as done in the section above, but the nodal regression, the perigee precession, and the variations of the mean anomaly of the orbit, all caused by the secular effects of the J2 perturbation, must be taken into account.
This is done through the following formula:

$$\frac{T_{S/C}}{T_{Earth}} = \frac{m}{k} = \frac{\omega_E - \dot{\Omega}}{n + \dot{\omega} + \dot{M}_0} \tag{3}$$

This equation has to be solved with respect to a_{rep} as an initial guess to find the semi-major axis for which the ground track will repeat. Equation 3 only takes account of secular variations of Keplerian elements, and the propagation with this new semi-major axis in reality continues to not provide an exactly repeating ground track. This is because in the equation, the eccentricity and the inclination are assumed to be constant, even though they are known to vary due to air drag and short-term oscillations of J2.
In Figure 10 below, it is clearly visible how the perturbed orbit is unable to generate a repeating ground track and how much it varies in a 15-day window.

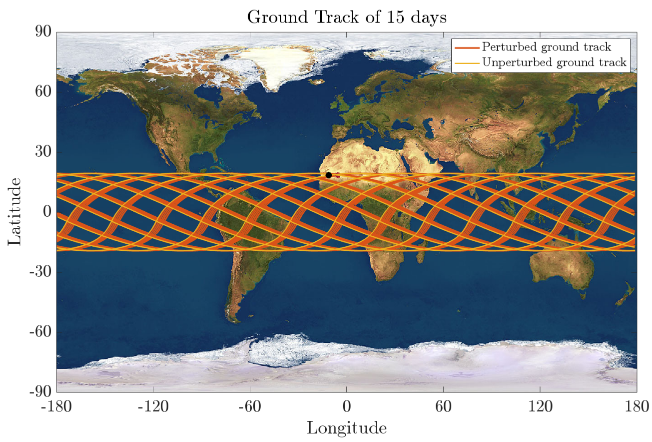


Fig. 10: Perturbed repeating ground track for 15 days

2.3 Orbit Propagation

An in-depth study of the evolution of the orbit and its Keplerian elements in time has been conducted using two distinct methods:

- The numerical integration of Cartesian equations of motion;
- The numerical integration of Gauss' planetary equations for non-conservative perturbations.

Both integrations were performed using the Matlab function *ode113.m*, with a relative tolerance of 10^{-12} and an absolute tolerance of 10^{-13}. The propagation has been performed from the starting position of the spacecraft for a total of 300 orbital periods, or roughly 76 days.

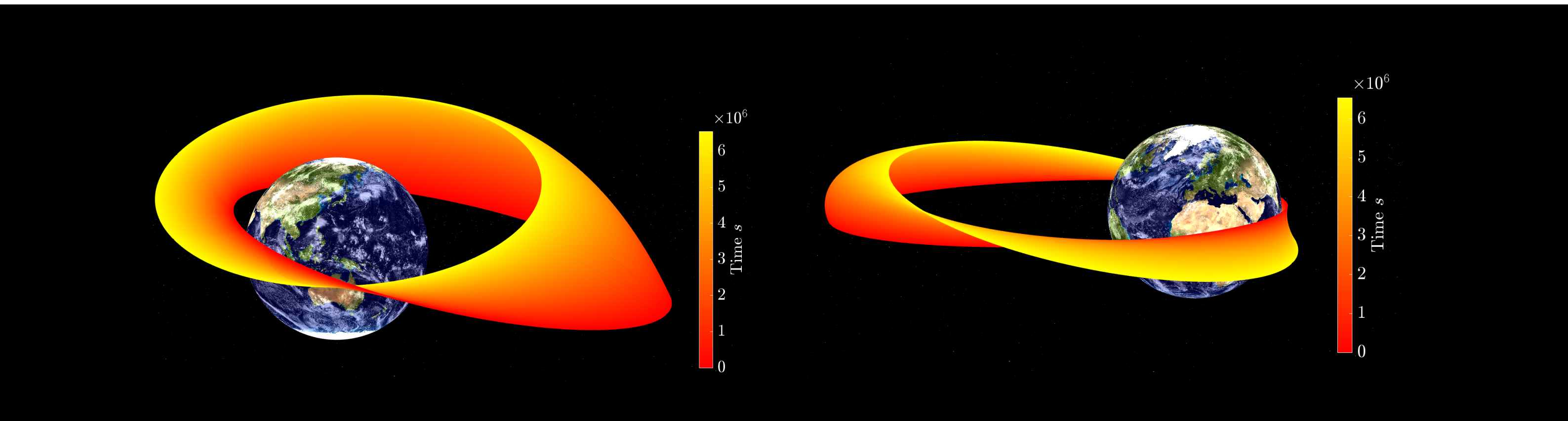
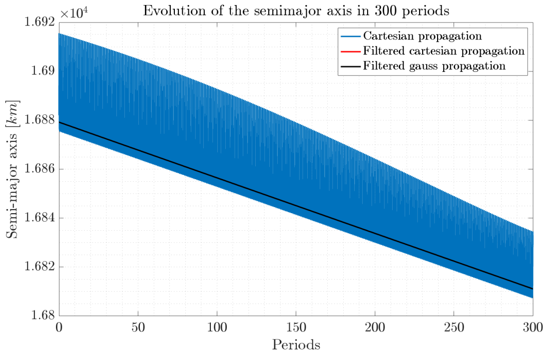


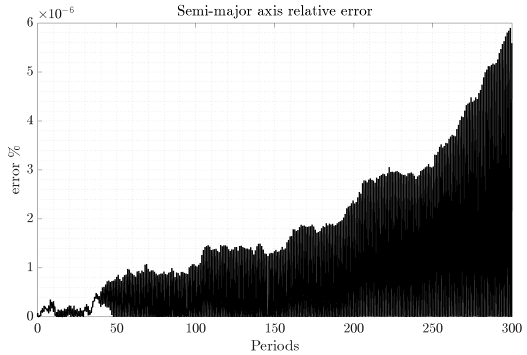
Fig. 11: Perturbed Orbit - 300 periods

2.3.1 Evolution of Keplerian Elements

The evolution of the Keplerian elements is shown in the figures below. Each figure contains the unfiltered Keplerian parameter obtained through the integration of Cartesian equations of motion and the filtered elements obtained using both methods.
Each representation of Keplerian elements is coupled with a graph showing the relative percentage error between the results obtained using the Cartesian and Gauss equations at each time step, with respect to the Cartesian equations. It is possible to observe that the curves of the propagated elements are almost superimposed, and the percentage error never reaches an order higher than 10^{-6}, which can be considered acceptable.

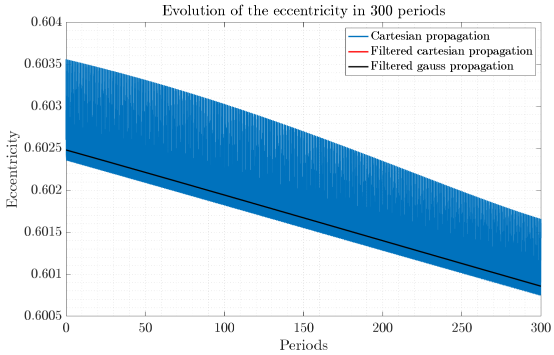


(a) Variation of the semi-major axis

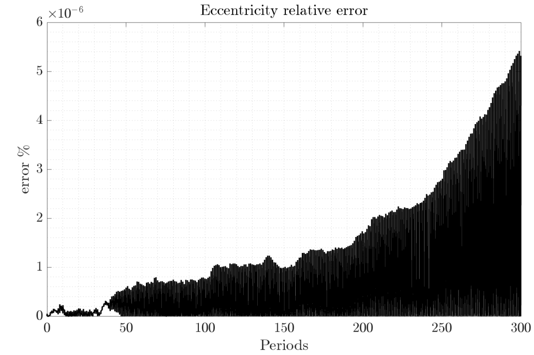


(b) Relative Error

Fig. 12: Semi-major axis

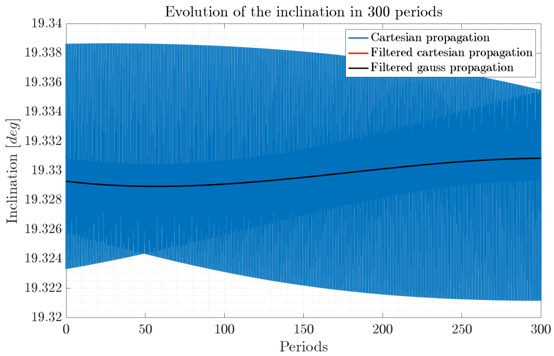


(a) Variation of the eccentricity

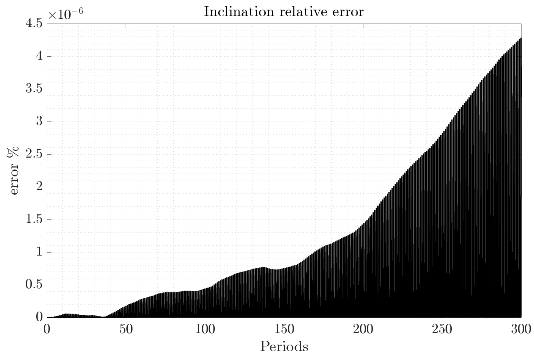


(b) Relative Error

Fig. 13: Eccentricity

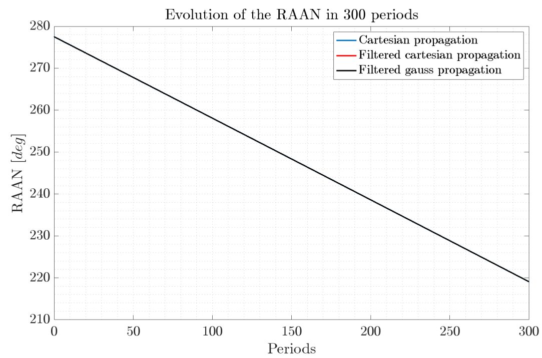


(a) Variation of the inclination

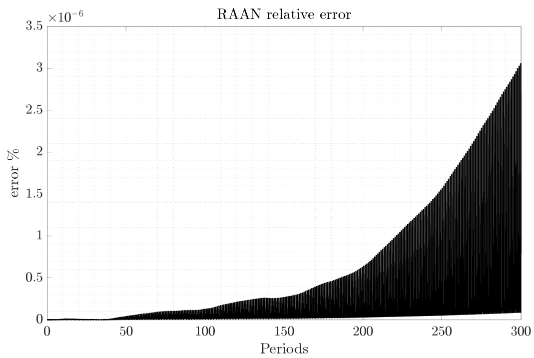


(b) Inclination

Fig. 14: Inclination

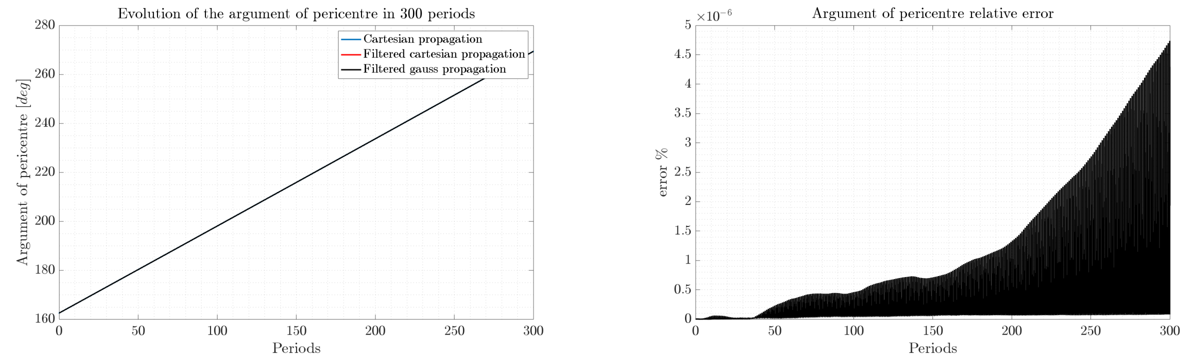


(a) Variation of the RAAN



(b) Relative Error

Fig. 15: RAAN

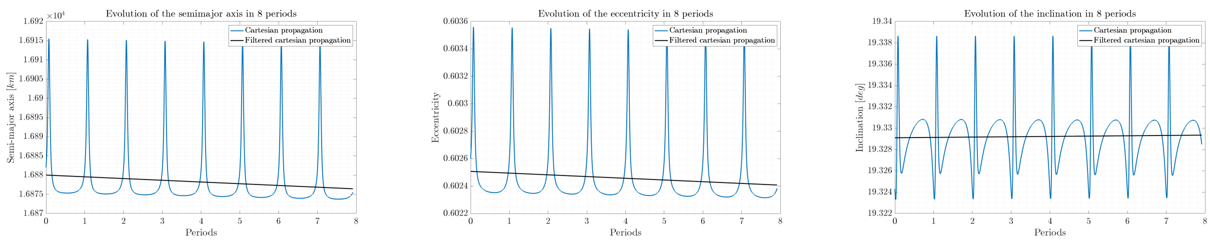


(a) Variation of the argument of pericentre

(b) Relative Error

Fig. 16: Argument of pericentre

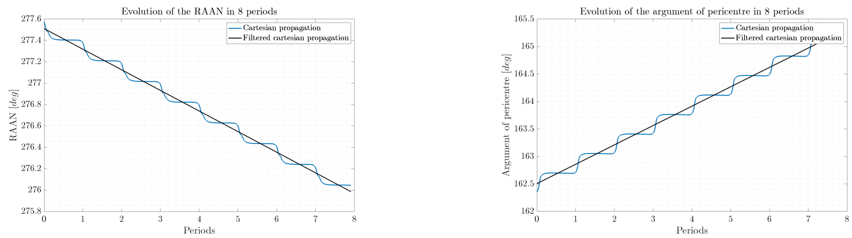
The filtering of the Keplerian elements has been performed using the Matlab functions *plyfit* and *polyval*. In Figure 18, the difference between the short-term oscillations of the perturbations and the secular variations is clearly visible.



(a) Variation of the semi-major axis

(b) Variation of the eccentricity

(c) Variation of the inclination



(d) Variation of the RAAN

(e) Variation of ω

Fig. 17: Short-term oscillations of the keplerian elements

2.4 Real Satellite Comparison

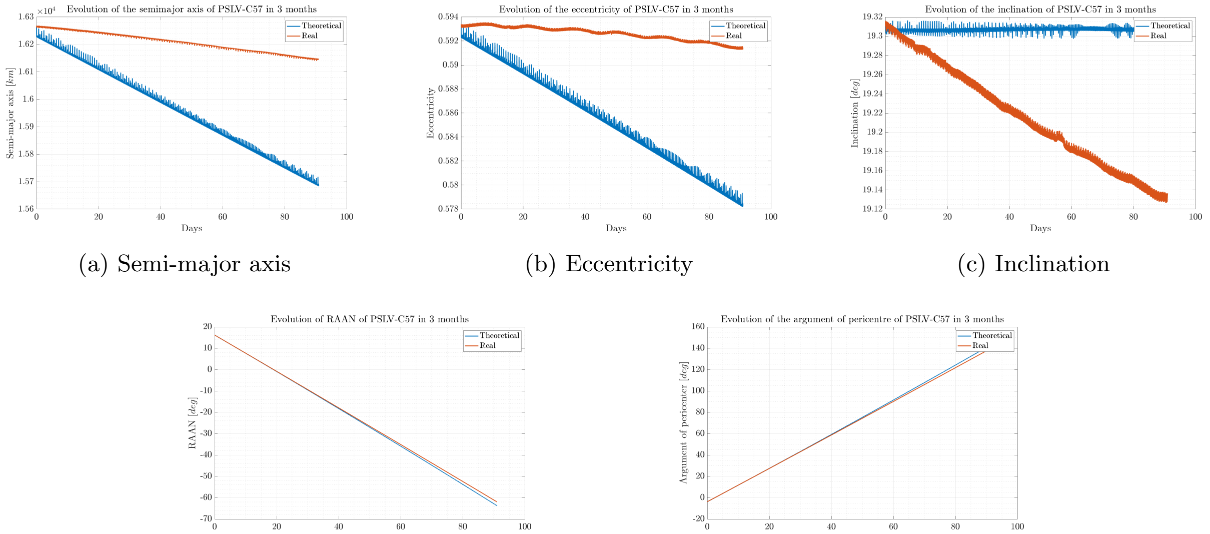
To verify the accuracy of the model used for this assignment, a comparison between numerical integration and real data is made.The orbit of the rocket body of the PSLV-C57 launcher, departed on the 2^{nd} of September 2023, was used as a reference due to the similarity of its orbit with the specific case of this assignment.

a $[km]$	e $[-]$	i $[deg]$	Ω $[deg]$	ω $[deg]$	θ $[deg]$
16 262	0.5934	19.3154	16.1782	356.1966	21.0312

Table 8: Orbital parameters of the PSLV-C57

Its known orbital parameters are extracted from the TLEs found on Space-Track [2] and confronted with the orbital parameters obtained from the Gauss propagation method. The time window analysed spans from the 9/9/2023 to the 9/12/2023 and the initial parameters used for the propagation are the same as in the real case. As shown in the figures below, the model does not suit reality. Other perturbations, such as the third body perturbation, solar radiation pressure, and magnetic field, not accounted for in this assignment, must be considered to get closer to the real behavior of the spacecraft.

Another big factor that could affect the error observed between the real and modeled behavior is the CD and mass over area ratio. These factors have been assumed and could greatly vary from the real spacecraft thus creating a significant error.
For this reasons, the real and theoretical graphs for eccentricity, inclination, and the semi-major axis differ considerably. The Ω and ω graphs vary slightly as the main effect on these parameters is the J2 perturbation, which is modeled.



(a) Semi-major axis (b) Eccentricity (c) Inclination

(d) RAAN (e) Argument of the pericentre

Fig. 18: Comparison between the real orbit and the propagated one

2.5 Conclusion

The modeling of the Planetary Explorer Mission highlights the difficulties faced when taking into account many different phenomena for control and prediction purposes. The Keplerian models are to be integrated with a variety of perturbations present in the real world if the results need to be accurate. Because of the complexity of the problem, an analytical approach is not suitable, and an accurate prediction of the trajectory of the spacecraft can be performed only with numerical methods, such as the previously mentioned Gauss and Cartesian propagation.
To have the best representation of reality, it is necessary to implement and consider all the possible disturbances that one orbit can be affected by.
In the end, it's important not to forget that the accuracy of the final results is limited by all the mistakes arising during the analysis, both numerical and modelling ones.

References

[1] List of earth flybys. https://en.wikipedia.org/wiki/List_of_Earth_flybys. Accessed: 03/01/2024.

[2] Space-track. https://www.space-track.org/.