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DEPARTMENT OF AEROSPACE ENGINEERING

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Launch Systems

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Professors: Filippo Maggi, Stefania Carlotti

Flipped Project - Optimal Staging

Person Code	Surname	Name
10726898	Juvara	Matteo Giovanni
10728962	Modelli	Simone
10973642	Separovic	Tomislav Marko

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Acronyms

GLOM Gross Lift-Off Mass. i, 1–4

TSTO Two-Stage-To-Orbit. 1–4

1 Introduction

In this project, an optimization method for the computation of the Gross Lift-Off Mass (GLOM) of a launcher will be implemented and analysed. In particular the method optimizes the GLOM required to bring a 5000 kg payload to an orbital speed of $\Delta V = 10\ km/s$. This is initially performed for a three stage launcher with fixed parameters and then repeated for a Two-Stage-To-Orbit (TSTO) vehicle. The gravitational acceleration g_0 was given by the problem as $g_0 = 9.80665\ m/s^2$.

Stage	1	2	3
Specific Impulse I_{sp} [s]	250	340	410
Structural Index ε_s [-]	0.07	0.11	0.15

Table 1: Parameters for the three-stage launcher

The requests of the problem are reported below:

1. Compute the optimal GLOM for a three-stage launcher, using the parameters reported in Table 1.
2. Compute the GLOM and the sensitivity of this parameter with respect to a 10-15% variation of the structural mass indexes. Analyse and discuss the results.
3. Provide reliable assumptions for ε_s and I_{sp} for the stages of a TSTO launcher and compute it's optimal GLOM. Analyse and discuss the results.
4. For the proposed TSTO vehicle, assume reliable gravity and drag losses values, considering that the provided ΔV budget already takes into consideration these effects. Use the **robust method** to compute the minimum GLOM, assuming the coefficient $\beta = 1$, which implies that all the losses occur during the first stage. Analyse and discuss the results.

2 Methods

2.1 Optimal Gross Lift-Off Mass Computation

To compute the optimal GLOM for a generic n-stage launcher, applying a constraint on the required ΔV and $m_{P/L}$, the minimization function J is defined as [2]:

$$J = ln\left(\frac{m_0}{m_{P/L}}\right) + \lambda \cdot \left(\Delta V - \sum_{i=1}^{n} c_i \cdot ln(n_i)\right) = ln\left(\frac{(1-\varepsilon_{s,i})\cdot n_i}{1-\varepsilon_{s,i}\cdot n_i}\right) + \lambda \cdot \left(\Delta V - \sum_{i=1}^{n} c_i \cdot ln(n_i)\right) \tag{1}$$

Where m_0 is the initial mass, c_i are the exhaust velocities of each stage computed as $c_i = I_{sp} \cdot g_0$, $1/n_i$ are the mass ratios for each stage, and λ is the Lagrangian multiplier. To minimize this function, the derivatives with respect to n_i and λ have to be imposed to be equal to zero. This creates two further equations, shown below, which are then used to compute the optimal GLOM.

$$\Delta V = \sum_{i=1}^{n} c_i \cdot ln(n_i) \tag{2}$$

$$n_i = \frac{c_i\lambda - 1}{\varepsilon_{s,i}\, c_i\, \lambda} \tag{3}$$

Equation (3) is substituted inside Equation (2), which is then solved for λ using Matlab's `fsolve` solver. Once the Lagrangian multiplier is computed the mass ratios of each stage can be computed using Equation (3). At this point the total masses as well as the propellant and structural masses of each stage can be computed using the equations reported below. The GLOM is simply the sum of the payload mass and the mass of every stage.

$$m_i = \frac{n_i - 1}{1 - n_i\, \varepsilon_{s,i}} \cdot (m_{P/L} + \sum_{j=i+1}^{n} m_j) \tag{4}$$

$$m_{s,i} = m_i \cdot \varepsilon_{s,i}$$

$$m_{p,i} = m_i \cdot (1 - \varepsilon_{s,i})$$

2.2 Robust Method

The robust method is an alternative method to compute the optimal GLOM of a launcher, which is usually used for TSTO vehicles. This method splits both the ΔV_{ideal} and the ΔV_{loss}, due to drag and gravity, between the two stages using the proportional coefficients α and β.

$$\Delta V_{S1} = \alpha \, \Delta V_{ideal} + \beta \, \Delta V_{loss} \quad (5)$$

$$\Delta V_{S2} = (1-\alpha) \, \Delta V_{ideal} + (1-\beta) \, \Delta V_{loss} \quad (6)$$

In particular the ΔV required by each stage can be computed using Equation (5) and Equation (6). Once the ΔV are computed, the masses of each stage can be computed by manipulating the Tsiolkovsky rocket equation, as seen in Equation (7) and Equation (8) .

$$m_1 = (m_{P/L} + m_2) \cdot \frac{e^{\Delta V_1 / I_{sp} \cdot g_0} - 1}{1 - \varepsilon_{s,1} \cdot e^{\Delta V_1 / I_{sp} \cdot g_0}} \quad (7)$$

$$m_2 = m_{P/L} \cdot \frac{e^{\Delta V_2 / I_{sp} \cdot g_0} - 1}{1 - \varepsilon_{s,2} \cdot e^{\Delta V_2 / I_{sp} \cdot g_0}} \quad (8)$$

3 Results

3.1 Point 1

The equations seen in Section 2.1 have been implemented in a Matlab function, `calculateGLOM`, and validated using launcher parameters with known solutions. The results of Point 1 were then computed using the `calculateGLOM` function and are reported in Table 2.

	m_1	m_2	m_3	GLOM
Mass [kg]	107417	88049	19955	220420

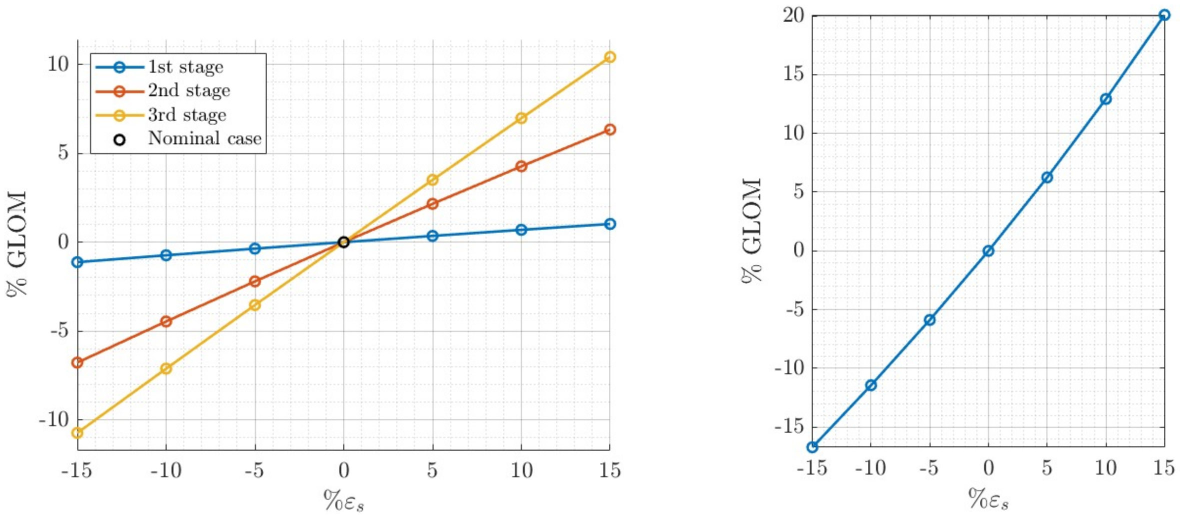
Table 2: Optimal masses for a three-stage launcher

3.2 Point 2

To compute the variation of the GLOM and its sensitivity with respect to variations of ε_s for each stage a Matlab function was created. This function utilizes the previously mentioned `calculateGLOM` function to compute a matrix of size $\prod_{i=1}^{N_{stages}} n_i$, where n is the chosen discretization and N_{stages} is the number of stages, containing all possible GLOM computed using the varied ε_s. This function computes the GLOM by calling `calculateGLOM` in multiple nested `for` loops, as many as the number of stages of the launcher. For every iteration of each `for` loop the corresponding stage ε_s is varied accordingly to the chosen discretization and ε_s percentage variation limits.

The parameters of the launcher were kept the same as the ones used in Section 3.1, while the ε_s variation limits were set to $\pm 15\%$ and the discretization n was set to 7, to assure a 5% interval between one point and another. After the computation of the $n \times n \times n$ GLOM matrix, the most representative values for our purposes can be extracted. First, it was decided to analyse the sensitivity of the GLOM for a varying ε_s of each stage, while keeping the ε_s of the other stages fixed. This was done to compare the effects of variations of each stage's ε_s on the GLOM.

This analysis is made through the use of Figure 1a. From the aforementioned matrix, three vectors are extracted, containing the variation of ε_s corresponding to each stage, while fixing the ε_s of the other stages as the values seen in Table 1.



(a) Variation percentage of the GLOM for each $\varepsilon_{s,i}$ variation

(b) Variation percentage of the GLOM for a fixed ε_s variation percentage for all stages

It can be clearly seen that the GLOM variation percentage is almost linear with respect to the ε_s variation for each single stage. In particular for the same ε_s variation percentage the GLOM is affected more when varying for higher stages. From this it can be concluded that the GLOM is more sensitive to changes of structural mass in higher stages. Varying each ε_s of the same amount simultaneously may represent another interesting analysis. This vector has been extracted from the GLOM matrix and it's results are presented in Figure 1b.

3.3 Point 3

The values of I_{sp} and ε_s for a TSTO Launcher were assumed using the same values of the Atlas V 401 [1] and have been reported in Table 3.

Stage	1	2
Specific Impulse I_{sp} [s]	311.3	450.5
Structural Index ε_s [-]	0.07411	0.1112

Table 3: Parameters for the TSTO launcher

Using the optimal staging equations shown in Section 2.1, the results shown in Table 5 are obtained. This values are considered feasible, as the GLOM obtained is lower than the actual mass of the Atlas V 401, considering that the payload mass is almost halved and that the actual required ΔV might be higher.

	m_1	m_2	GLOM
Mass [kg]	106398	31131	142529

Table 4: Optimal masses for a TSTO launcher

3.4 Point 4

To calculate the GLOM through the robust method a 20 % ΔV loss is assumed, leading to ΔV values of $\Delta V_{ideal} = 8000\ m/s$ and $\Delta V_{loss} = 2000\ m/s$. By using the I_{sp} and ε_s detailed in Section 3.3 for a TSTO, the m_1, m_2 and GLOM of the launcher are calculated for α spanning from 0.1 to 0.9, using the equations reported in Section 2.2. The results are presented in the following figure.

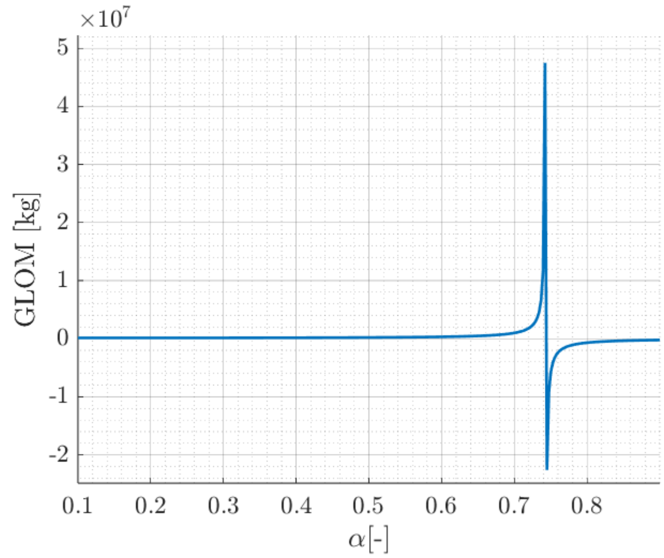


Fig. 2: GLOM for varying α

It is evident that for high values of α the result doesn't make any physical sense as the GLOM experiences a very sharp peak and then assumes negative values. When the chosen alpha value is too high, the required ΔV_{S1} is higher than what is possibly achievable for a first stage with the selected I_{sp} and ε_s. The maximum ΔV possible for any stage can be calculated with the equation $\Delta V_{max} = I_{sp} \cdot g_0 \cdot ln(\varepsilon^{-1})$. In particular for the first stage $\Delta V_{max} = 7944\ m/s$, which is obtained with $\alpha = 0.743$. For this reason the computation was performed again limiting α up to 0.6, enabling us to analyse and present valid results.

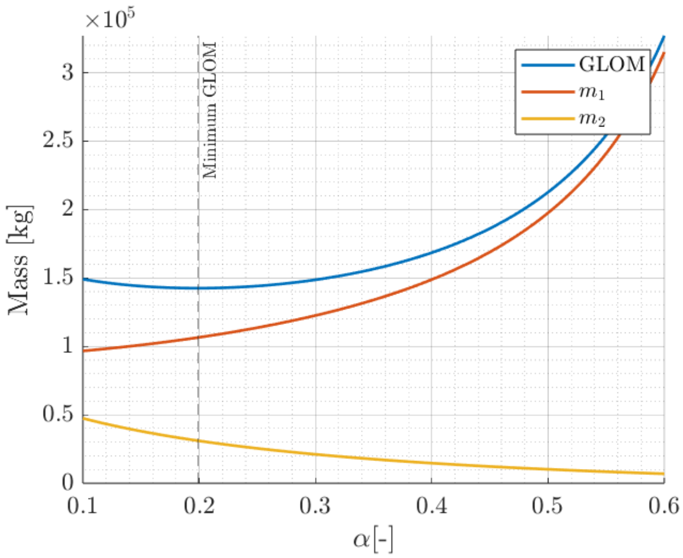


Fig. 3: GLOM, m_1 and m_2 for varying α

It can be noted that the minimum GLOM is obtained with an α value of $\alpha = 0.1987$. As expected, the results are very similar to the ones obtained through the optimal staging computation seen in Section 3.3. The masses of the singles stages and the GLOM are reported below.

	m_1	m_2	GLOM
Mass [kg]	106433	31096	142529

Table 5: Optimal masses for a TSTO launcher

References

[1] United Launch Alliance. Atlas V Launch Services User's Guide, 2010.

[2] Stefania Carlotti. Staging. Launch Systems Course 2024/25, Politecnico di Milano.