

Tiempo de Ejecución III

Ejemplo de recurrencia
(reescribiendo una parte
como sumatoria)

Recurrencia

$$T(n) = \begin{cases} c & , n \leq 1 \\ 2T\left(\frac{n}{2}\right) + 2n\log_2(n), & n > 1 \end{cases}$$

$$\text{paso 1, } T(n) = 2T\left(\frac{n}{2}\right) + 2n\log_2(n)$$

$$\begin{aligned} \text{paso 2, } T(n) &= 2\left[2T\left(\frac{n}{2^2}\right) + 2\frac{n}{2}\log_2\left(\frac{n}{2}\right)\right] + 2n\log_2(n) = \\ &= 2^2T\left(\frac{n}{2^2}\right) + 2^2\frac{n}{2}\log_2\left(\frac{n}{2}\right) + 2n\log_2(n) = \\ &= 2^2T\left(\frac{n}{2^2}\right) + 2n\log_2\left(\frac{n}{2}\right) + 2n\log_2(n) = \end{aligned}$$

$$\text{paso 3, } = 2^2\left[2T\left(\frac{n}{2^3}\right) + 2\frac{n}{2^2}\log_2\left(\frac{n}{2^2}\right)\right] + 2n\log_2\left(\frac{n}{2}\right) + 2n\log_2(n) =$$

$$\begin{aligned}
\text{paso 3,} &= 2^2 \left[2T\left(\frac{n}{2^3}\right) + 2\frac{n}{2^2} \log_2\left(\frac{n}{2^2}\right) \right] + 2n \log_2\left(\frac{n}{2}\right) + 2n \log_2(n) \\
&= 2^3 T\left(\frac{n}{2^3}\right) + 2^3 \frac{n}{2^2} \log_2\left(\frac{n}{2^2}\right) + 2n \log_2\left(\frac{n}{2}\right) + 2n \log_2(n) \\
&= 2^3 T\left(\frac{n}{2^3}\right) + 2n \log_2\left(\frac{n}{2^2}\right) + 2n \log_2\left(\frac{n}{2}\right) + 2n \log_2(n)
\end{aligned}$$

$$\text{paso } i = 2^i T\left(\frac{n}{2^i}\right) + 2n \sum_{j=0}^{i-1} \log_2\left(\frac{n}{2^j}\right) =$$

$$\frac{n}{2^i} = 1 \rightarrow n = 2^i \rightarrow i = \log_2 n$$

$$2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + 2n \sum_{j=0}^{\log_2 n - 1} \log_2\left(\frac{n}{2^j}\right) =$$

$$\begin{aligned}
& 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + 2n \sum_{j=0}^{\log_2 n - 1} \log_2\left(\frac{n}{2^j}\right) = \\
& = nT(1) + 2n \left(\sum_{j=0}^{\log_2 n - 1} \log_2 n - \sum_{j=0}^{\log_2 n - 1} \log_2 2^j \right) \\
& = nc + 2n \left((\log_2 n)^2 - \sum_{j=0}^{\log_2 n - 1} j \right) \quad \text{CUIDADO AL REESCRIBIR / RESOLVER} \\
& = nc + 2n \left((\log_2 n)^2 - \frac{(\log_2 n - 1)(\log_2 n - 1 + 1)}{2} \right) \\
& = nc + 2n \left((\log_2 n)^2 - \frac{(\log_2 n - 1)(\log_2 n)}{2} \right)
\end{aligned}$$

$$\begin{aligned}
&= nc + 2n \left((\log_2 n)^2 - \frac{(\log_2 n - 1)(\log_2 n)}{2} \right) \\
&= nc + 2n \left((\log_2 n)^2 - \frac{(\log_2 n)^2}{2} + \frac{(\log_2 n)}{2} \right) \\
&= nc + 2n \left(\frac{(\log_2 n)^2}{2} + \frac{(\log_2 n)}{2} \right) \\
&= nc + n(\log_2 n)^2 + n(\log_2 n)
\end{aligned}$$