Flight control system design for wind gust rejection based on an Unknown Input Observer and a Simple Adaptive Controller

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Flight control system design for wind gust rejection based on an Unknown Input Observer and a Simple Adaptive Controller

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Abstract— **This paper presents a design methodology for the rejection of wind gusts during flight. The proposed application is based on the Simple Adaptive Controller (SAC) methodology, but differs from other SAC applications in the sense that the reference model utilised is an Unknown Input Observer (UIO). As a consequence, the adaptive controller is fed by an estimate of the wind gust and will only act in the event of such a disturbance occurring, retaining nominal performance otherwise. Although stability guarantees of the flight control system proposed follows closely previous work, some differences are found and commented upon in this work. Success of the proposed scheme is demonstrated via simulation using a linear model of the MuPAL-α experimental aircraft of the Japanese Aerospace Exploration Agency (JAXA), and the results are compared with an implementation of the original SAC formulation.**

I. NOMENCLATURE

q	=	pitch rate
u_x	=	velocity along the x axis
w_z	=	velocity along the z axis
δ_e	=	elevator angle
θ	=	pitch attitude

II. INTRODUCTION

Modern flight control systems have contributed to improve the performance, safety and handling qualities of aircraft. However, these controllers are designed under nominal flight conditions (no faults or failures) and no external disturbances from the environment and typically cannot adapt themselves to adverse (or unaccounted) situations such as failure of the actuators and/or sensors, structural damage or exogenous excitations such as wind gust. This paper will be concerned with the latter situation, although the ideas presented here can be translated into a fault tolerant control scenario with little additional work.

Several studies have been performed in order to improve wind gust rejection using different control techniques, including H_∞ control [6] and adaptive control [15], [8], [22], [3]. A robust controller must guarantee performance of the closed-loop system even under the influence of disturbances and/or model uncertainties. In an aircraft system there are disturbances such as turbulence or wind gusts, and uncertainties such as those associated with the pilot and his/her interaction with the vehicle.

Adaptive control allows the controller to automatically adjust itself to changes in the plant or exogenous disturbances, and it has been studied as early as the 1950's [2]. Recently, $\mathcal{L}_1$ adaptive control has been proposed, and although its superiority has been challenged [16], a growing body of work has been presented since its introduction [7] (see for example [5]). A particular approach to adaptive control is so-called Simple Adaptive Control (SAC), which essentially consists of adaptive feedforward and feedback loops [10]. SAC facilitates the use of low order reference models, may be applied to MIMO plants and only the system's output is used (as opposed to the plant's states in most applications).

Flight tests using the MuPal-α experimental aircraft have successfully demonstrated the capabilities of SAC to improve wind gust rejection [18] capabilities. Also, the capability of SAC to improve handling of faults (and failures) has been demonstrated [3]. The capabilities of H_∞ robust control have also been tested in this aircraft, and this technique has proven to be successful in reducing the adverse effects of wind gusts, different types of faults and model uncertainties [19].

Nonetheless, adaptive techniques tend to change the nominal performance of the aircraft even in the case that no faults or external disturbances are present, and this may be undesirable in manned aircraft since this may impose additional and unexpected workload on the pilot. For this reason, this paper proposes a control structure in which SAC is activated only in the event that a wind gust disturbance is present; hence the SAC will not affect the nominal dynamics of the closed-loop system (in our case the nominal controller will be a PID controller). In order to achieve this goal, the disturbance must be detected via some kind of observer and then fed-back to the SAC.

In terms of disturbance estimation, developments in the area of fault detection and isolation area can be used, particularly in this case we will use the proposal in [21]. More generally, an Unknown Input Observer (UIO) may be used in oder to estimate the disturbance (the interested reader may consult [4]). The proposed technique will use a UIO, which has a general Luenberger structure (see [13]) and provides large-signal $\mathcal{L}_2$ performance guarantees, to provide an estimate of the wind gust. The observer structure facilitates its integration with the SAC and provides stability guarantees (ultimate boundedness to be more exact) that can be proven using existing results [11], [3]. The main difference in formulation lies in the fact that the UIO, which will be assume to be the reference model, is coupled with the plant through its output.

The paper is organised as follows: first, we will revisit some known studies; this is followed by the presentation of some preliminary results, including the basics of the SAC and the UIO used; the main result are then stated, where an outline of the proof of stability is given; finally, the effectiveness of the proposed SAC is shown via simulation using JAXA's MuPal-α model and compared with the original SAC formulation.

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III. Preliminaries

A. Simple Adaptive Control

Consider the plant described by:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \qquad y(t) = Cx(t) \tag{1}$$

where the signals $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$ and $f(t) \in \mathbb{R}^f$ are the plant's states, inputs, outputs, and external disturbances respectively. From now on we will drop the dependence on the time for compactness. The matrices A, B, C and F are constant, real and of suitable dimensions. Note that the input u and the output y have the same dimensions, hence the SAC technique can only be applied to square plants. The value of the disturbance vector f is unknown but bounded.

Consider a reference model given by:

$$\begin{array}{rcl} \dot{x}_m & = & A_m x_m + B_m u_m \\ y_m & = & C_m x_m \end{array} \tag{2}$$

where the signals $x_m(t) \in \mathbb{R}^{n_m}$, $u_m(t) \in \mathbb{R}^m$, and $y_m(t) \in \mathbb{R}^m$ are the states, inputs and outputs of the reference model respectively. The matrices A_m, B_m and C_m are constant, real and of suitable dimensions. The main objective is to design an adaptive controller such that the closed-loop system is stable and the plant output y tracks the reference model output y_m.

Now assume that there exists an ideal trajectory $x^* \in \mathbb{R}^n$ that is produced by an ideal control signal defined as

$$u^* = \tilde{k}_u u_m + \tilde{k}_{xm} x_m \tag{3}$$

where $\tilde{k}_u$ and $\tilde{k}_{xm}$ are constant gains.

Observe that the goal is to guarantee that there exist ideal trajectories such that if the plant stays on, or close, to these trajectories, then the plant will track the reference model. To achieve this goal we need to guarantee that the ideal output is equal to the reference output, which means that

$$y^* = y_m = Cx^* = C_m x_m \tag{4}$$

Define now the ideal trajectory as

$$x^* = X_1 x_m + X_2 u_m \tag{5}$$

Combining (4) and (5), we obtain

$$CX_1 = C_m \qquad CX_2 = 0 \tag{6}$$

An adaptive control law is proposed, where the input u can be calculated as:

$$u = k_u(t)u_m + k_{xm}(t)x_m + k_e(t)e_y \tag{7}$$

where the output tracking error is defined as $e_y = y - y_m$

Observe that both $k_u(t)$ and $k_{xm}(t)$ are feedforward gains, while $k_e(t)$ is a feedback gain (Figure 1 depicts the SAC interconnection). The main idea of SAC is to adapt these gains in an online manner in order to guarantee tracking of the reference model even in the event of uncertainties and disturbances. For compactness, define matrix k as

$$k = [k_e, k_{xm}, k_u] \tag{8}$$

The integral adaptation rule used to calculate gain k is:

$$\dot{k} = -e_y r^T \Gamma - \sigma k \tag{9}$$

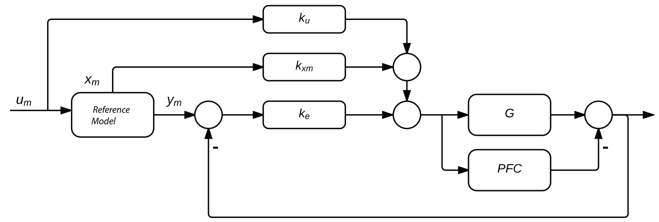

Fig. 1. Original SAC control architecture

$$r(t) = [e_y^T, x_m^T, u_m^T]^T \tag{10}$$

where $\Gamma \in \mathbb{R}^{(n_m+2m)\times(n_m+2m)}$ is a constant weighting matrix. The term $-\sigma k$ is used to avoid divergence (and guarantee a return to zero when $e_y = 0$) of the integral gains. A complete stability proof of the SAC approach may be found in [3], [11].

A necessary condition for existence of a SAC is for the plant to be Almost Strictly Positive Real (ASPR), i.e. there exists a constant gain K such that

$$G_{cl}(s) = [I - G_a(s)K]^{-1} G_a(s)$$

is asymptotically stable. This may be difficult to guarantee since most real systems are not ASPR, hence some techniques have been proposed in order to guarantee this condition (see for example the Parallel Feedforward Compensator (PFC)[8] and the feed forward design approach in [3]). The basic idea to circumvent this difficulty is to add some dynamics to the original plant such that the augmented system G_a is guaranteed ASPR. Hence, under some mild conditions, we can guarantee the ASPR condition for some augmented system, and apply SAC to the new system. In this paper we will use the PFC approach to ensure this necessary condition.

B. Unknown Input Observers

Consider a plant described by

$$\begin{array}{rcl} \dot{x} & = & Ax + Bu + Ff + N\delta \\ y & = & Cx + Gf + D\delta \end{array} \tag{11}$$

where x are the system states, u are the (known) control inputs, and f and δ are disturbance vectors. The system's measured outputs are given by y. The matrices A, B, F, N, C, G and D are all constant, real and of suitable dimensions.

The main objective is to design an (LTI) observer to estimate the vector f (i.e. an unknown input signal), while attenuating the effect of other disturbances represented by δ (for example measurement noise). To achieve this, a residual signal is constructed in such a way that it captures any inconsistencies between the plant and observer output signals produced by f, while being insensitive to other disturbances δ. The observer used has a state space representation given by:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + Bu + L(C\hat{x} - y) \\ \hat{y} & = & C\hat{x} \\ \tilde{f} & = & H(y - \hat{y}) \end{array} \tag{12}$$

where L is the observer gain and H is a matrix that maps output error to an estimate of the unknown signal f; both of these matrices need to be calculated *a priori*. The following standard assumptions are made [21],[4]:

1) $G(s)$ is stable and has no non-minimum phase zeros
2) The pair (A, C) is observable
3) $rank(CF) = rank(F)$
4) Exogenous signal are bounded, i.e. $\|\delta\| < \alpha$, $\|f\| < \eta$ for all $t \geq 0$ and some $\alpha > 0$ and $\eta > 0$

We will use the formulation proposed in [21], where an ultimate boundedness condition is obtained such that for $\delta = 0$, $\|\tilde{f} - f\|^2 < \gamma\|f\|^2$ for all $\|e_x\|^2 > \eta/\psi$, where $e_x = x - \hat{x}$, γ is a positive constant that bounds the norm of the unknown input, estimation error $\|\tilde{f} - f\|$. The interested reader may find more information on UIO in [4], [9].

A solution is obtained if there exist unstructured matrices Y and H, a symmetric positive definite (s.p.d.) matrix P_1, and positive real scalars γ and ψ such that LMI (13) holds. The UIO can be constructed as in equation (12), where the observer gain is $L = P_1^{-1}Y$.

$$\begin{bmatrix} A'P_1 + P_1A + 2YC + \psi I & P_1F - C'H' + YG & C'H' \\ \star & -\gamma^2 I - G'H' - HG & G'H' \\ \star & \star & -I \end{bmatrix} < 0 \tag{13}$$

In order to reject any other disturbance, i.e. vector δ, the bounded real lemma is applied such that $\|He_y\|^2 < \mu\|\delta\|^2$ is guaranteed if there exists unstructured matrices Y and H, a s.p.d matrix P_1, and a positive real scalar μ such that LMI (14) holds.

$$\begin{bmatrix} A'P_1 + P_1A + YC + C'Y' & P_1N + YD & C'H' \\ \star & -\mu I & D'H' \\ \star & \star & -\mu I \end{bmatrix} < 0 \tag{14}$$

Finally, an optimisation problem is proposed, where the objective is minimising $\gamma + W_1\mu$ subject to (13) and (14). The term W_1 is a user defined weighting variable used to offset disturbance rejection properties and estimation properties of the UIO.

By applying this synthesis method it is possible to obtain a UIO that estimates the disturbance signal f, while rejecting other disturbances δ. This design will be used as the reference model of the proposed SAC.

IV. Main Results

The main objective is to design an adaptive controller such that the detrimental effects of disturbances are minimised. Consider also, that there exists a robust UIO such that an estimate of the disturbance is available. Assume that the dynamics of the UIO are optimal in the sense that they represent nominal system dynamics, i.e. the expected dynamics if disturbances were not present. Hence we define the UIO as the reference model of the SAC. Given the system (1) and the reference model (12), find a simple adaptive control law such that the error dynamics are ultimately bounded for large enough adaptive gains $k = [k_e, k_{um}]$ and the gains k are bounded.

The feedback gain of the (original) SAC is driven by the output error, which means that it will try to compensate system dynamics even during nominal (unperturbed) operation (for example, wind gusts are not present) and drive it to follow some desired model dynamics. Compensation of the nominal controller even in absence of perturbations may not always be desirable, so we propose to use the estimated value of the unknown signal of interest to drive the SAC.

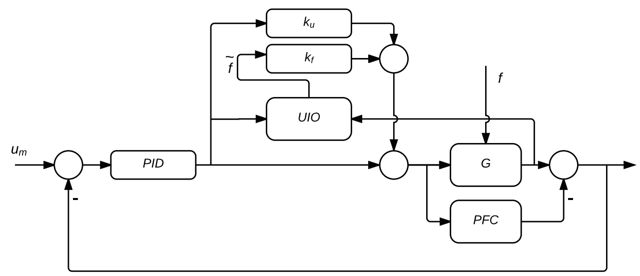

Fig. 2. Proposed structure of SAC

Consider the plant (1) where the term f represents the external disturbance that we want to explicitly reject, i.e. wind gusts in our case. Define the reference model as an UIO such that

$$\begin{aligned} \dot{x}_m &= (A + LC)x_m(t) + Bu_m(t) - Ly \\ y_m(t) &= Cx_m(t) \end{aligned} \tag{15}$$

Now consider the objective of tracking the ideal trajectory x^*, i.e. we want the tracking error $e_x^* = x^* - x$ to converge to zero. The proposed adaptive control law is given by

$$u = k_u u_m + k_f \tilde{f} \tag{16}$$

where $\tilde{f} = He_y = HCe_x$. Define $k = [k_u, \ k_f]$ and an update rule

$$\dot{k}(t) = -\tilde{f}r^T\Gamma - \sigma k \tag{17}$$

$$r(t) = [u_m^T, \ \tilde{f}^T]^T \tag{18}$$

After some calculations, the tracking error can be written as (see Appendix VII for details):

$$\dot{e}_x^* = (A + LC)e_x^* - B[k(t) - \tilde{k}(t)]r(t) - M(t) \tag{19}$$

where $\tilde{k}(t) = [I_{m\times m}, \tilde{k_f}]$ is a constant gain and

$$M(t) = (A + LC)X_2 u_m + Ff$$

It must be mentioned that in the proposed application, the feedforward gain k_{xm} is omitted because the reference model and the plant have the same A-matrix. In general, large control amplitude occurs during the transient response of the adaptation law, hence the use of an additional controller, in particular the use of derivative action, is needed in order to prevent this problem (see [1] for details). In this paper we implement a PID controller as our baseline control law, where Figure 2 depicts the proposed control structure.

Theorem 1: The states of the closed-loop interconnection of system (1), UIO (15), control signal (16) and adaptation law (17) are ultimately bounded if:

1) For given L and H designed as proposed in Section III-B, there exist positive matrices $P = P^T > 0$ and $Q > 0$, and control gain $\tilde{k}_f$ such that
$$(A + LC - B\tilde{k}_f HC)^T P + P(A + LC - B\tilde{k}_f HC) + Q < 0 \tag{20}$$
and $PB = (CH)^T$
2) $dim[y] \leq dim[u]$
3) External disturbances are bounded, i.e. $\|f(t)\| < \infty$ and $\|\delta\| < \infty$

4) $(A + LC)$ is Hurwitz

Even more, a quadratic Lyapunov function $V(e_x^*, k)$ such that its time derivative is negative ($\dot{V} < 0$) for large enough e_x^* and $\Delta_K = k - \tilde{k}$ can be constructed as

$$V(e_x^*, k) = e_x^{*T} P e_x^* + tr[(k - \tilde{k})^T \Gamma^{-1} (k - \tilde{k})] \quad (21)$$

□

Condition 1) in Theorem 1 is equivalent to requiring that the (augmented) plant is ASPR. An interesting fact is that the SAC implementation used does not need P or Q, hence the designer may opt to use other tools to provide ASPR guarantees for the plant, e.g. it is possible to use IQC [14] theory or other frequency techniques [12], [10], rather than solving explicitly (20).

Proof: The complete proof of Theorem 1 follows the results of [3], hence we will only outline the main idea. Considering plant (1), reference model (15), adaptive control law (16) and (17), and Lyapunov function (21), after some manipulations, it is possible to obtain the following inequality:

$$\begin{aligned}\dot{V}(e_x^*, k) < & -e_x^{*T} Q e_x^* - 2\tilde{f}^T \tilde{f}(\tilde{f}^T \Gamma_e \tilde{f}) - 2\tilde{f}^T \tilde{f}(u_m^T \Gamma_u u_m) \\ & -2\sigma tr[(k - \tilde{k})^T \Gamma^{-1} (k - \tilde{k})] - 2\sigma tr[(k - \tilde{k})^T \Gamma^{-1} \tilde{k}] \\ & -2 e_x^{*T} P B M(t)\end{aligned} \quad (22)$$

where Γ_e and Γ_u are positive diagonal matrices obtained by partitioning $\Gamma = diag\{\Gamma_u,\ \Gamma_e\}$. Noting that u_m and δ are bounded, it is possible to conclude that

$$\begin{aligned}\dot{V}(e_x^*, k) \quad < \quad & -\alpha_1 \|e_x^*\| - \alpha_2 \|e_x^*\|^4 - \alpha_3 \|e_x^*\|^2 \|u_m\|^2 \\ & -\alpha_4 \|k - \tilde{k}\|^2 + \alpha_5 \|k - \tilde{k}\| + \alpha_6 \|e_x^*\|\end{aligned} \quad (23)$$

where α_i for $i \in \{1, \ldots, 6\}$ are positive real constants. We can conclude that $\dot{V}(e_x^*, k)$ is negative for large enough $\|e_x^*\|$ and $\|k\|$ since the negative terms will dominate over the positive term. □

V. Numerical Example

The proposed SAC is tested on the (linear) longitudinal dynamics of JAXA's MuPAL-α experimental aircraft. The system is linearised at a levelled wings straight flight condition of altitude $1524\ m$, velocity $V_{TAS} = 66.5\ m/s$, angle of attack $\alpha = 4.98$ deg . Exact details of the MuPAL aircraft can be found in [15] and the references therein. The linearised plant system has a state-space model:

$$\dot{x} = Ax + Bu + Ff \qquad y = Cx + D\delta$$

The disturbance signal f represents the wind gust that perturbs nominal flight of the aircraft; δ represents measurement noise [20] and matrix D is used for the design of the UIO. The control input is assumed to be the elevator command such that $u = \delta_e$. The system states are given by $x = [u_x, w_z, \theta, q]^T$, and the measure output is $y^T = [\theta,\ q]$. It must be mentioned that since we are interested in tracking of the pitch angle reference, only θ will be used for the tracking controller, which in our case, is a PID controller as proposed in [15].

$$A = \begin{bmatrix} -0.0175 & 0.173 & -9.77 & -5.63 \\ -0.192 & -1.09 & -0.846 & 64.6 \\ 0 & 0 & 0 & 1 \\ 0.0081 & -0.0738 & 0.0062 & -1.90 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.428 \\ 4.91 \\ 0 \\ 4.22 \end{bmatrix} \qquad F = \begin{bmatrix} 0.1462 \\ -1.257 \\ 0 \\ -1.4209 \end{bmatrix}$$

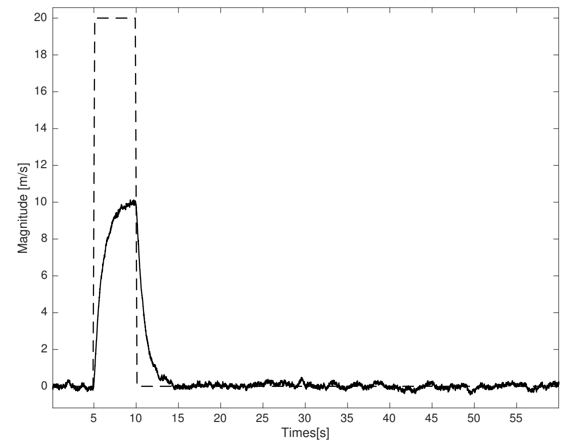

Fig. 3. Real and estimated wind gust profile in the y-axis. The estimate uses the UIO proposed in Section III-B with $W_1 = 0.5\mathrm{e}^{-10}$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = [0.1,\ 0; 0,\ 0.1]$$

In order to meet the condition that the plant is ASPR, we use the PFC approach as presented in [3]. Noise at the measured output is considered in the simulation.

The UIO is designed considering that we want to estimate the wind gust (i.e. signal f) and reject the effects of noise or other output disturbance (i.e. δ). Using LMI's (13), (14) and setting $W_1 = 0.5\mathrm{e}^{-10}$, we obtain the following observer parameters.

$$\gamma = 108.08 \quad \mu = 1.6377 \times 10^5$$

$$L = \begin{bmatrix} -0.0225 & 0.4785 \\ 0.7550 & 5.5092 \\ -0.0016 & -0.0005 \\ -0.0324 & -0.2345 \end{bmatrix} \times 10^3 \quad H = \begin{bmatrix} -0.2227 \\ 1.8425 \end{bmatrix}^T \times 10^4$$

The pitch angle PID controller is designed based on the results obtained in the MuPal-α flight tests documented in [20]. The PID gains used for simulation purposes are $k_P = 1$, $k_I = 1.2$, $k_D = 0.02$. The parameters chosen for the SAC with wind gust estimation design, i.e. Γ and σ, are:

$$\Gamma = \begin{bmatrix} 160 & 0 \\ 0 & 8 \end{bmatrix} \quad \sigma = [1 \quad 1]^T$$

The wind gust input matrix F and wind profile is generated following suggestions in [17]; we assume that the wind gusts is a pulse type signal with magnitude $20\ m/s$. Figure 3 shows the wind disturbance signal f, the estimate $\tilde{f}$. It can be observed that the (open-loop) observer has an error but it effectively estimates the wind signal f. In order to compare performance of the proposed strategy with that of the original SAC (i.e. the SAC in [22]), the weighting matrix Γ and the parameter σ are chosen as:

$$\Gamma = \begin{bmatrix} 4 \times 10^6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2000 \end{bmatrix} \qquad \sigma = [0.01 \quad 0.01 \quad 0.01]^T$$

The reference model used is assumed to be first order, with unit gain and pole at $20\ rad/s$.

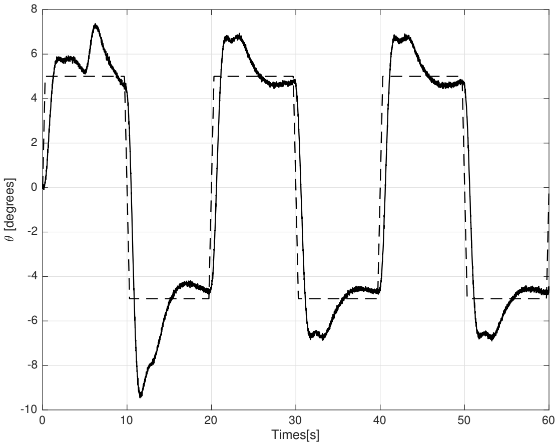

Fig. 4. Nominal PID controller closed-loop dynamics for the longitudinal axis under the occurrence of wind gusts: reference signal (solid line) and pitch angle response θ (dashed line)

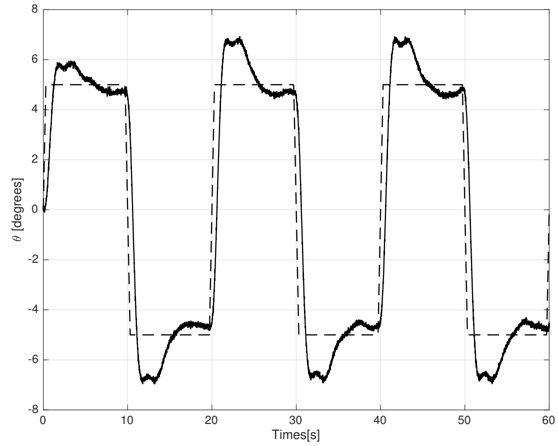

Fig. 5. PID controller with added SAC+UIO closed-loop dynamics for the longitudinal axis under the occurrence of wind gusts: reference signal (solid line) and pitch angle response θ (dashed line)

A. Simulation Results

In this section we compare the performance of the nominal controller (PID), the nominal controller augmented with the proposed SAC+UIO and the original SAC. A pulse train reference signal of magnitude ± 5 degrees and a period of 30 seconds is injected to the system; the wind gust disturbance is added as depicted in Figure 3. Figure 4 shows that the PID controller, under nominal conditions, renders the system stable and behaves according to the objectives set out by the designer; however, the presence of a wind gust causes undesired results, presenting a divergence from nominal dynamics of approximately ± 2 degrees in the positive and negative flank of the pulse. Using the PID with added original SAC (see Figure 7), the effect of the wind gust is reduced satisfactorily. However, the SAC acts even during normal flight conditions and modifies the nominal closed-loop dynamics considerably (note the additional damping is added by the SAC). Also, notice how the initial adaptation introduces undesired vibrations that are later damped. Although the SAC successfully reduces the effects of the wind gust on the pitch angle, it is observed that the adaptive gain k_e is required to grow excessively in order to be able to track the reference model, hence the total control signal is expected to saturate. Although saturation is not the main concern of this work, it is widely known that this hard nonlinearity is detrimental for system performance.

With the SAC+UIO acting only during events of wind gusts disturbance, the nominal dynamics of the PID are maintained and the wind gust disturbance is "actively" rejected almost entirely (see Figure 5) with lower gains that in the SAC case.

The additional term σ in (9) and (17) is intended to help the adaptive gain to return to zero once the system tracks the reference model (this effect is depicted in Figures 8 and 6). For the case of the SAC+UIO, observe how the gains grow in magnitude only in the event of a wind gust and returns to zero once this has ceased (Figure 6). For the case of the original SAC (Figure 8), note how the gains adapt for every tracking event error. It may be observed that the gains vary considerable from zero every time the reference signal varies, and this is what injects the additional damping to the nominal system response.

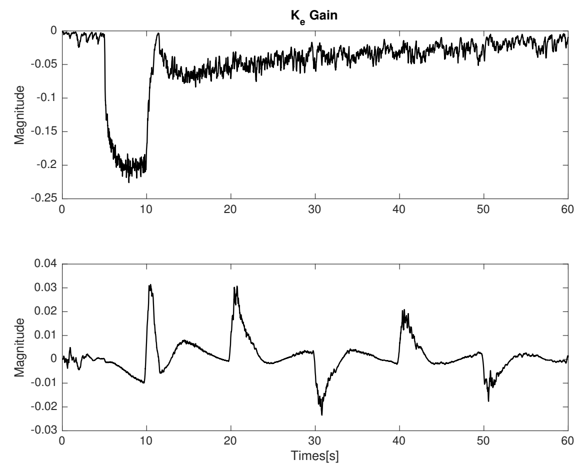

Fig. 6. Dynamic response of the adaptive gains k_f and k_u for the added SAC+UIO compensator.

VI. Conclusions

This paper has proposed an adaptive control law which uses the estimation of the undesired disturbances to actively reject them. By using an UIO as the reference model of the Simple Adaptive Control scheme, it is possible to construct a compensator that focusses on eliminating the effect of a specific disturbance and not on eliminating the tracking error as in the original SAC proposal. Sufficient boundedness conditions are given based on the work of Sobel et. al., and simulation results demonstrate the effectiveness. It must be mentioned that the construction of the UIO is independent of the SAC design, something that may present restrictions when trying to guarantee the ASPR condition for the estimated plant model. Future work will include the coupled design of the UIO and the SAC in order to guarantee integrity of the solution. Although we have concentrated on the wind gust rejection problem, similar architectures may be applied for Fault Tolerant Control.

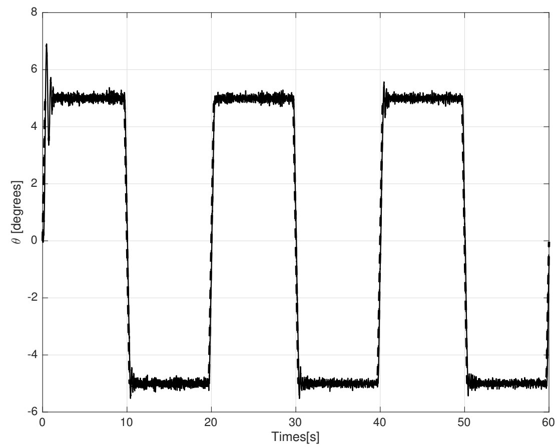

Fig. 7. PID controller with added SAC (original proposal) closed-loop dynamics for the longitudinal axis under the occurrence of wind gusts: reference signal (solid line) and pitch angle response θ (dashed line)

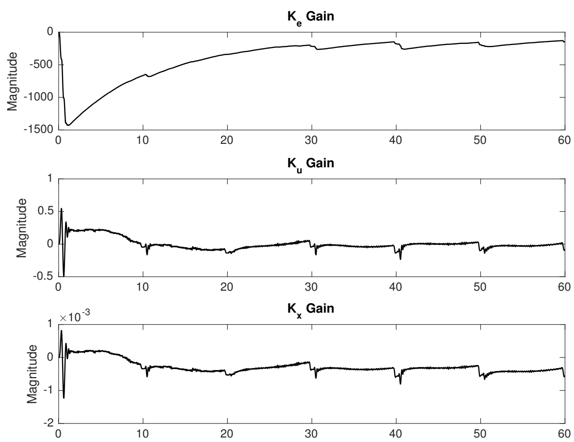

Fig. 8. Dynamic response of the adaptive gains k_e, k_{xm} and k_u for the added SAC (original proposal) compensator.

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VII. APPENDIX A

Consider the tracking error $e_x^* = x^* - x$. Note that if $e_x^* \to 0$, the plant's output tracks the ideal trajectory such that $y^* = y_m = y$. In this paper, and without any loss of, we assume that $\dot{u}_m = 0$; time-varying signals can be included in the analysis by using the unknown command generator approach presented in [11], [15], [3]. Observe that

$$\dot{e}_x^* = \dot{x}^* - \dot{x}$$

Using the definition (5), (1), setting $X_1 = I$ and the assumption $\dot{u_m} = 0$, we obtain

$$\begin{aligned} \dot{e}_x^* &= \dot{x}_m - \dot{x} \\ &= (A + LC)x_m + Bu_m - LCx - [Ax + Bu + Ff] \\ &= (A + LC)x_m + Bu_m - (A + LC)x - Bu - Ff \end{aligned}$$

Now adding and subtracting $(A + LC)e_x^*$ and grouping terms,

$$\begin{aligned} \dot{e}_x^* &= \dot{x}_m - \dot{x} \\ &= (A + LC)x_m + Bu_m - (A + LC)x - Bkr \\ &\quad +(A + LC)x^* - (A + LC)x^* - Ff \\ &= (A + LC)e_x^* - Bkr + (A + LC)x_m \\ &\quad +Bu_m - (A + LC)x^* - Ff \\ &= (A + LC)e_x^* - Bkr + (A + LC)x_m \\ &\quad +Bu_m - (A + LC)x_m - (A + LC)X_2u_m - Ff \end{aligned}$$

Now adding and subtracting $B\tilde{k}r$

$$\begin{aligned} \dot{e}_x^* &= (A + LC)e_x^* - B[k - \tilde{k}]r \\ &\quad +Bu_m - B\tilde{k}r - (A + LC)X_2u_m - Ff \end{aligned}$$

Defining $\tilde{k} = [I,\ \tilde{k}_f]$ and using (16), we obtain

$$\dot{e}_x^* = (A + LC - B\tilde{k}_f HC)e_x^* - B[k - \tilde{k}]r - M(t)$$

where $M(t) = (A + LC)X_2u_m + Ff$

$\square$